CAMBRIDGE AS & A LEVEL

Pure Mathematics (9709) 2023

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Presentación.

El Cambridge International **A Level** es normalmente un curso de dos años, mientras que el Cambridge International **AS Level** suele durar uno. En el caso de algunas asignaturas, es posible empezar en el Cambridge International AS Level y ampliarlas en el Cambridge International A Level.

Obtención del título.

Para la obtención de un título "**AS Level**":

Es necesario superar 2 Papers, siguiendo una de las siguientes opciones:

Ruta 1: Paper 1 & Paper 2
Ruta 2: Paper 1 & Paper 4
Ruta 3: Paper 1 & Paper 5

Para la obtención de un título "**A Level**":

Es necesario superar 4 Papers: (1,3,4,5), (1,3,5,6) o (1,3,4,5).

Para su distribución en dos años se aconseja seguir una de las siguientes rutas:

Ruta 1: Primer año: Paper 1 (Pure Mathematics)
Paper 4 (Mechanics)
Segundo año: Paper 3 (Pure Mathematics)
Paper 5 (Probability & Statistics 1)

Ruta 2: Primer año: Paper 1 (Pure Mathematics)
Paper 5 (Probability & Statistics 1)
Segundo año: Paper 3 (Pure Mathematics)
Paper 6 (Probability & Statistics 2)

Ruta 4: Primer año: Paper 1 (Pure Mathematics)
Paper 5 (Probability & Statistics 1)
Segundo año: Paper 3 (Pure Mathematics)
Paper 4 (Mechanics)

Observamos que el Paper 2 es exclusivo de la titulación "AS Level".

Nivel previo recomendable.

Cambridge IGCSE Mathematics 0580 (Extended curriculum)
o Cambridge International O Level (4024/4029)

Papers (Pruebas).

Paper 1 Pure Mathematics 1 (1 hour 50 minutes)

1.1 Quadratics
1.2 Functions
1.3 Coordinate geometry
1.4 Circular measure
1.5 Trigonometry
1.6 Series
1.7 Differentiation
1.8 Integration

Paper 2 Pure Mathematics 2 (1 hour 15 minutes)

2.1 Algebra
2.2 Logarithmic and exponential functions
2.3 Trigonometry
2.4 Differentiation
2.5 Integration
2.6 Numerical solution of equations

Paper 3 Pure Mathematics 3 (1 hour 50 minutes)

3.1 Algebra
3.2 Logarithmic and exponential functions
3.3 Trigonometry
3.4 Differentiation
3.5 Integration
3.6 Numerical solution of equations
3.7 Vectors
3.8 Differential equations
3.9 Complex numbers

Paper 4 Mechanics (1 hour 15 minutes)

4.1 Forces and equilibrium
4.2 Kinematics of motion in a straight line
4.3 Momentum
4.4 Newton's laws of motion
4.5 Energy, work and power

Paper 5 Probability & Statistics 1 (1 hour 15 minutes)

5.1 Representation of data
5.2 Permutations and combinations
5.3 Probability
5.4 Discrete random variables
5.5 The normal distribution

Paper 6 Probability & Statistics 2 (1 hour 15 minutes)

6.1 The Poisson distribution
6.2 Linear combinations of random variables
6.3 Continuous random variables
6.4 Sampling and estimation
6.5 Hypothesis tests

Índice.

MATHEMATICS

Paper 9709/12
Pure Mathematics 1

Key messages

Since the change in syllabus, the previous reports have each highlighted that the question paper contains a statement in the rubric on the front cover that ‘no marks will be given for unsupported answers from a calculator.’ Although this message has been taken on board by most candidates, there is still a significant minority whose working does not contain enough detail. Clear working must always be shown to justify solutions.

For quadratic equations, for example, it is necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for certain marks to be awarded. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Candidates should ensure that factors must always produce the coefficients of the quadratic equation when expanded. This message is particularly relevant to **Questions 5**, **7(a)** and **9(c)**.

General comments

Nearly all candidates were able to attempt most of the questions and many very good scripts were seen. The requirement that candidates know and can use the algebra, geometry and trigonometry techniques studied at GCSE level (or equivalent) seemed to be evident in the responses from most centres.

Comments on specific questions

Question 1

The need to form a quadratic equation, usually in x, was clear to most candidates, as was the need to use the discriminant of their quadratic equation. The necessary presentation of the discriminant as a perfect square was sometimes seen, but the correct interpretation of this was rarely seen. This, combined with sign errors, meant a minority of candidates were awarded more than half marks.

Question 2

Although it was acceptable to attempt the algebraic interpretation of the transformations in any order, or altogether, most candidates presented their interpretations in the order of transformations given in the question. Many favoured using the completed square form of the equation. The reflection proved to be most challenging of the three transformations, but other common errors included squaring $2x$ incorrectly and multiplying only some of the terms by 3. A few candidates who carried out the three transformations correctly did not leave the result in the required form, so could not be awarded the final A1.

Question 3

The differentiation of this type of function seemed to be well understood by most candidates. The interpretation of the rates of change to come to the realisation that $\frac{dy}{dx} = 1$ or a chain rule calculation using $\frac{dx}{dt} = \frac{dy}{dt}$ was not always evident. The false assumption $\frac{dy}{dx} = 0$ was followed in many incorrect solutions.

Question 4

(a) Most candidates did not take account of the fact that the first term should be 5.02 with $n = 20$ (or 5.00 with $n = 21$) and could only be awarded a mark for selection and use of the correct formula.

(b) Incorrect interpretation of the number of terms was a common error for most candidates. However, the correct value of the common ratio and the use of the appropriate formula were both evident in most answers.

Question 5

A variety of different methods were seen, the most common being to substitute the equation of the line into the equation of the circle and solve the resulting quadratic. The formation of the circle equation from its radius and centre was well understood by most who chose to use this approach. Other methods successfully found the equation of the perpendicular bisector of *AB* then its mid-point and used either the midpoint formula or a vector method to find the coordinates of *B*. Some candidates incorrectly used the coordinates of *A* when forming an equation of the circle. Very few diagrams were seen and, for those could not make a start with any strategy, a diagram might have helped them visualise a suitable method.

Question 6

Most candidates were able to identify the two terms either from an expansion or by considering the general term. They usually went on to form an equation using the given condition. Errors were seen in manipulating powers and some candidates omitted the negative solution when they found the square root of $\frac{1}{9}$.

Question 7

(a) This was a well answered question with both the relationship between tangent and sine and cosine, and the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$, being used accurately to find a quadratic equation in cosine. Many correct answers were seen, although some came from calculator solutions of the quadratic equation. As mentioned in the Key Messages, answers from a calculator must be supported with full working. Most candidates correctly gave answers in the first and fourth quadrants.

(b) This part only required use of the relationships used in **part (a)** and some very clear solutions were seen, albeit with a lot of variation in the number of steps used. Others made multiple attempts where, often, initially correct working was spoilt by poor division of fractions. Candidates should be reminded that, in this type of question, they should quote the relationships they are using in each stage.

Question 8

(a) Most candidates were able to successfully find the length of *AC* and angle *ADC* using combinations of simple trigonometry or the sine or cosine rule and Pythagoras' theorem. These were generally used correctly to find the arc length *AC*. These reports have often commented on the effect of premature rounding of intermediate answers on the final answer; this was particularly evident in both this part and **7(b)**. Use of the arc length formula with degrees or radians as appropriate was usually seen. There were a few instances of Pythagoras' theorem being applied to non-right angle triangles.

(b) Again, angles in degrees or radians were usually used with the appropriate sector area formula. Most correct answers involved subtraction of the area of triangle *ADC* from area of sector *ADC,* although a significant minority chose to use the formula of the area of a segment using angle *ADC*. Answers which were prematurely rounded in **part (a)** invariably resulted in an inaccurate answer in this part.

Question 9

(a) Most candidates presented their answers in a correct form. When $x \leqslant -1$ was presented as an answer no marks could be awarded as this could have been a restatement of the domain.

(b) Most candidates reached the square root of a correct expression. Some stated this as ± but few realised that the given domain could only be achieved using the negative root. Candidates should be reminded that examiners must be convinced that square root signs refer to the whole of a fraction where this is appropriate.

(c) Finding the correct expressions for fg(x) and gf(x) seemed to be understood by the majority of candidates. However, sign errors and algebraic errors when squaring and combining the expressions often prevented candidates reaching the correct quartic equation. It was essential to show a method of solving the quartic equation in x as solving a quadratic in x^2. Factorising, completing the square, or using the quadratic formula, gained full credit. Some candidates substituted $x = x^2$, but then did not take the square root of their answers. A minority stated that, given the domain of f(x), both the solutions needed to be negative.

Question 10

(a) As the answer was given, most candidates realised it was necessary to show all the steps leading to the result, including the evaluation of $4^{-\frac{1}{2}}$ as $\frac{1}{2}$.

(b) This integration process seemed to be well understood and many candidates scored full marks in this part. Errors in dealing with the division by $\frac{1}{2}$ and omission of the constant of integration were occasionally seen.

(c) This part was also well answered. Usually, candidates correctly set the given $\frac{dy}{dx}$ to 0, with -2 substituted for k. Most candidates dealt with the power of x correctly and found the required x-coordinate. However, some candidates ended their response after only having evaluated x, without also finding the value of the y-coordinate.

(d) This was another well answered part, with nearly all completely correct answers coming from evaluation of the second derivative at $x = \frac{1}{4}$. A few candidates looked at the gradient change from values of x either side of $x = \frac{1}{4}$ and those who chose a lower value between zero and $\frac{1}{4}$ usually gained both the available marks.

Question 11

(a) This part was answered well by nearly all candidates. The most common errors included inverting the gradient or swapping the x and y coordinates when using the straight-line equation.

(b) Although there were many correctly worked solutions there were also a lot of scripts where candidates gained few marks. The question clearly stated that the region was rotated about the y-axis, but a significant number of candidates did not take this into account in their working. Others did not square the two functions (so did not integrate x^2) and some integrated a function of y but used limits for x. In a number of cases, correct answers were found from a calculator after errors in the working. As mentioned in the Key Messages, it is not possible to gain credit for unsupported answers of this type. It was essential to show all steps, including the integration of both functions, the substitution of the correct limits, and the subtraction of the volume between the curve and the y-axis from the volume between the line and the y-axis. An alternative method, used successfully by a small number of candidates, involved finding the volume of a frustum of a cone, using the coordinates of the given points, followed by subtracting the volume between the curve and the y-axis.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2

There were too few candidates for a meaningful report to be produced.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3

Key Messages

Candidates need to:

- know what is meant by $\arg(z - z_0) = \alpha\pi$, $(-1 < \alpha \le 1)$, and how to draw this on an Argand diagram. Showing equal scales on both axes is essential. See **Question 2**.
- be able to divide a polynomial containing constants by a second polynomial. See **Question 3**.
- be able to take real and imaginary parts of an equation. See **Question 4**.
- know how to integrate $\dfrac{1}{e^{\alpha x}}$, $\sin^2(nx)$ and $\cos^2(nx)$, where α is a real number and n is an integer. See **Question 9**.
- understand that integrating $\dfrac{\alpha x + \beta}{\varepsilon x^2 + \delta}$ may require splitting this into two separate integrals, namely $\dfrac{\alpha x}{\varepsilon x^2 + \delta}$ and $\dfrac{\beta}{\varepsilon x^2 + \delta}$. See **Question 11b**.

General comments

The standard of work on this paper was variable. However, regardless of the candidate's ability, this made little difference to the level of presentation and candidates of all ability levels can improve in this area. Several candidates showed multiple different attempts at certain questions, often jumping between these different methods as they worked through their solutions. Candidates are advised to demonstrate a single solution worked clearly and methodically, deleting any early approach should they wish to change their mind. Often candidates did not present their work in this manner which made it challenging to follow. This, coupled with the numerous different methods being tried on a single question, such as in **Question 3** and **Question 11(b)**, meant work was often complicated to mark.

Candidates often gave good responses to **Question 1**, **Question 7(b)**, **Question 7(c), Question 8(a)**, **Question 8(b)**, **Question 10(a)** and **Question 11(a)**. However, **Question 2(a)**, **Question 2(b)**, **Question 3**, **Question 4**, **Question 7(a)**, **Question 9** and **Question 11(b)** proved challenging for many.

Comments on Specific Questions

Question 1

Most candidates scored full marks, however a few believed $\ln(a \pm b)$ could be replaced by $\ln a \pm \ln b$ and hence made no progress.

Question 2

(a) Few candidates scored more than a single mark, with the errors being wide ranging. These included having no scale or showing the scale on only one axis; drawing the line $y = 3$ instead of the line $x = 3$; having the line $x = 3$ confined to the first quadrant; drawing full lines instead of half-lines; showing half lines starting from the origin instead of $1 + 2i$; showing half lines being symmetrical about the vertical line instead of the horizontal line; drawing lines asymmetrical about the horizontal line; marking lines at an angle of $\frac{1}{3}\pi$ but clearly drawing an angle of $\frac{1}{4}\pi$ or less; showing a single line at $\frac{1}{3}\pi$; and finally the presence of a circle, were all very common.

(b) Approximately fewer than five percent of the candidates scored any marks here, since candidates required a correct sketch in **2(a)** before they could make any progress. This work should have been followed by recognising that the point needed was where $\arg(z - 1 - 2i) = -\frac{1}{3}\pi$ and the line $x = 3$ met, together with the calculation of the distance of this point below the x-axis, namely $2\sqrt{3} - 2$.

Question 3

This was another question where candidates experienced problems. Again, the reasons for this were wide ranging.

Candidates found it challenging to deal with the unknown constants in the polynomial when attempting long division, hence they rarely had a linear expression to equate to the given remainder. Some tried to subtract the given remainder from the polynomial and equate their remainder to zero, which is a perfectly acceptable approach, but fails to help with this long division issue. Many other candidates erroneously believed that the divisor times the remainder was equal to the polynomial.

Another common approach was to find a root of the divisor and then equate the polynomial to the remainder at that root. Again, an acceptable approach, but in this case the root was complex and when substituted it required taking real and imaginary parts. This is something which candidates found difficult, as seen by their work in **Question 4**. However, the algebra involved in evaluating this complex root up to the power 4, before substitution into the equation, was prone to numerous errors.

Others incorrectly believed that this required root came from setting the remainder to zero, namely $x = -\frac{2}{3}$.

The easiest approach, and one which most successful candidates chose, was to equate the polynomial to the divisor times a quotient $(Ax^2 + Bx + C)$ plus $3x + 2$. Equating the various powers of x then leads to five simple equations which are easily solved, even easier for those who sensibly commenced with $A = 2$.

Question 4

Unfortunately, this was the third question in a row which candidates found challenging. Here, this was mainly due to sign errors, omitting constants, x's or y's, or incorrect multiplication of simple terms.

Few candidates reached a correct simplified equation. Many finished with an imaginary part consisting of just a single variable, which made the final part of the question much easier. However, there were several candidates who succeeded in reaching the correct equations and then stopped, not realising that it was necessary to equate both the real parts and the imaginary parts to zero to make further progress.

Question 5

(a) Many candidates performed well on this question, showing clear details of their cancellation of the $(2t + 1)$ term. However, there were still too many who, when differentiating te^{2t}, only differentiated the e^{2t} part or did not recognise the need to use the product rule.

(b) This part required candidates to substitute $t = -1$ to find the coordinates of x and y, together with the negative inversion of the given answer in **5(a)**. Unfortunately, the gradient of the normal too often remained as e^{-2t} and candidates believed that the answer given should be used in the working to find the answer given, often resulting in candidates showing 0 = 0. Candidates should be aware that the answer given should never be used in trying to obtain that result, it is simply there to show the candidate the form that the examiner wishes the final answer to be presented in.

Question 6

(a) Candidates were expected to undertake the expansion of $R\cos(\theta - \alpha)$ and equate terms, leading to $R\cos\alpha = 12$ and $R\sin\alpha = 5$, before using Pythagoras and simple trigonometry to find R and α.

Whilst there is nothing wrong in quoting formulae, if this is the approach that candidates wish to adopt, then they need to know how to apply them correctly and not finish with $\tan\alpha = \frac{12}{5}$, or with $\cos\alpha = 12$ and $\sin\alpha = 5$, as many candidates did. In this question R is an exact value, however seeing statements such as $R = \dfrac{12}{\cos(0.395)} = 13$ did not necessarily convince that exact mathematics was being used. The exact answer should be gained from $R^2 = (R\sin\alpha)^2 + (R\cos\alpha)^2 = 5^2 + 12^2$, resulting in $R = 13$.

(b) Most candidates scored the mark for $\cos^{-1}\left(\dfrac{6}{R}\right)$ but then often continued to use θ instead of $2x$. Candidates using $2x$ were unable to obtain more than the method mark unless α obtained in **6(a)** was correct. Many candidates obtaining the correct answer of 0.743 were unsure as to whether there was another answer in the interval, and if there was, that it needed the use of $2\pi - \cos^{-1}\left(\dfrac{6}{R}\right)$ to generate it.

Question 7

(a) Candidates either had little trouble in scoring full marks or were unsure how to start the question.

For those that made progress, a factor of 3 was usually associated with the correct term, however some of the expressions for the area of major sector had π instead of 2π. Others dropped the value of $\frac{1}{2}$ in their working or omitted it from some of their statements regarding areas. Often the area of the triangle was quoted correctly as $r^2\cos\left(\dfrac{x}{2}\right)\sin\left(\dfrac{x}{2}\right)$, but candidates continued with this throughout, despite the answer given informing them that they should have used $\frac{1}{2}r^2\sin x$ from the start or introduce it from using the double angle formula.

(b) A few candidates omitted this section, but in most cases excellent solutions were produced.

(c) This good work was continued here, with most candidates scoring full marks. The only blemishes were a few candidates who, for example, believed that they had shown convergence to two decimal places when they reached 2.1865, 2.1831.

Question 8

(a) Candidates were well trained in the product rule and usually had little trouble in securing the first three marks, however many then began to use decimals despite the question requesting exact coordinates. Unfortunately, many who did stay with exact notation then muddled their final line by expressing the y coordinate as $e^{-1} - \frac{1}{3}$ instead of $-\dfrac{1}{3e}$, or an equivalent correct form.

(b) This was another question where the technique, namely integration by parts, was accurately applied, with most candidates securing the first four marks. However, only a few candidates realised that area must be positive, which meant candidates should have been dealing with the modulus of their integral. The fact that many had $\frac{1}{64}\ln\left(\frac{1}{2}\right)$ together with $-\frac{15}{256}$ meant candidates did not always recognise that this answer was negative, which was not the case for those who happened to have converted to $-\frac{1}{64}\ln 2$.

Question 9

This question proved extremely difficult for most. Despite much algebra and trigonometry being presented, few marks were scored.

Separation of variables was no problem, however the conversion of $\frac{1}{e^{3y}}$ and $\sin^2(2x)$ into something that they could integrate, namely e^{-3y} and $\frac{1}{2}(1 - \cos(4x))$, was a challenge for most. Some candidates had incorrect signs in the trigonometry formula and candidates should note that these are available in the formula book.

Question 10

(a) Many candidates scored full marks, however there were some basic misconceptions seen, such as believing $\sqrt{14}\sqrt{14}$ to be $2\sqrt{14}$ and not knowing what an obtuse angle was.

(b) Most candidates knew how to obtain the direction vector together with either of the two points, but often arithmetical or miscopying errors were seen. Sometimes, the direction vector and the point were interchanged within the vector equation of the line formula.

However, the most common error was the incorrect notation on the left-hand side of the equation. The left-hand side should be an underlined $\underline{r}$ i.e. not left blank, expressed in words or just denoted as *l*.

(c) Again, most candidates knew what to do for this section but had arithmetical or miscopying errors. Vector questions need great care with the arithmetic, checking as one proceeds, otherwise the loss of marks can be considerable. When solving the two line equations, both parameters should be found and used to establish the intersection point, as different points reveal an error in the solution of equations part of the question. However, it does not preclude there being errors when establishing the line equations, so constant checks are essential.

Question 11

(a) Many candidates scored full marks with little difficulty. However, several incorrect formulations of the partial fractions were often seen, with $\frac{A}{1+x} + \frac{B}{4+x^2}$ and $\frac{Ax+B}{1+x} + \frac{C}{4+x^2}$ being the most common.

(b) Virtually all candidates established the correct integral for $\frac{A}{1+x}$, however the integral of $\frac{Bx+C}{4+x^2}$ proved challenging for most. This integral should have been subdivided into $\frac{Bx}{4+x^2}$ and $\frac{C}{4+x^2}$, as opposed to being treated as $(Bx + C)$ multiplied by the integral of $\frac{1}{4+x^2}$. Following the subdivision, the first integral is easily undertaken by setting $z = 2x$, whilst the latter is available in the formula book. Finally, to achieve the last accuracy mark, it was necessary to see some detailed ln work and not simply go from $3\ln 3 + \ln 8 - \ln 2 - \frac{1}{8}\pi$ to the answer given.

MATHEMATICS

Paper 9709/42
Mechanics

Key messages

When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation, or a Newton's law equation. Such a diagram would have been particularly useful in **Questions 4(c)**, **5** and **6**.

Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

General comments

The paper was well attempted and candidates at all levels were able to show their knowledge of the subject. **Questions 1**, **3(a)** and **6(a)** were found to be the most accessible questions, whilst **Questions 5**, **6(b)** and **7(c)** proved to be the most challenging.

Comments on specific questions

Question 1

(a) This question was answered well by most candidates using one of two methods. The most efficient method being to use $Pt = W$, giving 4 500 × t = 600 × 15 and leading to a time of 2 seconds. However, the most popular method seen was to use $P = Fv$ to obtain a speed of 7.5 m s^{-1} and then using this speed to get the required time.

(b) There were many good answers seen for this part, with most finding an acceleration of –3 m s^{-2} from the use of Newton's second law and then using this in the constant acceleration formula $v = u + at$. The two most common errors seen were to use the distance from the first part of the motion, and to have an equation such as 0 = 7.5 + 3t that would lead to a negative time.

Question 2

(a) This question was answered well by the majority of candidates. Very occasionally, an acceleration of 10 m s^{-2} rather than –10 m s^{-2} was used.

(b) Only a minority of candidates achieved a fully correct answer. Most answers did have a distance travelled for P and Q in terms of t using $s = ut + \frac{1}{2}at^2$. The method then required the use of the fact that the total distance travelled by P and Q is 18 m to form an equation in t, and then use this t value to find the required distance. However, it was more often seen for candidates to equate the distance for P and the distance for Q without any use of the fact that the particles were initially 18 m apart.

Question 3

The concept of variable acceleration being related to calculus rather than constant acceleration was well understood and this question was well answered by many candidates.

(a) Many candidates integrated correctly and evaluated the resulting expression at $t = 9$.

(b) Again, many candidates integrated correctly here and equated this to the expression found in **part (a)**. The majority then completed the question by solving the equation for t. However, some candidates found difficulty in dealing with the fractional indices when solving. Another common error seen was for the expression for displacement to be equated to 72 m, which was the speed found in **part (a)**.

Question 4

(a) No resistance on the truck and the truck moving with constant speed should have indicated that the tension in the coupling was zero. A common incorrect answer often seen was 0.2 N.

(b) This request was often correct from the use of $P = Fv$.

(c) Many good answers were seen, with the use of two of the three possible Newton's second law equations and a calculation to find the driving force. However, a significant number of candidates seemed confused about the driving force, using it with both the equations for the locomotive and the truck. Another common error was to include the tension in the coupling when attempting the equation for the whole system.

Question 5

Many candidates confused tension and thrust in this question.

(a) Only a significant minority of candidates appreciated that the force in the struts were thrusts with a direction towards point D. A significant number of candidates had the force in the struts as a tension, which did lead to a negative force. As the request was for the magnitude of the force, the majority of answers seen were given as positive. Only a minority of candidates gave a negative answer.

(b) Again, the majority who made a valid attempt at this question had the force in the strut BD as a tension. However, this did not have any effect on the solution by resolving forces vertically and horizontally and solving for F. A common error seen was to obtain a negative tension from the vertical equation and making this positive before substituting into the horizontal equation. Also, F was sometimes assumed to be acting horizontally to the left. This resulted in a negative value which was not always made positive, given that the question stated that F was the magnitude of the horizontal force applied.

Question 6

(a) (i) This question was well answered by the majority of candidates. Common errors included the omission of a weight component, no use of the given acceleration, or not resolving the tension in the rope.

(ii) Many good attempts to evaluate the coefficient of friction were seen. The main error, as is common in these types of tasks, was to omit the component of the tension when resolving perpendicular to the plane to find the normal reaction, so getting $R = 2g\cos 30$.

(b) The approach usually seen was to find the maximum possible friction of $0.8 \times (2g\cos 30 - 15\sin 20)$ and compare this with the friction required to stop the block moving up the plane of $15\cos 20 - 2g\sin 30$. This shows that the friction needed to stop the block moving up the plane is less than the maximum friction and hence the block does not move. Even though many candidates have correct values to compare, the incorrect conclusion often stated was that the block moved down the plane. This conclusion was not appreciating that this would have to arise from the friction acting up the plane, and so a comparison with $2g\sin 30 - 15\cos 20$ would be required. In this scenario this value is negative.

Question 7

Many candidates appeared to find this a demanding question in parts.

(a) A significant number of candidates scored well on this part. The method did require the speed of particle *P* when it arrived at *B*, to be found using energy, and then to use the principle of conservation of momentum to show the speed of *Q* to be $10\,\text{m s}^{-1}$ after the collision. The main error seen by examiners was to use constant acceleration to find the speed of particle *P* when it arrived at the point *B*.

(b) Most candidates used momentum correctly to find the speed of the combined particles after particle *Q* collided with particle *R*. Then candidates used one of two different methods to complete the request, both with equal success. Some used energy to find the vertical height of point F above point C, before finding the required angle from use of trigonometry. Others used constant acceleration to find the acceleration on the slope before then equating this to $g\sin\theta$ to find the required angle.

(c) Very few candidates scored full marks on this part. The most commonly seen approach was to work with time. Firstly finding the time for particle *Q* to reach the point *C* (0.7 s). Then finding the time for the combined particle to travel up the slope and return to the point *C* (0.8 s). Then determining that the total time that particle *P* has been in motion since it collided with particle *Q* is 1.5 s. Using this total time, the distance of between the particle *P* and the combined particle can be found to be 1 metre. Finally, the remaining time until *P* collides with the combined particle can be found as *P* moves $4t$ metres and the combined particle moves $2t$ metres, so the total distance moved is $6t$ metres, which equates to 1 metre. This gives an additional time of $\frac{1}{6}$ s, leading to a distance of $\frac{20}{3}$ metres from *B*. The most common error seen was to omit the time that the combined particle takes to come back down the slope.

MATHEMATICS

Paper 9709/52
Probability & Statistics 1

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when working is not fully correct earlier in their solution. The use of algebra to communicate processes is anticipated at this level, as this enables candidates to review their method effectively and is an essential tool when showing given statements are true. When errors are corrected, candidates would be well advised to clearly cross these through and replace the original working. Where a candidate’s workings are overwritten, it is often difficult to interpret these accurately.

As stated in the paper instructions, candidates should state only non-exact answers to three significant figures. There should be a clear understanding of how significant figures work for decimal values less than 1. It is important that candidates realise the need to work to at least four significant figures throughout to justify a value correct to three significant figures. Many candidates rounded prematurely in normal approximation questions which produced inaccurate values from the tables and lost accuracy in their solutions. It is not an efficient use of time to convert an exact fractional value to an inexact decimal equivalent, as there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. This is particularly the case where the answer should be presented with reference to the context of the question. Candidates would be well advised to include a focus on this within their preparation.

General comments

Many well-structured responses were seen, however some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the processes which were being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. Candidates should be aware that cumulative frequency graphs are constructed with a curve, and that these need to be accurately drawn. It was pleasing to see an improvement in the labelling of the statistical diagrams from the previous series.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. The majority of candidates performed well across a range of topics. However, a few candidates did find questions more challenging when more than one technique was required within a solution. Many good solutions were seen for **Questions 1, 4** and **6**. The context in **Questions 5** and **7** was found to be challenging for many.

Comments on specific questions

Question 1

Many candidates used the area around the data table both to calculate the cumulative frequency and to state the mid-class values. In these cases, it is important that these calculations are presented in a manner so that the working is allocated to the relevant part.

(a) Almost all candidates calculated the cumulative frequency accurately. The majority of solutions used the vertical axis for the cumulative frequency, which is the normal convention. Most candidates plotted the required points on the class upper boundary, although a very small number used a continuity correction. This suggests that only integer value times had been recorded, which was not implied by the data table or the question. It was also noted that some candidates plotted at the mid-class or class lower boundary values.

A common misunderstanding was to assume that the number of hours recorded started at 0, so the cumulative frequency graph commenced at $(0, 0)$ and not $(30, 0)$ as indicated by the data.

Candidates should be aware that a curve is required for a cumulative frequency graph, as the use of line segments creates a cumulative frequency polygon which is not acceptable.

It was encouraging to see many graphs with correctly labelled axes, although a few candidates used scales which made it difficult to plot points to the required degree of accuracy.

Weaker candidates often plotted a frequency graph or bar chart.

(b) The majority of candidates interpreted the question correctly and calculated 70% of the number of years. It was pleasing to see that most candidates provided evidence on their graph of where they were obtaining their value. Good solutions included ruled lines perpendicular to the axes to enable the values to be read accurately. A surprising number of candidates did not read values from their graphs to the expected accuracy. Candidates who had used more challenging scales in **1(a)** often struggled to interpret their graph accurately.

(c) Many good solutions to this part were seen. The best of these initially stated the mid-class values, provided a clear calculation for the estimated mean, and then evaluated an exact answer. As stated in the Key messages, candidates should be aware that there is no requirement to convert an exact answer to a three significant figure. Some candidates chose to present their solutions within a table structure. While this is clear, it provides more detail than is strictly required compared to calculating individual values. Common errors included using either the class upper boundary or the class width in the mean calculation. A small number of candidates applied a continuity correction to the mid-class value.

Question 2

Several solutions included a tree diagram in **2(a)** to clarify the possible outcomes of the coins. This was often helpful in **2(b)**.

(a) Candidates were required to provide evidence that the given statement was true. The majority of candidates were able to find a calculation that gave a result of 0.225 but did not provide a justification for their expression. Good solutions often identified the possible scenarios that produced exactly one head, then stated the probability calculation for each scenario prior to showing that these summed to 0.225. The best solutions clearly identified which coin was biased, rather than assuming that the probabilities alone were sufficient.

An alternative combinations approach was successfully utilised by a small number of candidates, which was sufficient justification for the different scenarios where the head was on a fair coin.

(b) Almost all candidates determined that $\mathrm{P}(X = 2) + \mathrm{P}(X = 3) = 0.650$ and this was utilised to find their second probability. The most successful solutions considered $\mathrm{P}(X = 3)$, which was a similar calculation to **part (a)**. Candidates who first found $\mathrm{P}(X = 2)$ often omitted other possible scenarios. Candidates who had drawn a tree diagram in **part (a)** were able to identify the required scenarios efficiently. Weaker candidates simply stated that each probability was 0.325. Several solutions were seen where the total probability was not equal to 1, which should be a prompt that an error has been made.

(c) Most candidates who completed the probability distribution table attempted to find the variance. The best solutions included a clear, unevaluated expression using the standard variance formula, before stating the answer to three significant figures. Weaker solutions failed to subtract the square of the given expected value.

A few candidates recalculated the given expected value, although not always accurately.

Question 3

Almost all candidates identified that the binomial approximation was the appropriate approach for **part (a),** and most identified that the geometric approximation was required in **parts (b)** and **(c)**.

(a) This standard binomial approximation question was answered well by most candidates. Almost all stated at least one correct binomial term, but a common error was to include 17 residents as meeting the stated success criteria. Candidates are well advised to consider the difference of the success criteria ‘at least 17’, which includes 17, and ‘more than 17’, which does not.

A significant number of solutions were inaccurate due to a premature approximation of values in the calculation. As highlighted in the Key messages, candidates should be aware that at least four significant figures must be used to obtain an answer that will be accurate to three significant figures. Good practice is to state the unevaluated expression, then use the calculator efficiently to provide an answer to at least four significant figures, before finally rounding to three significant figures. There is no expectation of intermediate arithmetical steps to be shown at this level.

(b) Most solutions used the geometric approximation. Good solutions evaluated the solution exactly and did not round the answer to three significant figures. As highlighted in the Key messages, candidates should be aware of the syllabus requirement leave exact values unrounded. Weaker solutions failed to interpret the success criteria accurately, often finding the probability for the sixth person not being in favour.

(c) Most candidates found this part challenging, with a significant number of candidates omitting it altogether. The success criteria required both the seventh person to not be in favour, as well as exactly one of the previous six people not to be in favour. The most successful solutions listed the six possible scenarios and calculated the probabilities of each before summing. More able candidates recognised that combinations could be used efficiently for this.

The most common misconception was to assume that multiplying the probabilities for five residents in favour and two residents not in favour was all that was required.

Question 4

Many good solutions were seen for this probability question. The best included a tree diagram which clarified the information provided. Candidates should be aware that a visual representation of probability data and the related scenarios often leads to efficient solutions. Good solutions then identified the relevant branch and formed a linear equation which was solved accurately. Weaker solutions often used the probability complement for one of the criteria. Again, some candidates unnecessarily converted their exact fractional probability to a rounded three significant figure answer.

A small number of candidates misinterpreted the success criteria, assuming that Aran was not wearing a hat, or not wearing a scarf or not wearing both of them, and then solved the more complex equation.

Question 5

Candidates found this probability question challenging and few made an attempt. Many candidates were able to identify the probability of event *A* as $\frac{1}{2}$, but made little progress in determining the probability of event *B*. The most successful solutions used a simple diagram of the four boxes and listed the possible scenarios that could be obtained, before identifying those that fulfilled the given criteria. A few failed to realise that the yellow and green marbles could simply swap places and the red marble would still fulfil the criteria. Less successful approaches were to attempt to use permutations to calculate possible outcomes, or to assume that the green and yellow marbles were always in boxes *M* and *N*.

Where candidates had found probabilities for events *A* and *B*, most used the relationship $\mathrm{P}(A)\times\mathrm{P}(B)=\mathrm{P}(A\cap B)$ to determine independence and interpreted their findings appropriately. A very small number of candidates used the conditional probability approach and also interpreted their data appropriately.

Question 6

This was a relatively standard normal approximation question. It was encouraging to see sketches of the normal distribution being used to identify the appropriate probability area curve in many solutions. The number of candidates who did not attempt **6(c)** was higher than anticipated.

(a) Almost all candidates were able to find the anticipated probability. Good solutions clearly stated the normal standardisation formula before evaluating to find the *z*-value and then stating the appropriate probability accurately. A small number of solutions assumed that time was not continuous and used a continuity correction. An unexpectedly high number of solutions did not include an unevaluated normal standardisation formula with values substituted, which is essential working for this component.

A few candidates found the complementary probability, which may have been avoided by using a simple sketch of the normal distribution curve.

(b) Most candidates used the normal standardisation formula with both of the times given, and accurately found the probability that a single cyclist met the criteria. As the upper criteria was the complement to **6(a)**, it was anticipated that candidates would use their previous work here. A significant number of candidates did not make any further progress and did not find the probability that all four cyclists fulfilled the criteria. The best solutions used a four or more significant figure probability in the calculation and then rounded the final answer to three significant figures. A common error was to approximate prematurely when finding the probability for a single cyclist and then using the three significant figure value, resulting in an inaccurate solution.

(c) An unexpected number of candidates omitted this question, although it involved a fairly standard process. Most candidates were able to state the required *z*-values for the given probabilities. The use of critical values is expected in this component. A common error was not to state the critical value linked to a probability of 10%, but instead to find a value using an alternative approach. The best solutions then formed the two simultaneous equations and showed a clear algebraic method to find the values of the mean and standard deviation. Any correct method for solving simultaneous equations was acceptable, but if a calculator is used, then there is an expectation that the two equations will be in the same form.

The most common error was to link 36 minutes with 0.739, an error which the use of a sketch of the normal distribution curve could have helped avoid. A small number of solutions included a time continuity correction.

Question 7

Most candidates used an appropriate combinations approach to this question. Solutions with simple ‘diagrams’ illustrating possible scenarios were often more successful.

(a) The criteria used in this part was found challenging by some, but many good solutions were seen. The first approach was to consider the arrangements of the letters with the Es together and the Ds removed. The best solutions used a simple diagram to illustrate the scenario and found the number of arrangements for these seven letters, then considered how the Ds would be inserted and multiplied by the correct factor. The most able candidates used ${}^6C_2$ at this stage. A common error was not dividing by 2 as the Ds could not be identified.

The second approach was considering the total number of arrangements that all the letters could make with the Es together and then subtracting the number of arrangements with the Es and the Ds both being grouped together. The best solutions clearly identified the scenario being considered and then used the separate answers to calculate the required value. Again, a common error with this method was not dividing the first term by 2 as the Ds could not be identified.

(b) Most candidates did not appreciate that a different approach was required for this part as a probability was requested, so the repeated letters should be considered identifiable to ensure that the total number of possible outcomes could be obtained.

The most successful solutions illustrated the possible scenarios that fulfilled the given condition and then calculated the number of arrangements for each scenario. The values were then summed to produce the numerator of the probability. The denominator required the calculation of the total possible number of arrangements and, because a probability was required, repeats were not considered.

The majority of candidates did assume that the letters were not identifiable and used a similar method but divided through consistently by 2! for the Ds and 3! for the Es and obtained the same final answer.

A small number of candidates assumed that the condition in **7(a)** also applied. Candidates should be aware that the question would be structured on the paper so that any requirement for both parts would be stated before the **part (a)** information.

(c) This question had a more standard success criteria and most candidates were able to make some progress to a solution. The best solutions listed the possible scenarios in a systematic manner, then calculated the number of selections which fulfilled each scenario before showing a clear addition. Many solutions omitted one or more possible scenarios and some solutions included scenarios with three Ds, which is not possible from the word DELIVERED. A common misunderstanding was that the Ds and Es needed to be included in the calculation of the selections, and so multiplication by additional combinational terms was seen frequently.

Some more able candidates recognised that, as all possible scenarios involved DE, they were effectively looking at selections from the remaining seven letters. Therefore, as only the E was repeated, the scenarios consisted of assuming zero, one or two Es are selected, with the remaining letters picked from the letters DLIVR.

MATHEMATICS

Paper 9709/62
Probability & Statistics 2

Key messages

- Candidates are reminded of the need to present work clearly with just one solution offered.
- Full working must be clearly shown. A final answer with no justification is not sufficient.
- If questions are continued on additional pages, the question number must be clearly identified.
- It is important that candidates refer to the context given in the question when requested to do so, as general statements (such as those found in the course textbook) will not be sufficient.
- Candidates should appreciate the difference between rounding to three significant figures and three decimal places, as the latter will not always give the accuracy required on this paper.

General comments

This was a reasonably well attempted paper. There were some very good and well-presented scripts, but equally there were some candidates who seemed to not be fully prepared for the demands of the paper. Questions that were well attempted were **Questions 1**, **2(b)(i)**, **2(c)**, **3(b)** and **5(a),** whilst **Questions 5(b)**, **6(b)(c)** and **(d)** and **2(b)(ii)** were found to be more demanding. On the whole, adequate working was given, but there were occasions where candidates did not fully justify their answers.

Comments on individual questions follow, but it should be noted that there were many good, fully correct solutions offered as well as the common errors highlighted below.

Comments on specific questions

Question 1

(a) This was a good, standard question and was well attempted. Most candidates found the correct z-value and substituted it into an equation of the correct form. However, some candidates set up an incorrect confidence interval using 49 rather than $\frac{49}{140}$.

(b) Some candidates realised that reducing the percentage of the confidence interval was required here, though some thought it should increase. Others did not realise that the question required use of Anita's survey results, so changing the number of students in the sample was not an option here.

Question 2

(a) Many candidates found two correct assumptions, but as highlighted in the Key messages, these were not always put into the context of the question. Others were confused by what the question was asking and gave answers which were not assumptions, such as 'mean and variance must be equal'.

(b) (i) This question was well attempted with most candidates showing the required working and reaching the correct answer. It is important that the Poisson expression is clearly written out i.e. all relevant working is shown. As highlighted in the Key messages, some candidates gave an answer of 0.087, possibly confusing three significant figures with three decimal places.

(ii) A variety of errors were seen here. Two Poisson probabilities were required, which needed to be multiplied together and then doubled. Very few candidates realised the need to double and gave a final answer of 0.111 . Other candidates found the two probabilities but added them and often end errors were seen when calculating 'at least two'.

(c) This part was well attempted. Common errors seen were standardising without a continuity correction, standardising with an incorrect one, or choosing the wrong probability area. Many candidates gained full marks.

Question 3

(a) In the past, questions like this have not been well attempted, so it was pleasing to note here that more candidates were able to use the symmetry of the diagram to work out the required probability. It is important that candidates have an understanding of probability density functions and their graphs rather than just relying on formulae to find probabilities.

(b)(i) This part was particularly well attempted, and most candidates were able to show by integration that k was $\frac{10}{81}$. Few errors were seen here, though weaker candidates occasionally did not clearly equate their integral to 1, and errors attempting to integrate f(x) were seen.

(ii) Many candidates calculated E(X) rather than realising that, from given information, it must equal $\frac{3}{2}$. Some candidates merely calculated E(X^2) and left their answer as $\frac{18}{7}$, and errors attempting to integrate x^2f(x) were seen, but overall, this part was well attempted.

Question 4

(a) It was pleasing to note that this part, requiring the probability of a Type I error, was well attempted. Many candidates successfully calculated the correct Poisson probability and fully justified their answer. Some candidates used an incorrect λ, or incorrectly included P(X = 3) in their expression, but the most common error was to calculate 1 – P(X = 0,1,2) rather than P(X = 0,1,2). Occasionally full working was omitted, which as highlighted in the Key messages, is not acceptable. It was expected here that the full Poisson expression was clearly shown, and not just a final answer given. Occasionally a final answer of 0.077 was seen, which once again shows a possible confusion between three significant figures (as required) and three decimal places.

(b) Again, it was pleasing to note that many candidates correctly found the probability of a Type II error in this context, as questions on Type I and Type II errors have not always been well answered in the past. As in **part (a)**, end errors and lack of working were noted. The most common error was to calculate P(0,1,2) rather than 1 – P(0,1,2). Again, a final answer of 0.063 was occasionally seen with no previous working to justify three significant figures of accuracy.

Question 5

(a) This part was well attempted. Errors included incorrect values for the variance (commonly using $3.6^2 + 2 \times 3.7^2$ rather than $3.6^2 + 4 \times 3.7^2$ or subtracting the two variances rather than adding) and finding an incorrect probability area (>0.5 rather than <0.5).

(b) This was a more unusual question of its type and was a challenge for many candidates. Candidates had difficulty with how to deal with the 60 per cent and the 40 per cent of the boxes containing different packets. It was necessary to deal with the large packets and find the probability of the total mass being greater than 4080 separately from the small packets and their probability of the total mass being greater than 4080. These two probabilities with the relative proportions of 60 per cent and 40 per cent were then to be combined for the final probability. Few candidates scored full marks.

Question 6

(a) Most candidates answered this question well, showing understanding of the difference between one and two tailed tests.

(b) Many candidates did not give a valid comparison here, which was required to explain the conclusion drawn. The conclusion should have been in context and candidates did not always give their conclusion in this form.

(c) Many candidates calculated $\Phi(2.14)$ rather than $1 - \Phi(2.14)$, whereas others left their answer as 0.0162 or did not find the correct inequality. Some candidates did not know how to approach the question.

(d) The most common error here was inconsistent use of signs. Many candidates standardised correctly, though some omitted $\sqrt{100}$. Many realised that this standardisation should be equated to the appropriate z-value, but used +1.645 when –1.645 was required. The sign needed to be consistent with the standardisation attempt. Other errors included using 1.96 or sometimes not equating to a z-value at all. After reaching 25.4, many candidates thought a region was required rather than a value for m.

Cambridge Assessment
International Education

Grade thresholds – March 2023

Cambridge International AS & A Level Mathematics (9709)

Grade thresholds taken for Syllabus 9709 (Mathematics) in the March 2023 examination.

		Minimum raw mark required for grade:				
	Maximum raw mark available	A	B	C	D	E
Component 12	75	60	52	42	33	24
Component 22	50	38	34	28	22	16
Component 32	75	58	51	42	33	24
Component 42	50	37	30	24	17	10
Component 52	50	36	29	24	18	12
Component 62	50	39	34	28	22	16

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows. Options with an additional letter, e.g. BYJ, refer to A2-only options.

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AY	250	12, 32, 42, 52	220	191	162	131	100	70
CY	250	12, 32, 52, 62	220	193	166	136	106	76
DY	250	32, 52, 85	216	185	154	122	90	58
DYJ	125	32, 52	109	93	71	52	33	14
EY	250	32, 42, 88	216	185	154	120	86	53
EYJ	125	32, 42	109	94	72	52	32	13
GY	250	32, 62, 88	216	187	158	125	92	59
GYJ	125	32, 62	110	96	76	56	37	18
HY	250	32, 52, 95	216	189	162	129	96	64
HYN	125	32, 52	109	93	71	52	33	14
IY	250	32, 42, 98	217	191	165	130	95	61
IYN	125	32, 42	109	94	72	52	32	13
KY	250	32, 62, 98	217	193	169	135	101	67
KYN	125	32, 62	110	96	76	56	37	18
OV	250	32, 52, 75	218	189	160	128	96	65
OVM	125	32, 52	109	93	71	52	33	14

Grade thresholds continued
Cambridge AS & A Level Mathematics (9709)

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
PV	250	32, 42, 78	215	186	157	124	92	60
PVM	125	32, 42	109	94	72	52	32	13
RV	250	32, 62, 78	215	188	161	129	97	66
RVM	125	32, 62	110	96	76	56	37	18
S4	125	12, 22	–	98	86	70	55	40
S5	125	12, 42	–	97	82	66	50	34
S6	125	12, 52	–	96	81	66	51	36

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 4 6 1 4 6 4 9 4 1 7 *

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

Show that the line and the curve meet for all values of k. [4]

2 A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

Stretch parallel to the x-axis with scale factor $\frac{1}{2}$

Reflection in the y-axis

Stretch parallel to the y-axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a, b and c are constants. [4]

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3 A curve has equation $y = \frac{1}{60}(3x + 1)^2$ and a point is moving along the curve.

Find the x-coordinate of the point on the curve at which the x- and y-coordinates are increasing at the same rate. [4]

4 The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

(a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

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(b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

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5 Points $A\ (7,\ 12)$ and B lie on a circle with centre $(-2,\ 5)$. The line AB has equation $y = -2x + 26$.

Find the coordinates of B. [6]

6 In the expansion of $\left(\frac{x}{a} + \frac{a}{x^2}\right)^7$, it is given that

$$\frac{\text{the coefficient of } x^4}{\text{the coefficient of } x} = 3.$$

Find the possible values of the constant a. [6]

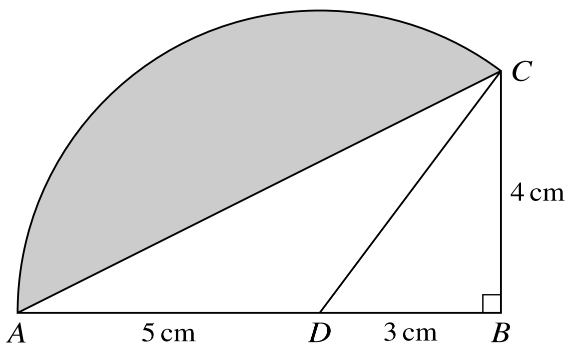
7 **(a)** By first obtaining a quadratic equation in $\cos\theta$, solve the equation

$$\tan\theta\sin\theta = 1$$

for $0° < \theta < 360°$. [5]

(b) Show that $\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta} \equiv \tan\theta\sin\theta$. [3]

8



The diagram shows triangle ABC in which angle B is a right angle. The length of AB is 8 cm and the length of BC is 4 cm. The point D on AB is such that $AD = 5$ cm. The sector DAC is part of a circle with centre D.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

9 The function f is defined by $f(x) = -3x^2 + 2$ for $x \leqslant -1$.

(a) State the range of f. [1]

(b) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = -x^2 - 1$ for $x \leqslant -1$.

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

10 At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} + k$, where k is a constant.

(a) Show that $k = -2$. [1]

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(b) Find the equation of the curve. [4]

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(c) Find the coordinates of the stationary point. [3]

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(d) Determine the nature of the stationary point. [2]

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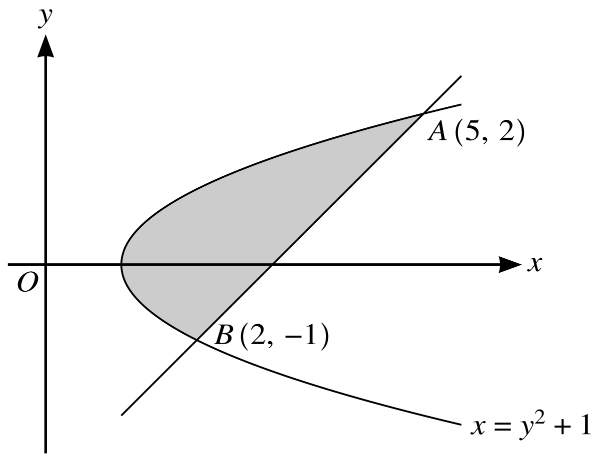

The diagram shows the curve with equation $x = y^2 + 1$. The points $A\,(5,\ 2)$ and $B\,(2,\ -1)$ lie on the curve.

(a) Find an equation of the line AB. [2]

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(b) Find the volume of revolution when the region between the curve and the line AB is rotated through 360° about the ***y*-axis**. [9]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Generic Marking Principles

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GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$x^2 - kx + 2 = 3x - 2k$ leading to $x^2 - x(k+3) + (2+2k)$ $[=0]$	**M1**	3-term quadratic, may be implied in the discriminant.
	$b^2 - 4ac = (k+3)^2 - 8(1+k)$ (ignore '= 0' at this stage)	**DM1**	Cannot just be seen in the quadratic formula.
	$=(k-1)^2$ accept $(k-1)(k-1)$	**A1**	Or use of calculus to show minimum of zero at $k = 1$ or sketch of $\mathrm{f}(k) = k^2 - 2k + 1$.
	$\geqslant 0$ Hence will meet for all values of k	**A1**	Clear conclusion.
		4	

Question	Answer	Marks	Guidance
2	Stretch: $(2x)^2 - 2(2x) + 5$ or $(x-1)^2 + 4$ leading to $(2x-1)^2 + 4$	**M1**	Replacing x by $2x$.
	Reflection: $(-2x)^2 - 2(-2x) + 5$ or $(-2x-1)^2 + 4$	**M1**	Replacing x by $-x$. FT on *their* stretch.
	Stretch: $3\left\{(-2x)^2 - 2(-2x) + 5\right\}$ or $3\left\{(-2x-1)^2 + 4\right\}$	**M1**	Multiplying the whole function by 3. FT on *their* (stretch plus reflection).
	$12x^2 + 12x + 15$	**A1**	
		4	

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx} = \left\{\frac{1}{60}(3x+1)\times 2\right\}\times\{3\}$	B1 B1	May see $\frac{1}{60}(18x+6)$.
	$\frac{1}{10}(3x+1)=1$	M1	Equate *their* $\frac{dy}{dx}$ to 1.
	$x=3$	A1	
		4	

Question	Answer	Marks	Guidance
4(a)	$5.00+20\times 0.02$ or $5.02+19\times 0.02$	M1	Allow for a = 5, n = 20 with d = 0.02 only. a = 5, n = 21(OE) with d = 0.2 gets M1 only.
	5.40	A1	
		2	
4(b)	$r=\frac{5.02}{5}=1.004$ or $\frac{251}{250}$	B1	
	$5.00\times(their 1.004)^{20}$ or $5.02\times(their 1.004)^{19}$	M1	Allow a = 5, n = 20.
	5.42	A1	Any correct rounding of 5.41557108 .
		3	

Question	Answer	Marks	Guidance
5	$r^2 = (7+2)^2 + (12-5)^2$	**B1**	Expect 130, may use *AC* rather than *r*.
	Equation of circle is $(x+2)^2 + (y-5)^2 = 130$	**B1 FT**	OE FT *their* 130, may use distance *BC* rather than circle.
	$(x+2)^2 + (-2x+21)^2 = 130$	**M1**	Substitute $y = -2x + 26$ into a circle equation.
	$5x^2 - 80x + 315\ [=0]$ leading to $[5](x-9)(x-7)$	**M1**	Factorisation OE must be seen.
	$x = 9$	**A1**	With or without $x = 7$.
	$y = 8$ OR (9, 8)	**A1**	$y = 8 \text{ or } (9,8)$ only. Both A1's dependent on the first M1.
		6	

Question	Answer	Marks	Guidance
6	$7C1\left(\frac{x}{a}\right)^6\left(\frac{a}{x^2}\right)$ or $7C6\left(\frac{x}{a}\right)^6\left(\frac{a}{x^2}\right)$ $7C2\left(\frac{x}{a}\right)^5\left(\frac{a}{x^2}\right)^2$ or $7C5\left(\frac{x}{a}\right)^5\left(\frac{a}{x^2}\right)^2$	**B1 B1**	Coefficients x^4 & x. Can be seen in an expansion.
	$\frac{\left(\frac{7}{a^5}\right)}{\left(\frac{21}{a^3}\right)} = 3$	**M1**	OE. Allow extraneous x^4 and x at this stage; numerator and denominator must be functions of a. Allow errors in evaluation of the combinations.
		A1	Completely correct.
	$a^2 = \frac{1}{9}$	**A1**	SOI (implied by $a = \frac{1}{3}$).
	$a = \pm\frac{1}{3}$	**A1**	Allow ± 0.333 .
		6	

Question	Answer	Marks	Guidance
7(a)	$\tan\theta\sin\theta = 1$ leading to $\sin^2\theta = \cos\theta$	M1	Use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and multiplication by $\cos\theta$.
	$1-\cos^2\theta = \cos\theta$ or $\cos^2\theta + \cos\theta - 1$ $[=0]$	M1	Use of trig identity to form a 3-term quadratic.
	$[\cos\theta =]$ $\dfrac{-1\pm\sqrt{5}}{2}$	M1	Use of formula or completion of the square must be seen on a 3-term quadratic. Expect 0.6180 .
	51.8º,	A1	Both A marks dependent on the 2nd M1.
	308.2º	A1 FT	FT for (360º – 1st soln), A0 if extra solutions in range. Radians 0.905 and 5.38, A1 only for both.
		5	
7(b)	$\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta} = \dfrac{\sin\theta}{\sin\theta\cos\theta} - \dfrac{\sin\theta\cos\theta}{\sin\theta} = \dfrac{1}{\cos\theta} - \cos\theta$	M1	Use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ twice with correct use of fractions.
	$= \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$	M1	Use $1-\cos^2\theta = \sin^2\theta$ with correct use of fractions.
	$= \tan\theta\sin\theta$	A1	WWW
		3	

Question	Answer	Marks	Guidance
8(a)	$\tan BDC = \frac{4}{3}$ or $\sin BDC = \frac{4}{5}$ or $\cos BDC = \frac{3}{5}$ **used** to find ADC	**M1**	May use cosine rule or $CAD = \tan^{-1}\frac{4}{8}$.
	$BDC = 0.927[3] \rightarrow ADC = \pi - 0.927[3]$ $[= 2.214 \text{ to } 2.215]$	**A1**	Allow degrees, 126.87, and 0.7048π or 0.705π.
	$Arc\ AC = 5 \times their\ 2.214$	**M1**	Use of $r\theta$ or $\frac{\theta}{360}.2\pi r$ Expect 11.07 .
	$AC = \sqrt{8^2 + 4^2}$ *or* $2 \times 5 \times \sin 1.107$	**M1**	Expect 8.94 .
	$[\text{Perimeter} = 11.07 + 8.94 =]\ 20.0$	**A1**	Accept AWRT [20.01, 20.02].
		5	
8(b)	Sector $ACD = ½ \times 5^2 \times their\ 2.214$	**M1**	See use of $\frac{1}{2}r^2\theta$ or $\frac{\theta}{360}.\pi r^2$. Expect 27.7 .
	Subtracting the area of $\Delta ADC = ½ \times 5 \times 4$ or $\frac{1}{2}5^2 \sin their\ 2.214$ or $\frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 3 \times 4$	**M1**	Subtracting the area of ΔADC, expect -10.
	$\text{Shaded area} = 27.7 - 10 = 17.7$	**A1**	Accept AWRT [17.67, 17.68]. Correct answer cannot come from an angle of 2.215 .
		3	

Question	Answer	Marks	Guidance
9(a)	$[y] \leqslant -1$	B1	Accept f or $\mathrm{f}(x) \leqslant -1$, $-\infty < y \leqslant -1$, $(-\infty, -1]$. Do not accept $x \leqslant -1$.
		1	
9(b)	$y = -3x^2 + 2$ rearranged to $3x^2 = 2 - y$, leading to $x^2 = \dfrac{2-y}{3}$ or $y^2 = \dfrac{2-x}{3}$	M1	
	$x = [\pm]\sqrt{\dfrac{2-y}{3}} \rightarrow [\mathrm{f}^{-1}(x)] = \{-\}\left\{\sqrt{\dfrac{2-x}{3}}\right\}$	A1 A1	A1 for minus, A1 for $\sqrt{\dfrac{2-x}{3}}$, allow $-\sqrt{\dfrac{x-2}{-3}}$.
		3	
9(c)	$\mathrm{fg}(x) = -3\left(-x^2 - 1\right)^2 + 2$	M1	SOI expect $-3x^4 - 6x^2 - 1$.
	$\mathrm{gf}(x) = -\left(-3x^2 + 2\right)^2 - 1$	M1	SOI expect $-9x^4 + 12x^2 - 5$.
	$\mathrm{fg}(x) - \mathrm{gf}(x) + 8 = 0$ leading to $6x^4 - 18x^2 + 12$ $[= 0]$	A1	OE
	$[6]\left(x^2 - 1\right)\left(x^2 - 2\right)$ $[= 0]$ or formula or completion of the square	M1	Solving a 3-term quadratic equation in x^2 must be seen.
	$x = -1$, $-\sqrt{2}$ only these **two** solutions	A1	Allow $-\sqrt{}1$, $-1.41[4]$ Answers only **SC B1**.
		5	

Question	Answer	Marks	Guidance
10(a)	$-\frac{3}{2}=\frac{1}{2}+k$ leading to $k=-2$	B1	**AG** Need to see $4^{-\frac{1}{2}}$ evaluated as $\frac{1}{4^{\frac{1}{2}}}$ or better.
		1	
10(b)	$[y]=2x^{\frac{1}{2}}-2x \quad [+c]$	M1 A1	Allow $\frac{x^{\frac{1}{2}}}{1/2}-2x$.
	$-1=4-8+c$	M1	Substitute x = 4, y = -1 (c present) Expect c = 3.
	$y=2x^{\frac{1}{2}}-2x+3$ or $y=2\sqrt{x}-2x+3$	A1	Allow if f(x) = or y = anywhere in the solution.
		4	
10(c)	$x^{-1/2}-2=0$	M1	Set *their* $\frac{dy}{dx}$ to zero.
	$x=\frac{1}{4}$	A1	If $\left(\frac{1}{2}\right)^2=\pm\frac{1}{4}$ max of M1A1 if $\left(\frac{1}{4},3\frac{1}{2}\right)$ seen.
	(¼, 3½)	A1	
		3	

Question	Answer	Marks	Guidance
10(d)	$\dfrac{d^2y}{dx^2} = -\dfrac{1}{2}x^{-\frac{3}{2}}$	**B1**	
	< 0 (or -4) hence Maximum	**DB1**	WWW Ignore extra solutions from $x = -\dfrac{1}{4}$.
		2	

Question	Answer	Marks	Guidance
11(a)	Gradient of AB = $\dfrac{2-(-1)}{5-2}$	**M1**	Expect 1, must be from $\Delta y / \Delta x$.
	Equation of AB is $y-2=1(x-5)$ or $y+1=1(x-2)$	**A1**	OE. Expect $y = x-3$.
		2	

Question	Answer	Marks	Guidance
11(b)	$[\pi]\int x^2 dy = [\pi]\int \left(y^2+1\right)^2 dy = [\pi]\int \left(y^4+2y^2+1\right) dy$	**M1**	For curve: Attempt to square y^2+1 and attempt integration. Subtracting curve equation from line equation before squaring is M0. Integration before squaring M0.
	$[\pi]\left(\frac{y^5}{5}+\frac{2y^3}{3}+y\right)$	**A2, 1, 0**	
	$[\pi]\int (y+3)^2 dy = [\pi]\int \left(y^2+6y+9\right) dy$	**M1**	For line: Attempt to square *their* $y+3$ and attempt integration.
	$[\pi]\left(\frac{y^3}{3}+3y^2+9y\right)$ or $[\pi]\left(\frac{(y+3)^3}{3}\right)$	**A2, 1, 0**	Not available for incorrect line equations.
	$[\pi]\left\{\frac{8}{3}+12+18-\left(-\frac{1}{3}+3-9\right)\right\}$ or $[\pi]\left\{\frac{32}{5}+\frac{16}{3}+2-\left(-\frac{1}{5}-\frac{2}{3}-1\right)\right\}$	**DM1**	Apply limits $-1 \to 2$ to either integral providing they have been awarded M1. Expect $15\frac{3}{5}$ $[\pi]$ and/or $39[\pi]$. Some evidence of substitution of both -1 and 2 must be seen. Dependent on at least one of the first 2 M1 marks.
	Volume = $[\pi](39 - 15\frac{3}{5})$	**DM1**	Appropriate subtraction. Dependent on at least one of the first 2 M1 marks.
	$= 23\frac{2}{5}\pi$ or $\frac{117}{5}\pi$ or awrt 73.5[1327]	**A1**	
		9	

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 8 0 1 7 8 8 9 7 4 6 *

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **February/March 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 Find the exact value of $\int_0^{\frac{1}{2}\pi} 2\tan^2\left(\frac{1}{2}x\right)\,dx$. [4]

2 Solve the equation $\tan(\theta - 60°) = 3\cot\theta$ for $-90° < \theta < 90°$. [5]

3 The polynomial $p(x)$ is defined by

$$p(x) = ax^3 - ax^2 + ax + b,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $p(x)$, and that the remainder is 35 when $p(x)$ is divided by $(x - 3)$.

(a) Find the values of a and b. [5]

(b) Hence factorise $p(x)$ and show that the equation $p(x) = 0$ has exactly one real root. [3]

4 **(a)** Sketch, on the same diagram, the graphs of $y = |2x - 11|$ and $y = 3x - 3$. [2]

(b) Solve the inequality $|2x - 11| < 3x - 3$. [3]

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(c) Find the smallest integer N satisfying the inequality $|2\ln N - 11| < 3\ln N - 3$. [2]

5 It is given that $\int_1^a \left(\frac{4}{1+2x} + \frac{3}{x}\right) dx = \ln 10$, where a is a constant greater than 1.

(a) Show that $a = \sqrt[3]{90(1+2a)^{-2}}$. [5]

(b) Use an iterative formula, based on the equation in **(a)**, to find the value of a correct to 3 significant figures. Use an initial value of 1.7 and give the result of each iteration to 5 significant figures. [3]

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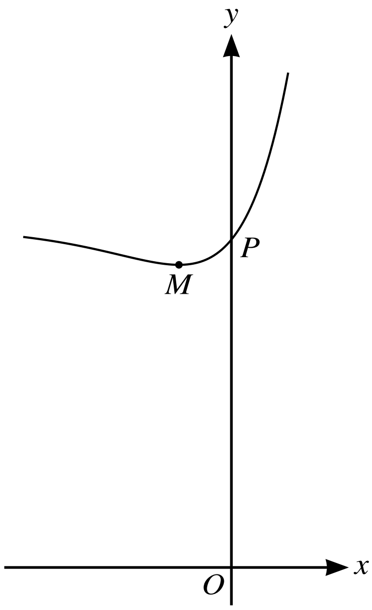

The diagram shows the curve with equation $y = \dfrac{4e^{2x} + 9}{e^x + 2}$. The curve has a minimum point M and crosses the y-axis at the point P.

(a) Find the exact value of the gradient of the curve at P. [4]

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(b) Find the exact coordinates of M. [4]

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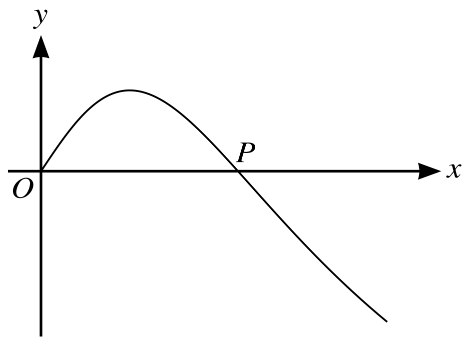

The diagram shows the curve with parametric equations

$$x = k \tan t, \qquad y = 3 \sin 2t - 4 \sin t,$$

for $0 < t < \frac{1}{2}\pi$. It is given that k is a positive constant. The curve crosses the x-axis at the point P.

(a) Find the value of $\cos t$ at P, giving your answer as an exact fraction. [3]

(b) Express $\dfrac{dy}{dx}$ in terms of k and $\cos t$. [4]

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(c) Given that the normal to the curve at P has gradient $\frac{9}{10}$, find the value of k, giving your answer as an exact fraction. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS Level

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **February/March 2023**

MARK SCHEME

Maximum Mark: 50

Published

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GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Express integrand as $2\sec^2\frac{1}{2}x-2$	B1	
	Integrate to obtain form $a\tan\frac{1}{2}x+bx$	M1	where $ab\neq 0$.
	Obtain correct $4\tan\frac{1}{2}x-2x$	A1	
	Obtain $4-\pi$	A1	
		4	

Question	Answer	Marks	Guidance
2	State $\dfrac{\tan\theta-\tan 60}{1+\tan\theta\tan 60}=\dfrac{3}{\tan\theta}$	B1	OE involving $\tan\theta$ only.
	Attempt arrangement of equation to quadratic form	M1	Condone sign errors in first step and retention of $\tan 60$.
	Obtain $\tan^2\theta-4\sqrt{3}\tan\theta-3=0$	A1	OE involving decimals.
	Solve 3-term quadratic equation to obtain at least one value of θ	M1	
	Obtain –22.2 and 82.2	A1	Or greater accuracy; and no others between –90 and 90.
		5	

Question	Answer	Marks	Guidance
3(a)	Substitute $x=-2$ and equate to zero	***M1**	
	Substitute $x=3$ and equate to 35	***M1**	
	Obtain $-8a-4a-2a+b=0$ and $27a-9a+3a+b=35$	**A1**	
	Solve a pair of relevant simultaneous linear equations to find a or b	**DM1**	Dependent at least one M mark.
	Obtain $a=1$ and $b=14$	**A1**	
		5	
3(b)	Divide by $x+2$ at least as far as the x term	**M1**	
	Obtain $\left[(x+2)\right](x^2-3x+7)$	**A1**	
	Conclude with reference to -2, and discriminant is $9-28$ and hence no root	**A1**	OE
		3	

Question	Answer	Marks	Guidance
4(a)	Draw V-shaped graph with vertex on positive x-axis	**B1**	
	Draw approximately correct graph of $y=3x-3$ with greater gradient	**B1**	Crossing x-axis between origin and vertex of first graph.
		2	

Question	Answer	Marks	Guidance
4(b)	Attempt solution of linear equation where signs of $2x$ and $3x$ are different	**M1**	
	Solve $-2x+11=3x-3$ to obtain $x=\frac{14}{5}$	**A1**	OE
	Conclude $x>\frac{14}{5}$	**A1**	OE
	Alternative method for Question 4(b)		
	Attempt solution of 3-term equation $(2x-11)^2=(3x-3)^2$ to obtain at least one value of x	**M1**	Or equivalent inequality.
	Obtain at least $x=\frac{14}{5}$	**A1**	OE
	Conclude $x>\frac{14}{5}$	**A1**	OE
		3	
4(c)	Attempt value of N (maybe non-integer at this stage) using logarithms and *their* answer to part **(b)**.	**M1**	
	Conclude with single integer 17	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	Integrate to obtain form $k_1 \ln(1+2x)+k_2 \ln x$	***M1**	$k_1 \neq 0,\ k_2 \neq 0$.
	Obtain correct $2\ln(1+2x)+3\ln x$	**A1**	
	Use limits correctly and equate to $\ln 10$	**DM1**	
	Apply relevant logarithm properties correctly and arrange as far as $a^3 = \ldots$	**DM1**	
	Confirm given result $a=\sqrt[3]{90(1+2a)^{-2}}$ with sufficient detail	**A1**	AG
		5	
5(b)	Use iteration process correctly at least once	**M1**	Need to see 1.6848 .
	Obtain final answer 1.68	**A1**	Answer required to exactly 3 sf.
	Show sufficient iterations to 5 sf to justify answer or show a sign change in the interval $[1.675,\ 1.685]$	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	Differentiate using quotient rule	**M1**	OE
	Obtain $\dfrac{(e^x+2)\,8e^{2x}-(4e^{2x}+9)\,e^x}{(e^x+2)^2}$	**A1**	OE
	Substitute $x=0$ in first derivative and attempt evaluation	**M1**	
	Obtain $\frac{11}{9}$	**A1**	
		4	
6(b)	Equate first derivative to zero and attempt factorisation or equivalent	**M1**	
	Solve a three-term quadratic equation in e^x to obtain $e^x=...$	**M1**	$(2e^x+9)(2e^x-1)=0$.
	Obtain *x*-coordinate $\ln\frac{1}{2}$ or $-\ln 2$	**A1**	
	Obtain *y*-coordinate 4	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Use identity $\sin 2t = 2\sin t\cos t$	**B1**	
	Attempt solution of $y=0$ for $\cos t$	**M1**	
	Obtain $\cos t = \frac{2}{3}$	**A1**	Or exact equivalent.
		3	

Question	Answer	Marks	Guidance
7(b)	Differentiate to obtain at least one of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct	*M1	
	Obtain $\frac{dy}{dx} = \frac{6\cos 2t - 4\cos t}{k\sec^2 t}$	A1	
	Attempt to express $\frac{dy}{dx}$ in terms of $\cos t$	DM1	Using correct identities.
	Obtain $\frac{dy}{dx} = \frac{6(2\cos^2 t - 1)\cos^2 t - 4\cos^3 t}{k}$	A1	OE
		4	
7(c)	Substitute value from part **(a)** in expression for $\frac{dy}{dx}$ involving k and $\cos t$	*M1	
	Equate to $-\frac{10}{9}$ and solve for k	DM1	
	Obtain $k = \frac{4}{3}$	A1	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER					
CANDIDATE NUMBER					

* 9 3 3 7 4 9 5 7 5 6 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

1 It is given that $x = \ln(2y - 3) - \ln(y + 4)$.

Express y in terms of x. [3]

2 **(a)** On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $-\frac{1}{3}\pi \leqslant \arg(z - 1 - 2\mathrm{i}) \leqslant \frac{1}{3}\pi$ and $\operatorname{Re} z \leqslant 3$. [3]

(b) Calculate the least value of $\arg z$ for points in the region from **(a)**. Give your answer in radians correct to 3 decimal places. [2]

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3 The polynomial $2x^4 + ax^3 + bx - 1$, where a and b are constants, is denoted by $\mathrm{p}(x)$. When $\mathrm{p}(x)$ is divided by $x^2 - x + 1$ the remainder is $3x + 2$.

Find the values of a and b. [5]

4 Solve the equation

$$\frac{5z}{1+2\mathrm{i}} - zz^* + 30 + 10\mathrm{i} = 0,$$

giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

5 The parametric equations of a curve are

$$x = te^{2t}, \qquad y = t^2 + t + 3.$$

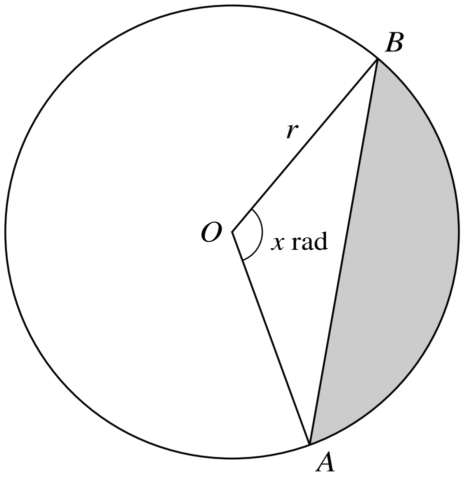
(a) Show that $\dfrac{dy}{dx} = e^{-2t}$. [3]

(b) Hence show that the normal to the curve, where $t = -1$, passes through the point $\left(0,\ 3 - \dfrac{1}{e^4}\right)$. [3]

6 **(a)** Express $5\sin\theta + 12\cos\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]

(b) Hence solve the equation $5\sin 2x + 12\cos 2x = 6$ for $0 \leqslant x \leqslant \pi$. [4]

7



The diagram shows a circle with centre O and radius r. The angle of the **minor** sector AOB of the circle is x radians. The area of the **major** sector of the circle is 3 times the area of the shaded region.

(a) Show that $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$. [4]

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(b) Show by calculation that the root of the equation in **(a)** lies between 2 and 2.5. [2]

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(c) Use an iterative formula based on the equation in **(a)** to calculate this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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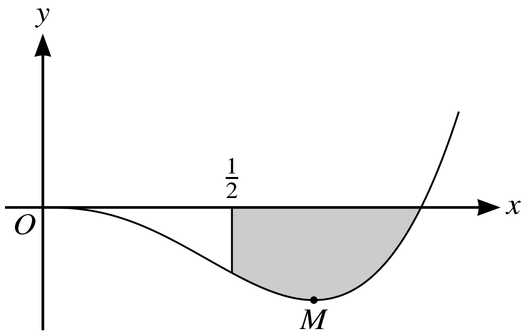

The diagram shows the curve $y = x^3 \ln x$, for $x > 0$, and its minimum point M.

(a) Find the exact coordinates of M. [4]

(b) Find the exact area of the shaded region bounded by the curve, the x-axis and the line $x = \frac{1}{2}$. [5]

9 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{3y} \sin^2 2x.$$

It is given that $y = 0$ when $x = 0$.

Solve the differential equation and find the value of y when $x = \frac{1}{2}$. [7]

10 With respect to the origin O, the points A, B, C and D have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 5 \\ -6 \\ 11 \end{pmatrix}.$$

(a) Find the obtuse angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. [3]

The line l passes through the points A and B.

(b) Find a vector equation for the line l. [2]

(c) Find the position vector of the point of intersection of the line l and the line passing through C and D. [4]

11 Let $f(x) = \dfrac{5x^2 + x + 11}{(4 + x^2)(1 + x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^2 f(x)\,dx = \ln 54 - \frac{1}{8}\pi$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2023 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **17** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
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3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
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5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
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Mark Scheme Notes

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Types of mark

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
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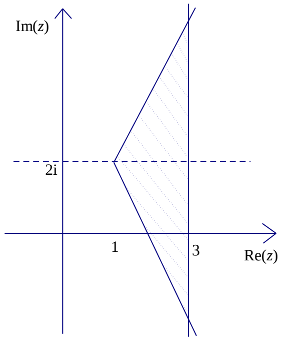
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a quotient or express x as ln e^x	**M1**	$x = \ln[(2y-3)/(y+4)]$ or $\ln e^x = \ln(2y-3) - \ln(y+4)]$.
	Remove logarithms and obtain a correct equation e.g. $e^x = \dfrac{2y-3}{y+4}$	**A1**	
	Obtain answer $y = \dfrac{3+4e^x}{2-e^x}$	**A1**	OE ISW
		3	

Question	Answer	Marks	Guidance
2(a)	Show correct half-lines from $1 + 2i$, symmetrical about $y = 2i$ (drawn between $\dfrac{\pi}{4}$ and $\dfrac{5\pi}{12}$).	**B1**	
	Show the line $x = 3$ extending in both quadrants.	**B1**	
	Shade the correct region. Allow dashes on axes as scale. FT If only error is one of following: FULL lines or $x \neq 3$ or one sign error in $1 + 2i$ or angle outside tolerance or scale missing on one axis.	**B1 FT**	
			SC No scale on either axis allow **B1 FT** for otherwise correct figure in correct position.
		3	

Question	Answer	Marks	Guidance
2(b)	Carry out a complete method for finding the least value of arg z	**M1**	e.g. $-\tan^{-1}\dfrac{\left(2\sqrt{3}-2\right)}{3}$ or $\tan^{-1}\dfrac{\left(-2\sqrt{3}+2\right)}{3}$.
	Obtain answer –0.454 3dp	**A1**	**SC B1** 0.454 .
		2	

Question	Answer	Marks	Guidance
3	Commence division and reach partial quotient $2x^2 + (a \pm 2)x$	**M1**	$2x^2 + (a + 2)x + a$ need $2x^2 + (a \pm 2)x$ $(x^2 - x + 1)\ 2x^4 + ax^3 + 0x^2 + bx - 1$ $2x^4 - 2x^3 + 2x^2$ $(a + 2)x^3 - 2x^2 + bx$ $(a + 2)x^3 - (a + 2)x^2 + (a + 2)x$ $ax^2 + (b - (a + 2))x - 1$ $ax^2 - ax + a$ $(b - 2)x - (1 + a)$ $3x + 2$ Working backwards from remainder: $2x^2 + (\ldots.)x \pm 3$ **M1** $2x^2 - x - 3$ **A1**
	Obtain correct quotient $2x^2 + (a + 2)x + a$	**A1**	Allow sign error e.g. in $b - 2$.
	Set *their* linear remainder equal to part of "$3x + 2$" and solve for a or for b	**M1**	Remainder = $3x + 2 = (b - 2)x - 1 - a$. Allow for just equating x term or constant term.
	Obtain answer $a = -3$	**A1**	
	Obtain answer $b = 5$	**A1**	

Question	Answer	Marks	Guidance
3	**Alternative method for Question 3**		
	State $2x^4+ax^3+0x^2+bx-1=\left(x^2-x+1\right)\left(2x^2+Ax+B\right)+3x+2$ and form and solve equation(s) to obtain A or B	**M1**	e.g. $0 = B - A + 2$ and $-1 = B + 2$.
	Obtain $A = -1$, $B = -3$	**A1**	
	Form and solve equations for a or for b	**M1**	e.g. $a = A - 2$ or $b = -B + A + 3$.
	Obtain answer $a = -3$	**A1**	
	Obtain answer $b = 5$	**A1**	
	Alternative method for Question 3		
	Use remainder theorem with $x=\dfrac{1\pm\sqrt{-3}}{2}$ or $x=\dfrac{1\pm i\sqrt{3}}{2}$	**M1**	Allow for correct use of a reasonable attempt at either root in exact or decimal form in the remainder theorem $x^2=\dfrac{-1+\sqrt{-3}}{2}$ $x^3=-1$ $x^4=\dfrac{-1-\sqrt{-3}}{2}$.
	Obtain $-a+\dfrac{b}{2}\pm\dfrac{b\sqrt{-3}}{2}\mp\sqrt{-3}-2=\dfrac{7}{2}\pm\dfrac{3\sqrt{-3}}{2}$ or $-a+\dfrac{b}{2}\pm\dfrac{bi\sqrt{3}}{2}\mp i\sqrt{3}-2=\dfrac{7}{2}\pm\dfrac{3i\sqrt{3}}{2}$	**A1**	Expand brackets and obtain exact equation for either root. Accept exact equivalent.
	Solve simultaneous equations, or single equation, for a or for b	**M1**	
	Obtain answer $a = -3$ from exact working	**A1**	
	Obtain answer $b = 5$ from exact working	**A1**	
		5	

Question	Answer	Marks	Guidance
4	Substitute $z = x + \mathrm{i}y$ and $z^* = x - \mathrm{i}y$ to obtain a correct equation, horizontal or with $(1-2\mathrm{i})/(1-2\mathrm{i})$ seen, in x and y	**B1**	$5(x + \mathrm{i}y) - (x + \mathrm{i}y)(x - \mathrm{i}y)(1 + 2\mathrm{i}) + (30 + 10\mathrm{i})(1 + 2\mathrm{i}) = 0$ $5(x + \mathrm{i}y)(1 - 2\mathrm{i})/[(1 + 2\mathrm{i})(1 - 2\mathrm{i})] - (x + \mathrm{i}y)(x - \mathrm{i}y) + (30 + 10\mathrm{i}) = 0$ $x - 2\mathrm{i}x + \mathrm{i}y + 2y - x^2 - y^2 + 30 + 10\mathrm{i} = 0$.
	Use $\mathrm{i}^2 = -1$ at least once and equate real and imaginary parts to zero	***M1**	OE For their horizontal equation.
	Obtain two correct equations e.g. $x + 2y - x^2 - y^2 + 30 = 0$ and $-2x + y + 10 = 0$	**A1**	$5x - (x^2 + y^2) + 10 = 0$ $5y - 2(x^2 + y^2) + 70 = 0$ $5y - 10x + 50 = 0$ $x + 2y - (x^2 + y^2) + 30 = 0$ Allow $-2\mathrm{i}x + \mathrm{i}y + 10\mathrm{i} = 0$.
	Solve quadratic equation for x or for y	**DM1**	$x^2 - 9x + 18 = (x - 3)(x - 6) = 0$ $y^2 + 2y - 8 = (y + 4)(y - 2) = 0$ **DM0** If x or y imaginary.
	Obtain answers $3 - 4\mathrm{i}$ and $6 + 2\mathrm{i}$	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	Obtain $\dfrac{dx}{dt} = e^{2t} + 2te^{2t}$	**B1**	OE
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$	**M1**	$\dfrac{dy}{dx} = \dfrac{2t+1}{e^{2t}\left(1+2t\right)}$.
	Obtain the given answer $\dfrac{dy}{dx} = e^{-2t}$	**A1**	AG Need to see e^{2t} $(1 + 2t)$ in denominator.
		3	
5(b)	Obtain $x = -e^{-2}$ or $-\dfrac{1}{e^2}$ and $y = 3$ at $t = -1$	**B1**	
	Obtain gradient of normal = $-e^{-2}$ or $-\dfrac{1}{e^2}$	**B1**	
	$x = 0$ substituted into equation of normal or use of gradients to give $y = 3 - \dfrac{1}{e^4}$ with no errors	**B1**	Equation of normal $y - 3 = -e^{-2}\left(x - -e^{-2}\right)$. AG **SC** Decimals **B0 B1 B0** $-$ 0.135 .
		3	

Question	Answer	Marks	Guidance
6(a)	State $R = 13$	**B1**	Allow if $\sqrt{(12^2 + (-5)^2)}$ seen.
	Use correct trig formulae to find $\alpha = \tan^{-1}(\pm 5/12) = \cos^{-1}(\pm 12/13) = \sin^{-1}(\pm 5/13)$	**M1**	$\cos(\alpha) = 12$ and $\sin(\alpha) = 5$ M0 However, $\sin(\alpha)/\cos(\alpha) = 5/12$ or $-5/12$ with no error seen, or $\tan(\alpha) = 5/12$ or $-5/12$ quoted then allow.
	Obtain $\alpha = 0.395$	**A1**	CWO If negative sign seen when finding R then A0 here. If degrees 22.6 A0 MR. Only penalise degrees once in **(a)** and **(b)**. Note $\alpha = 0.39479\ldots$
		3	
6(b)	$\cos^{-1}\left(\frac{6}{R}\right)$	**B1FT**	SOI 1.0910… FT *their* incorrect R.
	Use correct method to find a value of $2x$ in the interval	**M1**	$2x = \cos^{-1}\left(\frac{6}{R}\right) + \alpha$ or $2\pi - \cos^{-1}\left(\frac{6}{R}\right) + \alpha$. Allow if $\cos(2x + 0.395)$ seen
	Obtain answer, e.g. $x = 0.743$ or 0.742	**A1**	42.5 or 42.6 degrees.
	Obtain second answer, e.g. $x = 2.79$ and no others in the interval	**A1**	159.8, 159.9 or 160.0 degrees all possible depending whether using 3 dp or 4 dp.
		4	

Question	Answer	Marks	Guidance
7(a)	State or imply area of major sector = $\frac{1}{2}r^2(2\pi - x)$	**B1**	OE
	State or imply area of shaded segment = $\frac{1}{2}r^2x - \frac{1}{2}r^2\sin x$	**B1**	OE $r^2\sin(x/2)\cos(x/2)$ B0 until changed to $(1/2)r^2\sin x$.
	State $\frac{1}{2}r^2(2\pi - x) = 3\left(\frac{1}{2}r^2x - \frac{1}{2}r^2\sin x\right)$	**M1**	OE Area of major sector = 3 times (area of minor sector – area of triangle). Allow $r^2\sin(x/2)\cos(x/2)$.
	Obtain the given answer $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$ after full and correct working	**A1**	AG Allow rectified slip if before penultimate line.
		4	
7(b)	Calculate the values of a relevant expression or pair of expressions at $x = 2$ and $x = 2.5$	**M1**	$x = 2$ $\quad$ $x = 2.5$ $(3/4)\sin x + (1/2)\pi$ 2.2(5277) $\quad$ 2.0(197) $2 < 2.2$ or 2.3 $\quad$ $2.5 > 2.0$ $x - (3/4)\sin x - (1/2)\pi$ $-0.2(5277) < 0$ $\quad$ $+0.4(803) > 0$ or change of sign Attempt both values and one correct for M1.
	Complete the argument correctly with correct calculated values	**A1**	Degrees award 0/2
		2	

Question	Answer	Marks	Guidance
7(c)	Use the iterative formula correctly at least twice	**M1**	
	Obtain final answer 2.18	**A1**	
	Show sufficient iterations to 4 d.p. to justify 2.18 to 2 d.p. or show there is a sign change in the interval (2.175, 2.185)	**A1**	2 2.25 2.5 2.2528 2.1543(5) 2.0196(5) 2.1530 2.1967 2.2465 2.1972 2.1786 2.1560 2.1784 2.1865 2.1960 2.1866 2.1831 2.1789 2.1830 2.1845 2.1863 2.1846 2.1831 2.1845 Degrees award 0/3
		3	

Question	Answer	Marks	Guidance
8(a)	Use the product rule correctly	***M1**	$x^3\ \mathrm{d}/\mathrm{d}x(\ln x) + \mathrm{d}/\mathrm{d}x(x^3)\ \ln x$.
	Obtain the correct derivative in any form	**A1**	e.g. $\dfrac{x^3}{x} + 3x^2 \ln x$.
	Equate derivative to zero and solve exactly for x	**DM1**	Reaching $x = \mathrm{e}^a$.
	Obtain answer $\left(\dfrac{1}{\sqrt[3]{\mathrm{e}}}, -\dfrac{1}{3\mathrm{e}}\right)$ or exact equivalent	**A1**	ISW
		4	

Question	Answer	Marks	Guidance
8(b)	Integrate by parts and reach $ax^4 \ln x + b\int(x^4 / x)\mathrm{d}x$	*M1	
	Obtain $\dfrac{x^4}{4}\ln x - \dfrac{1}{4}\int(x^4 / x)\mathrm{d}x$	A1	OE
	Complete integration and obtain $\dfrac{x^4}{4}\ln x - \dfrac{x^4}{16}$	A1	OE
	Use limits of $x = \dfrac{1}{2}$ and $x = 1$ in the correct order, having integrated twice	DM1	Correct substitution [(1/4)ln1 or 0 − 1/16] – [(1/64)ln(1/2) – $(1/16)^2$] or minus this value CWO. Allow omission of (1/4)ln1 or 0.
	Obtain answer $\dfrac{15}{256} - \dfrac{1}{64}\ln 2$ or exact equivalent final answer	A1	
		5	

Question	Answer	Marks	Guidance
9	Separate variables correctly and obtain e^{-3y} and $\sin^2 2x$ on the opposite sides	**B1**	
	Obtain term $-\frac{1}{3}e^{-3y}$	**B1**	
	Use correct double angle formula for $\sin^2 2x = (1/2)[1 - \cos 4x]$	**M1**	
	Obtain terms $\frac{1}{2}\left[x - \frac{1}{4}\sin 4x\right]$ oe	**A1**	
	Use $x = 0$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form ax and $b\sin 4x$ and $ce^{\pm 3y}$	**M1**	
	Obtain correct answer in any form e.g. $-\frac{1}{3}e^{-3y} = \frac{1}{2}\left[x - \frac{1}{4}\sin 4x\right] - \frac{1}{3}$	**A1**	
	Substitute $x = \frac{1}{2}$ and obtain $y = 0.175$ or $-\frac{1}{3}\ln\left(\frac{1}{4} + \frac{3}{8}\sin 2\right)$	**A1**	OE ISW
		7	

Question	Answer	Marks	Guidance
10(a)	Carry out correct process for evaluating the scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OB}$	**M1**	$\pm(3, -1, 2).(1, 2, -3) = \pm(3 - 2 - 6) = [-5]$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain $\cos^{-1}\{\pm(3-2-6)/[\sqrt{(3^2+(-1)^2+2^2)}\ \sqrt{(1^2+2^2+(-3)^2)}]\}$	**A1**	
	Obtain answer 110.9° or 1.94^c	**A1**	
		3	
10(b)	Use a correct method to form an equation for line through *AB*	**M1**	
	Obtain $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu_1(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	**A1**	OE e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \mu_2(-2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})$. Need $\mathbf{r}$ or (x, y, z).
		2	
10(c)	Obtain a correct equation for line through *CD* e.g. $[\mathbf{r} =]\ \mathbf{i} - 2\mathbf{j} + 5\mathbf{k} + \lambda_1(-4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$	**B1**	OE e.g. $[\mathbf{r} =]\ 5\mathbf{i} - 6\mathbf{j} + 11\mathbf{k} + \lambda_2(-4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$. $\mathbf{r}$ can be omitted or another symbol used.
	Equate two pairs of components of general points on *their l* and *their CD* and solve for λ or for μ	**M1**	
	Obtain e.g. $\lambda_1 = -2$ or $\mu_1 = 3$ or $\lambda_2 = -1$ or $\mu_2 = -4$	**A1**	
	Obtain position vector $9\mathbf{i} - 10\mathbf{j} + 17\mathbf{k}$	**A1**	Condone $(9, -10, 17)$ but not $(9\mathbf{i}, -10\mathbf{j}, 17\mathbf{k})$.
		4	

Question	Answer	Marks	Guidance
11(a)	State or imply the form $\dfrac{Ax+B}{4+x^2}+\dfrac{C}{1+x}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	$(Ax+B)(1+x)+C(4+x^2)=5x^2+x+11$.
	Obtain one of $A=2$, $B=-1$ and $C=3$	**A1**	If error present in above still allow A1 for C.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	If $A=0$ then max M1 A1 (for C).
		5	
11(b)	Integrate and obtain terms $\left(\dfrac{A}{2}\right)\ln\left(4+x^2\right)+\dfrac{B}{2}\tan^{-1}\left(\dfrac{x}{2}\right)+C\ln\left(1+x\right)$	**B1FT + B1FT + B1FT**	The FT is on A, B and C. Integral of $\dfrac{Ax+B}{4+x^2}=\dfrac{B}{2}\tan^{-1}\left(\dfrac{x}{2}\right)$ or $(A/2)\ln(4+x^2)$ only. B0FT unless clearly from single term.
	Substitute limits 0 and 2 correctly in an integral of the form $a\ln\left(4+x^2\right)+b\tan^{-1}\left(\dfrac{x}{2}\right)+c\ln\left(1+x\right)$, where $abc\neq 0$	**M1**	$a\ln\left(4+4\right)+b\tan^{-1}\left(\dfrac{2}{2}\right)+c\ln\left(1+2\right)-[$ $a\ln 4+b\tan^{-1}0+c\ln\left(1\right)]$. Allow one sign or substitution error. Allow omission of $b\tan^{-1}0+c\ln\left(1\right)$.
	Obtain answer $\ln 54-\dfrac{\pi}{8}$ after full and correct working	**A1**	AG – work to combine or simplify at least 2 of ln terms is required CWO. A0 if any non exact value(s) seen.
		5	

Cambridge Assessment International Education

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CENTRE NUMBER						CANDIDATE NUMBER			

* 7 3 5 0 2 8 5 0 0 7 *

MATHEMATICS **9709/42**

Paper 4 Mechanics **February/March 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 A crate of mass 200 kg is being pulled at constant speed along horizontal ground by a horizontal rope attached to a winch. The winch is working at a constant rate of 4.5 kW and there is a constant resistance to the motion of the crate of magnitude 600 N.

(a) Find the time that it takes for the crate to move a distance of 15 m. [2]

The rope breaks after the crate has moved 15 m.

(b) Find the time taken, after the rope breaks, for the crate to come to rest. [3]

2 A particle P is projected vertically upwards from horizontal ground with speed $15\,\mathrm{m\,s^{-1}}$.

(a) Find the speed of P when it is $10\,\mathrm{m}$ above the ground. [2]

At the same instant that P is projected, a second particle Q is dropped from a height of $18\,\mathrm{m}$ above the ground in the same vertical line as P.

(b) Find the height above the ground at which the two particles collide. [3]

3 A particle moves in a straight line starting from rest from a point O. The acceleration of the particle at time t s after leaving O is a m s^{-2}, where $a = 4t^{\frac{1}{2}}$.

(a) Find the speed of the particle when $t = 9$. [2]

(b) Find the time after leaving O at which the speed (in metres per second) and the distance travelled (in metres) are numerically equal. [3]

4 A toy railway locomotive of mass 0.8 kg is towing a truck of mass 0.4 kg on a straight horizontal track at a constant speed of $2\,\mathrm{m\,s^{-1}}$. There is a constant resistance force of magnitude 0.2 N on the locomotive, but no resistance force on the truck. There is a light rigid horizontal coupling connecting the locomotive and the truck.

(a) State the tension in the coupling. [1]

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(b) Find the power produced by the locomotive's engine. [1]

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The power produced by the locomotive's engine is now changed to 1.2 W.

(c) Find the magnitude of the tension in the coupling at the instant that the locomotive begins to accelerate. [5]

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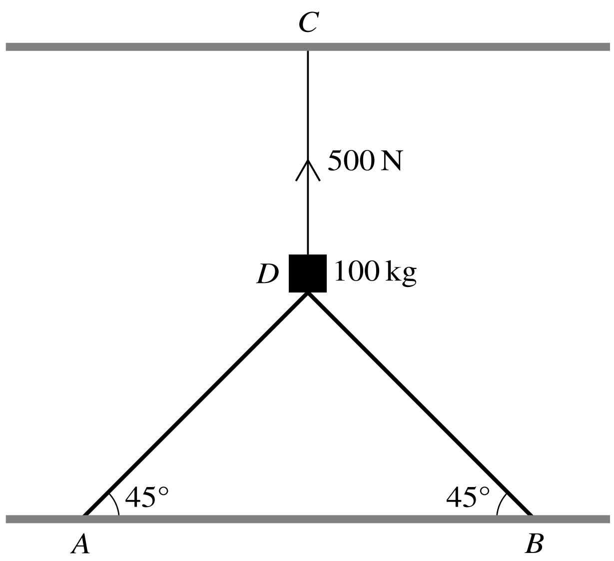

The diagram shows a block D of mass 100 kg supported by two sloping struts AD and BD, each attached at an angle of 45° to fixed points A and B respectively on a horizontal floor. The block is also held in place by a vertical rope CD attached to a fixed point C on a horizontal ceiling. The tension in the rope CD is 500 N and the block rests in equilibrium.

(a) Find the magnitude of the force in each of the struts AD and BD. [3]

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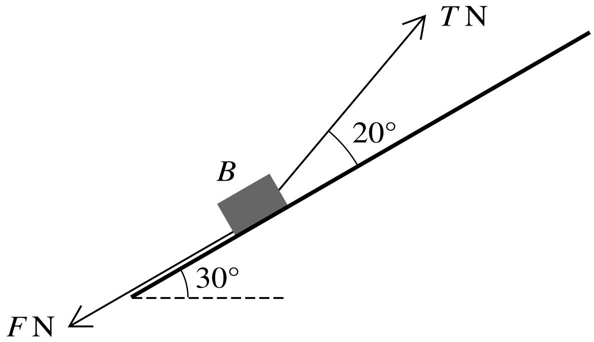
A horizontal force of magnitude F N is applied to the block in a direction parallel to AB.

(b) Find the value of F for which the magnitude of the force in the strut AD is zero. [3]

6



A block B, of mass 2 kg, lies on a rough inclined plane sloping at 30° to the horizontal. A light rope, inclined at an angle of 20° above a line of greatest slope, is attached to B. The tension in the rope is T N. There is a friction force of F N acting on B (see diagram). The coefficient of friction between B and the plane is μ.

(a) It is given that $F = 5$ and that the acceleration of B up the plane is $1.2\,\mathrm{m\,s^{-2}}$.

(i) Find the value of T. [3]

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(ii) Find the value of μ. [3]

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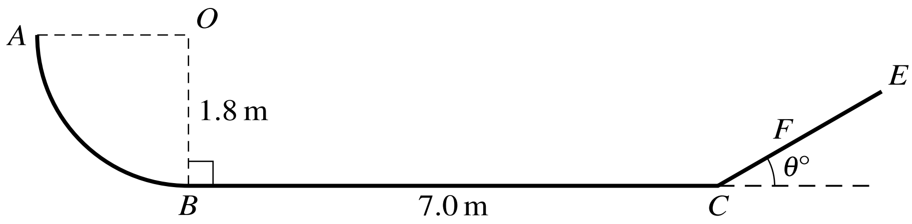
(b) It is given instead that $\mu = 0.8$ and $T = 15$.

Determine whether B will move up the plane. [3]

7



The diagram shows a smooth track which lies in a vertical plane. The section AB is a quarter circle of radius 1.8 m with centre O. The section BC is a horizontal straight line of length 7.0 m and OB is perpendicular to BC. The section CFE is a straight line inclined at an angle of $\theta°$ above the horizontal.

A particle P of mass 0.5 kg is released from rest at A. Particle P collides with a particle Q of mass 0.1 kg which is at rest at B. Immediately after the collision, the speed of P is $4\,\text{m s}^{-1}$ in the direction BC. You should assume that P is moving horizontally when it collides with Q.

(a) Show that the speed of Q immediately after the collision is $10\,\text{m s}^{-1}$. [4]

When Q reaches C, it collides with a particle R of mass 0.4 kg which is at rest at C. The two particles coalesce. The combined particle comes instantaneously to rest at F. You should assume that there is no instantaneous change in speed as the combined particle leaves C, nor when it passes through C again as it returns down the slope.

(b) Given that the distance CF is 0.4 m, find the value of θ. [4]

[Question 7 continues on the next page.]

(c) Find the distance from B at which P collides with the combined particle. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/42**

Paper 4 Mechanics **February/March 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2023 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **19** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$4500 = \frac{600 \times 15}{t}$	**M1**	Use of power = ΔW / Δt to get an equation in t. May see $600v = 4500 \Rightarrow v = 7.5$ followed by $7.5t = 15$.
	$t = 2$ s	**A1**	
		2	
1(b)	$\pm 600 = 200a\left[\Rightarrow a = \pm 3\right]$	***M1**	Use of Newton's second law; 2 terms only.
	$0 = \frac{15}{their\,2} + \left(their - 3\right)t$	**DM1**	Use of constant acceleration to set up an equation that would lead to a positive t, e.g. $v = u + at$ with *their* $t = 2$ and *their* negative a (and possibly *their* 7.5 from **(a)**).
	$t = 2.5$s	**A1**	
		3	

Question	Answer	Marks	Guidance
2(a)	Use constant acceleration in an attempt to find v or v^2 $[v^2 = 15^2 - 2g \times 10]$	**M1**	e.g. $v^2 = u^2 + 2as$ with $a = \pm g$.
	Speed = $5\,\mathrm{m\,s^{-1}}$	**A1**	
		2	

Question	Answer	Marks	Guidance
2(b)	$(s_P =)\pm\left(15t - \frac{1}{2}gt^2\right)$, $(s_Q =)\pm\frac{1}{2}gt^2$	*B1	Use of $s = ut + \frac{1}{2}at^2$ for either. Allow if a not substituted, need both expressions with opposite sign of t^2 term and the same a.
	Use $s_P + s_Q = 18$ and solve for t	DM1	Allow $s_P + s_Q = \pm 18$. Must have s_P and s_Q of the correct form.
	So height = $10.8\,\text{m}$	A1	
	Alternative method for Question 2(b): Using relative velocity		
	$\pm 15t$	*B1	Use of relative velocity (no acceleration).
	Use $15t = 18$ and solve for t	DM1	Allow $15t = \pm 18$.
	So height = $10.8\,\text{m}$	A1	Not from $t = -1.2$ made positive without justification.
		3	

Question	Answer	Marks	Guidance
3(a)	Attempt to integrate $\left[(v =)\frac{4}{1.5}t^{\frac{3}{2}} = \frac{8}{3}t^{\frac{3}{2}}(+c)\right]$	M1	Increasing power by 1 and a change in coefficient in at least one term; may be unsimplified. $v = at$ M0.
	Substitute $t = 9$ to get speed = $72\,\text{m}\,\text{s}^{-1}$	A1	Or use limits $t = 0$ and $t = 9$.
		2	

Question	Answer	Marks	Guidance
3(b)	Attempt at integration of *their* v $\left[(s=)=\dfrac{8/3}{2.5}t^{2}=\dfrac{16}{15}t^{\frac{5}{2}}(+c')\right]$	*M1	Increasing power by 1 and a change in coefficient in at least one term; may be unsimplified. $s=vt$ M0 *Their* v, which has come from integration in part **(a)**.
	Equate *their* v and *their* s and attempt to solve for t $\left[\dfrac{16}{15}t^{\frac{5}{2}}=\dfrac{8}{3}t^{\frac{3}{2}}\Rightarrow\dfrac{16}{15}t-\dfrac{8}{3}=0\right]$	DM1	*Their* v must have come from integration. Allow if *their* c from **(a)** is not 0.
	time $=\dfrac{5}{2}$ s	A1	Must discard $t=0$ and $t=-\dfrac{5}{2}$.
		3	

Question	Answer	Marks	Guidance
4(a)	Tension = 0 N	B1	May be implied.
		1	
4(b)	Power $[=0.2\times 2]=0.4$ W	B1	Use of power = Fv. Allow without units.
		1	

Question	Answer	Marks	Guidance
4(c)	Driving force = 1.2/2 [= 0.6 N]	**B1**	
	Use of Newton's second law for locomotive or truck or system	**M1**	Correct number of relevant terms.
	For locomotive: $\text{DF} - 0.2 - T = 0.8a$ For truck: $T = 0.4a$ For system: $\text{DF} - 0.2 = 1.2a$	**A1**	For any two correct.
	For attempt to solve for T	**M1**	From equations with correct number of relevant terms. Using *their* dimensionally correct DF. May see $a = \frac{1}{3}$.
	$T = \frac{2}{15}\text{N}$	**A1**	Allow awrt 0.133 .
		5	

Question	Answer	Marks	Guidance
5(a)	Attempt to resolve vertically	**M1**	4 terms; allow with T_A and T_B; allow sign errors; allow g missing.
	$500 + T\cos 45 + T\cos 45 - 100g = 0$ OR $500 + T_A\cos 45 + T_B\cos 45 - 100g = 0$ AND $T_A(\sin 45) = T_B(\sin 45)$	**A1**	Must have $T_A = T_B = T$. Allow if $500 - 2T\cos 45 - 100g = 0$. Allow $500 - T_A\cos 45 - T_B\cos 45 - 100g = 0$ AND $T_A(\sin 45) = T_B(\sin 45)$.
	$T = 354\text{N}$	**A1**	Allow $250\sqrt{2}$, $\frac{500}{\sqrt{2}}$. Allow if $500 - 2T\cos 45 - 100g = 0$ to obtain $T = -354$ and then state magnitude is 354. If T_A and T_B are different values then A0.
	Alternative Method 1 for Question 5(a): Resolving perpendicular to a strut		
	Resolve perpendicular to T_A or T_B	**M1**	3 terms; allow sign errors; allow g missing.
	$T_A(\text{or}\, T_B) + 500\cos 45 = 100g\cos 45$	**A1**	Allow $T_A(\text{or}\, T_B) + 100g\cos 45 = 500\cos 45$.
	$T_A = T_B = 354$	**A1**	Allow $250\sqrt{2}$, $\frac{500}{\sqrt{2}}$.

Question	Answer	Marks	Guidance
5(a)	**Alternative Method 2 for Question 5(a): Using triangle of forces**		
	Attempt Pythagoras on a right-angled triangle of forces or use of trigonometry $\left[\begin{array}{l} {T_A}^2 + {T_B}^2 = (100g - 500)^2 \\ \text{OR } \sin 45 \text{ or } \cos 45 = \dfrac{100g-500}{T_A} \text{ or } \dfrac{100g-500}{T_B} \end{array}\right]$	**M1**	4 terms; allow with T_A and T_B; allow sign errors; allow g missing.
	$T^2 + T^2 = (100g - 500)^2$ OR ${T_A}^2 + {T_B}^2 = (100g-500)^2$ AND $T_A(\sin 45) = T_B(\sin 45)$ OR $\sin 45 = \dfrac{T_A(\text{or } T_B)}{100g-500}$ OR $\cos 45 = \dfrac{T_A(\text{or } T_B)}{100g-500}$	**A1**	Allow $\sin 45 = \dfrac{T_A(\text{or } T_B)}{500-100g}$ OR $\cos 45 = \dfrac{T_A(\text{or } T_B)}{500-100g}$.
	$T_A = T_B = 354$	**A1**	Allow $250\sqrt{2}$, $\dfrac{500}{\sqrt{2}}$.
	Alternative Method 3 for Question 5(a): Using Lami's Theorem		
	Attempt at Lami	**M1**	Allow with T_A and T_B; allow sign errors; allow g missing.
	$\dfrac{100g-500}{\sin 90} = \dfrac{T_A(\text{or } T_B)}{\sin 135}$	**A1**	Allow $\dfrac{500-100g}{\sin 90} = \dfrac{T_A(\text{or } T_B)}{\sin 135}$. Allow $\dfrac{100g-500}{\sin 270} = \dfrac{T_A(\text{or } T_B)}{\sin 45}$.
	$T_A = T_B = 354$	**A1**	Allow $250\sqrt{2}$, $\dfrac{500}{\sqrt{2}}$.
		3	

Question	Answer	Marks	Guidance
5(b)	Attempt to resolve vertically **and** horizontally	**M1**	3 terms vertically and 2 terms horizontally; allow sign errors; allow g missing. Must have $T_A = 0$.
	$T_B \cos 45 + 500 - 100g = 0$ and $F - T_B \sin 45 = 0$	**A1**	Allow $-T_B \cos 45 + 500 - 100g = 0$ and $F + T_B \sin 45 = 0$ OR $T_B \cos 45 + 500 - 100g = 0$ and $F + T_B \sin 45 = 0$ OR $-T_B \cos 45 + 500 - 100g = 0$ and $F - T_B \sin 45 = 0$. For both equations correct.
	$F = 500$	**A1**	awrt 500 to 3sf.
	Alternative Method 1 for Question 5(b): Resolving perpendicular to T_B		
	Attempt to resolve perpendicular to T_B	**M1**	3 terms; allow sign errors; allow g missing. Must have $T_A = 0$.
	$F \cos 45 + 500 \cos 45 = 100g \cos 45$	**A1**	Allow $-F \cos 45 + 500 \cos 45 = 100g \cos 45$.
	$F = 500$	**A1**	awrt 500 to 3sf.

Question	Answer	Marks	Guidance
5(b)	**Alternative Method 2 for Question 5(b): Using Lami's Theorem**		
	Attempt at Lami	**M1**	Allow sign errors.
	$\frac{100g-500}{\sin 135}=\frac{F}{\sin 135}\left(=\frac{T_B}{\sin 90}\right)$	**A1**	Allow $\frac{500-100g}{\sin 135}=\frac{F}{\sin 135}$ or $\frac{100g-500}{\sin 45}=\frac{F}{\sin 45}$ or $\frac{100g-500}{\sin 45}=\frac{F}{\sin 225}$ or $\frac{100g-500}{\sin 225}=\frac{F}{\sin 45}$.
	$F=500$	**A1**	awrt 500 to 3sf.
	Alternative Method 3 for Question 5(b): Using triangle of forces		
	Attempt use of trigonometry on right angled triangle	**M1**	Allow sign errors; allow g missing.
	$\tan 45=\frac{F}{100g-500}$	**A1**	Allow $\tan 45=\frac{F}{500-100g}$
	$F=500$	**A1**	awrt 500 to 3sf.
		3	

Question	Answer	Marks	Guidance
6(a)(i)	Attempt to resolve parallel to the plane	**M1**	4 terms; allow sin/cos mix; allow sign errors; allow g missing.
	$T\cos 20-5-2g\sin 30=2\times 1.2$	**A1**	Correct equation.
	$T=18.5$	**A1**	awrt 18.5 .
		3	

Question	Answer	Marks	Guidance
6(a)(ii)	Attempt resolve perpendicular to the plane $R = 2g\cos 30 - T\sin 20$	**M1**	3 terms; allow sin/cos mix: allow sign errors; allow with T or their T; allow g missing.
	Use of $5 = \mu R$ to get an equation in μ only $\left[5 = \mu \times (2g\ \cos 30 - T\sin 20)\right]$	**M1**	Where R is a two term expression with a component of $2g$ and a component of *their* T; allow g missing.
	$\mu = 0.455$	**A1**	awrt 0.455; allow 0.46 or 0.45; do not allow $\frac{5}{11}$.
		3	
6(b)	Max $F = 0.8 \times (2g\ \cos 30 - 15\sin 20)$ $[= 0.8 \times 12.1902\ldots\ldots = 9.7521\ldots\ldots]$	***B1**	
	Net force up the plane $= 15\cos 20 - 2g\sin 30 [= 4.0953\ldots\ldots]$	***B1**	OR $15\cos 20 - 2g\sin 30 - 0.8 \times (2g\ \cos 30 - 15\sin 20)$ $[= 2a] \Rightarrow -5.6567\ldots\ldots [= 2a] \Rightarrow a = -2.8283\ldots\ldots$ If max F incorrect and use $F = ma$ then allow B1 for $15\cos 20 - 2g\sin 30 - \textit{their}\ \max F$.
	[State $4.0953\ldots\ldots < 9.7521\ldots\ldots$,] hence the block does not move [up the plane]	**DB1**	Must have correct values (to at least 1 sf) to compare for this mark. No incorrect statement seen.
	Alternative Method 1 for Question 6(b)		
	Max force down plane $= 0.8 \times (2g\ \cos 30 - 15\sin 20) + 2g\sin 30$ $[= 0.8 \times 12.1902 + 10\ldots\ldots = 19.7521\ldots\ldots]$	***B1**	
	Force up plane $= 15\cos 20 [= 14.0953\ldots\ldots]$	***B1**	i.e. using it to compare with their max force down the plane.
	[State 1 $4.0953\ldots\ldots < 19.7521\ldots\ldots$,] hence the block does not move [up the plane]	**DB1**	Must have correct values (to at least 1 sf) to compare for this mark. No incorrect statement seen.

Question	Answer	Marks	Guidance
6(b)	**Alternative Method 2 for Question 6(b)**		
	$F = 15\cos 20 - 2g\sin 30 [= 4.9053\ldots\ldots]$ Or $R = 2g\ \cos 30 - 15\sin 20 [= 12.1902\ldots\ldots]$	***B1**	
	Get $\mu = \dfrac{15\cos 20 - 2g\sin 30}{2g\ \cos 30 - 15\sin 20} [= 0.3359\ldots\ldots]$	***B1**	
	[State $0.3359\ldots\ldots < 0.8$,] hence the block does not move [up the plane]	**DB1**	Must have correct value of μ (to at least 1 sf) to compare for this mark. No incorrect statement seen.
		3	

Question	Answer	Marks	Guidance
7(a)	Attempt to use conservation of energy $\left[\frac{1}{2}\times 0.5v^2 = 0.5g\times 1.8\right]$ or $\left[\frac{1}{2}\times mv^2 = mg\times 1.8\right]$	**M1**	2 terms, dimensionally correct. Do not allow from use of constant acceleration.
	$v = 6$	**A1**	Do not allow from use of constant acceleration.
	Attempt at conservation of momentum $\left[0.5\times 6(+0) = 0.5\times 4 + 0.1w\right]$	**M1**	3 terms; allow sign errors; allow *their* $v = 6$ or just v; allow if using mgv (consistently in all terms).
	Speed of Q $(=w) = 10\ \mathrm{m\,s^{-1}}$	**A1**	AG Do not allow from use of constant acceleration. Do not allow if using mgv. Use of constant acceleration gets M0 A0 M1 A0 maximum.
		4	**SC Assuming elastic collision** **M1A1** $0.5g\times 1.8 = \frac{1}{2}\times 0.1w^2 + \frac{1}{2}\times 0.5\times 4^2$ **M1** For attempt at conservation of energy, 3 terms; allow sign errors. **B1** Speed of Q $(=w) = 10\ \mathrm{m\,s^{-1}}$

Question	Answer	Marks	Guidance
7(b)	Attempt at conservation of momentum $\left[0.1\times10=(0.1+0.4)\times z\ (\Rightarrow z=2)\right]$	*M1	3 terms, allow sign errors, allow if using mgv.
	Attempt to use conservation of energy $\left[\frac{1}{2}\times(0.1+0.4)\times(\mathit{their}\ 2)^2=(0.1+0.4)gh\ \ (\Rightarrow h=0.2)\right]$	*DM1	Dependent on previous M mark. 4 terms, dimensionally correct. Do not allow from use of constant acceleration. $\mathit{their}\ 2\neq10$.
	Use trigonometry to get an equation in θ and solve for θ $\left[\theta=\sin^{-1}\left(\frac{\mathit{their}\ 0.2}{0.4}\right)\right]$	DM1	Dependent on previous 2 M marks. Using *their* h and 0.4 . Allow sin/cos mix.
	θ = 30	A1	Do not allow if using mgv.
	Alternative method for Question 7(b): Using constant acceleration		
	Attempt at conservation of momentum $\left[0.1\times10=0.5\times z\ (\Rightarrow z=2)\right]$	*M1	2 terms, allow sign errors, allow if using mgv.
	Attempt at use of constant acceleration $\left[0^2=(\mathit{their}\ 2)^2\pm2\times a\times0.4\ \ (\Rightarrow a=\mp5)\right]$	*DM1	Dependent on previous M mark. Uses constant acceleration with $u=\mathit{their}\ 2$ and $s=0.4$ to get an equation in a; $\mathit{their}\ 2\neq10$.
	Use N2L to get an equation in θ leading to a positive value of θ and solve for θ $\left[(0.5)\lvert\mathit{their}\ a\rvert=(0.5)g\sin\theta\right]$	DM1	Dependent on previous 2 M marks. Using *their* a; May have m for 0.5 . Allow sin/cos mix.
	θ = 30	A1	Do not allow if using mgv.
		4	

Question	Answer	Marks	Guidance
7(c)	Q takes 0.7 s to travel from B to C	**B1**	
	$0.4 = \frac{(their\ 2)+0}{2}t \Rightarrow t = 0.4$	**B1FT**	SOI FT *their* 2 from (b), $t = \frac{0.8}{their\ 2}$. For use of $s = \left(\frac{u+v}{2}\right)t$ to get a time up the slope. Allow for total time on slope from $0 = (their\ 2)t - \frac{1}{2}(their\ a)t^2 \Rightarrow t = 0.8$.
	Distance between P moved is $(0.7+0.8)\times 4(=6)$	**B1**	Allow 1 m from point *C*.
	Set up equation in t using $4t$, $(their\ 2)t$ and *their* 6 and solve for t $\left[4t + (their\ 2)t = (their\ 1) \text{OR} (their\ 6) + 4t + (their\ 2)t = 7\right]$	**M1**	Must have considered all parts of motion to find times from relevant equations.
	Distance from B = $6\frac{2}{3}$ m	**A1**	

Question	Answer	Marks	Guidance
7(c)	**Alternative method for last 3 marks of Question 7(c)**		
	[Time for P =]$\frac{b}{4}$ **and** [Time for QR =]$\frac{7-b}{2}$ OR [Time for P =]$\frac{7-c}{4}$ **and** [Time for QR =]$\frac{c}{2}$	**B1**	Where b is distance from B OR Where c is distance from C.
	Attempt to form an equation from use of total time and solve for b (or c) $\left[\frac{7-b}{2}+0.7+0.4+0.4=\frac{b}{4}\left[\Rightarrow b=6\frac{2}{3}\right]\right.$ $\left.OR\ \frac{c}{2}+0.7+0.4+0.4=\frac{7-c}{4}\left[\Rightarrow c=\frac{1}{3}\right]\right]$	**M1**	Where b is distance from B OR Where c is distance from C. Must have considered all parts of motion to find times from relevant equations.
	Distance from B = $6\frac{2}{3}$ m	**A1**	
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 0 7 7 6 2 6 5 8 7 1 *

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **February/March 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

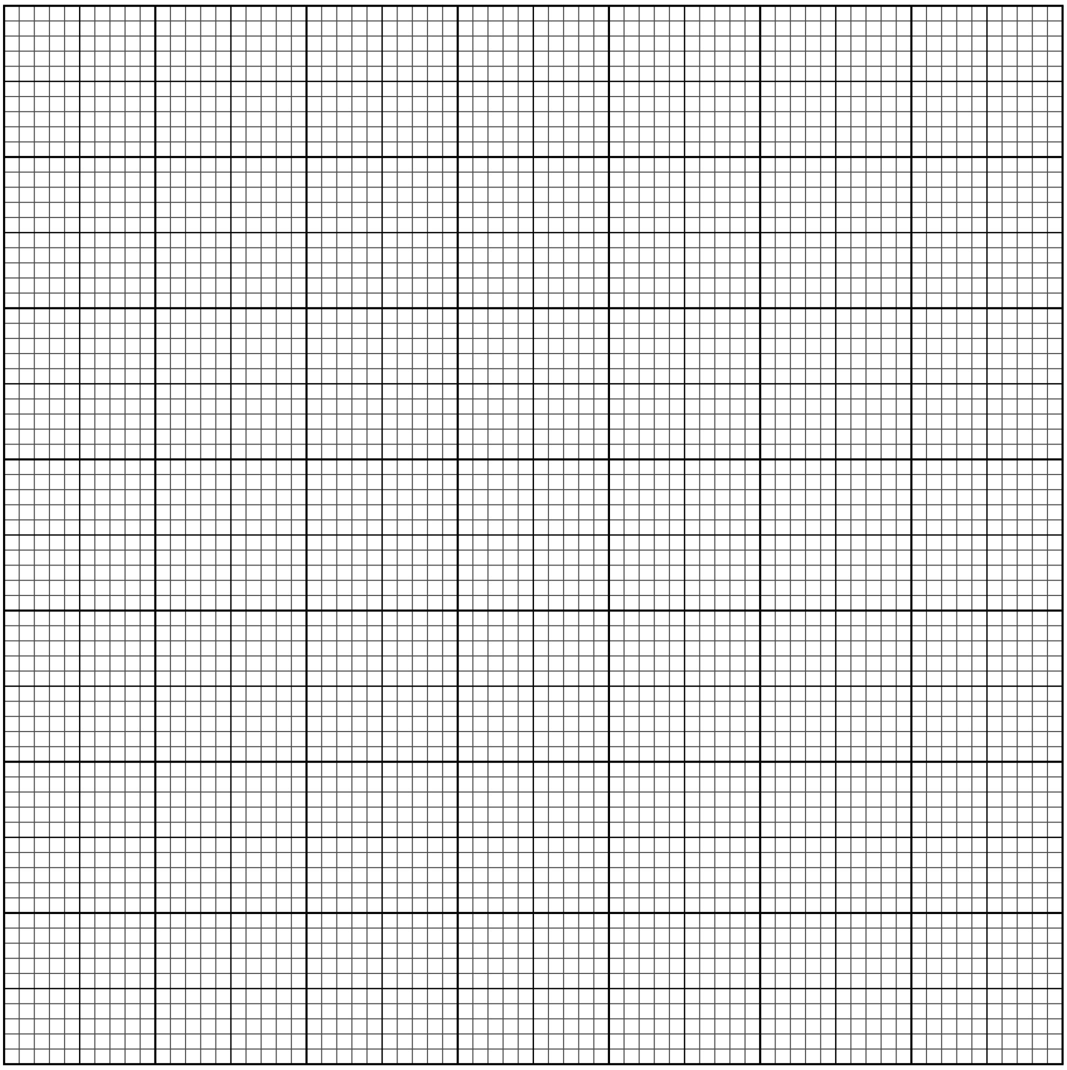
INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 Each year the total number of hours, x, of sunshine in Kintoo is recorded during the month of June. The results for the last 60 years are summarised in the table.

x	$30 \leqslant x < 60$	$60 \leqslant x < 90$	$90 \leqslant x < 110$	$110 \leqslant x < 140$	$140 \leqslant x < 180$	$180 \leqslant x \leqslant 240$
Number of years	4	8	14	25	7	2

(a) Draw a cumulative frequency graph to illustrate the data. [3]



(b) Use your graph to estimate the 70th percentile of the data. [2]

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(c) Calculate an estimate for the mean number of hours of sunshine in Kintoo during June over the last 60 years. [3]

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2 Alisha has four coins. One of these coins is biased so that the probability of obtaining a head is 0.6. The other three coins are fair. Alisha throws the four coins at the same time. The random variable X denotes the number of heads obtained.

(a) Show that the probability of obtaining exactly one head is 0.225. [3]

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(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	0.05	0.225			0.075

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(c) Given that $E(X) = 2.1$, find the value of $Var(X)$. [2]

3 80% of the residents of Kinwawa are in favour of a leisure centre being built in the town.

20 residents of Kinwawa are chosen at random and asked, in turn, whether they are in favour of the leisure centre.

(a) Find the probability that more than 17 of these residents are in favour of the leisure centre. [3]

(b) Find the probability that the 5th person asked is the first person who is **not** in favour of the leisure centre. [1]

(c) Find the probability that the 7th person asked is the second person who is **not** in favour of the leisure centre. [2]

4 The probability that it will rain on any given day is x. If it is raining, the probability that Aran wears a hat is 0.8 and if it is not raining, the probability that he wears a hat is 0.3. Whether it is raining or not, if Aran wears a hat, the probability that he wears a scarf is 0.4. If he does not wear a hat, the probability that he wears a scarf is 0.1. The probability that on a randomly chosen day it is not raining and Aran is not wearing a hat or a scarf is 0.36.

Find the value of x. [3]

5 Marco has four boxes labelled K, L, M and N. He places them in a straight line in the order K, L, M, N with K on the left. Marco also has four coloured marbles: one is red, one is green, one is white and one is yellow. He places a single marble in each box, at random. Events A and B are defined as follows.

A: The white marble is in either box L or box M.

B: The red marble is to the left of both the green marble and the yellow marble.

Determine whether or not events A and B are independent. [3]

6 In a cycling event the times taken to complete a course are modelled by a normal distribution with mean 62.3 minutes and standard deviation 8.4 minutes.

(a) Find the probability that a randomly chosen cyclist has a time less than 74 minutes. [2]

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(b) Find the probability that 4 randomly chosen cyclists all have times between 50 and 74 minutes. [4]

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In a different cycling event, the times can also be modelled by a normal distribution. 23% of the cyclists have times less than 36 minutes and 10% of the cyclists have times greater than 54 minutes.

(c) Find estimates for the mean and standard deviation of this distribution. [5]

7 **(a)** Find the number of different arrangements of the 9 letters in the word DELIVERED in which the three Es are together and the two Ds are **not** next to each other. [4]

(b) Find the probability that a randomly chosen arrangement of the 9 letters in the word DELIVERED has exactly 4 letters between the two Ds. [5]

Five letters are selected from the 9 letters in the word DELIVERED.

(c) Find the number of different selections if the 5 letters include at least one D and at least one E. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

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MATHEMATICS **9709/52**

Paper 5 Probability and Statistics 1 **February/March 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2023 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **15** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

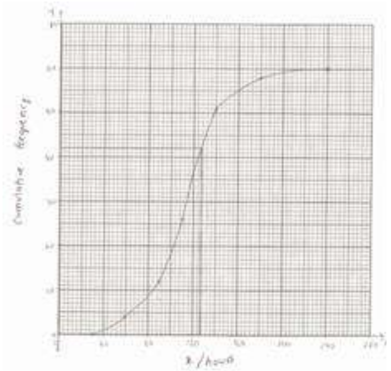
DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>1(a)</td><td><table><tr><td>Upper value</td><td>60</td><td>90</td><td>110</td><td>140</td><td>180</td><td>240</td></tr><tr><td>cf</td><td>4</td><td>12</td><td>26</td><td>51</td><td>58</td><td>60</td></tr></table></td><td>B1</td><td>All cumulative frequencies stated.
May be under data table, condone omission of 4.
May be read accurately from graph, must include 4.</td></tr>
<tr><td></td><td></td><td>M1</td><td>At least 5 points plotted at class upper end points, daylight rule tolerance.
Linear cf scale $0 \leqslant \text{cf} \leqslant 60$, linear time scale $30 \leqslant \text{time} \leqslant 240$ with at least 3 values identified on each axis.</td></tr>
<tr><td></td><td></td><td>A1</td><td>All points plotted correctly. Curve drawn (within tolerance), no ruled segments, and joined to $(30, 0)$.
Axes labelled ‘cumulative frequency’ and ‘hours [of sunshine]’ (OE including appropriate title).</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
<tr><td>1(b)</td><td>[60 × 0.7 =] 42</td><td>M1</td><td>42 may be implied by clear use on graph.</td></tr>
<tr><td></td><td>126</td><td>A1 FT</td><td>Must be clear evidence on graph of use of 42, e.g. an appropriate mark on either axis, appropriate mark on curve.
FT from increasing cf graph only read at 42 only.</td></tr>
<tr><td></td><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
1(c)	Midpoints: 45, 75, 100, 125, 160, 210	**B1**	At least 5 correct mid-points seen, check by data table or used in formula.
	[Mean =] $\frac{4\times45+8\times75+14\times100+25\times125+7\times160+2\times210}{60}$ $\left[=\frac{6845}{60}\right]$	**M1**	Correct mean formula using their 6 midpoints (must be within class, not upper bound, lower bound), condone 1 data error If correct midpoints seen accept $\frac{180+600+1400+3125+1120+420}{60}$.
	$= 114,\ 114\frac{1}{12}$	**A1**	Accept 114.1, 114.08[3…] If A1 not awarded, **SC B1** for 114, $114\frac{1}{12}$, 114.1 or 114.08[3…].
		3	

Question	Answer	Marks	Guidance
2(a)	$0.6(0.5)^3 + 0.4(0.5)^3\times3$	**B1**	Either $0.6(0.5)^3 + a$ or $b + 0.4(0.5)^3\times(3 \text{ or } {}^3C_1)$, $0 < a, b < 1$ seen.
		M1	$0.6(0.5)^3 + 0.4(0.5)^3\times d$ seen, $d = 1, 3$. Condone $0.075 + 0.05\times d$, $d = 1, 3$.
	= 0.225	**A1**	AG full supporting working required. Scenarios identified and linked to calculations.
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="3">2(b)</td><td><table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X = x)$</td><td>0.05</td><td>0.225</td><td>0.375</td><td>0.275</td><td>0.075</td></tr></table></td><td>B1</td><td>Either [P(2) =] 0.375, $\frac{3}{8}$
or [P(3) =] 0.275, $\frac{11}{40}$ seen.
Condone not in table if identified.</td></tr>
<tr><td></td><td>B1 FT</td><td>Both values in table.
FT P(2) + P(3) = 0.650 .</td></tr>
<tr><td></td><td>2</td><td></td></tr>
<tr><td rowspan="3">2(c)</td><td>Var(X)
$= [1^2 \times]0.225 + 2^2 \times \textit{their}\, 0.375 + 3^2 \times \textit{their}\, 0.275 + 4^2 \times 0.075 - 2.1^2$</td><td>M1</td><td>Appropriate variance formula from their probability distribution table with at least 4 terms, $0 < \textit{their}$ P(x) < 1.
Condone 4.41 for 2.1^2 .
Condone mean clearly recalculated inaccurately.
Or
$0.225 + 4 \times \textit{their}\, 0.375 + 9 \times \textit{their}\, 0.275 + 16 \times 0.075 - 2.1^2$
Condone 2.1^2 for 4.41.</td></tr>
<tr><td>$\left[5.4 - 2.1^2\right] = 0.99[0]$</td><td>A1</td><td>If M0 awarded
SC B1 for 0.99[0] WWW.</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
3(a)	**Method 1 for Question 3(a)**		
	[P(X > 17) = P(18, 19, 20) =] ${}^{20}C_{18}\,(0.8)^{18}(0.2)^{2} + {}^{20}C_{19}\,(0.8)^{19}(0.2)^{1} + {}^{20}C_{20}\,(0.8)^{20}$ = 0.13691 + 0.05765 + 0.01153	M1	One term ${}^{20}C_x\,(p)^x(1-p)^{20-x},\ 0<p<1, 0<x<20$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.206	B1	Mark the final answer at the most accurate value $0.206 \leqslant p \leqslant 0.2061$.
	Method 2 for Question 3(a)		
	[P(X > 17) = 1 – P(0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17) =] $1 - ({}^{20}C_0\,(0.8)^0(0.2)^{20} + {}^{20}C_1\,(0.8)^1(0.2)^{19} + {}^{20}C_2\,(0.8)^2(0.2)^{18} + \ldots + {}^{20}C_{16}\,(0.8)^{16}(0.2)^4 + {}^{20}C_{17}\,(0.8)^{17}(0.2)^3)$ $= 1 - (1.048\times10^{-14} + 8.389\times10^{-13} + 3.188\times10^{11} + \ldots + 0.2182 + 0.2054)$	M1	One term ${}^{20}C_x\,(p)^x(1-p)^{20-x},\ 0<p<1, 0<x<20$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer. If answer correct, condone omission of any 15 of the 16 middle terms.
	0.206	B1	Mark the final answer at the most accurate value $0.206 \leqslant p \leqslant 0.2061$. Condone omission of brackets.
		3	
3(b)	$\left[(0.8)^4(0.2) = \right] 0.08192, \dfrac{256}{3125}$	B1	Accept $\dfrac{8192}{100000}$ OE.
		1	

Question	Answer	Marks	Guidance
3(c)	$(0.8)^5(0.2)^2 \times 6$	**M1**	$(0.8)^5(0.2)^2 \times k$ or $(0.8)^5(0.2) \times k \times 0.2$, $2 \leqslant k \leqslant 7$.
	$= 0.0786, \dfrac{8144}{78125}$	**A1**	$0.0786 \leqslant p < 0.07865$, $\dfrac{786432}{10000000}$. If A0 awarded, **SC B1** for correct answer WWW.
		2	

Question	Answer	Marks	Guidance
4	$(1 - x) \times 0.7 \times 0.9 = 0.36$	**M1**	$(1-x) \times a \times b = 0.36$, $a = 0.7$ or 0.3, $b = 0.9$ or 0.1
		B1	$(1\text{-}x)\times0.7\times0.9=0.36$, $(1\text{-}x)\times0.63=0.36$, $0.63 - 0.63x = 0.36$ or $1 - x = \dfrac{0.36}{0.63}$ seen. Condone recovery from omission of brackets.
	$x = \dfrac{3}{7}$	**A1**	Accept 0.428571 to at least 3 sf. Condone 0.4285 rounding to 0.429 . If M0 awarded, **SC B1** for $x = \dfrac{3}{7}$ or 0.428571 to at least 3 sf.
		3	

Question	Answer	Marks	Guidance
5	$\text{P}(A) = \frac{1}{2}$, $\text{P}(B) = \frac{8}{24} = \frac{1}{3}$,	**B1**	Both stated, accept unsimplified.
	$\text{P}(A \cap B) = \frac{1}{6}$	**M1**	Evidence that independence properties not used.
	$\text{P}(A) \times \text{P}(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ so events are independent	**A1**	Evaluated and conclusion stated. $\text{P}(A) \times \text{P}(B)$ and $\text{P}(A \cap B)$ seen.
		3	

Question	Answer	Marks	Guidance
6(a)	$[\text{P}(X < 74) =]\ \text{P}\left(Z < \frac{74 - 62.3}{8.4}\right)\left[= \text{P}(Z < 1.393)\right]$	**M1**	Use of ± standardisation formula with 74, 62.3 and 8.4 substituted appropriately, not 8.4^2, not $\sqrt{8.4}$, no continuity correction.
	= 0.918	**A1**	$0.918 \leqslant p \leqslant 0.9185$.
		2	

Question	Answer	Marks	Guidance
6(b)	[$\mathrm{P}(50 < X < 74)$ = P]$\left(\frac{50-62.3}{8.4} < Z < \frac{74-62.3}{8.4}\right)$ [P $(-1.464 < Z < 1.393)$]	**M1**	Use of ± standardisation formula with both 74 (may be seen in 6(a) if *their* value seen) & 50, 62.3 and 8.4 substituted appropriately. Condone use of $8.4^2, \sqrt{8.4}$ and continuity correction ±0.5 (73.5 or 74.5 and 49.5 or 50.5).
	[$\Phi(1.464) + \Phi(1.393) - 1$] 0.9285 + 0.9182 – 1	**M1**	Calculating the appropriate probability area from stated Φ of z-values (leading to *their* final answer > 0.5) but not symmetrical values.
	= 0.847	**A1**	$0.8465 \leqslant p < 0.8475$. **SC B1** for $0.8465 \leqslant p < 0.8475$ if M0A0 awarded.
	$(0.8467)^4 = 0.514$	**B1 FT**	Accept $0.513 \leqslant p \leqslant 0.514$. FT (*their* 4-figure $p)^4$, $0 < p < 1$.
		4	
6(c)	$z_1 = \frac{36-\mu}{\sigma} = -0.739$ $z_2 = \frac{54-\mu}{\sigma} = 1.282$	**B1**	$-0.740 < z_1 < -0.738$ or $0.738 < z_1 < 0.740$.
		B1	$z_2 = \pm 1.282$ (critical value).
		M1	Use of the ±standardisation formula once with μ, σ and a z-value (not 0.23, 0.77, 0.90, 0.10, ±0.261, ±0.282…). Condone continuity correction ±0.5, not $\sigma^2, \sqrt{\sigma}$.
	Solve, obtaining values for μ and σ $\mu = 42.6$, $\sigma = 8.91$	**M1**	Solve using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ.
		A1	$42.58 \leqslant \mu \leqslant 42.6$, $8.90 \leqslant \sigma \leqslant 8.91$.
		5	

Question	Answer	Marks	Guidance
7(a)	**Method 1: Arrangements with 3 Es together – arrangements with 3 Es together and 2 Ds together**		
	$\frac{7!}{2!}-6!$	**B1**	$\frac{7!}{2!}-e$, e a positive integer (including 0).
		M1	$f-6!$, $f>6!$
		M1	$\frac{7!}{a!b!}-\frac{6!}{c!d!}$, a,c = 1, 2 and b,d = 1, 3.
	1800	**A1**	
	Method 2: Identified scenarios ^ EEE ^ ^ ^		
	$5!\times\frac{6\times5}{2}$	**B1**	$5!\times j$, j a positive integer (j = 1 may be implied).
		M1	$\frac{k!}{m!}\times\frac{6\times5}{2}$, $\frac{k!}{m!}\times{}^6C_2$, $\frac{k!}{m!}\times\frac{{}^6P_2}{2}$ or $k!\times\frac{7\times6}{n}$, k a positive integer (k = 1 may be implied), m = 1, 2 n = 1, 2, 3.
		M1	$k!\times\frac{m\times(m-1)}{n}$ k a positive integer > 1, m = 10, 9, 8, 7, 6 and n = 1, 2.
	1800	**A1**	
		4	

Question	Answer	Marks	Guidance
7(b)	**First 2 marks: Method 1 – Number of arrangements with 2 Ds in one position with 4 letters in between – repeats allowed**		
	$7! \times 4 \times 2$	**M1**	$7! \times s$, s = positive integer > 1.
		M1	$t! \times 4 \times 2$, $t = 8, 7, 6$. Condone $t! \times 8$.
	First 2 marks: Method 2 – Picking 2Ds, arranging 4 letters from remaining letters between and then arranging terms		
	${}^7P_4 \times 4! \times 2!$	**M1**	${}^7P_4 \times a! \times b!$, $1 \leqslant a \leqslant 6$ and $b = 1, 2, 3$.
		M1	${}^7P_c \times 4! \times 2!$, $c = 3, 4, 5$.
	First 2 marks: Method 3 – Identified scenarios involving Es between Ds		
	D ^ ^ ^ ^ D E E E = ${}^4C_4 \times 4! \times 4! \times 2! = 1152$ D E ^ ^ ^ D E E ^ = ${}^4C_3 \times 4! \times 4! \times 3 \times 2! = 13824$ D E E ^ ^ D E ^ ^ = ${}^4C_2 \times 4! \times 4! \times 3 \times 2! = 20736$ D E E E ^ D ^ ^ ^ = ${}^4C_1 \times 4! \times 4! \times 2! = 4608$	**M1**	1 identified scenario value correct.
		M1	4 appropriate scenarios added, no incorrect.

Question	Answer	Marks	Guidance
7(b)	**Final 3 marks for Methods 1, 2 and 3**		
	40320	**A1**	If A0 scored, **SC B1** for 40320 WWW.
	[Total number of arrangements =] $[9!=]$ 362880	**B1**	Accept unsimplified. May be seen as denominator of probability.
	Probability = $\frac{40320}{362880}=\frac{1}{9}$	**B1FT**	$\frac{\textit{their}\ 40320}{\textit{their}\ 362880}$, accept unsimplified. B1FT if *their* 40320 and *their* 362880 supported by work in this part. Condone *their* 362880 supported by calculation in **7(a)**.
		5	
7(c)	Scenarios D E _ _ _ $^4C_3$ 4 D E E _ _ $^4C_2$ 6 D E E E _ $^4C_1$ 4 D D E _ _ $^4C_2$ 6 D D E E _ $^4C_1$ 4 D D E E E $[^4C_0]$ 1	**B1**	1 correct unsimplified outcome/value for one identified scenario excluding DDEEE. Note: $^4C_1$ cannot be used for $^4C_3$.
		M1	Add values of 6 appropriate scenarios, no additional, incorrect or repeated scenarios. Accept unsimplified.
	[Total =] 25	**A1**	
		3	

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 7 2 8 8 7 5 1 6 3 7 *

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **February/March 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 Anita carried out a survey of 140 randomly selected students at her college. She found that 49 of these students watched a TV programme called *Bunch*.

(a) Calculate an approximate 98% confidence interval for the proportion, p, of students at Anita's college who watch *Bunch*. [3]

Carlos says that the confidence interval found in **(a)** is not useful because it is too wide.

(b) Without calculation, explain briefly how Carlos can use the results of Anita's survey to find a narrower confidence interval for p. [1]

2 The number of orders arriving at a shop during an 8-hour working day is modelled by the random variable X with distribution Po(25.2).

(a) State **two** assumptions that are required for the Poisson model to be valid in this context. [2]

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(b) **(i)** Find the probability that the number of orders that arrive in a randomly chosen 3-hour period is between 3 and 5 inclusive. [3]

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(ii) Find the probability that, in two randomly chosen 1-hour periods, exactly 1 order will arrive in one of the 1-hour periods, and at least 2 orders will arrive in the other 1-hour period. [4]

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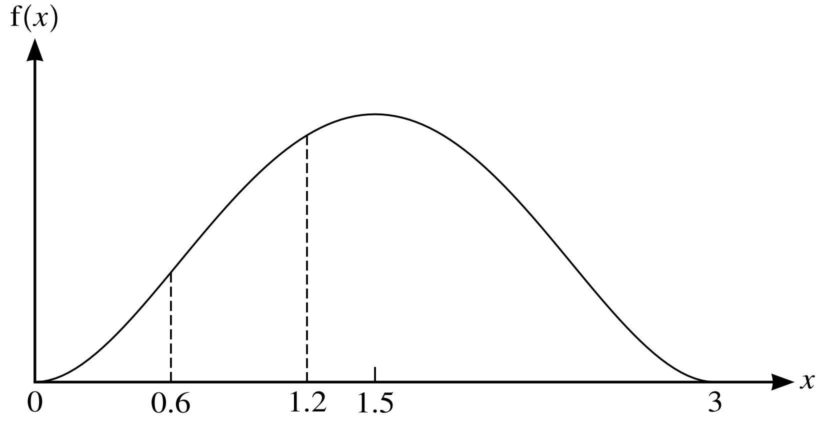
(c) The shop can only deal with a maximum of 120 orders during any 36-hour period.

Use a suitable approximating distribution to find the probability that, in a randomly chosen 36-hour period, there will be too many orders for the shop to deal with. [4]

3



The diagram shows the graph of the probability density function, f, of a random variable X that takes values between $x = 0$ and $x = 3$ only. The graph is symmetrical about the line $x = 1.5$.

(a) It is given that $P(X < 0.6) = a$ and $P(0.6 < X < 1.2) = b$.

Find $P(0.6 < X < 1.8)$ in terms of a and b. [2]

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(b) It is now given that the equation of the probability density function of X is

$$f(x) = \begin{cases} kx^2(3-x)^2 & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{10}{81}$. [3]

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(ii) Find Var(X). [3]

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4 The number of accidents per 3-month period on a certain road has the distribution $\mathrm{Po}(\lambda)$. In the past the value of λ has been 5.7. Following some changes to the road, the council carries out a hypothesis test to determine whether the value of λ has decreased. If there are fewer than 3 accidents in a randomly chosen 3-month period, the council will conclude that the value of λ has decreased.

(a) Find the probability of a Type I error. [2]

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(b) Find the probability of a Type II error if the mean number of accidents per 3-month period is now actually 0.9. [3]

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5 The masses, in grams, of large and small packets of Maxwheat cereal have the independent distributions $N(410.0, 3.6^2)$ and $N(206.0, 3.7^2)$ respectively.

(a) Find the probability that a randomly chosen large packet has a mass that is more than double the mass of a randomly chosen small packet. [5]

The packets are placed in boxes. The boxes are identical in appearance. 60% of the boxes contain exactly 10 randomly chosen large packets. 40% of the boxes contain exactly 20 randomly chosen small packets.

(b) Find the probability that a randomly chosen box contains packets with a total mass of more than 4080 grams. [6]

6 Last year, the mean time taken by students at a school to complete a certain test was 25 minutes. Akash believes that the mean time taken by this year's students was less than 25 minutes. In order to test this belief, he takes a large random sample of this year's students and he notes the time taken by each student. He carries out a test, at the 2.5% significance level, for the population mean time, μ minutes. Akash uses the null hypothesis H_0: $\mu = 25$.

(a) Give a reason why Akash should use a one-tailed test. [1]

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Akash finds that the value of the test statistic is $z = -2.02$.

(b) Explain what conclusion he should draw. [2]

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In a different one-tailed hypothesis test the z-value was found to be 2.14.

(c) Given that this value would lead to a rejection of the null hypothesis at the $\alpha\%$ significance level, find the set of possible values of α. [3]

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The population mean time taken by students at another school to complete a test last year was m minutes. Sorin carries out a one-tailed test to determine whether the population mean this year is less than m, using a random sample of 100 students. He assumes that the population standard deviation of the times is 3.9 minutes. The sample mean is 24.8 minutes, and this result just leads to the rejection of the null hypothesis at the 5% significance level.

(d) Find the value of m. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

MATHEMATICS **9709/62**

Paper 6 Probability and Statistics 2 **February/March 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2023 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\left[\frac{49}{140}=0.35\right]$		
	$0.35 \pm z\sqrt{\frac{0.35(1-0.35)}{140}}$	**M1**	Use of formula of correct form, ft *their* $\frac{49}{140}$, any z (not a probability).
	$z = 2.326$	**B1**	Accept 2.326 to 2.329 .
	Confidence interval = 0.256 to 0.444 (3 sf)	**A1**	Must be an interval.
		3	
1(b)	Find a smaller **percentage** confidence interval/ lower **level** of confidence	**B1**	ISW if 2 reasons given. Just saying 'use smaller z' oe B0. Accept a correct example e.g. 90% (even if not qualified with statement).
		1	

Question	Answer	Marks	Guidance
2(a)	Orders arrive at constant mean rate (must say mean or rate) Orders arrive at random Orders arrive independently Orders arrive singly		Must be in context (accept 25.2 as context).
		B1	Any one reason correctly stated.
		B1	A second reason correctly stated.
			SC B1: both correct, not in context.
		2	

Question	Answer	Marks	Guidance
2(b)(i)	$\lambda = \frac{3}{8} \times 25.2$ [= 9.45]	**B1**	
	$e^{-'9.45'}(\frac{9.45^3}{3!} + \frac{9.45^4}{4!} + \frac{9.45^5}{5!})$ **or** $e^{-'9.45'}$ (140.65 + 332.29 + 628.03) **or** 0.01107 + 0.02615 + 0.04942	**M1**	Allow any λ. Allow end errors. Expression must be seen.
	= 0.0866 (3 sf)	**A1**	If M0 allow **SC B1** for 0.0866 no working seen.
		3	
2(b)(ii)	$e^{-3.15} \times 3.15$ **or** $(1 - e^{-3.15}(1 + 3.15))$ **or** 0.135 **or** 0.822 (3 sf)	**B1**	
	$e^{-3.15} \times 3.15 \times (1 - e^{-3.15}(1 + 3.15))$	**M1**	M1 for product of two Poisson probabilities P(1) × (1– P(0,1)) (no end errors accepted). Accept any λ.
	× 2 or 0.111 × 2	**M1**	M1 for *their* product of two Poisson probabilities (accept end errors) × 2. Accept any λ
	0.222 (3 sf)	**A1**	
		4	
2(c)	N(113.4, 113.4)	**B1**	SOI
	$\frac{120.5-113.4}{\sqrt{113.4}}$ [= 0.667]	**M1**	Standardise with *their* values. Allow wrong or no cc. Must have √.
	1 – ϕ(*their* '0.667')	**M1**	For probability area consistent with *their* values.
	= 0.252 (3 sf)	**A1**	
		4	

Question	Answer	Marks	Guidance
3(a)	$1 - 2(a + b)$ **or** $1 - 2a$ **or** $0.5 - a - b$ **or** 1-(a+b) **or** a+a+b	**M1**	OE. Seen or implied – may be on the diagram (or for correct un-simplified final expression).
	$P(0.6 \le X \le 1.8) = 1 - 2a - b$	**A1**	Accept 1 – (2a + b).
		2	
3(b)(i)	$k\int_0^3 (9x^2 - 6x^3 + x^4)dx = 1$	**M1**	Attempt integrate f(x) ignore limits and '= 1'.
	$k\left[\frac{9x^3}{3} - \frac{6x^4}{4} + \frac{x^5}{5}\right]_0^3 = 1$	**A1**	Correct integration seen, correct limits.
	$k \times \frac{81}{10} = 1,\ k = \frac{10}{81}$	**A1**	AG. Convincingly obtained. No errors seen. (Must see integration).
		3	
3(b)(ii)	$\frac{10}{81}\int_0^3 (9x^4 - 6x^5 + x^6)dx$ $\left[\frac{10}{81}\left[\frac{9x^5}{5} - x^6 + \frac{x^7}{7}\right]_0^3\right]$ [= $\frac{18}{7}$ or 2.57...]	**M1**	Attempt integrate x^2f(x) between 0 and 3 condone missing k. Must see integration or correct answer of 18/7 seen or implied.
	$\frac{18}{7}$ – '1.5'^{2}	**M1**	Their integral of x^2f(x) – 1.5^2 (or their mean^2).
	$= \frac{9}{28}$ or 0.321	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	$e^{-5.7}(1 + 5.7 + \frac{5.7^2}{2!})$ **or** $e^{-5.7}(1 + 5.7 + 16.245)$ **or** 0.003346 + 0.01907 + 0.05436	**M1**	Allow one end error. Must see this expression.
	= 0.0768 (3 sf)	**A1**	**SC B1** for unsupported answer of 0.0768 .
		2	
4(b)	$e^{-0.9}(1 + 0.9 + \frac{0.9^2}{2!})$	**M1**	Attempted; allow one end error (must see expression).
	$= 1 - e^{-0.9}(1 + 0.9 + \frac{0.9^2}{2!}) = 1 - e^{-0.9}(1 + 0.9 + 0.405) = 1 - (0.4066 + 3659 + 0.1647)$	**A1**	Correct expression $P(X \geqslant 3)$ no end errors (must see expression).
	= 0.0629 (3 sf)	**A1**	**SC B2** for unsupported answer of 0.0629 .
		3	

Question	Answer	Marks	Guidance
5(a)	D = L – 2S E(D) = 410 – 2(206) = –2	**B1**	SOI. OE using 2S–L.
	$\text{Var}(D) = 3.6^2 + 4 \times 3.7^2$ [= 67.72]	**B1**	SOI
	$\frac{0-(-2)}{\sqrt{'67.72'}}$ [= 0.243]	**M1**	For standardising using *their* values.
	1 – ϕ(*their* ‘0.243’)	**M1**	For probability area consistent with *their* values.
	= 0.404 (3 sf)	**A1**	As final answer.
		5	

Question	Answer		Marks	Guidance
5(b)	$T_L \sim N(4100, 10\times3.6^2)$	$T_S \sim N(4120, 20\times3.7^2)$	B1	One of N(4100, 129.6) or N(4120, 273.8) USED (unchanged) in a standardising equation.
	$\frac{4080-4100}{\sqrt{'129.6'}}$ (= –1.757)	$\frac{4080-4120}{\sqrt{'273.8'}}$ (= –2.417)	M1	Standardising with either *their* N(4100, 129.6) or N(4120, 273.8) or their N(…,…) (could be from a combination).
	1 – ϕ(‘–1.757’) = ϕ(1.757)	1 – ϕ(‘–2.417’) = ϕ(2.417)	M1	One area consistent with *their* working (could be from a combination). Do not ISW.
	= 0.9605 **or** 0.961	= 0.9921 **or** 0.9922 **or** 0.992	A1	Both of these correct. Do not ISW.
	0.6 × ‘*their* 0.9605’ + 0.4 × ‘*their* 0.9921’		M1	Must be using probabilities.
	= 0.973 (3 sf)		A1	
			6	

Question	Answer	Marks	Guidance
6(a)	He is expecting a **decrease** (in μ)	B1	OE
		1	
6(b)	$-2.02 < -1.96$	M1	For valid comparison. Allow 2.02 > 1.96 **or** 0.0217 < 0.025 **or** 0.9783 > 0.975
	(Reject H_0) There is evidence to suggest that this year’s (mean) time is less than 25	A1	OE (such as evidence to support Akash’s belief), in context, not definite. No contradictions.
		2	

Question	Answer	Marks	Guidance
6(c)	$1 - \phi(2.14)$ [= 0.0162]	**M1**	
	1.62	**A1**	Allow 1.62% or 1.6 or 1.6%.
	$\alpha \geqslant 1.62$ (3 sf)	**A1ft**	FT *their* 1.62 . Allow $\alpha \geqslant 1.62\%$ or 1.6 or 1.6%. Condone >.
		3	
6(d)	$\frac{24.8-m}{3.9 \div 10}$	**M1**	For standardising.
	$\frac{24.8-m}{3.9 \div 10} = -1.645$	**M1**	Equate *their* standardised value to −1.645 (signs must be consistent).
	$m = 25.4$ (3 sf)	**A1**	
		3	

MATHEMATICS

Paper 9709/11
Pure Mathematics 1 (11)

Key messages

Previous reports have highlighted that the rubric on the front cover of the question paper states 'no marks will be given for unsupported answers from a calculator.' Although most candidates have taken this message on board, there is still a significant minority who have not. To justify solutions, clear working must always be shown.

For quadratic equations, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Factors must also always produce the coefficients of the quadratic when expanded.

Similarly, with definite integrals, a detailed method showing limits substituted must support the final answer. Using a calculator for integration is not sufficient although it can be helpful for checking.

It is always advisable to read the question carefully to see whether later parts are linked with earlier parts. In some questions it was necessary to use values found earlier.

General comments

Some very good scripts were seen, although generally the paper was found to be quite challenging. Success in this paper requires candidates to be familiar with the mathematical methods specified in the syllabus and to understand when to apply them. While some questions involved interpretation and problem-solving, several of these questions were easily accessible to candidates who had practised these methods.

Candidates should be reminded to take care with their presentation. When candidates make several attempts at a question, they should clearly indicate which solution is to be marked by crossing out unwanted working. If there is insufficient space, they should use the extra page at the end instead of writing in small spaces within another solution. Clear working will minimise the risk of making errors through misreading their own work. Some of the answers were written in pencil which is then superimposed with ink giving a very unclear image when the script is scanned. Centres are strongly advised that candidates should be told not to do this.

Comments on specific questions

Question 1

Most candidates started by rewriting $\tan\theta$ as $\dfrac{\sin\theta}{\cos\theta}$ but many could make little or no further progress. Those who found the solution often did so without factorising so could not be awarded the second M1, only the SC mark for a correct answer. Some candidates multiplied through by $\sin\theta$ to generate $\sin^2\theta$ in order to use the relevant identity. Weaker candidates used incorrect trigonometric identities such as $\sin\theta = 1 - \cos\theta$. A small number solved $\cos\theta = \dfrac{1}{4}$, leading to 75.5, or gave their answer as 104 which was not rounded to 1 decimal place, so could not be awarded the final mark.

Question 2

Candidates who were familiar with the binomial expansion generally produced accurate answers in **(a)** and **(b)**, though sign errors were seen in the second and third terms in **(b)**. Those who attempted to multiply brackets instead were prone to error. There was some confusion over which terms were needed, with the constant term omitted in some cases. In **(c)**, some candidates were awarded both marks, having found the expansions correctly then identifying the terms that contributed to the coefficient in x^2. Those who had incorrect expansions but successfully multiplied the correct pairs of terms were awarded M1. Weaker candidates wrote the x^2 terms from both expansions and either added them or chose the bigger coefficient.

Question 3

This is one of the new topics introduced in 2019 and there is evidence that candidates are better prepared for this type of question, with some using correct terminology in a fully correct solution. A translation should be described using a vector or a clear statement that it was translated left horizontally (or in the negative x-direction) by 6. For the stretch, it was necessary to specify the direction (in this case the y-direction or vertically) and the scale factor (2). A number of candidates misinterpreted the question and provided instructions for the reverse transformation, g(x) to f(x). Often there was more description present than needed which led to multiple answers, some of which were contradictory.

Question 4

Most candidates found the correct value of θ in radians and the arc length. Stronger responses used correct trigonometry, splitting triangle ABC into two right-angled triangles and using $\sin\frac{1}{12}\pi$ to find half the length of BC. An alternative method was to use the cosine or sine rule to find the length of BC. Candidates should learn these rules and use them correctly in order to be awarded the method mark. Some candidates attempted to find the area of segment BCD rather than its perimeter. Others assumed that ABC was a right-angled triangle and wrote $BC = 8\tan\theta$.

Question 5

Candidates appeared to find this question challenging, with many of them unable to proceed with a method or introducing multiple algebraic and sign errors. Successful candidates generally equated y for the line and the curve to reach $2kx^2 - 2kx + 1 = 0$ or similar, then used $b^2 - 4ac = 0$ to find k. It was common to see $-\frac{1}{2x}$ becoming $-\frac{1}{2}x$ in the working, leading to incorrect equations, in many cases not quadratic equations. Despite this, a number of candidates attempted to use the discriminant. Some candidates did not reject $k = 0$. When candidates solve a quadratic to find two possible answers, they should check that both answers make sense in the context of the question.

In the alternative method, some candidates found $\frac{dy}{dx} = \frac{1}{2x^2}$ correctly and a few of them realised $k = \frac{1}{2x^2}$ and so formed the equation $\frac{1}{2x^2}x - \frac{1}{2x^2} = -\frac{1}{2x}$ which they solved to find x. Incorrect differentiation was common, e.g. $y = 2x^{-1}$ leading to $\frac{dy}{dx} = -2x^{-2}$ or $y = (2x)^{-1}$ leading to $\frac{dy}{dx} = -(2x)^{-2}$.

Question 6

(a) Candidates who used $(2p-6) - \frac{p^2}{6} = p - (2p-6)$ or $2(2p-6) = p + \frac{p^2}{6}$ were often able to arrive at $p^2 - 18p + 72 = 0$ or an equivalent correct quadratic equation. Some candidates found both solutions but did not reject $p = 6$. Poor presentation was a handicap in this question. Many candidates made several attempts, or misread their own work, and multiple errors were seen in manipulating the algebra.

(b) Successful candidates found a, the second term and r using $p = 12$ then calculated the sum to infinity from $\frac{a}{1-r}$. A follow through mark was available for candidates who used this formula with an incorrect value of p, provided it resulted in $|r| < 1$.

Question 7

(a) Those who considered the maximum and minimum values of $\sin\theta$ could deduce that the maximum and minimum values of the overall expression were 5 and –1. A large number of candidates stated 2 as the least and 2.33 as the greatest simply by substituting lower and upper values of the domain. For some, there was some confusion between range and domain. Other candidates seemed to not understand the question: their values in **(a)** were incorrect but their graph in **(b)** showed the correct maximum and minimum for which credit was given.

(b) Most candidates sketched a sinusoidal curve but the period was often incorrect. Others supplied a non-periodic graph. Sketching graphs is a necessary skill and should be practised without first drawing up a table of values. Candidates who used their incorrect maximum and minimum values from **(a)** in a periodic graph were awarded follow through marks.

(c) The majority of candidates either attempted to solve an equation or made a guess at the number of solutions. More successful candidates used their sketch to consider intersections on the graph, concluding that there would be only one solution.

Question 8

(a) Most candidates wrote $1+\dfrac{2a}{7-a}=\dfrac{5}{2}$ but this was often followed by poor algebra leading to an incorrect value of a. A common error was omitting to multiply the 1 when multiplying through by $7 - a$ to clear the denominator. This meant the method mark could not be awarded. Finding b proved to be more difficult for those who seemed unsure how a composite function is formed. Those who found f(5) first were generally more successful, though a common error was to omit the '1 +' in the formula for f(x).

(b) A small number of candidates found the correct answer of $x > 1$. Many confused the range and the domain. Others gave the answer $x > 3$, the domain of f(x). A large number gave $x \neq 1$ as their answer. Poor notation was often seen.

(c) Successful responses swapped the variables and made y the subject, reaching a correct expression for $f^{-1}(x)$ in a few lines of working. Many algebraic errors were seen, and unclear working was a hindrance to some candidates.

Question 9

(a) The majority of candidates did not appreciate that this question about rates of change required differentiation so could be awarded no marks. Some did not attempt the question at all. Of those who did, a common error was to substitute $h = 10$ into the volume formula.

(b) Only the strongest candidates successfully used their $\dfrac{dh}{dt}$ from **(a)** to form an equation in h using the chain rule. After solving to find h, this value was substituted into the volume formula and the volume given to at least 3 significant figures. Many candidates did not answer this question, but those who did attempt it often did not realise differentiation was needed. The most common error was to substitute $h = 0.075$.

Question 10

(a) In this question, candidates needed to show all the steps in integrating y^2, including substituting the limits of $\frac{3}{2}$ and 1. Using the calculator for integration was not sufficient. Some fully correct solutions were seen, with the volume of the cylinder subtracted correctly and the final answer given in exact form with π included. Some candidates found the integral of y rather than y^2 while others did not attempt to find the volume of the cylinder. Errors were seen in integration, with the power reduced rather than increased, or the chain rule omitted. Some candidates appeared to be unsure whether to integrate or differentiate.

(b) A minority of candidates produced fully correct solutions assisted by a clear strategy, logical working and a diagram. Those who found the normal gradient often went on to find the equations of the normal and tangent then their intercepts with the *x*-axis. Some candidates found the lengths of the sides of the triangle unnecessarily, not realising the height of the triangle is 1 (the *y*-coordinate of *B*).

Some found the problem-solving aspects of this part more challenging, with many errors introduced, often through having an incorrect or no diagram. It is often advisable to sketch a new diagram rather than working with unclear annotations on the given graph. Many candidates omitted to use the chain rule when differentiating and, as with **(a)**, power errors were common.

Question 11

(a) Successful candidates used the given information to substitute $x = a$ into $\frac{dy}{dx}$ then set it equal to 0, leading to $6a^2 - 30a + 6a = 0$. From this they obtained *a* = 4. Some attempted to use the quadratic formula to solve a two-term quadratic. Other candidates did not read the question carefully and used $\frac{dy}{dx} = -15$.

(b) Most candidates found $\frac{d^2y}{dx^2} = 12x - 30$ correctly, so could be awarded the method mark. Some candidates had no *a*-value to substitute while others made arithmetical errors that led to the wrong conclusion about the nature of the stationary point.

(c) Many candidates integrated correctly to find $y = 2x^3 - 15x^2 + 6(\textit{their } a)x + c$ and proceeded to substitute *x* = *their a* and $y = -15$ for which they could be awarded B1 FT and M1. In this type of question, candidates should take care to write an equation correctly: $y = 2x^3 - 15x^2 + 24x + 1$ not $2x^3 - 15x^2 + 24x + 1$ to be awarded the final A mark.

(d) In this question, it was necessary to demonstrate a method of solving the quadratic from the given $\frac{dy}{dx} = 0$ leading to *x* = 1 and *x* = 4, then to reject *x* = 4. It was then straightforward to substitute *x* = 1 into the expression for *y* to obtain (1, 12). A large number of arithmetical errors were seen in obtaining the coordinates, even from correct equations.

Question 12

(a) Most candidates found the correct equation of a circle but '= 10' or '= √10' were common. Other errors included omitting to square the brackets, swapping *x* and *y*, and sign errors.

(b) Some pleasing coordinate geometry work was seen. Many candidates successfully found the gradient of the radius using $\frac{y_2 - y_1}{x_2 - x_1}$ and then the correct equation of the radius. Others inverted the gradient or omitted to find the perpendicular gradient. Candidates who attempted to find the gradient of the tangent by differentiation were generally less successful.

(c) Many candidates used their answer to **(b)** to find the equation of circle *Q*. Others did not link the two parts of the question so were unable to form an equation. Most candidates did not attempt to verify the *y*-coordinate of the points of intersection of the two circles. Those who did often equated x^2 or y^2 from the two circle equations and solved to obtain *y* = 11. A quicker method was to substitute *y* = 11 into both circle equations to show that the intersections occurred at the same *x* (or x^2) value in both cases.

(d) Stronger candidates substituted the equation of their tangent into the equation of their circle then solved to obtain $x = \pm\sqrt{20}$. Some were able to find the correct exact values of *y* also. A number of candidates seemed to not understand the meaning of 'surd form' which is mentioned as required prior knowledge in the syllabus.

MATHEMATICS

Paper 9709/12
Pure Mathematics 1 (12)

Key messages

The syllabus for this paper changed in 2019 and new topics including circles and transformations were introduced. Many candidates seemed very confident with these new topics, but others may have required further practice on them.

The previous three reports have each highlighted that the question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' This message has been taken on board by most candidates, but there are still a significant minority for whom it has not. Clear working must always be shown to justify solutions. For quadratic equations, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Factors must also produce the coefficients of the quadratic when expanded.

General comments

Many very good scripts were seen, although the paper was found to be challenging for candidates generally. Candidates seemed to have sufficient time to finish the paper. Presentation of work was mostly good, although some of the answers were initially written in pencil and then superimposed with ink giving a very unclear image when the script is scanned. Centres are strongly advised that candidates should be told not to do this.

Comments on specific questions

Question 1

This question was a good start to the paper for many candidates. The vast majority realised that integration was required although some weaker candidates either differentiated or used the equation of a straight line. The negative power was well dealt with by most as was the evaluation of the constant of integration. Some candidates did not cancel $\frac{4}{-2}$ to -2 or include '$y =$' and so did not score full marks.

Question 2

This question was also very well completed by most candidates and full marks was often awarded. The binomial expansion was generally very well understood although the coefficient of x^2 was sometimes given as $54a$ rather than $54a^2$. Candidates were confident forming and solving the resulting equation although the negative answer was sometimes lost.

Question 3

In **part (a)** most candidates were confident completing the square although the '4' was a challenge for some. In **part (b)** only a few saw the link with **part (a)**, and instead usually restarted using the discriminant. These attempts were often successful initially, but simplifying the resulting inequality correctly proved challenging for many.

Question 4

Most candidates recognised the question as a quadratic in x^3 and were able to solve it either directly, or by using a suitable substitution. A small number of candidates attempted to use *x* as a substitution for x^3 and this often then caused confusion. To avoid this, candidates should be encouraged to use a different letter in this type of question. Some candidates correctly found answers for x^3 but then incorrectly discarded the negative solution. Others found the cube root of $\frac{1}{8}$ to obtain $\pm\frac{1}{2}$. The most common error however occurred through using a calculator to solve the quadratic, rather than factorisation or other similar method. As stated in the 'Key messages', this should be discouraged.

Question 5

This question was the most successful one on the whole paper for candidates in general. A very large majority realised that definite integration was needed although a few applied the rules for differentiation instead. The $\frac{5}{2}$ proved a challenge for some but most were able to integrate successfully. Some candidates did not show the *x* values clearly substituted into their integrand and therefore were not awarded full marks.

Question 6

This question was the point where the paper started to be more challenging for many candidates**.** In **part (a)** many used the sine or cosine rule correctly but some were then unable to find the given expression. Angle *OPA* was sometimes stated to be θ or $2\pi - 2\theta$ rather than $\pi - 2\theta$. Those who used angle *AOP* with the cosine rule were usually more successful in finding the given answer although possibly the easiest method was to split the triangle in half and use the resulting right-angle triangle. It is very important that all necessary working is shown in the answer space, especially when the required answer is given in the question. In **part (b)** for the sector area most candidates seemed to realise a need to use the length of *OA* from the given answer in **part (a)**, although some used the arc length or their answer for *OA* instead. For the area of the triangle *OPA* those using right-angle triangles or angle *OPA* as $\pi - 2\theta$ were equally successful.

Question 7

Part (b) was generally well completed and **part (a)(i)** reasonably well, but the other two parts proved to be more challenging. In **part (a)(i)** most candidates followed the instruction to expand although the resulting $2\sin\theta\cos\theta$ term was sometimes missing. Good responses showed a simplification of the resulting equation to $2\sin\theta\cos\theta = 0$ and then stated that either $\sin\theta = 0$ or $\cos\theta = 0$. Sometimes only two answers were given rather than the three asked for in the question. A minority seemed to be expecting the question to be more complex than it actually was. Similarly in **part (ii)**, many responses seemed to over complicate the question rather than simply verifying which of the 3 solutions were valid for this changed equation; 30 per cent omitted it altogether. **Part (b)** was more familiar for candidates and many correct proofs were seen. In **part (c)** many candidates seemed unsure of what was required, and it was omitted by about 20 per cent. For those who did gain credit it was usually after cancelling $\sin\theta + \cos\theta - 1$ rather than taking this out as a common factor. This usually led only to two of the correct solutions. A few responses only considered $\sin\theta + \cos\theta - 1 = 0$ and some, when cancelled, obtained 0 rather than 1. To get full marks four correct answers were required.

Question 8

The response to this question was varied with many fully correct solutions seen but also many responses scoring few or no marks at all. In **part (a)** many candidates seemed to know that a reflection in the line $y = x$ was required but often they assumed that this line could be plotted by joining the origin to the point in the top right hand corner of the grid given. Weaker responses often involved reflecting the given graph in the horizontal line where it intercepted the *y*-axis. **Part (b)** was completed better with many candidates seemingly familiar with the required process and able to reach $\sin\frac{1}{4}x = \frac{y-3}{2}$. A significant number then tried to divide by $\sin\frac{1}{4}$ rather than taking the inverse sine. **Part (c)** proved challenging with many responses not showing a graph 'levelling out'. A significant number of responses also did not recognise the connection between their sketch and whether or not the function had an inverse. Many fully correct solutions were seen for **part (d)**, including the required order, but a significant number seemed to find the transformations topic a significant challenge.

Question 9

Part (a) of this question was very well done by most candidates, but **part (b)** was much more challenging for many and about 20 per cent omitted it entirely. In **part (a)** only the weakest responses did not show a formation of the two required equations from the given information and then a combination of them into a quadratic equation in one variable. Again, there was significant use of calculator functions to solve this equation and so full marks were often not scored. In **part (b)** some candidates seemed to not recognise that their answers from **part (a)** were meant to be used. Many who were able to form the *n*th terms were unable to go on to prove the given statement. Those who were successful often divided the *n*th terms and separated $(\frac{4}{5})^{n-1}$ successfully. Incorrect statements such as $20\times(\frac{4}{5})^{n-1}=16^{n-1}$ were common.

Question 10

Part (a) was reasonably well done with most candidates substituting for *y* from the equation of the tangent into the circle although a few did replace *a* with 4. Many were able to rearrange to obtain a quadratic and then find the discriminant. The resulting quadratic in *a* was usually solved correctly although, again, there was a significant amount of calculator usage. Some candidates who found **part (a)** challenging omitted **part (b)** entirely but many more realised that the two parts were independent and were more successful in the second part. Most realised that the gradient of the normal would be -2 and that it would pass through the centre of the circle, although a significant minority did extra work and found the point *P*. **Part (c)** proved to be much more challenging: 43 per cent of candidates omitted it completely and many others just repeated some of their work from **part (b)**. Successful responses tended to either replace *y* in the original equation of the circle with $-2x+c$ and follow a similar approach to **part (a)** or found the equation of the diameter parallel to the tangent and then the points where this intersected the circle.

Question 11

Similarly, to question 10, the first two parts were completed quite well but **part (c)** proved much more challenging. Most candidates were confident in differentiating the given function although some forgot to multiply by 4 and others multiplied the -1 by 4 as well. In **part (b)** many realised that $\frac{dy}{dx}$ needed to be equated to 0 and most of these were able to successfully make *x* the subject of the resulting equation. **Part (c)** relied upon candidates knowing that the gradient of a line is equivalent to the tangent of the angle that the line makes with the horizontal. Many seemed unaware of this and omitted this part or made an incorrect statement such as $\frac{dy}{dx}=0$ instead and were therefore unable to make any meaningful progress. Those who realised that $\frac{dy}{dx}=2$ were often then able to solve the resulting equation to find *x,* then *y,* and then the required equation of the normal.

MATHEMATICS

Paper 9709/13
Pure Mathematics 1 (13)

Key messages

The syllabus for this paper changed in 2019 and new topics including circles and transformations were introduced. Many candidates seemed very confident with these new topics, but others may have required further practice on them.

Previous reports have highlighted that the question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' Although this message has been taken on board by most candidates, there are still a significant minority for whom it has not. Clear working must always be shown to justify solutions. For quadratic equations, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Factors must also always produce the coefficients of the quadratic when expanded.

Similarly, whist some calculators can give the answer to a definite integration, to gain credit for a definite integration the algebraic result and substitution of appropriate limits must be shown.

General comments

It is worthwhile reminding candidates that when providing answers to 3 significant figures, the working should be of at least 4 significant figure accuracy to ensure their final answers are of acceptable accuracy.

Most candidates appeared able to finish this paper and there was evidence that questions had been checked and altered where appropriate. In such cases candidates should ensure their alterations are consistent throughout their solution and any content that they don't want marking is clearly crossed out.

Comments on specific questions

Question 1

The transformation shown was the combination of a stretch parallel to the x-axis by scale factor 2 and a translation of $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$. Most responses correctly detailed the transformations but the standard of description was variable. Those who chose not to use a vector to describe the translation often struggled to describe it using acceptable language and the direction of the stretch and its magnitude were regularly given incorrectly.

Question 2

Those who expressed $f(x)$ in completed square form and appreciated that the minimum value of $(x-3)^2$ is zero produced many completely correct solutions, as did those who were able to show by calculus or the properties of quadratic functions that the minimum point on the curve $y = f(x)$ is $(3, c-9)$. Setting the discriminant of $f(x) - 2$ to equal less than zero also worked well although many responses that pursued this route involved setting the discriminant to equal more than zero and no further progress was made.

Question 3

(a) This question part was amongst the best answered on the paper. The expansion of this type of expression was well understood. Minor errors in dealing with combinations and powers were rare and nearly all answers were presented in the expected simplified form.

(b) With so many correct answers seen in **part (a)** the majority of candidates were able to calculate and find the required terms in x and $\frac{1}{x}$ then obtain the required equations in *a* and *b*. When correctly obtained these were almost always solved correctly.

Question 4

(a) Several approaches were seen, with the most popular being to express $\tan^2 x$ in terms of $\sin^2 x$ and $\cos^2 x$, multiply through by $\cos^2 x$ and apply $\sin^2 x = 1 - \cos^2 x$. There were many correct versions of the required result, although some errors in the use of brackets and powers were seen.

(b) The correct use of a quadratic equation in $\cos^2 x$ was seen in most solutions and most of these used the square roots of the only valid solution to obtain 2 angles in the required range. A significant number of answers for $\cos^2 x$ were obtained without any method being shown; as mentioned in the 'Key messages' all answers must be supported by working.

Question 5

(a) The majority of solutions successfully used substitution for *y* in the circle equation to obtain a quadratic equation in *x* whose solutions allowed *y* values to be found from the line equation. Again a significant number of answers for *x* were obtained without any method of solution shown for the quadratic equation.

(b) The use of the mid-point of *AB* as the centre of the required circle was appreciated by most candidates and, although sign errors were common, most found half the length of *AB* as the required radius. The standard equation of a circle, $(x-a)^2 + (y-b)^2 = r^2$, was nearly always used. It was expected that r^2 would not be left in surd form in the final answer.

Question 6

(a) There were many excellent solutions presented for this question, with most candidates using the correct formulae for arc length and sector area. Those who chose to use $\frac{1}{2}ab\sin C$ to find the area of triangle *OAB* or the formula for segment area generally produced more concise, accurate solutions than those who used $\frac{1}{2}bh$ for the area of the triangle. In these cases, calculations tended to lose the accuracy necessary to be able to give the final answer to 3 significant figures. Many candidates were confident in using radians and few instances were seen where angles in degree measure were used. Although many candidates used correct methods in this question, final accuracy marks were often not awarded as final answers were not written to 3 significant figures.

(b) Many correct methods were seen leading to the length *AB*. Occasionally this had been found in **part (a)** and was simply quoted here. Achieving 3 significant figure accuracy was less of a problem here and more correct answers were seen.

Question 7

(a) Given the form of the function and the domain it was surprising to see only around half of candidates finding and expressing the range correctly.

(b) The change of subject method was universally favoured by candidates and most were able to reach a correct inverse function without sign or algebraic errors. Using their range from **part (a)** correctly expressed as a domain for this part was sufficient and often seen.

(c) The formation of a composite function and the subsequent combination of algebraic fractions was nearly always presented correctly. There were very few instances of the wrong order of functions or errors in algebraic manipulation and this was the most successfully answered question part on the paper.

Question 8

(a) The relationship between the terms of a geometric progression appeared to be well understood. Calculation of the common ratio in terms of the first term and substitution into the formula for the sum to infinity was the most frequently used method with accurate algebraic manipulation often leading to a correct quadratic or cubic equation. A visible method of solution for these equations was required together with selection of the positive solution for full credit.

(b) The process of obtaining the common difference from the first two terms appeared to be well understood as did the use of the formulae for the sum to *n* terms of an arithmetic progression. Here it was more usual for candidates to work with an equality rather than an inequality. Whilst most solutions showed the algebra used to reach a quadratic expression very few showed how this expression was solved to obtain values of *n*. Those who had preserved the inequality were able to show the appropriate integer solution which was apparent from their positive solution.

Question 9

(a) The need to integrate and the process of integration and the calculation of the constant of integration were well understood and applied. Most attempts were successful with few errors reported in the manipulation of the negative power and the calculation of the constant of integration.

(b) The better answers justified why the tangent gradient at $x = 0$ was 3 and why the equation of the tangent was $y = 3x + 3$. The tangent was then, almost always, equated to the candidate's equation of the curve. Whilst the required result was then only three algebraic steps away the quickest route was often missed. Some found the value of x which satisfied their equation and showed this then satisfied the given equation.

(c) Solutions which showed a full calculation of the left-hand side of the equation when $x = \frac{3}{2}$ made more progress with this part. The value of y at $x = \frac{3}{2}$ was most easily found from the tangent and this was usually seen in the better answers.

Question 10

(a) The procedure for finding the turning points on a curve was well understood and many completely correct solutions to this part were seen. Common errors were sign errors and mistakes in squaring. The differentiation of this type of function did not seem to cause undue problems for candidates.

(b) The quickest verification came from finding the gradient of the curve at *A* and the gradient of *AB* and showing they had a product of -1. Some candidates chose to find one of the gradients, predict the other using $m_1 m_2 = -1$ and then verify it by calculation of the second gradient. When explained clearly this was given full credit.

(c) This proved to be a very accessible final question part, approached equally successfully by finding the area under the curve and subtracting the area under the line or by subtracting the area of the equivalent trapezium. Correct forms of the required integration of the curve equation were often seen and these were sometimes combined with the integration of the line equation. Only answers which showed how the limits of integration were applied gained full credit. Stating the required integration and then quoting an answer from a calculator without showing any of the integration and substitution processes gained no credit.

MATHEMATICS

Paper 9709/21
Pure Mathematics 2 (21)

Key messages

It is important that candidates check that they are giving their answers in the form specified in the question and to the correct level of accuracy if appropriate.

Each question should be read carefully to ensure that all the relevant facts have been taken into account and that the demands of the question have been met.

Candidates should also ensure that they are familiar with the rubric on the front of the examination paper.

General comments

There did not appear to be any timing issues and most candidates had more than sufficient space in which to write their answers.

It was evident that many candidates had prepared for the examination by revision and full coverage of the syllabus, although some had prepared less as evidenced by low marks obtained.

Comments on specific questions

Question 1

Most candidates attempted to use logarithms with varying levels of success. Although many responses involved taking logarithms of both sides, brackets were frequently omitted, which then lead to errors in the subsequent attempts at simplification. Errors with the application of the laws of logarithms were also common.

Question 2

It was essential that differentiation of a quotient or equivalent product was attempted to make any progress. Again, poor use of brackets led to errors in the calculation of the numerical value of the gradient. Most candidates were able to attempt the equation of the tangent using either their incorrect gradient or the correct gradient and the given point. It is important that candidates check that they are giving their answers in the required form. Too many candidates obtained a correct equation for the tangent but did not simplify to integer coefficients and subsequently the form specified in the question.

Question 3

(a) Most candidates were able to make a reasonable attempt at the integration and subsequent substitution of the limits. Some candidates did have difficulties with the correct integration of e^{2x}. Even though many responses detailed the correct equation $12 = \frac{3}{2}e^{3a} - a - \frac{3}{2}$, or equivalent, most were then unable to show a rearrangement of this equation to obtain the given result. By looking at the form of the given answer, candidates should have been able to determine that there is no exponential term and so isolate the exponential term to obtain $12 + \frac{3}{2} + a = \frac{3}{2}e^{3a}$, or equivalent, as a subsequent step in the re-arrangement process.

(b) A surprisingly large number of candidates did not attempt this question at all. For the candidates that did attempt this part, most were successful, gaining full marks. It should be noted that some candidates did not give their final answer to the required level of accuracy, but most candidates did give sufficient iterations to justify their final answer.

Question 4

(a) Incorrect answers were rare as most candidates realised that they needed to use the factor formula which provided the required answer easily. Some candidates did attempt to use algebraic long division, usually with less success.

(b) It was intended that algebraic long division be used to obtain a quadratic factor. Synthetic division was also acceptable providing sufficient detail was shown. The demand of the question was to find the quotient and factorise the polynomial completely, but a significant number gave their final answer as $(x-3)\left(2x^2+9x+10\right)$, not realising that the quotient could also be factorised.

(c) Many candidates gained full marks here, although some did seem to not understand the question demand and were awarded no marks. Problems occurred when candidates equated the modulus expression to a negative root obtained in **part (b)** and obtained a solution which was not discounted. It was essential that candidates realised that the only value that *a* could take was 3. Candidates were not penalised if they used an incorrect positive value obtained in **part (b)**.

Question 5

(a) It was essential that *y* be differentiated as a product for any progress to be made and many candidates did not recognise *y* as a product. Of those candidates that did attempt differentiation of a product, many omitted the 2 from the differentiation of the trigonometric term.

(b) Although candidates realised that their answer to **part (a)** needed to be equated to zero, most derivatives were not in the correct form needed to yield an equation in $\tan 2t$ and so there were very few correct solutions. Some candidates, having obtained an equation in $\tan 2t$ did not find a negative value for *t*.

Question 6

In questions of this type, it is better for candidates to not work backwards from the given answer as often they would obtain method marks which are usually unavailable once alterations are made to match the given answer. In this case it was necessary to use the appropriate double angle formula for $4\cos^2 2x$ to obtain an expression in terms of $\cos 4x$, thus enabling integration. Unfortunately few candidates were able to do this and often the only mark which many candidates did gain was for recognising that $\dfrac{1}{\cos^2 x} = \sec^2 x$.

Question 7

(a) Many candidates were able to obtain the value of *R* correctly and also the value of α, although some candidates found this angle in radians.

(b) Many candidates recognised that they needed to use their result from **part (a)** to solve the given equation. Of the candidates using a correct method, most were able to obtain the answer of 117.7°, but the answer of 29.8° was less common as candidates did not consider using a negative angle to obtain this result. Many candidates also obtained the answer of 389.8°, but of course this was out of range.

(c) Completely correct solutions were very rare as most candidates did not relate the question to the result obtained in **part (a)**. If the result from **part (a)** was not used to write the given expression as $\dfrac{150}{25\cos\left(\dfrac{1}{2}\beta - 73.74^\circ\right) + 50}$, then little progress could be made. The question then depended upon the candidate realising that greatest possible value of V was obtained when $25\cos\left(\dfrac{1}{2}\beta - 73.74^\circ\right) = -1$. From this, the maximum value of V and the corresponding value of β could have been obtained.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2 (22)

Key messages

It is important that candidates check that they are giving their answers in the form specified in the question and to the correct level of accuracy if appropriate.

Each question should be read carefully to ensure that all the relevant facts have been taken into account and that the demands of the question have been met.

Candidates should also ensure that they are familiar with the rubric on the front of the examination paper as well as the formulae that are given in the List of Formulae and Statistical Tables MF19.

General comments

There did not appear to be any timing issues and most candidates had more than sufficient space in which to write their answers.

It was evident that many candidates had prepared for the examination by revision and full coverage of the syllabus, although some had prepared less as evidenced by low marks obtained.

Comments on specific questions

Question 1

Very few completely correct responses were seen. It was important that candidates recognised that they needed to obtain an equation in terms of $\sec\theta$ or $\cos\theta$ only. It was essential that candidates expressed $5\tan^2\theta$ as $5\left(\sec^2\theta - 1\right)$. Errors included the omission of 5 and sign errors. It was also acceptable to attempt to obtain the given equation in terms of sine and cosine initially and then attempt an equation in $\cos\theta$ only. A large number of responses did not mention either the trigonometric identities, even though the relevant identity is given in the List of Formulae MF19, or the fact that $\sec\theta = \dfrac{1}{\cos\theta}$.

Question 2

Again very few correct responses were seen. It was essential that the given equation $y = Ae^{(A-B)x}$ be written in the form $\ln y = \ln A + (A-B)x$. Many candidates found the gradient of the straight line but were unable to relate it correctly to $A - B$. The solution of the resulting simultaneous equations also proved problematic as the term involving $(A-B)$ needed to be eliminated so that an equation in terms of $\ln A$ remained.

Substitution of the correct coordinates into $y = Ae^{(A-B)x}$ was also acceptable, but very few candidates realised that the coordinates that needed to be used were $\left(0.4, e^{3.6}\right)$ and $\left(2.9, e^{14.1}\right)$.

Question 3

It was pleasing to see that most candidates realised that an integration of $\frac{6}{2x+3}$, with respect to *x* was needed and obtained a result of the form $k\ln(2x+3)$ before the substitution of the limits 6 and zero. The fact that the question specified what form the answer needed to be given in did help some candidates realise that logarithms were involved in the integration process. Many fully correct responses were seen.

Question 4

(a) Most candidates produced a correct sketch of $y=|5x-4|$. It was essential that some indication be made of the two points of intersection with the given curve implying that there are exactly two solutions to the equation $3-e^{-\frac{1}{2}x}=|5x-4|$. A comment with words to the effect that there were two points of intersection, so two solutions, or some indication on the graph was expected.

(b) The candidates needed to consider the left-hand side of the modulus graph which was equivalent to the equation $y=4-5x$. The best way to show by calculation that α lies between 0.36 and 0.37, is to make use of the expression $3-e^{-\frac{1}{2}x}+5x-4$ or $3+e^{-\frac{1}{2}x}-5x+4$ and substitute in the values of 0.36 and 0.37. Many candidates did just this. It is essential that the resulting decimal values are shown and not just indications of the expression having a positive or negative value. It is also expected that a comment is made about the change of sign indicating that the root lies between the two value.

(c) There were many correct solutions gaining full marks to this part, but also a reasonable number of blank answer spaces. It was expected that the first value used in the iteration process was such that $0.36 \leqslant x_1 \leqslant 0.37$.

Question 5

(a) Those candidates that realised that they needed to use differentiation of a product were able to gain some marks. It was also essential that the coordinates of the point *B* were calculated. This was done correctly by most candidates that attempted this question. It should be noted that an exact value of the gradient was required.

(b) It was expected that candidates make use of their derivative from **part (a)** and equate it to zero in order to find the exact coordinates of the point *C*. Many candidates did not factorise the resulting equation, perhaps not realising that $e^{-\frac{1}{2}x}\neq 0$ and so a quadratic equation in *x* only could be obtained. Again, it was essential that exact answers be given, with some candidates having obtained the correct $x=7$, and then giving their *y* coordinate in decimal form.

Question 6

(a) In a question of this type, it is essential that each step of working be shown clearly and in full. The compound angle formulae are shown in the List of Formulae MF19, but there were still a number of responses which did not utilise these formulae. Some sign errors were made, and some candidates multiplied each bracket by 4. Use of $\sin^2\theta+\cos^2\theta=1$ and $2\sin\theta\cos\theta=\sin 2\theta$ was expected.

(b) Many correct solutions were seen making use of the given result in **part (a)**, but candidates needed to indicate that they were making a substitution of $\frac{3\pi}{8}$ or equivalent in order to gain full marks.

(c) It was expected that candidates again make use of the result from **part (a)**. Although many attempted this, a number did not realise that an angle of 2*x* rather than θ was being used.

Question 7

(a) With a given result to show, it was particularly essential that each step of working be shown clearly and in full. An attempt at the quotient rule, or equivalent product rule, in order to find $\frac{\mathrm{d}x}{\mathrm{d}t}$ was expected. Most candidates made correct use of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x}$, but some then equated to zero rather than 1.

(b) Many correct responses were seen, with candidates realising that a substitution of $t = -1$ and use of the factor theorem was needed in order to find the value of a. It was not necessary that candidates had successfully attempted **part (a)** as it was expected that the given result be used.

(c) A small number of correct responses were seen for this part. Candidates were expected to factorise their expression using their value and obtain the equation $(t+1)\left(2t^2 + 9t + 11\right) = 0$, the solution of which would give the x values of any stationary points. The equation $\left(2t^2 + 9t + 11\right) = 0$ has no real roots and an indication of this using the discriminant or equivalent was expected.

MATHEMATICS

Paper 9709/23
Pure Mathematics 2 (23)

Key messages

It is important that candidates check that they are giving their answers in the form specified in the question and to the correct level of accuracy if appropriate.

Each question should be read carefully to ensure that all the relevant facts have been taken into account and that the demands of the question have been met.

Candidates should also ensure that they are familiar with the rubric on the front of the examination paper as well as the formulae that are given in the List of Formulae and Statistical Tables MF19.

General comments

There did not appear to be any timing issues and most candidates had more than sufficient space in which to write their answers.

It was evident that many candidates had prepared for the examination by revision and full coverage of the syllabus, although some had prepared less as evidenced by low marks obtained.

Comments on specific questions

Question 1

Very few completely correct responses were seen. It was important that candidates recognised that they needed to obtain an equation in terms of $\sec\theta$ or $\cos\theta$ only. It was essential that candidates expressed $5\tan^2\theta$ as $5\left(\sec^2\theta-1\right)$. Errors included the omission of 5 and sign errors. It was also acceptable to attempt to obtain the given equation in terms of sine and cosine initially and then attempt an equation in $\cos\theta$ only. A large number of responses did not mention either the trigonometric identities, even though the relevant identity is given in the List of Formulae MF19, or the fact that $\sec\theta=\dfrac{1}{\cos\theta}$.

Question 2

Again very few correct responses were seen. It was essential that the given equation $y=Ae^{(A-B)x}$ be written in the form $\ln y=\ln A+\left(A-B\right)x$. Many candidates found the gradient of the straight line but were unable to relate it correctly to $A-B$. The solution of the resulting simultaneous equations also proved problematic as the term involving $\left(A-B\right)$ needed to be eliminated so that an equation in terms of $\ln A$ remained.

Substitution of the correct coordinates into $y=Ae^{(A-B)x}$ was also acceptable, but very few candidates realised that the coordinates that needed to be used were $\left(0.4,e^{3.6}\right)$ and $\left(2.9,e^{14.1}\right)$.

Question 3

It was pleasing to see that most candidates realised that an integration of $\frac{6}{2x+3}$, with respect to *x* was needed and obtained a result of the form $k\ln(2x+3)$ before the substitution of the limits 6 and zero. The fact that the question specified what form the answer needed to be given in did help some candidates realise that logarithms were involved in the integration process. Many fully correct responses were seen.

Question 4

(a) Most candidates produced a correct sketch of $y=|5x-4|$. It was essential that some indication be made of the two points of intersection with the given curve implying that there are exactly two solutions to the equation $3-e^{-\frac{1}{2}x}=|5x-4|$. A comment with words to the effect that there were two points of intersection, so two solutions, or some indication on the graph was expected.

(b) The candidates needed to consider the left-hand side of the modulus graph which was equivalent to the equation $y=4-5x$. The best way to show by calculation that α lies between 0.36 and 0.37, is to make use of the expression $3-e^{-\frac{1}{2}x}+5x-4$ or $3+e^{-\frac{1}{2}x}-5x+4$ and substitute in the values of 0.36 and 0.37. Many candidates did just this. It is essential that the resulting decimal values are shown and not just indications of the expression having a positive or negative value. It is also expected that a comment is made about the change of sign indicating that the root lies between the two value.

(c) There were many correct solutions gaining full marks to this part, but also a reasonable number of blank answer spaces. It was expected that the first value used in the iteration process was such that $0.36 \leqslant x_1 \leqslant 0.37$.

Question 5

(a) Those candidates that realised that they needed to use differentiation of a product were able to gain some marks. It was also essential that the coordinates of the point *B* were calculated. This was done correctly by most candidates that attempted this question. It should be noted that an exact value of the gradient was required.

(b) It was expected that candidates make use of their derivative from **part (a)** and equate it to zero in order to find the exact coordinates of the point *C*. Many candidates did not factorise the resulting equation, perhaps not realising that $e^{-\frac{1}{2}x}\neq 0$ and so a quadratic equation in *x* only could be obtained. Again, it was essential that exact answers be given, with some candidates having obtained the correct $x=7$, and then giving their *y* coordinate in decimal form.

Question 6

(a) In a question of this type, it is essential that each step of working be shown clearly and in full. The compound angle formulae are shown in the List of Formulae MF19, but there were still a number of responses which did not utilise these formulae. Some sign errors were made, and some candidates multiplied each bracket by 4. Use of $\sin^2\theta+\cos^2\theta=1$ and $2\sin\theta\cos\theta=\sin 2\theta$ was expected.

(b) Many correct solutions were seen making use of the given result in **part (a)**, but candidates needed to indicate that they were making a substitution of $\frac{3\pi}{8}$ or equivalent in order to gain full marks.

(c) It was expected that candidates again make use of the result from **part (a)**. Although many attempted this, a number did not realise that an angle of 2*x* rather than θ was being used.

Question 7

(a) With a given result to show, it was particularly essential that each step of working be shown clearly and in full. An attempt at the quotient rule, or equivalent product rule, in order to find $\frac{dx}{dt}$ was expected. Most candidates made correct use of $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$, but some then equated to zero rather than 1.

(b) Many correct responses were seen, with candidates realising that a substitution of $t = -1$ and use of the factor theorem was needed in order to find the value of *a*. It was not necessary that candidates had successfully attempted **part (a)** as it was expected that the given result be used.

(c) A small number of correct responses were seen for this part. Candidates were expected to factorise their expression using their value and obtain the equation $(t+1)\left(2t^2 + 9t + 11\right) = 0$, the solution of which would give the *x* values of any stationary points. The equation $\left(2t^2 + 9t + 11\right) = 0$ has no real roots and an indication of this using the discriminant or equivalent was expected.

MATHEMATICS

Paper 9709/31
Pure Mathematics 3 (31)

Key messages

- Read the question carefully and make sure that the answer matches the demand.
- Take care with basic algebra and arithmetic because many marks were not awarded due to basic slips.
- If a question asks for an exact answer, such as **Questions 6(b)** and **8(b)**, then a decimal approximation is not an acceptable substitute.
- In a 'show that' question, such as **Questions 4(a), 5(a), 9(a), 10(a)** and **10(b)**, take extra care with giving a full and clear explanation.
- Do not overwrite one solution with another as this makes it difficult to read when scanned.
- Your work needs to be legible – in particular, the numerals need to be clear and not ambiguous.

General comments

Some candidates demonstrated a strong understanding of the topics examined. However, for the majority, there were many blank responses. There did not appear to be any particular pattern to this – for example, candidates started **Question 6** (vectors), but about 70 percent gave no response to **part (c)**. In **Question 9**, many candidates attempted the integration by parts (**part (a)**), but more than one third of the responses to **parts (b)** and **(c)** (numerical methods) were blank. In **Question 10**, the majority of candidates attempted **part (a)** (factor theorem), but about one third offered no response to **part (b)** (on the same topic) and more than half gave no response to **part (c)**. In general, the marks were low, with a mean score of 22.6 and a quarter of all candidates were awarded fewer than 12 marks in total.

Comments on specific questions

Where numerical and other answers are given in the comments on individual questions that follow, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct' answer.

Question 1

For the candidates who rearranged this as a quadratic equation in e^x, this was a very straight-forward question. There were many candidates who attempted to rewrite the laws of logarithms and worked through to obtain incorrect equations such as $3\ln 2x - 4\ln(-2x) = \ln 5$. In solutions with a correct initial method, the most common error was to try to process a negative value for e^x.

Question 2

(a) The sketches drawn were of varying degrees of quality. In many cases the sketch showed a curve. In sketches composed of straight lines, it was unusual for the vertex to be at $\left(-\frac{3}{2}, 0\right)$. Many had the vertex on the y-axis.

(b) **Part (a)** was intended as a hint to candidates so that they would be aware that they were only looking for a single critical value. A minority of candidates did approach this part by considering linear inequalities, but the majority chose to square the inequality to remove the modulus sign. This usually led to an incorrect critical value at $x = -5$ which formed part of the final answer.

Question 3

There were several correct responses to this question. The majority of candidates were aware of what was required for the binomial expansion. Some expanded in powers of x, rather than $4x$, and there were several slips in the arithmetic.

Question 4

(a) Many candidates made correct use of double angle formulae. The question asked candidates to form an equation in $\sin\theta$ and $\cos\theta$, but many responses did not conclude with an equation.

(b) A few candidates recognised that the equation in **part (a)** could be factorised as $(\cos\theta - \sin\theta)(\cos\theta + 3\sin\theta) = 0$. From this form of the equation they often obtained the correct answers. Other correct solutions involved rewriting the equation as a quadratic in $\tan\theta$, or completing the square to obtain $(\cos\theta + \sin\theta)^2 = (2\sin\theta)^2$. Several candidates obtained a solvable equation by using incorrect identities.

Question 5

(a) This was a routine task that many candidates completed correctly. Several candidates treated a as a variable, so they differentiated ay^2 as a product, and several included $12a^2$ as part of their derivative. The given answers helped some candidates to identify and correct their errors.

(b) There was some confusion about how to use the information about the tangent. Some candidates used the original equation and substituted $y = 0$. Some looked for a tangent parallel to the x-axis. Several candidates obtained the correct equation $2ay = x^2$, and a minority used this correctly to find the required co-ordinates. A few solutions included incorrect points because candidates overlooked the fact that if $2ay = x^2$, then $y = -2a$ is not possible.

Question 6

(a) There were few correct solutions to this question because many candidates incorrectly assumed that $\overrightarrow{AB} = \overrightarrow{CD}$. Many appeared to be unaware of the convention that the vertices are named in sequence. Solutions were often poorly set out, with very little explanation of what the candidates were trying to do.

(b) This part of the question did not depend on the position vector of D, so most candidates started again with the given position vectors. Many demonstrated a good understanding of how to use the scalar product to find an angle. In this case the angle did not need to be found, but the exact value for the cosine of the angle was required. In several instances the only exact answer seen was $\frac{6}{3\sqrt{32}}$, which should have been simplified.

(c) The geometry required here is relatively simple: the area of the parallelogram is twice the area of triangle ABC and that can be found easily by using the formula $\frac{1}{2}ac\sin B$. The value of $\sin B$ follows from the answer to **part (b)**. Very few candidates used this approach. There were a small number of correct solutions using the perpendicular distance of a vertex from the opposite side, but the majority of candidates offered no response.

Question 7

Many solutions were awarded the first mark for the correct separation of the variables. Some recognised that the integral in y required the double angle formula and they often obtained an integral of the correct form, albeit with some errors in the coefficients. The alternative was to use integration by parts, but very few candidates completed this process correctly. There are several methods available for the integral in x. The most straight-forward was to recognise $\int \sec 2x \tan 2x \, dx$ as $k\sec 2x$. Some candidates used $\int \frac{\sin 2x}{\cos^2 2x} dx$

and integration by parts. A minority of candidates did obtain the correct forms for both integrals, but fully correct integration was unusual.

Question 8

(a) The majority of candidates demonstrated a good understanding of partial fractions. Most opted for the decomposition into three terms, but the two-term form was also accepted. The candidates who only split the fraction into two terms were not making this part any easier, and were also leaving themselves with additional work to do in **part (b)**. For candidates starting with a correct form, the most common errors were due to slips in the arithmetic.

(b) Most candidates recognised the correct forms for the integrals of some of their terms. Candidates with only two fractions often dealt with one correctly and made no progress with the second. For the logarithms, errors seen were in the signs and the coefficients. The correct answer for $\int \frac{1}{(x+2)^2}\,dx$ was less common, with many candidates giving a third log term. Several candidates did substitute the correct limits correctly, but there were few fully correct answers.

Question 9

(a) The majority of candidates demonstrated a good understanding of the process for integration by parts, and a number were awarded the first three marks. Using the limits to obtain the given equation proved to be more challenging, with several candidates not attempting to rearrange their equation.

(b) This was a familiar task, and several candidates completed it correctly. There were many possible approaches, some used the given equation, and some went back to the definite integral. Some candidates made vague statements, not supported by numerical evidence, and a large minority offered no response at all.

(c) There were a number of fully correct responses, but here again a large minority offered no response at all. For candidates using the iterative process correctly, the most common errors were due to not working to the required accuracy, or drawing the incorrect conclusion from correct work.

Question 10

(a) The majority of candidates gave a correct solution. The simplest approach was to demonstrate that $\mathrm{p}(-3)=0$. In terms of the rest of the question, dividing through by $(x+3)$ to obtain the quadratic factor $(x^2+2x+25)$ was a very helpful start.

(b) Starting from the quadratic factor it is straightforward to demonstrate the given result. Substituting $z=-1+2\sqrt{6}\mathrm{i}$ into the cubic equation and simplifying was more complicated. Using $z=-1+2\sqrt{6}\mathrm{i}$ and its conjugate to form the quadratic factor was simpler, but few candidates tried this. Candidates who used their calculators to write down the three roots of the cubic had not demonstrated that any of the values were roots of the equation and consequently were awarded no marks. There were several blank responses.

(c) Very few candidates recognised that the earlier parts of the question had been leading them to the roots of this equation. The first mark, for stating that the square roots of –3 were solutions of the equation, should have been accessible to all. Most of the candidates who made any attempt at all started by squaring $-1+2\sqrt{6}\mathrm{i}$ and made no useful progress. The majority of candidates offered no response.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3 (32)

Key messages

- Read the question carefully and make sure that your answer matches the demand.
- Take care with the basic algebra and arithmetic because many marks were not awarded due to basic slips.
- If a question asks for an exact answer, then a decimal approximation is not an acceptable substitute.
- In a 'show that' question, take extra care with giving a full and clear explanation.
- Do not overwrite one solution with another as this makes it difficult to read when scanned.
- Your work needs to be legible – in particular, the numerals need to be clear and not ambiguous.

General comments

The majority of candidates showed an understanding of all the topics examined. As usual, candidates approached algebraic tasks such as **Question 1** (the inequality) and **Question 9(a)** (partial fractions) with confidence. Other topics of strength were **Question 6(c)** (iteration) and **Question 7(a) (**implicit differentiation). The trigonometry question, **Question 4**, proved to be more challenging than usual. Candidates should be reminded that in questions with a given answer, such as **Question 9(b)**, time is better spent checking and correcting errors in working rather than trying to reverse engineer an answer.

Comments on specific questions

Where numerical and other answers are given in the comments on individual questions that follow, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct' answer.

Question 1

Most candidates demonstrated a good understanding of how to solve the inequality. There were some errors in the arithmetic, but the majority of errors were in the form of the final answer. The two common incorrect answers were $\frac{17}{11} < x < 11$ and $\frac{17}{11} > x > 11$. Some candidates with an otherwise correct solution stated $x < \frac{17}{11}$ and $x > 11$, which is impossible.

Question 2

Those candidates who applied the laws of logarithms correctly usually obtained $x^2 = 2$. Many did not reject the negative root as being impossible. The question asks for an exact answer, so those candidates who only stated the decimal equivalent were not awarded the final mark.

Question 3

(a) The standard of most answers was very good. The majority of diagrams had a circle of the correct size in the correct place. There were a few circles of the correct size in the wrong quadrants, and in some diagrams the negative real axis was not a tangent to the circle at $(-3, 0)$. A small number of candidates did not score the final mark because they shaded the interior of their circle.

(b) The question asks for a length, but many candidates responded with an angle. There were a minority of correct answers. Some candidates recognised that they needed to find the distance from the centre of the circle to the origin, but did not then subtract the radius.

Question 4

Many candidates understood that they needed to convert the given equation to an equation in a single cosine function. A minority of candidates obtained a correct equation. The common incorrect approaches were:

– to use an incorrect 'half angle' formula thinking that $\cos\frac{1}{2}x = \frac{1}{2}\cos^2 x - 1$.

– to use the double angle formula and covert to expressions using $\cos x$ and $\cos\frac{1}{4}x$.

– to use $u = \cos\frac{1}{2}x$ and state the incorrect quadratic $2u^2 - u - 1 = 0$.

– to use the correct substitution $\cos 2x = 2\cos^2 x - 1$ but not double the formula.

Those candidates who obtained a correct quadratic equation usually solved it correctly and obtained at least one correct value for x. There were quite a few responses that included at least one incorrect solution in the required interval, most commonly 2π. A few used degrees rather than radians.

Question 5

(a) The minority of candidates who factorised to obtain $f(a) = (a-2)a(a+1)$ before making the substitution had a relatively simple substitution and usually reached the correct answer. Of those candidates who started by finding the square and the cube of $2 + y\mathrm{i}$, the majority demonstrated a correct method, but there were many slips in the arithmetic and algebra. The error $(\mathrm{i}y)^2 = -y$ was common. A few candidates showed insufficient working to make it clear that they had expanded the cube for themselves, rather than using a calculator, and these responses were not awarded full marks.

(b) The first mark was available to any candidate whose expression for $f(a)$ had a non-zero real part. Many who had the correct term, $-5y^2$, rejected the negative root, so they made no further progress. Many of those who scored the first two marks then went on to find $\arg f(a)$ rather than $\arg a$.

Question 6

(a) Successful candidates used a variety of different calculations and comparisons. The most common approach was to calculate $f(0.5)$ and $f(1)$ for $f(x) = \cot\left(\frac{1}{2}x\right) - 3x$ and note the change of sign. In some cases, candidates did not score the A mark because they did not give a clear conclusion with either a comment or suitable inequality statement.

(b) Many candidates appeared not to understand what this question was asking them to do. Several tried to apply the iterative formula to obtain numerical values rather than using the required algebraic argument. Those starting with the iterative formula and working towards $3x = \cot\left(\frac{1}{2}x\right)$ were much more likely to be successful than those who tried to work in reverse. Several candidates appeared to be confused between $\tan^{-1}\left(\frac{1}{2}x\right)$ and $\cot\left(\frac{1}{2}x\right)$.

(c) Many candidates scored full marks for this part of the question and were clearly well practised in using their calculator to carry out an iterative process. Most gave their iterations to the required 4 decimal places and the final answer to the required 2 decimal places. A minority of candidates worked in degrees, earning no marks. A common error was to round the final answer to 0.80 rather than 0.79.

Question 7

(a) This was a straight-forward question for many candidates. Some candidates did not take sufficient care in placing the minus sign, leading to ambiguous or incorrect answers. Some candidates did not use the notation for derivatives correctly.

(b) Many candidates attempted to equate the given derivative to –2 and those who did this correctly invariably obtained the correct answers. Sign errors were a problem in this part, however. The most common error was to substitute $y = -2x$ into the original equation. The question asks for exact values, so decimal approximations were not accepted.

Question 8

(a) The majority of responses were awarded the first mark for correct separation of the variables. Common errors included writing e^{2x+1} as $e^{2x}+1$ or obtaining $\int \frac{e^{2x+1}}{dx} = \int \frac{4+9y^2}{dy}$. There were also many errors in dealing e^{-2x-1}, which often became e^{-2x+1} before integration. The majority of candidates recognised the *y* integral as requiring an inverse tangent, but many obtained an incorrect coefficient: $\frac{1}{2}$ in place of $\frac{1}{6}$ was very common. Some candidates seemed unsure about how to integrate either side of the equation and the presence of fractions in the integrands caused some to introduce logarithms to obtain expressions such as $\ln\left(4+9y^2\right)$ and $\ln\left(e^{-2x-1}\right)$. Those candidates who integrated correctly were usually able to find the constant successfully, although care should be taken when rearranging before substituting: the error of going from $tan^{-1}\left(\frac{3y}{2}\right) = -\frac{1}{2}e^{-2x-1}+c$ to $\frac{3y}{2} = \tan\left(-\frac{1}{2}e^{-2x-1}\right)+c$ was common.

(b) Candidates are becoming more proficient at this type of question. Those with a correct or nearly correct solution in **part (a)** usually gained the mark in **part (b)**. Some guessing was in evidence, with '*y* tends to zero' or '*y* tends to infinity' being common answers, and candidates are reminded that justification for their statements is required.

Question 9

(a) This seems to be a popular topic with the candidates, and many scored full marks here. A large majority chose the form of 3 fractions and fully correct solutions were common. Those candidates who found a common denominator and then substituted $x = 2$ and $x = -\frac{1}{2}$ were usually more successful than those who chose to compare coefficients, the latter method being more prone to sign errors and arithmetic slips.

A significant minority of candidates approached the square factor as a quadratic, and attempted a decomposition with $\frac{Dx+E}{(2-x)^2}$. Usually full marks was awarded in **part (a),** but then little progress was made in **(b)** as this was not in a form that could be easily integrated.

(b) This part proved to be more of a challenge and there were several errors with signs and coefficients. Many recognised that the two fractions with linear denominators integrated to log terms, but they struggled with $\int \frac{k}{(2-x)^2}dx$. The majority of those who completed the integration were able to substitute the limits in the correct order and some proceeded correctly to the given answer. As is often the case when an answer is given, many seemed to attempt to reverse engineer the required form from an incorrect integral: candidates should be advised that their time is better spent reviewing their solution to find an error, rather than trying to defy the laws of algebra.

Candidates who could correctly complete the integration often did not show sufficient working: candidates should be aware that a 'show that' question requires full and thorough steps of working to secure full marks.

As mentioned in **part (b)**, candidates with a fraction of the form $\frac{Dx+E}{(2-x)^2}$ often made no progress with the integration, but a few were successful in using integration by parts.

Question 10

(a) The majority of candidates recognised the need to use the product rule. This was often carried out correctly. The most common errors occurred in using the chain rule to differentiate the square root. Candidates then attempted to set the derivative equal to zero and solve for *x*. Some struggled with the two terms involving the square root. In the resulting linear equation, the common error was to obtain $-x+5$ instead of $-x-5$, resulting from an error in removing brackets. Several candidates who obtained $x=-\frac{2}{3}$ did not go on to obtain the corresponding value of *y*, or they stated *y* as a decimal. A minority of candidates avoided differentiating the square root by squaring the whole expression before attempting to differentiate.

(b) This part provided a real challenge for candidates and there were very few fully correct solutions. Many scored the B1 for the correct values for the limits for *x*, but this was commonly followed by using the limits for *u* in the wrong order. The majority of candidates substituted for all of the parts of the integrand, including correct substitution for d*x*. Errors then occurred in the attempts to tidy up the integral, which then made the subsequent integration more complicated than it needed to have been. It proved difficult for candidates to gain any further credit, either because the integration went wrong, or the limits were used the wrong way round.

Question 11

(a) Many candidates understood what they needed to do, and several scored full marks. A common error was to find the vector $\overrightarrow{AB}$ rather than the equation of the line *AB*, which then caused an absence of simultaneous equations. There were several slips in attempting to solve the simultaneous equations. A minority of candidates did all the working correctly but did not draw a clear conclusion.

(b) There were several clear and concise solutions. Early errors from some candidates who knew what was required often led to only the M1 being scored. Many candidates did not know how to get started – they often understood that a scalar product was required, but then did not use the correct vectors. There were also a large number of blank responses.

MATHEMATICS

Paper 9709/33
Pure Mathematics 3 (33)

Key messages

Candidates need to:

(i) ensure that they are prepared when they enter the examination room, that is they have a pencil, black biro, ruler, compasses and protractor. This applied particularly to **Question 3**.

(ii) be able to include the extra detail required in a proof question, such as **Question 7a**.

(iii) ensure that they retain throughout their working the independent variable given in the question, for example using θ not x **(Question 4)**, and that their handwriting clearly distinguishes between x and y (**Question 8**). This helps avoid confusion that can lead to later errors in working.

(iv) know what is meant by arg $z = -\frac{\pi}{4}$ (**Question 11**), i.e. $y = -x$, and **not** assume values such as $x = \sqrt{2}$ and $y = -\sqrt{2}$.

General comments

Generally the standard of work seen was of high quality, with only a very small number of candidates finding the paper very challenging. In addition it was good to see candidates appeared to have read recent reports and were improving their presentation skills. The earlier short questions appeared to give most candidates confidence and the time to tackle the longer later questions.

Most candidates found **Question 1** to **Question 7**, and **Question 10a** and **Question 10b**, relatively straight forward, however **Question 8**, **Question 9a** and **Question 9b**, and **Question 11a** and **Question 11b** proved more challenging.

Comments on specific questions

Question 1

Most responses were awarded full marks. The logarithms were generally removed successfully, either by subtracting first or by using exponentials, to establish a correct equation. The main error here was not giving the final answer to 3 decimal places.

Question 2

Most candidates opted to solve via long division as opposed to finding the quotient and remainder by inspection. This was usually very well done, although a few responses had the incorrect remainder by expressing the $6x$ term, after subtraction, with the incorrect sign, giving a final remainder of $-2x - 15$.

Question 3

This question was answered well by the majority of candidates. The most common error seen was in omitting a scale from the diagram or showing the scale on only one axis. Responses needed to show evidence for the centre or radius of the circle drawn. Often the second inequality was shown as a circle instead of the straight line $y = 2$, and some replaced the line $y = 2$ with $y = 4$. A few wrongly shaded the area below $y = 2$. There were a handful of freehand sketches, which were sometimes good enough to earn marks. Those who drew their diagram accurately and with correct scales, see **Key messages**, were able to obtain full marks. Candidates must draw their Argand Diagram showing equal scales on the two axes.

Question 4

This question was answered well by most candidates. Almost all were awarded the first B1, although a few missed differentiating the θ term within y. The vast majority also achieved success using either the product or the quotient rule. Again virtually all were able to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$ from their $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ and $\frac{\mathrm{d}x}{\mathrm{d}\theta}$. There were some notation errors where t or x were used instead of θ, and some omitted the independent variable completely. Candidates should ensure that, when they are asked to obtain a given answer, they take particular care to show full working and conclude with the answer printed on the question paper.

Question 5

(a) Nearly all candidates applied the product rule correctly, although there were some errors with the differential of $\cos 3x$, for instance $-\frac{1}{3}\sin 3x$ instead of $-3\sin 3x$ or $\sin x$ instead of $\sin 3x$. However, reaching the given answer took many algebraic steps, and candidates often either omitted to show that they were using $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ at $x = a$ or suggested that $\tan^{-1} 3a$ was the reciprocal of $\tan 3a$.

(b) Most candidates produced correct answers and were awarded full marks, with a very small number using degrees instead of radians. Many candidates listed more iterations than were required for convergence to the stated degree of accuracy, and some even showed a sign change as well.

Question 6

(a) The majority of candidates reached $4\cos x + \sqrt{3}\sin x$ and then continued with a fully correct answer. The main errors were either writing $-\sqrt{3}\sin x$ instead of $\sqrt{3}\sin x$ or writing $\frac{\sqrt{3}}{2}$ instead of $\sqrt{3}$. Some candidates did not replace $\cos 60$ with 0.5.

(b) There were many correct solutions seen although the second value of θ was sometimes missing or found incorrectly, for example by using $\theta = 180 - 39.2$ or $2\theta = 360 - 2 \times 39.2$. Using θ instead of 2θ or $2(\theta - 23.41)$ instead of $2\theta - 23.41$ were common errors. Occasionally degrees from **Question 6a** were combined with radians here, or vice versa.

Question 7

(a) A small number presented the proof fully correctly, with many missing dx or du or a minus sign that disappeared without the order of the limits being changed. Candidates are reminded of the need to structure carefully their answers to such questions. For example, they should change the limits and find $\frac{\mathrm{d}u}{\mathrm{d}x}$ and hence $\frac{\mathrm{d}x}{\mathrm{d}u}$ separately from the working for their integrand. Then each piece of information should be substituted singly: the expression for dx, followed by the limits and then reverse the limits due to the presence of the minus sign from differentiating $\cos x$, taking three lines of working in all. Candidates who introduced dx and reversed limits within a single line of working did not show clearly that they were carrying out all of the operations correctly.

(b) The majority of candidates answered this question well but several removed the 2 at the start and forgot to return it later. The most common errors were integrating e^{2u} incorrectly to obtain $2e^{2u}$ or making one or more sign errors. Several candidates did not spot the linkage between the two parts of the question. This meant they were faced with a far more difficult integral, and had to attempt to integrate by parts twice, often making little progress.

Question 8

There were many complete solutions seen, however a number of candidates found this question challenging and could only separate the variables correctly and integrate $\frac{1}{x}$. Of those that did recognise that the integrand in *y* was best dealt with by subdividing into two fractions, the majority completed the question correctly. However, some did not recognise that the integrand $\frac{4}{y^2+4}$ led to a standard arctan result and some wrote $\arctan\left(\frac{x}{2}\right)$. See **Key messages**. Some good solutions were marred by using degrees to find the constant of integration. A small number chose to complete the *y* integrand by parts, but after undertaking this correctly once, for which M1 was available, little or no progress was made in the next integration step. For the final mark, candidates needed to simplify their answer; $\exp^{\ln(\ldots)}$ was not sufficiently simplified as it was necessary to remove the ln.

Question 9

(a) There were many fully correct solutions, often completed efficiently in a few lines of working. However, the common error was for candidates to assume a point of intersection and solve by equating lines. The lines do meet but this is not stated in the question and should not be assumed. Often this followed work using the given point *P* on the line *l* producing $\lambda = -2$, but the fact that the intersection approach led to $\lambda = -1$, and thus what appeared to be a contradiction should have been spotted and corrected by candidates who followed this approach. The difference in the λ values arose simply because the two approaches are actually looking at different points on the line.

(b) Candidates found this part more difficult and many did not attempt it. Equating the lines was a common approach but this was only awarded full marks if the equations were checked for consistency. The most obvious and simplest approach was to create a vector from point *P* to a point on the line *m* and then to use the fact that this vector was perpendicular to the direction vector of *m*, hence producing the components of vectors $\overrightarrow{OQ}$ and $\overrightarrow{PQ}$, both needed in the final part of the question. Common errors were using $\frac{2}{3}$ instead of $\frac{3}{2}$ (or $\frac{3}{2}$ instead of $\frac{5}{2}$), direction errors in the relevant vectors, not squaring $\frac{3}{2}$ when using magnitudes, or finding vector $\overrightarrow{QR}$ not $\overrightarrow{OR}$. There were a number of very good correct solutions that were produced neatly within an efficient number of lines.

Question 10

(a) This question was very well done by the majority of candidates, with most scoring full marks with the correct 3 fraction format usually preferred over the 2 fraction alternative. Incorrect formats such as $\frac{A}{1+2x}+\frac{B}{3-x}$, $\frac{A}{1+2x}+\frac{C}{(3-x)^2}$ or $\frac{A}{1+2x}+\frac{B}{3-x}+\frac{C+Dx}{(3-x)^2}$ were seen. Incorrect working seen included multiplying throughout by $(1 + 2x)(3 – x)^3$ or inverting term by term to produce $A(1 + 2x) + B(3 – x) + C(3 - x)^2$. The less common approach for finding *A*, *B* and *C* was expanding to form simultaneous equations then solving them. This method resulted in errors more often than the substitution method.

(b) Almost all candidates obtained the first two terms of one of the relevant expansions and hence could be awarded M1. The most common errors were extracting 3^{-1} instead of 3^{-2}, extracting 3 and 9 instead of 3^{-1} and 3^{-2} or not evaluating the squared term(s) correctly. In the latter case the term was often written correctly in the initial expansion but not squared correctly when tidying up.

Question 11

(a) Most candidates recognised the need to convert to $x + \mathrm{i}y$ form but errors were often made when doing this; $3 + a^2$, $9 + a$ and $9 – a^2$ were seen in the denominator and 15 or 2 seen instead of $15a$ or $2a$ in the numerator. The correct sign was not always used in the final part and the second values of *a* and *z* were not always rejected. Some candidates opted to convert to $x + \mathrm{i}y$ and attempted to use $\pm\tan\frac{1}{4}\pi$ with arguments of $5a - 2i$ and $3 + ai$, but made little progress. Both alternative solutions were seen very infrequently.

(b) A minority of candidates attempted this part. Some responses were partially correct but used two values of z from **Question 11a**. Many used $|z|^3$ and 3θ and so achieved correct values easily. Others evaluated $(2 - 2i)^3$, not always correctly, then found the magnitude of the result. This often led to $\arg z^3 = \frac{1}{4}\pi$ from $\tan\theta = 1$. It was necessary to consider the fact that both real and imaginary parts are negative not positive.

MATHEMATICS

Paper 9709/34
Pure Mathematics 3 (34)

Key messages

- Read the question carefully and make sure that your answer matches the demand.
- Take care with the basic algebra and arithmetic because many marks were not awarded due to basic slips.
- If a question asks for an exact answer, then a decimal approximation is not an acceptable substitute.
- In a 'show that' question, take extra care with giving a full and clear explanation.
- Do not overwrite one solution with another as this makes it difficult to read when scanned.
- Your work needs to be legible – in particular, the numerals need to be clear and not ambiguous.

General comments

The majority of the candidates for this paper produced responses to parts of all the questions. Candidate showed confidence with the algebraic topics, such as **Question 2** (binomial expansion), **Question 7** (roots of a polynomial) and **Question 11** (if they recognised the partial fractions). In the two questions on Complex. Numbers candidates showed good skill with the Argand diagram, but they struggled with using the numbers. A sound knowledge of the basic methods in all topics was often undermined by slips in very basic algebra and arithmetic.

Candidates need to pay attention to the presentation of their solutions. The best work is clearly set out, and carefully explained. At the other extreme, some work was barely legible, standard notation was not used correctly and it was not clear what the written equations represented.

Comments on specific questions

Where numerical and other answers are given in the comments on individual questions that follow, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct' answer.

Question 1

Many candidates were successful in combining the terms and removing the logarithms. The error $\ln(2x+1)+\ln(2x-1)=\ln\left(\frac{2x+1}{2x-1}\right)$ was common. The question asks for the final answer to be given to 3 decimal places, but several candidates left their answer in exact form or to 3 significant figures. Several candidates did not reject the invalid negative solution.

Question 2

Most candidates demonstrated familiarity with the binomial expansion. Many candidates found the x^3 term in the expansion of $(1+2x)^{-2}$ but did not go on to find the term in x^4. Those who did calculate the required terms were usually successful in combining them to obtain the correct answer. A small minority of candidates attempted to use partial fractions before proceeding with the expansion; this approach involved unnecessary work and errors were common. The most common error in the binomial expansion was to work with powers of *x*, rather than 2*x*. Some candidates tried to expand $(1+3x)^1$, with not all of them obtaining $1+3x$. The question asks for the coefficient of x^4 but many candidates stated the term in x^4 or the expansion up to and including the term in x^4.

Question 3

There were many fully correct solutions for this question. Common errors included using the incorrect sign in the expansion of $\cos(x-60°)$, losing the 3 in one or both terms on the right-hand side of the equation, sign errors when rearranging the equation, and errors in processing the surds. The majority of candidates knew that the graph of $\tan x$ repeats every $180°$, but $180° - x$ and $360° - x$ were common errors. Several candidates included $16.8°$ as part of their final answer.

Question 4

This was the first question that presented a real challenge to the candidates. A significant minority did not appear to understand the notation *z**, with several simply ignoring the *. Many candidates made a correct start, either simplifying the first term, or multiplying the whole equation by i. However, some did not get as far as an expression in *x* and *y*. Many candidates with an expression in *x* and *y* did not go on to form equations for the real and imaginary parts. A common error was to equate the expression to $x + \mathrm{i}y$ rather than to zero. There were many slips in the working, so fully correct solutions were unusual.

Question 5

(a) When the candidates used rulers and compasses, they often produced good diagrams. Most understood that a circle was required with a centre that involved coordinates of ±3 and ±2. Of those who attempted this part, the majority did have the correct centre. Candidates need to take care with the scales on their axes – in many cases the radius of the circle drawn did not match the scale shown. It the scales on the two axes are different, then the 'circle' should be an ellipse. The half line for $\arg z = -\frac{\pi}{4}$ did not always match the scale on the axes and did not pass through the point on the circle representing $2 - 2\mathrm{i}$. Several correct diagrams had the wrong region shaded or the half line drawn as a full line.

(b) A minority of responses were fully correct for this part. Some candidates did find the size of a relevant angle, but did not appreciate that it was only part of the required angle. Some candidates found the correct size for the required angle but gave a positive final answer. A minority of candidates found a length, not an angle.

Question 6

(a) Many candidates understood how to use the chain rule to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$. The work for $\frac{\mathrm{d}y}{\mathrm{d}t}$ showed that most recognised the need for the product or quotient rule. The error $\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{-t} = -t\mathrm{e}^{-t}$ was common, along with sign errors in simplifying this derivative. The work for $\frac{\mathrm{d}x}{\mathrm{d}t}$ was often incorrect, with the derivative of *t* sometimes being given as 0, and errors being made with the coefficients in the fraction $\frac{2}{1+4t^2}$. A significant minority did not use the correct form for the derivative of $\tan^{-1}(2t)$, often producing answers involving $\sec(2t)$ or $\sec^{-1}(2t)$. The attempts at obtaining $\frac{\mathrm{d}t}{\mathrm{d}x}$ often contained algebraic errors – a common mistake being $\frac{\mathrm{d}t}{\mathrm{d}x} = 1 + \frac{1+4t^2}{2}$, or the equivalent for the candidate's derivative. The question asked for the answer to be simplified – several candidates did not get as far as collecting like terms within a bracket, and a number did not identify 'simplified' with 'factorised', making **part (b)** more difficult.

(b) A minority of responses were awarded full marks here. Many stopped when they had given a value for *t*. Having started with a factorised form for the derivative, several candidates expanded the numerator before trying to solve for *t*, and did not then obtain the correct value. Others did not give the exact value for x. Some candidates worked in degrees and obtained the incorrect final answer of 45.5.

Question 7

(a) There were many fully correct solutions to this part of the question. Most of the mistakes were due to slips in the arithmetic, although some candidates used p(–1) = 0 instead of p(–1) = 5. A few started by equating $x+1$ to 5, so used $x = 4$ and p(4) = 0.

(b) Many candidates started by dividing their p(x) by $2x + 1$. There were several candidates who obtained the correct quadratic, but did not go on to factorise it. Candidates who used a calculator to factorise the quadratic often gave the two corresponding linear factors as $\left(x-\frac{3}{2}\right)$ and $\left(x+\frac{4}{3}\right)$, making their final answer incorrect unless they included the factor 6.

Question 8

(a) This question proved challenging for the majority of candidates. Some ignored the instruction to use integration by substitution and attempted to use integration by parts. Of those who did substitute, there were several who did not attempt to use $\frac{dx}{d\theta} = \frac{1}{\sqrt{3}}\cos\theta.$ There were also many errors in obtaining $\frac{\sin^2\theta}{\cos^2\theta}$. Another very common error was the incorrect sequence $\left(\sqrt{1-\sin^2\theta}\right)^3 = 1-\sin^3\theta = \cos^3\theta.$ The justification of the new limits was often absent or incorrectly shown with several candidates writing statements involving $\sin\frac{1}{2}$.

(b) For those candidates who used the substitution $\tan^2\theta = \sec^2\theta - 1$, this was a straight-forward task, and there were several concise and correct solutions. Only a minority of candidates stated the given answer without showing the full substitution of the limits. There were a few candidates who rewrote the integrand as $\sin^2\theta\sec^2\theta$ and used integration by parts – most of these attempts did not progress beyond the first stage of the integration. The most common incorrect method was to claim that $\int \tan^2 d\theta = \frac{1}{3}\tan^3\theta.$

Question 9

(a) There were several blank responses to this question, and many sketches were unrecognisable. The two graphs often had no labels and the axes showed either no scale or poor scales. The sketch of $y = \ln x$ tended to be the more recognisable of the two sketches shown, but it frequently crossed the x-axis at a point other than 1 and often appeared to have a horizontal asymptote or to exist for $x < 0$. The sketch of $y = 1+\cot\frac{1}{2}x$ often crossed the x-axis at a value less than π. Some attempts at $\cot\frac{1}{2}x$ looked more like $\cot x$, and for some there was a possible confusion with $\operatorname{cosec}\frac{1}{2}x$ or $\sec\frac{1}{2}x$. Where a sketch showed two intersecting curves, it was unusual for the point of intersection to be highlighted as being of any importance.

(b) The majority of candidates understood what was required for this question. There were many correct approaches. Those who rewrote the equation in the form f(x) = 0 usually produced the most complete answers.

(c) Many candidates seemed to not understand the difference between this part of the question, requiring an algebraic approach, and **part (d)** requiring a numerical approach. There was also evidence of confusion between $\tan^{-1}(x)$ and $\frac{1}{\tan x}$. There were several instances of the meaningless notation $\frac{1}{\tan}(x)$. Those candidates who worked from the iterative formula towards the equation given in **part (a)** were more successful than those who worked in the other direction.

(d) The majority of candidates were able to use their calculators to complete the iterative process, with only a small minority working in degrees rather than in radians. Most candidates worked to the required degree of accuracy. This sequence converged quite slowly, and some candidates did not give sufficient iterations to confirm the value of the root.

Question 10

(a) The majority of candidates understood the structure of the vector equation of a line, but a minority did not use the correct form **r** = … . There were several slips in the arithmetic when finding $\overrightarrow{AB}$ and several candidates used the incorrect form $\mathbf{r} = \overrightarrow{OA} + \lambda\overrightarrow{OB}$. Some candidates found the vector $\overrightarrow{AB}$ but did not go on to find the equation of the line.

(b) The majority of candidates used their line and the given line correctly. There were many fully correct solutions. Most errors were due to arithmetic slips or to miscopying from one line to the next.

(c) The most successful responses to this question were the ones where it was clearly stated what the vectors being used represented – this often led to the correct vectors being used. Many of the errors were due to sign errors and to arithmetic slips. Several candidates attempted to use the scalar product of their $\overrightarrow{AP}$ with the wrong direction vector. At the final step, some candidates found the length OP, rather than AP.

Question 11

This question should have been a good source of marks for the candidates, but many found the question very challenging. The two key steps were the correct separation of variables, and to recognise that the fraction needed to be split into partial fractions. If the 25 remained with the terms in x then the numbers were simpler. Those candidates who used a decomposition into three separate fractions were usually more successful with the integration at the next stage. Those who obtained the correct partial fractions often went on to complete the question successfully. After partial fractions, the most common errors were sign errors in the course of integration. A large number of responses were awarded just one mark for correctly separating the variables and obtaining t or $\frac{1}{25}t$, as appropriate. The most common error after separation of the variables was to claim that $\int \frac{1}{5x^2 + x^3}\,dx = \int \frac{1}{5x^2} + \frac{1}{x^3}\,dx$.

MATHEMATICS

Paper 9709/41
Mechanics (41)

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium situation or a Newton's law equation. Such a diagram would have been particularly useful in **Questions 5** and **6**.
- Non-exact numerical answers are required correct to three significant figures or angles correct to one decimal place as stated on the front of the question paper. Candidates are strongly advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

General comments

The questions were well answered by many candidates, and candidates at all levels were able to show their knowledge of the subject. **Questions 2(a)** and **5(a)** were found to be the easiest questions whilst **Questions 4(c), 5(b)** and **7(c)** proved to be the most challenging.

In **Question 7(c)**, the angle was given exactly as $\sin^{-1}\left(\frac{1}{60}\right)$. There is no need to evaluate the angle in this case and problems such as this can often lead to inexact answers; any approximation of the angle can lead to a loss of accuracy.

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

(a) This question was answered well by most candidates. It is necessary to use the principle of conservation of linear momentum for the collision between particle P and particle Q. Most candidates used the correct form of the equation, but several candidates incorrectly either gave an answer not in terms of m or gave a negative answer. On a few occasions, conservation of kinetic energy was used instead of conservation of momentum.

(b) Most candidates gained some credit for a second application of the principle of conservation of linear momentum between particles R and Q, although a number incorrectly included particle P too. A few candidates incorrectly implied that Q continued to move after its collision with R.

Question 2

(a) This part was answered extremely well by many candidates who correctly found the greatest height above the ground reached by P. The most common error was to use $v = 10$ and $g = 10$ in $v^2 = u^2 + 2as$, which led to the 'correct' answer but from clearly incorrect working. Although rare, some candidates used an energy approach by equating the loss of potential energy $(0.4 \times g \times h)$ to the gain in kinetic energy $\left(\frac{1}{2} \times 0.4 \times 10^2\right)$.

(b) The responses to this part were mixed. Many candidates made a correct start by working out either the kinetic energy before impact (or the equivalent loss of potential energy) and then using the given value of 7.2 to work out either the speed after impact or the maximum height achieved after the first impact. The most common error when finding the time between the first and second instants at which P hit the ground was to only calculate the time from the first impact to the maximum height and so implying a time of 0.8 rather than the correct 1.6 seconds.

Question 3

This was a relatively straightforward variable acceleration question. A few candidates incorrectly integrated the given expression or attempted to use constant acceleration formulae, but most differentiated correctly and set their differentiated expression equal to zero. Many candidates struggled with solving the equation $\frac{5}{2}t^{\frac{3}{2}}-\frac{45}{8}t^{\frac{1}{2}}=0$, with the most common errors occurring in those that attempted to square this equation; those that factorised this equation were far more successful. Some candidates, after correctly finding the time when the particle was next at rest, failed to complete the question and calculate the corresponding displacement.

Question 4

(a) This first part of the question was answered extremely well. Most candidates worked out the distance travelled by the particle in the first 10 seconds by considering the area below the line segments as two triangles and a rectangle, rather than considering the more obvious trapezium. Some candidates attempted to use the equations for constant acceleration, but these attempts were rarely successful. Where errors occurred in calculating the different areas below the line segments it was usually a slip in one (or more) value(s) or forgetting the half in the formula for the area of a triangle.

(b) It was clear in this part that many candidates failed to interact with the velocity-time graph and so were unsure how to find the minimum velocity of the particle. Even of those that did, many gave the incorrect answer of 7.2 (its speed) rather than the correct –7.2. It should also be noted that in this, as well as in **part (c)**, many candidates assumed that the two line segments between $t = 10$ and $t = T$ formed an isosceles triangle, which was not necessarily correct (but did lead to the correct answers in both these parts).

(c) Very few candidates scored full marks here and many left this part blank. The most common error, which appeared more often than the correct method, was to assume that the information given about the greatest speed of the particle was referring to the speed before time $t = 10$ and not after. Of those that did realise that the greatest speed was referring to the interval between $t = 10$ and $t = T$, many did not account for the time from 10 and implied instead that T satisfied the equation $\frac{1}{2}\times T\times 3=7.2$. Of those that did correctly work out that $T = 14.8$, most went on to correctly work out the average speed of the particle for the whole of its motion.

Question 5

(a) Many candidates gained most of the marks in this first part, with most not scoring full marks as they failed to correctly state the values of F and θ to at least three significant figures. Most achieved the first three marks for obtaining correct equations by resolving vertically and horizontally, although the layout was often poor.

(b) Most candidates scored the first two marks in this part for resolving vertically and horizontally and most gave the correct direction of the resultant force. However, a large number of candidates did not give the exact magnitude of the resultant force. Many gave an answer of 5.358… or left their answer as $20\sqrt{3}-40$ without realising that this was clearly negative.

Question 6

(a) This was another question in which many candidates struggled, and it was clear that many were unsure where to begin with such a non-standard question. Many candidates did score the first two marks for correctly working out the maximum possible magnitude of the friction force at Q. Most then tried to apply either Newton's second law for the two particles either separately or for the entire system, but it was unclear at times if they knew what they were trying to achieve; the setting out of their work was often challenging to follow. The two correct approaches seen were to either consider the net force in the direction BA, which was given by the expression $0.2g\sin 60 - 0.1g\sin 60 - F_{\max}$, and to indicate that this value was positive (and hence the particles were moving), or to assume that the system was in motion and show that the acceleration of the system in the direction BA would be 1.72… which again is consistent with the idea that the particles are in fact moving.

(b) Responses here were considerably better than in **part (a)**. Many applied Newton's second law correctly for P to work out the magnitude of the acceleration, before again applying Newton's second law for either the entire system or for Q only to work out the corresponding value of θ. When errors occurred, they were usually the standard sign errors or using the incorrect trigonometric ratio when resolving the weight components parallel to the two planes.

Question 7

(a) This part was answered extremely well with most candidates applying $P = F \times v$ correctly to find the driving force of the car and then using Newton's second law to find the required acceleration of the car. When errors occurred, they were usually in not using a value of 16 000 for the power, or sign errors when applying Newton's second law.

(b) Similarly to **part (a)**, this part was relatively straight-forward and therefore a well-prepared candidate did not hesitate in calculating the correct steady speed of the car.

(c) Many candidates struggled with this part. A significant number of candidates incorrectly thought that the equations for constant acceleration could be used in this problem. This is not the case as mechanically with a constant power the speed of the car is changing, hence the driving force produced by the engine of the car is variable and hence so is the car's acceleration. Furthermore, the question specifically stated that the resistance force was no longer constant. The work-energy principle had to be used here. It is necessary to find the work done by the engine, an expression for the increase in kinetic energy and the increase in potential energy. Combining these correctly with the work done by the resistive forces in the work energy equation will give an equation which can be solved to find the speed of the car at the top of the hill. Some errors were seen in trying to find a driving force from the given power and to use the given speed to find the work done by the engine. Some failed to find the correct change in height used in the potential energy calculation, and several candidates incorrectly used the change in kinetic energy as $\frac{1}{2} \times 1200 \times (v - 20)^2$ instead of the correct $\frac{1}{2} \times 1200 \times \left(v^2 - 20^2\right)$.

MATHEMATICS

Paper 9709/42
Mechanics (42)

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. Such a diagram would have been particularly useful here in **Questions 3**, **5, 7(a)(i)** and **7(a)(ii)**.
- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1, 2** and **4(a)** were found to be the most accessible questions whilst **Questions 3(a), 4(b)** and **6(d)** proved to be the most challenging.

In **Question 3(a)**, the angle α was given exactly as $\tan\alpha = \frac{4}{3}$. There is no need to evaluate the angle in situations such as this as it would not lead to the given exact answer being obtained.

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

The majority of candidates earned some credit for considering either potential energy or kinetic energy or both. Those who considered both energy terms usually went on to achieve a full correct answer, with only a very small minority making sign errors. An alternative, but less successful, approach seen was to find the acceleration of the particle and then use Newton's second law to find the air resistance to the motion of the particle, subsequently multiplying this by 9 m to find the work done against air resistance.

Question 2

(a) Candidates have become increasingly confident in applying conservation of momentum to a given situation. However, a significant number did not consider that the particles were initially moving towards each other by including a negative sign with one of the initial velocities. This often resulted in a value of v as –4.5. Some subsequently made this a positive value without justification.

(b) Only a few candidates did not know or correctly apply the formula for kinetic energy here. An incorrect answer of 10.8 J was seen almost as often as the correct answer of 75.6 J. The reasons for this were twofold. Some thought that, as the request was for the loss of kinetic energy, they had to find a difference between the initial kinetic energy of each particle, rather than the difference between the total initial kinetic energy and the final kinetic energy. The second reason for this incorrect answer was for instances of the square of –6 being evaluated as –36.

Question 3

(a) This question proved a challenge for many candidates. Resolving in two directions was performed well by the majority, obtaining two correct equations in terms of P, θ and α. Only a minority of candidates did not substitute a value for α. Many candidates then used an approximate value for α such as $\alpha = 53.1°$. This created an issue of using an approximate value to obtain an exact value, which is not mathematically robust. The given $\tan\alpha = \frac{4}{3}$ should have been used so that $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$ are substituted and then exact values are used. The other problem encountered was, as this is a 'show that' question, candidates had to show sufficient detail to convince Examiners that their method is correct. Hence those who had $P\cos\theta + 30\cos\theta = 48\cos\alpha$ immediately followed by $P\cos\theta + 30\cos\theta = 28.8$, have not shown Examiners where the 28.8 has come from, which may have been obtained from the given answer. An intermediate step of $P\cos\theta + 30\cos\theta = 48\times\frac{3}{5}$ would have been required. This is also the case for the other equation.

(b) The first request was to verify that $P = 6$. Only writing $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2 = 1$ is insufficient. Examiners needed to see at least one intermediate step to show verification rather than just inputting $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2$ into a calculator. Most candidates then went on to find θ correctly. The main error seen here was to truncate the value to 36.8, without a more accurate version being seen.

Question 4

(a) This question was answered well by most candidates, with only a very small minority giving the resistive force as $\frac{60}{3}$ and not evaluating it as 20.

(b) Almost all candidates attempted this question by using a work-energy method as was stated in the question. Although a good number of perfect solutions were seen, many solutions did not gain full credit due to prematurely approximating, particularly for the potential energy term. Others failed to gain marks by using potential energy and kinetic energy correctly, but then including the force terms 13 and 24 in their energy equation rather than the work done by these forces. There were a few candidates who used constant acceleration methods but they were rarely worthy of credit, usually due to not including the weight component in their Newton's second law equation.

Question 5

This question was well completed by many candidates. Almost all candidates resolved forces parallel and perpendicular to the inclined plane. This gave two-term expressions for both the friction, F, and the normal reaction, R. Some errors with signs were seen, as well as mixing the sine and cosine components. Some wrongly thought that the normal reaction was either $R = 0.6g\cos 35$ or $R = 0.6g$. However, candidates scored well overall on this question.

Question 6

(a) This question was answered well by many candidates.

(b) This was well attempted by most candidates who successfully differentiated the given expression for velocity and used $t = 1$ to find the required acceleration.

(c) Most knew that instantaneous rest occurs when $v = 0$ but many struggled to solve the equation which involved fractional powers. A number of candidates incorrectly believed that instantaneous rest meant solving $a = 0$ instead. Integration of the velocity to find the displacement was very well attempted by the majority of candidates.

(d) Although some very good solutions to this problem were seen by Examiners, candidates once again struggled to solve an equation with fractional powers. A surprisingly large number of candidates did not use the context of the question and incorrectly resorted to use of constant acceleration equations for this question. Many who had solved correctly went on to give the speed as a negative value.

Question 7

(a) (i) Those candidates who wrote down the two equations for the motion of P and Q separately seemed to enjoy more success than those who chose to consider the system equation, since many who attempted this method used an incorrect mass for the system. Some good answers were seen, however many candidates made errors such as omitting either the friction term or the tension in the string.

(ii) Most candidates used constant acceleration methods for this part and even those who had found an incorrect acceleration in **part (a)(i)** were still able to score the majority of the marks available. When calculating the acceleration in the section BC, many again used a system equation with the incorrect mass. A significant number of candidates did not appreciate that section BC being smooth meant that the situation is different for the rough section AB. These candidates proceeded to use a single particle equation, with the tension in the string while moving in the section AB being used for the motion in the section BC. A minority wrongly believed that the acceleration was the same for both the rough and smooth section.

(b) Some excellent answers were seen here. Even those who did not obtain full marks in the earlier parts usually used correct methods to find two times with their values.

MATHEMATICS

Paper 9709/43
Mechanics (43)

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.
- When answering questions involving forces in equilibrium, or Newton's Second Law or an energy approach, a complete force diagram can be helpful to ensure that all relevant terms are included in the equations formed. E.g., **Question 2**, **Question 4(a),** and **Question 6(c), (d).**
- In questions with a given answer, where equations must be solved in order to find that answer, candidates are advised not to use an equation solver on their calculator since this does not explicitly 'show' the given answer. E.g., **Question 4(a)** and **Question 5(a), 5(b)**.

General comments

This paper provided the opportunity for candidates at all levels to show their knowledge of the subject, whilst providing challenge for the stronger candidates. Much work of a very high standard was seen. **Question 1**, **Question 2, Question 5(a), 5(b)** and **Question 6(a), 6(b)** were found to be the most accessible questions, whilst **Question 6(d)** and **Question 7(b)** were found to be the most challenging.

In **Question 7**, the angle was given exactly as $\sin\alpha = 0.28$. There is no need to evaluate the angle in problems such as this as any approximation of the angle can lead to a loss of accuracy in the answer.

Comments on specific questions

Question 1

This first question was a straightforward conservation of momentum problem and was well attempted by most candidates, almost all of whom found a correct equation leading to the value of $0.8\,\mathrm{m\,s^{-1}}$. A few candidates made an error in solving, usually in the form of dividing the wrong way around and ending up with an answer of $1.25\,\mathrm{m\,s^{-1}}$. Many candidates also found the speed when P rebounds, but some of these gave an answer of the velocity $-1.33\,\mathrm{m\,s^{-1}}$ rather than the speed $1.33\,\mathrm{m\,s^{-1}}$.

Question 2

This question on connected systems was again straightforward, with the acceleration, driving force, tension in the tow-bar and resistance force on the trailer given. Candidates could therefore simply apply Newton's second law to the car and then to the trailer to find the resistance force on the car F and the mass of the trailer m, with no need to use simultaneous equations. Those who took this approach almost always found the correct answers. Many candidates did not realise this and applied Newton's second law to the system, together with either the car or the trailer. Although many of these later came to the correct solutions, this approach was less successful as it was slightly more complicated, and a variety of errors were seen with this method. Some candidates had an extra term in at least one of their equations, usually including both the tension in the tow-bar and the resistance force on the trailer in the equation for the whole system.

Question 3

Many candidates found this question on resolving forces rather more challenging than the first two questions. Most candidates resolved horizontally and vertically rather than parallel and perpendicular to X, although the with the latter method there was no need to solve simultaneous equations since the value of the tension could be obtained directly. Those who used this latter method usually gained full credit. Of those who used the former method, most candidates resolved correctly in the vertical direction. However, some candidates only included one tension in their horizontal equation and many others thought that there were two different tensions, not recognising it was a single string.

Question 4

(a) This question also proved to be rather demanding despite the given answer and there was a wide variety of approaches, all involving Newton's second law. Most candidates found two equations for either $\frac{P}{v}$ or for F in terms of a and $\frac{1}{2}a$. These were usually correct, although there were often sign errors seen. Some candidates then used a calculator to solve the equations, which was not satisfactory as it was required to 'show' that the power was 200 kW. One of the most popular methods was to express P in terms of a for both equations and equate the two expressions. Once candidates had solved for a, they then had to find P. Some did not have an explicit equation for P but instead simply wrote $\frac{P}{20} - 6000 = 15000a$ or similar, followed by $P = 200\,\text{kW}$ without first showing a substitution of the value of $a = \frac{4}{15}$. Omitting this substitution meant they could not be awarded the final mark. Some candidates tried to use the given value of P to find a but most of these only had one equation, usually $\frac{200\,000}{20} - 6000 = 15000a$, and unless they had a second equation they could only gain a maximum of two marks.

(b) Candidates were slightly more successful with this part, although despite the question asking for the steady speed that the lorry could maintain, a significant number included a non-zero acceleration in their work.

Question 5

(a) In this question on calculus, this first part was almost always answered correctly.

(b) This part was also very well answered. Most candidates integrated correctly and used the correct limits. Some made extra work for themselves by finding the constant of integration rather than simply using a definite integral. A few found both areas correctly but forgot to state that one was a tenth of the other. A few used a calculator to integrate which was not a satisfactory method for a 'show' question.

(c) There were many correct responses to this part, but there were also quite a few different errors which resulted in candidates not gaining full marks. A few candidates made an error in differentiation, some differentiated twice and stated that the maximum acceleration was 0.4 ms^{-2}, since this was the value of the second differential. Some only worked out the value of 0.4 without also finding the maximum acceleration of 0.3 ms^{-2} in the first 9 seconds, and others, although they found this value, did not clearly state which was the maximum.

Question 6

(a) The first two parts of this question were straightforward problems involving constant acceleration and both parts were very well done, although some candidates thought that the constant speed section ended at $t = 25$ rather than $t = 30$. Such candidates usually still had the correct time of 10 seconds for the deceleration.

(b) This part was usually fully correct apart from those candidates who had the error mentioned in **part (a)**. These could still get a mark for the correct method. Some candidates who had made this error in **part (a)** gained full marks in **part (b)** as they used equations for constant acceleration to find the total distance travelled, rather than using their diagram from **part (a)**.

(c) In this part, candidates had to use Newton's second law to find the value of the mass. This question was found to be quite challenging, with relatively few fully correct responses. Of those who made some progress, some omitted one of the terms or included an extra term. Rather more had an incorrect sign in their equation, often getting a final answer of 0.98 kg.

(d) There were very few correct responses to this part, again involving Newton's second law. Many candidates had no idea of how to proceed. Some used the wrong value for deceleration, usually 0.2 or g, and others included the mass of the elevator. Some had an equation with a sign error, but which was otherwise correct.

Question 7

(a) This question involved using a work-energy and was found to be somewhat challenging. Most candidates correctly found the loss in potential energy and some then went on to correctly find the velocity. However, many candidates had an incorrect sign in their work-energy equation or omitted the work done by the child, so had a two-term equation only. A few candidates tried to use an acceleration method which was not correct, since the work done was not stated to be constant and the child was moving on a curve.

(b) This part, which involved using an energy method to find the coefficient of friction, was again found to be challenging. Many candidates correctly found the loss in potential energy and the normal reaction force. However, often the equation for the work done by the friction force was not correct. Many candidates either had an incorrect sign, or more often omitted the distance term in the friction force and so had an equation which was not dimensionally correct, since one the terms in the work-energy equation was just a force. Candidates usually realised that they had to use the relationship $F = \mu R$ and some correctly found the value of the coefficient of friction, but some did not notice that the answer had to be given as a fraction in its simplest form. Some candidates gave, for example, the normal reaction force as $25g\cos\alpha$, but then did not evaluate this expression. Candidates should be aware that marks are only awarded once a value is substituted for α, or when $\cos\alpha$ is evaluated (in this case as 0.96).

MATHEMATICS

Paper 9709/51
Probability & Statistics 1 (51)

Key messages

Candidates need to be aware that workings and explanations are required to support their answers. It is especially important to include all the required steps when the proof of a given result is called for, including the mathematical operations. Good solutions were characterised by clear communication, particularly when combining different results or scenarios.

Where a diagram is required, it should be clear, accurate and appropriately labelled.

Candidates should only state non-exact answers correct to 3sf; exact answers should be stated exactly. To justify a final answer correct to 3sf, working values correct to at least 4sf should be used in the calculations throughout. There is no requirement for fractions to be converted to decimals.

General comments

Most candidates used the response space effectively. Where there is more than one attempt at a question, candidates should ensure that they clearly identify which one they intend to present for marking. When extra space is required, candidates should use the additional page in the first instance.

The use of helpful diagrams, sketches and tables were frequently seen in good solutions. They often supported and efficiently organised the explanations. Many candidates were able to tackle the earlier parts of **Questions 1, 2, 3** and **4**. Frequently, the latter parts of **Questions 1, 2, 3** and **7** seemed challenging for many.

Comments on specific questions

Question 1

Many candidates found this question challenging and were unsure how to deal with the summary statistics given in coded form. There were a significant number of scripts with no response to this question.

(a) Good solutions often found the mean of $x-q$ as 14 first and subtracted its square from $\frac{14235}{50}$ before square rooting to find the standard deviation. The correct answer of 9.42 often followed. A few answers were given only to 2sf. Weaker solutions simply found the square root of $\frac{14235}{50}$ or tried to expand the brackets, whilst others squared the 700 but not the 50.

(b) Successful candidates understood that $\sum q = 50q$ and formed the correct equation with $\sum x$ and $\sum(x-q)$, whilst others found the mean of x and subtracted the mean of $(x-q)$ from it. Many did not attempt this part or simply found $\frac{2865}{50}$ or 2165, not understanding how to use the coding.

Question 2

Many candidates were able to tackle **part (a),** but many found the restrictions in **part (b)** more difficult.

(a) Most candidates identified correctly that combinations were required to find the number of ways that the committee was chosen. The correct combinations of men, ${}^{6}C_{3}$, and women, ${}^{8}C_{3}$, were frequently seen. Many successfully found their product and gave the right answer. A significant number of candidates found their sum.

(b) Stronger candidates often gave an indication that the number of brothers required were 0, 1 or 2 and found the number of ways of selecting the remaining candidates from 11 people in each case. This situation, which requires the multiplication of combinations and then the addition of scenarios, is quite common. A few who adopted this method selected the remaining committee members from 8. Some omitted one of the scenarios (most often 0 brothers) when adding to get the required answer. A few successfully listed all the 12 different scenarios of men and women for each number of brothers and found the associated combinations. Many of those who attempted this method did not get all the scenarios and would have been well advised to list their options in a logical manner. Some used the efficient method of subtracting the number of ways that all 3 brothers could be selected from the total number of ways the committee could be selected. Most of these found the total number of ways of selecting the committee, ${}^{14}C_{6}$, but were unsure what to subtract for 3 brothers.

Question 3

There were a lot of good solutions to **parts (a)** and **(b)** which demonstrated understanding of arrangements with repeated Os and Cs. Many candidates found **part (c)** less accessible and were unable to deal with the conditional probability.

(a) The number of arrangements of 8 letters where 3 are Os and 2 are Cs is a fairly standard application involving factorials and was completed successfully by many. Some solutions indicated that candidates thought that the Os and Cs were distinguishable and just gave $8!$.

(b) Most candidates realised that they had only 6 letters to arrange. Stronger solutions were often accompanied by a simple diagram which showed the number of spaces to be filled and the letters that were left to fill them. Some omitted the denominator of $2! \times 2!$ to deal with the repetition of the remaining Os and Cs.

(c) A significant number of candidates found this part more demanding. Good candidates realised that this was an application of conditional probability where there were 5 items to arrange once the Os and Cs had been grouped together and 7 items to arrange (3 of which were Os) where the 2 Cs were next to each other. Weaker solutions simply found the number of arrangements with 3 Os together and 2 Cs together and divided the result by 3360, their answer to **part (a)**.

Question 4

The vast majority of candidates were able to use the Normal standardisation formula correctly. Most candidates correctly identified that, as time is a continuous variable, no continuity correction was necessary.

(a) The best solutions were often accompanied by a simple diagram to inform the method. In a few instances a continuity correction was applied, or the variance was used. Candidates must appreciate the need to use the full value obtained from the Normal distribution table to obtain an accurate probability. Some stopped after finding the probability; candidates would be well advised to ensure that they read through each question carefully to ensure that they fulfil all its requirements. Some better responses showed calculations of the number of students by multiplying by 250 and appreciated the need to give their answer as an integer without extra solutions or any reference to rounding.

(b) The finding of μ and σ from given information is a familiar application of the Normal distribution. Strong candidates often provided a supporting diagram identifying the probabilities given in the question. They went on to give the correct *z*-values corresponding to those probabilities in 2 standardisation formulae. Information in their diagram allowed them to see that both *z*-values should be negative. Most candidates who obtained the formulae were able to solve their system of equations to provide answers for both μ and σ. Some candidates prematurely rounded their *z* values leading to inaccurate solutions. Weaker solutions used the given probabilities as *z* values in the formulae.

Question 5

Candidates would be well advised to consider the nature of the data groups provided in the question with relation to the boundaries for each class before drawing their diagram. A large proportion did not fulfil the requirements of the question and drew a bar chart using the original data.

(a) Good solutions stated the frequency densities (using the correct boundaries for each class) before drawing the graph. This assisted the correct selection of a suitable scale. Scales must be selected to permit all the data to be represented, the population scale ending at 4800 was often seen. The data allowed a simple scale of 2 cm representing 1000 on the population axis and 2 cm representing 0.01 on the frequency density axis. Many candidates used the correct class intervals, but a significant number were inaccurate when marking them along the axis. Those who calculated the frequency densities correctly were often unable to plot 0.0625 in the correct place. The careful use of a ruler is essential to ensure that lines drawn are on the grid lines when necessary and along part squares where necessary. Graphs should always be drawn with a sharp pencil to make this clear. Where the bars are coloured in, it is not always possible to check if the class boundaries are correct. If an error occurs, careful erasing should ensure that the necessary correction can be made. Many candidates left gaps between the bars, not realising the significance of the words ‘to the nearest 100’ or drew a bar chart with the original data. The frequency density axis was often labelled correctly, but in many cases candidates did not reference the data table to select the label of ‘village population’ for the horizontal axis.

(b) Many candidates were able to select the correct interval for the median. A few weaker solutions gave 1300 – 2000, which was the middle interval, or gave 75, which was half of the number of villages.

(c) Good solutions identified the classes in which the upper and lower quartiles lay and a few of these appreciated that they needed to subtract the lowest value for the lower quartile from the highest value for the upper quartile. The calculation 3200 – 1300 was very often seen, not allowing for the correct boundaries in which the quartiles lay. Weaker solutions used the values needed to find the quartiles and calculated 112.5 – 37.5.

Question 6

Good solutions were not often seen in this question. Those who multiplied the correct probabilities for the number of 2s often neglected the number of ways this could be achieved.

(a) Good solutions were often accompanied by a list showing all the possible outcomes which ensured that the product of the probabilities was multiplied by 4 or ${}^4C_3$. Some good candidates did not appreciate the rigour required in a ‘show that’ question and needed to include the multiplication of probabilities to justify their answer. Weaker solutions, where the score on only 3 of the dice was considered to manufacture the given answer, were often seen.

(b) Stronger candidates were able to apply the method suggested in **part (a)** to find both missing values in the table. Many were able to score a mark by realising that the two missing values should sum to $\frac{81}{128}$, as the sum of the probabilities must be 1. In a few cases the probabilities found were not placed in the table; candidates would be well advised to read the requirements of the question carefully.

(c) Many candidates were able to use the correct method for calculating the expectation from their probability distribution table. Candidates are reminded to show where their values come from in order to score the method marks, even if they are using incorrect values from **part (b)**.

(d) The strongest candidates used the appropriate probability of scoring at least two 2s of $\frac{67}{256}$ to calculate the mean and variance of the approximating Normal distribution. Some candidates used the probability of scoring 2; reading the question carefully is always important. The best solutions included a helpful diagram and an appreciation that a continuity correction was necessary since the original data was discrete. Candidates are reminded of the need to work accurately, as premature rounding denied some the final mark. Fully correct solutions were rarely seen with a significant number of candidates not attempting this part or attempting to use the Binomial distribution in some form.

Question 7

Many candidates were uncertain about using the Geometric distribution and probability theory in this question, but many were able to use the Binomial distribution in **part (b)**.

(a) The approach most often seen was the addition of the probabilities of obtaining the elephant for the first time on the 1st, 2nd, 3rd, 4th or 5th day. Those who used the Geometric distribution often produced correct and efficient solutions. Occasionally, $1 - 0.8^6$ was seen. Many candidates found this part challenging, with some attempting to use the Binomial distribution. Solutions only finding the probability of getting an elephant for the first time on the 5th day were often seen.

(b) This part was completed successfully by a good number of candidates. The efficient approach of summing the probabilities of 0, 1 and 2 leopards and then subtracting from 1 was used by the majority of candidates. Those who attempted to sum the probabilities of 3 – 12 leopards were rarely successful due to errors in accuracy or omitting one or more of the outcomes. In a few instances the binomial coefficients were incorrect or missing. Candidates would be well advised of the need to work to at least 4sf to justify an answer to 3sf, as highlighted in the Key messages. Some gave a final answer to 2sf only.

(c) Correct solutions to this part were rarely seen. Many candidates realised that to get one of each animal 0.2^5 had to be calculated, but few understood that the arrangements of the animals had to be considered as they were all different. The product of 0.2^5 and $5!$ was rarely seen. Candidates who considered the number of ways of selecting the 1st, 2nd, 3rd, 4th and 5th animals, then dividing by the total number of ways of arranging the animals, were more likely to include the $5!$. This approach was not often used.

MATHEMATICS

Paper 9709/52
Probability & Statistics 1 (52)

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially if there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively and is an essential tool when showing given statements are true. When errors are corrected, candidates would be well advised to cross through and replace the term. It is extremely difficult to accurately interpret terms that are overwritten.

Candidates should state only non-exact answers to 3 significant figures, exact answers should be stated exactly. In particular there should be a clear understanding of how significant figures work for decimal values less than 1. It is important that candidates realise the need to work to at least 4 significant figures throughout to justify a 3 significant figures value. Many candidates rounded prematurely in normal approximation questions which produced inaccurate values from the tables and lost accuracy in their solutions. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. Candidates should be aware that a ruler should be used to construct box-and-whisker plots and that the scale should enable to accurate plotting of the five key-values to be achieved.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. The vast majority of candidates were well prepared, however candidates found it more challenging when more than one technique was required within a solution. Many good solutions were seen for **Questions 1** and **3**. The context in **Questions 2, 5** and **6** was found to be challenging for many.

Comments on specific questions

Question 1

Many good solutions to this question were seen.

(a) A probability distribution table was present in nearly all solutions. A large number of candidates formed an equation for the probabilities and calculated k before entering values into the table. Better answers did not find a numerical probability at this stage, but left the probabilities in terms of k. Weaker solutions included probabilities in the table which did not sum to 1, or were negative. Even though the value for the random variable X were stated in the question, some additional outcomes with probabilities were seen.

(b) Most candidates who had formed a probability distribution table in **part (a)** made good progress here. Many solutions included the full, unsimplified calculations for both $\mathrm{E}(X)$ and $\mathrm{Var}(X)$, which is good practice. The efficient use of a calculator allows these to be evaluated without further simplification while clearly communicating the method. A small number of solutions failed to use $(\mathrm{E}(X))^2$ in the variance calculation. As the question required two values to be found, candidates should be aware of the expectation that appropriate identification is provided.

Candidates who had not found a value for k in **part (a)** often solved here, but would not gain any credit, and complete the question accurately.

Question 2

A tree diagram was present in many solutions. This clarified the probability information significantly and, with effective labelling, enabled accurate interpretation of the success criteria throughout the question.

(a) The appropriate probability calculation was seen in most solutions. The most common error was not recognising that an exact value had been calculated and providing only a three significant figure answer. Candidates should be aware that the instruction on the front of the paper is that 'non-exact' answers are rounded, so exact values should be stated.

(b) Solutions which included a tree diagram were often successful. Most candidates recognised that they needed to consider only Sunday, Monday and Tuesday in determining the required probability. Those who included Wednesday often used 0.8 and 0.2 as the probabilities, rather than 0.7 and 0.3, since it had rained on Tuesday. A small number of solutions ignored the context and simply stated a probability of 0.2 that it rained after a day without rain.

(c) A significant number of candidates made little or no attempt at this part. The most successful solutions clearly identified the possible scenarios which fulfilled the criteria and then calculated the probability of each. Many candidates simply stated calculations and did not clearly communicate the logic that was being used. There were an unexpectedly high number of arithmetical errors in the evaluation of the expressions stated. As the probability was an exact value, candidates should not round to 3 significant figures.

Question 3

Almost all candidates were able to interpret the back-to-back stem-and-leaf accurately.

(a) The value for the median was found accurately by most candidates. As more than one item was demanded by the question, candidates should be aware of the need to identify each answer appropriately. The calculation of the interquartile range was less consistent, especially in determining the value of the upper quartile. Most candidates used the anticipated method of determining the middle value between the maximum or minimum value and the median to find the quartiles. There is an expectation that a calculation will be present or clearly implied for the interquartile range. A small number of candidates did not use the information from the stem-and-leaf key to scale their answers.

(b) The majority of candidates used an appropriate scale for their box-and-whisker plots. This enabled the key values to be plotted accurately, with the majority of values on grid lines. Many plots were drawn without a ruler, which is not appropriate at this level as an accurate representation of the data is expected. A small number of candidates failed to label the separate companies, so their comparison was unclear. Candidates should be aware that a linear scale is required and that it should be labelled with both the variable and units (e.g. salary in \$). Several candidates presented solutions with the plots of company *A* and *B* combined, which gained no credit.

(c) Many general, theoretical comments were noted. Candidates should be aware that comments need to be within the context of the question, and specific to the data presented. Reference to the n extreme value of \$3090 in company *B* was expected, or to be clearly implied in the comment. There appeared to be some confusion about whether the inclusion of the extreme value in the central tendency was appropriate or not.

Question 4

Many candidates found this probability question challenging. **Part (d)** was omitted by a significant number of candidates but was often completed successfully by those who attempted it, since much of the process was fairly standard.

(a) Almost all candidates recognised the geometric approximation was appropriate for the context, and stated the anticipated calculation, which was evaluated accurately. As this produced an exact answer, no rounding should have been undertaken for the final answer.

(b) Many good solutions were noted for this part. These often used the less efficient process of adding the probabilities of obtaining a 2 in each of the possible acceptable scenarios. The more efficient process using $1 - 0.8^5$ was used effectively from more confident candidates. As has been highlighted in previous reports, misinterpreting the success criteria is a very common error, and many solutions included the 6th spin as well.

(c) This conditional probability was found challenging by many candidates. A surprisingly high number of candidates omitted this part entirely. The most successful solutions used an outcome table, which identified all the possible scores that could be obtained. The conditional probability could then be stated directly by identifying the appropriate values. However, candidates were usually more successful when they used the outcome table to support the probabilities to substitute into the appropriate conditional probability fraction. Weaker solutions often used a listing approach, with outcomes omitted, or tried to use logical reasoning to find the required probabilities.

(d) This part was also omitted by a significant number of candidates. Good solutions were noted frequently when attempted. Many candidates found the context challenging, although when an outcome table had been used in **part (c)**, the required probability could be identified. The mathematical process was a fairly standard binomial approximation context. As in **part (b)**, misinterpreting the success criteria was a common error, with three occasions being included. Poor arithmetical accuracy was seen, not always linked with premature approximation as can be expected. Candidates should be aware that the efficient use of the calculator should enable them to evaluate the entire unsimplified expression without any intermediate values being stated, which can avoid both premature approximation and rekeying errors.

Question 5

This was a relatively standard normal approximation question. The number of candidates who did not attempt **part (c)** was higher than anticipated.

(a) Almost all candidates used the normal standardisation formula correctly at least once. The best solutions often had a sketch of the normal distribution curve to help identify the required probability area. Very few candidates used a continuity correction, which was not required as the data is continuous. A common misinterpretation was that the required probability area was symmetrical and, although the correct standardisation formulas were stated, the second evaluation was assumed and not calculated.

Many solutions finished when the probability area had been calculated, with the final process required to find the expected number of birds that fulfilled the criteria not attempted. Candidates are well advised to read the question again once they reach their final answer to check that it is both reasonable and does answer the question. Because of the context, the final answer needed to be an integer value and there should be no indication of rounding as the value chosen is a decision from the candidate. Candidates should also be aware that evidence of a probability to at least four significant figures is required to justify their decision.

(b) A fairly standard normal approximation technique was required for this question. Good solutions often used a sketch of the normal distribution curve to identify the magnitude of the anticipated z-value. Candidates who formed an appropriate equation using the normal standardisation formula were frequently successful in determining the value of σ. Many candidates found a probability value rather than a z-value from the information, so were unable to form an appropriate equation.

Candidates should be aware that their supporting work needs to be consistent throughout the solution. A number of candidates realised that the standard deviation could not be negative so stated a positive value for their final answer, but did not amend their initial error of using the incorrect z-value.

(c) Again, even though this question was not attempted by many candidates, several good attempts at this standard normal approximation question were seen. The best solutions had calculations of the mean and variance initially and then a substitution of these values in the normal approximation formula. Most candidates recognised that the data was discrete and so required a continuity correction, although a few used the upper rather than the lower bound. The most common error was to use the incorrect probability area, which may have been avoided if a simple sketch of the normal curve was used to clarify the success criteria.

Question 6

This permutations and combinations question was found challenging by many, but candidates who listed logically possible scenarios often achieved good solutions. Candidates should be aware that how the question is presented provides some additional guidance for their solution. As additional information was presented before **part (b)**, this is additional context for the remainder of the question and may be required not only in **part (b)** but also in **part (c)**.

(a) Candidates who listed the three possible scenarios that fulfilled the criteria were frequently successful. The best solutions included the unsimplified calculations involving the combinations linked with each scenario. A misconception was that as the number of swimmers was fixed at 1, which would have no effect on the ways the teams could be selected, so ${}^6C_1$ was omitted from each product. Weaker solutions summed the combinations to find the number of outcomes for each scenario.

(b) Candidates who used a simple diagram to clarify the requirements given in the question were often able to identify that not only could the team be arranged within their groups, but that the groups themselves could be arranged in different orders. Common errors were either not to arrange the groups, with an answer of 288, or not to arrange the people within the groups, with an answer of 6. A number of candidates assumed that they needed to select the identified people from the original group, and so increased the complexity of the question. As the additional information was given before the question was started, this is an indication that the previous conditions would no longer be applied.

(c) Many candidates found this part challenging, and the question was not attempted by a surprising number. Two main approaches were used to answer the question. The most successful was to consider how the people who were not cyclists were arranged and then how the cyclists could be placed. Good solutions often used a simple diagram to clarify the context. A common less successful method included deducting the number of arrangements where the cyclists were standing together from the total number of arrangements for the team. Most candidates who used this approach only attempted to deduct the cases where all three cyclists were together, and failed to consider that just two cyclists could be together with the final cyclist being placed elsewhere. Again, more successful solutions used simple diagrams to clarify the criteria and more explanation was included in the work to inform what was being considered.

MATHEMATICS

Paper 9709/53
Probability & Statistics 1 (53)

Key messages

More marks were awarded this session due to a larger number of candidates showing their method. However, a minority still need to be reminded that they must show their working and justify their answers. In **Question 2** we needed to see the standardisation formula with the correct values substituted, in **Question 3** we needed to see how they derived their values for a and k, in **Question 4a** we needed to see the subtraction of the lower quartile from the upper quartile and in **Question 6b** we needed to see the binomial terms in full.

General comments

A surprising number of candidates struggled with **Question 1** and did not seem comfortable with the geometric distribution. Conversely, **Question 7**, which required the use of Permutations and Combinations and is a topic which normally causes problems, was confidently dealt with by most candidates. A significant number of candidates struggled with the algebra in **Question 3**, especially expanding the brackets.

Comments on specific questions

Question 1

(a) This proved to be a challenging starter question for the many candidates who did not recognise a geometric distribution and remember that $\mathrm{E}(X) = \frac{1}{p}$. Some formed a probability distribution table and incorrectly used the formula $\mathrm{E}(X) = \sum x \cdot p(x)$, not appreciating the significance of the word 'State', i.e., that no working should be needed. Many worked with $\frac{1}{2}$ as the probability while the most common error was to give $\frac{1}{4}$ as the final answer.

(b) This question was answered more confidently, although a significant number of candidates confused three decimal places with three significant figures and dropped the last digit in their final answer. Those who worked with $\frac{1}{2}$ as the probability in **part (a)** often continued to do so in this part as well.

(c) Strong candidates knew to raise the probability of not getting a pair of heads to the power of 6 and subtract the result from 1. Others used the longer method of summing the probabilities of getting a pair of heads in 1, 2, 3, 4, 5 or 6 throws. Both methods were equally valid, but the second method was more prone to arithmetic errors. As in the previous parts of the question, a significant number worked with a probability of $\frac{1}{2}$ instead of $\frac{1}{4}$.

Question 2

This question was answered well with most candidates recognising what was required from a normal approximation to the binomial. However, the words 'between 36 and 54 inclusive' seem to have been disregarded or misunderstood by a significant number of candidates, with many of those who did remember the continuity correction using it in the wrong direction. Almost all candidates found the correct mean and variance and used them correctly in at least one standardisation expression. Evidence of standardisation was required here and candidates who used their calculators and went straight to the z-values were penalised. Most found the correct area between their two z-values, often using a sketch to help.

Question 3

(a) The question prompted candidates to form their first equation from the given information, $P(X = 4) = 3P(X = 2)$. Most correctly stated that $4k(4 + a) = 3 \times 2k(2 + a)$ and stronger candidates quickly cancelled the k on both sides of the equation and worked out that $a = 2$. Others expanded the brackets and simplified until reaching the equation $4k = 2ak$. Most then cancelled the k and arrived at the correct value for 'a', but in several cases they stalled at this late stage and never found the value of 'a'. A disappointing number made careless algebraic mistakes while expanding the brackets.

Many candidates never formed a second equation by summing the probabilities to 1. Some substituted $a = 2$ back into their original equation, giving $24k = 24k$, and decided that $k = 1$. Those who did remember the more usual way of dealing with a probability function generally obtained the correct value for 'k'.

(b) This was well answered with almost all knowing how to form a probability distribution. Despite the instruction to give the probabilities as 'numerical fractions', a significant number gave these algebraically.

(c) Most candidates were familiar with the formula for calculating the variance from a probability distribution and knew to show the sum of their squared x-values multiplied by their probability before they subtracted 3.2^2. Some ignored the fact that they were given the value of $E(X)$ and calculated it for themselves. Only a few still did not realise that they need to show their numerical substitution into the variance formula if they are to be awarded the marks.

Question 4

(a) Finding the median and interquartile range from a back-to-back stem-and-leaf diagram was a familiar task for most and accurately answered. A few were confused by the backward left hand Cheetahs figures and gave the upper quartile or 15th item of data as 101 rather than 106.

(b) When asked to make comparisons between sets of data, candidates should be encouraged to write answers in context and with an appropriate interpretation. This question challenged even the strongest candidates, with many simply comparing numerical values. When comparing the medians, we needed to read that generally the Cheetahs were faster, or completed the race in a shorter time, or that generally the Panthers were slower or took longer to complete the race. When comparing the interquartile ranges, we needed to read that the Panthers' times were more consistent or less spread out, or that the Cheetahs' times were more varied or more spread out.

Any comparisons of particular values, e.g., medians, averages, ranges or interquartile ranges, were not awarded marks.

(c) This question was answered well by most candidates. Strong candidates found the total time of the 20 including Kenny by multiplying 99 by 20 and then subtracted the total of the 19 that feature in the stem-and-leaf diagram. Some chose to find the average of the 19 which was 98 and then added 20 × 1, where 1 is the difference between the average of 19 and the average of 20. Apart from some who made arithmetic errors there were very few incorrect responses.

Question 5

(a) The most successful approach to finding the probabilities of A and B seemed to be using two grids showing the different outcomes. Without this, even the initial step in finding the total number of outcomes caused problems. Notation was generally very good although use of the intersection caused the most problems, with alternatives seen including: $P(A \text{ and } B)$, $P(A + B)$, or simply $P(AB)$.

The vast majority chose to use the independence condition $P(A \cap B) = P(A) \times P(B)$. Even weaker responses showed an attempt to multiply the two probabilities, demonstrating an awareness that this was linked with independent events.

(b) Strong candidates who had correctly evaluated the required probabilities in the previous part knew to divide $P(A \cap B')$ by $P(A')$ and usually obtained the correct final answer. Most explained that they had subtracted $P(A)$ from 1 to find $P(A')$ and some even explained that they had subtracted $P(A \cap B)$ from $P(B)$ to find $P(B \cap A')$. Weaker candidates often knew to divide by $P(A')$ but were less successful with the numerator, often assuming independence and multiplying $P(B)$ by $P(A')$ or just using $P(B)$.

The few who used the second method in the mark scheme and worked directly from the outcome tables were generally successful.

Question 6

(a) It was imperative that candidates used the critical value of 1.282 for this part; use of 1.2815, 1.281 or 1.28 was not accepted. Most candidates knew to equate a standardised expression using 16, 28 and sigma to a *z*-value with only a small number using the tables backwards or equating to 0.1 or 0.9.

(b) The majority recognised this as a binomial question and only a few incorrectly tried to work with the normal distribution. Understanding the words 'more than 2' was an issue for some, with a number omitting the probability of '2 days' from their calculation. A few found the probability of 0, 1 or 2 days and then did not go on to subtract from 1 and a few others only gave the answer to two significant figures.

It was pleasing to see that most candidates realised we need to see the method with all the binomial terms.

(c) This proved to be a challenging question and only the candidates with a firm grasp of the normal distribution were confident enough to jump directly to the correct *z*-values of +/– 1.3. Many others obtained the correct *z*-values after standardising using 28 and 9.36 but, unless they realised that the expressions easily simplified to +/–1.3, they often obtained inaccurate results. A common misconception was to treat +/– 1.3/9.36 as the required *z*-values.

Most candidates did obtain a probability, but many forgot to do the final part and answer the question about how many days they would expect the mass of grapes to be within 1.3 standard deviations of the mean. We insisted on an integer value for the final answer and that they multiplied 365 by a 4-figure probability.

Question 7

(a) The two methods in the mark scheme were seen and both were generally performed well.

In Method 1, they calculated the total number of ways of arranging the 10 letters, i.e. $\frac{10!}{2!4!}$, and subtracted the number of ways the letters could be arranged with the two Cs together, i.e. $\frac{9!}{4!}$.

In Method 2, they calculated the number of ways the 8 letters apart from the Cs could be arranged, i.e. $\frac{8!}{4!}$, and then multiplied by the number of ways the Cs could be inserted into the arrangement without being together ($^9C_2$).

(b) This part of the question was more challenging. Strong candidates quickly realised that there were $\frac{6!}{2!}$ ways of arranging the 6 remaining letters, after disregarding the As at the beginning and the end and the Cs. They then multiplied by 4 as there are four ways of positioning the block of 5 letters containing the two Cs and three other letters.

A slightly more complicated way was to see that there are $\frac{^6P_3}{2!}$ ways of arranging the 3 letters between the two Cs, $3!$ ways of arranging the other three letters and four ways of positioning the block of five letters. Candidates who thought about the problem in this way were more prone to error.

A significant number of candidates unnecessarily complicated the problem by thinking that they had to consider how many of the remaining two As were between the Cs. They made three calculations, considering 0, 1 or 2 As between the Cs and then added them. An impressive number of candidates used this complicated approach and arrived at the correct final answer, but many others went wrong.

Only a few made a start at the highly inefficient method of finding the number of ways with an A at the beginning and the end, namely $\frac{8!}{2!2!}$, and then subtracting the number of ways without three letters between the Cs. This would involve considering the six scenarios of 0, 1, 2, 4, 5 or 6 letters between the Cs and most gave up very quickly.

(c) This question was very well answered by many candidates. Very few used Method 1 where they only considered the number of As. With this method, they can disregard one of the Cs and select from 5 letters (C, S, B, L, N) to go with each of 2, 3 or 4 As. They then total the number of ways for the 3 scenarios.

With Method 2, most candidates successfully identified the six scenarios with 2, 3 or 4 As and 0 or 1 C and only a few omitted to show that essential stage of their working. Most appreciated that once they had determined how many Cs and As were in a scenario that only left four other letters (S, B, L, N) from which the remaining letters should be selected. Having found the number of ways for each of the six scenarios they then totalled them.

The most common error was to think that each number of As or Cs can be selected in multiple ways. For example: to find the number of ways of AAAC which should be $^4C_1$, i.e. the number of ways of selecting the remaining letter, they multiplied $^4C_3$ by $^2C_1$ by $^4C_1$.

MATHEMATICS

Paper 9709/61
Probability & Statistics 2 (61)

Key messages

- In all questions, sufficient method must be shown to justify answers.
- It is important that candidates read the question carefully and refer back to it when they have completed the question to ensure they have answered it in full.
- Candidates are strongly advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.
- For answers that are required 'in context', quoting general textbook statements will not be sufficient.
- All working should be done in the correct question space of the answer booklet. If answers need to be continued on the Additional page, it must be clearly labelled with the correct question number.
- Candidates should make corrections by crossing through and replacing the work, not by over-writing their answer.
- Only one solution should be offered.

General comments

Candidates did not always seem fully prepared for the demands of this paper. Questions where candidates performed well were **Questions 1, 3, 7(a)** and **7(e),** and questions which candidates found more demanding were **2(a)**, **2(b)**, **4(b)**, **4(c)**, and **7(d)**. There were a few places on the paper where it appeared that candidates had not read the question carefully, namely **Question 3** and **Question 7(c)**.

Candidates must note that the conclusion to a hypothesis test must be written in context and with a level of uncertainty in the language used.

Comments on specific questions follow which identify common errors, though it should be noted that there were many good and fully correct solutions seen as well.

Comments on specific questions

Question 1

(a) In general, this part was well attempted. The approximating distribution required was a Poisson distribution $\mathrm{Po}(3.4163)$. Many candidates calculated the correct proportion of adults but did not realise that a Poisson distribution was required. Some candidates used a binomial distribution or attempted a normal distribution, and of those who did attempt a Poisson distribution, accuracy was often lost (caused by premature rounding of 3.4163). It should be noted that when calculating a probability such as this sufficient method must be shown, so the Poisson expression needs to be clearly written in full.

(b) To justify an approximation, it is important that the context of the question is used. As highlighted in the Key messages, merely quoting from a textbook ($n > 50$, $np < 5$) is not sufficient. The values of n and np in the given situation need to be clearly stated to demonstrate that they do fulfil the requirements.

Question 2

(a) Some candidates successfully calculated the area under $\mathrm{f}(x)$ and showed it was equal to 1, as required for a probability density function. However, very few candidates stated $\mathrm{f}(x) \geqslant 0$, which is also a requirement.

(b) Many candidates omitted this question, and of those that did make an attempt, many chose to use a set method by integration rather than to use the formula for the area of a semi-circle and equate it to 1. It is important that candidates have an understanding of the methods they are using, and that sometimes use of integration is not always the best method to find an area. In general, there were few good attempts seen for this part.

(c) (i) A few candidates were able to give an acceptable explanation here. Candidates who discussed the skewed nature of the curve were not always successful in their reasoning. Mentioning area to the left and right of 15 was generally a more successful approach.

(ii) Integration attempts on this part were not always successful. Some candidates integrated $\mathrm{f}(x)$ rather than $x\mathrm{f}(x)$ and some confused the mean and median.

Question 3

Many candidates made a reasonable attempt at this question. There were some good solutions but equally some that did not carry out all the required steps fully. Some candidates omitted or gave incorrect hypotheses. Comparisons were not always valid or clearly stated, and very few conclusions were written with the required context and with a level of uncertainty in the language used. As highlighted in the Key messages, many candidates did not state a necessary assumption, highlighting the importance of reading the question carefully and checking back when finished to ensure that the question has been fully answered.

Question 4

(a) Candidates who successfully set up the correct equation were usually successful in finding n. Errors setting up the equation included using an incorrect z value, and more commonly, to omit the factor of 2.

(b) This was well attempted by only a few candidates. Many candidates stated only the first 3 months of the year had been chosen, but this was not sufficient to explain why this made the sample unsuitable. Further comments that this sample was not typical of the whole year, possibly including comments about weather or similar, needed to be made. Some candidates made comments about the sample size which were not accepted.

(c) Only a few candidates realised what was needed here, with a large number of candidates giving no response at all. Some candidates stated part of the expression but there were very few fully correct answers.

Question 5

(a) Many candidates were unsure of how to calculate the variance. Of those that did, a common error was to use 20^2 rather than 20.

(b) Some candidates successfully found the correct mean and variance and standardised to find the correct probability area. Errors were seen in both finding the variance and the correct probability area (> 0.5 rather than < 0.5), but in general reasonable attempts were made.

Question 6

This question received a wide range of responses, from totally correct to little or no response. Setting up the initial equation which required the correct formula for the unbiased estimate of the variance was reasonably well attempted, but many errors were made in solving this equation. A common algebraic error was expanding $(10 + a)^2$ incorrectly as $100 + a^2$.

Question 7

(a) There were many correct responses, but some candidates incorrectly used p or $\bar{x}$ rather than λ (or μ). On occasions this was omitted completely, with candidates stating $H_0 = 1.9$ and $H_1 < 1.9$ or $H_0 = 7.6$ and $H_1 < 7.6$.

(b) Candidates were required to evaluate $\mathrm{P}(X \leqslant 2)$ and $\mathrm{P}(X \leqslant 3)$ in order to find that the critical region was $X \leqslant 2$. Some candidates did not know how to approach the question, others used an incorrect value for λ (often 1.9) or merely found individual point probabilities. Some candidates correctly identified the critical region but gave their final answer for the region as a probability or thought the region was 2 rather than $\leqslant 2$. Candidates, having correctly evaluated the probabilities, were usually successful in finding the probability of a type 1 error.

(c) Again, it is important here that candidates do not quote textbook definitions but answer in the context of the question. It is also important that the two parts to the explanation are clear, i.e., what is concluded and what is the actuality. There were some good answers but many candidates did not answer in context.

(d) The question asked for a reason for the conclusion that the manager made as well as the conclusion itself. Many candidates merely gave a conclusion not realising that a comparison was required, i.e. either to compare 3 with the critical region or to compare $\mathrm{P}(X \leqslant 3)$ with 0.05. The conclusion needed to be in context and using non-definite language. In general, this part was well attempted by only a few candidates.

(e) A normal approximation was required here. Many candidates realised this and made a good attempt at the question. Errors included an incorrect, or omission of, a continuity correction and an incorrect probability area.

MATHEMATICS

Paper 9709/62
Probability & Statistics 2 (62)

Key messages

- In all questions, sufficient method must be shown to justify answers.
- Candidates need to work to the required level of accuracy; it is important that accuracy is not lost due to rounding answers to three decimal places rather than three significant figures, or to round too early in the question. If the final answer needs to be given to three significant figures, then all previous numerical answers need to be to at least four significant figures.
- For answers that are required 'in context', quoting general textbook statements will not be sufficient.
- All working should be done in the correct question space of the answer booklet. If answers need to be continued on the Additional page, they must be clearly labelled with the correct question number.
- Candidates should make corrections by crossing through and replacing the work, not by over-writing their answer.

General comments

This was a reasonably well attempted paper, with candidates able to demonstrate their knowledge and application of statistical techniques. Questions that were well attempted were **Questions 2(c), 3(a), 4(c)** and **4(d),** whilst **Question 7(b)** and **Question 5(b)(ii)** were not as well attempted.

Candidates generally presented their answers well and with sufficient working. It is important that sufficient method is shown; for example, if a probability is being calculated using a Poisson distribution, then the term or terms in the Poisson expression must be seen. Candidates must note that conclusion to a hypothesis test must be written in context and with a level of uncertainty in the language used.

Comments on specific questions follow which identify common errors, though it should be noted that there were also many good and fully correct solutions seen as well.

Comments on specific questions

Question 1

This was a reasonably well attempted question. Incorrect values for z were occasionally seen and a common error was to centre the interval around 46 rather than $\frac{46}{200}$. The final answer should have been written as an interval and not as two separate values. Generally, answers were given to the required accuracy level of three significant figures.

Question 2

(a) Many candidates realised that if the random variable W has a Poisson distribution, then $\mathrm{E}(W) = \mathrm{Var}(W)$. However, some candidates did not begin to attempt the question or gave an incorrect relationship.

(b) This part was not as successfully answered as **part (a)**. Some candidates quoted correct formulae for $\mathrm{E}(X)$ and $\mathrm{Var}(X)$, but very few were able to use these to correctly deduce that $(1 - p)$ needed to be close to 1 (so that np and $np(1 - p)$ were approximately equal) and thus p needed to be close to 0. The majority of candidates quoted $np < 5$, n large and p small, or similar conditions rather than using the formulae as requested in the question.

(c) This was a well attempted question. The most common error was to omit the $\mathrm{P}(Y = 2)$ term and occasional calculation errors were seen.

Question 3

(a) This question was well attempted with many candidates able to correctly calculate the values required. As has been the case in the past, there was some confusion between the two formulae for the unbiased estimate for the variance, but it was pleasing to note that very few candidates found the biased estimate for the variance.

(b) Some candidates were able to carry out the hypothesis test fully and to give a conclusion which was both in context and using non-definite language. However, many candidates omitted or gave incorrect hypotheses, the comparison statement was sometimes invalid or omitted and the conclusion was not always in context using appropriate non-definite language. Most candidates attempted a two-tailed test, however area comparisons with 0.01, rather than 0.005 after a two-tail test had been correctly declared, were occasionally seen.

Question 4

(a) Some candidates merely stated information that was given in the question (i.e., that the books were received at a constant rate). It was important that the answer was in context, so just stating 'independently' or 'singly' or 'randomly', or 'events should occur independently/singly/randomly', was not sufficient.

(b) This was generally well attempted, though a common error was to give the answer to two significant figures only; 0.044 was often seen as the final answer. This is potentially the result of a confusion between three significant figures and three decimal places. It is important that candidates know how to round to three significant figures, as if only 0.044 is seen here (with no pre-rounded figures) then marks would not be gained.

(c) This part was well attempted. Most candidates used the normal approximation correctly. The most common errors included an incorrect or missing continuity correction, or a probability area that was greater than 0.5 rather than less than 0.5.

(d) This part was also well attempted. Common errors here were using an incorrect value for λ (often 2.5 rather than 5.1 + 2.5) and omission of the $\mathrm{P}(3)$ term or omission of '1–….' in the Poisson expression.

Question 5

(a) The majority of candidates scored part marks on this question with very few achieving full marks. Many candidates used $\mathrm{E}(X - Y) = 1$ and $\mathrm{Var}(X - Y) = 5$ or equivalent (though some subtracted the variances using 3 – 2 rather than 3 + 2). Many also successfully calculated the probability of $(X - Y) > 2$ but did not identify the other possible case of $(Y - X) > 2$ i.e. $(X - Y) < -2$. Of those who did find the two required probabilities, some multiplied these together rather than adding them.

(b)(i) This part was quite well attempted. The most common error noted was an incorrect value for the variance (using 1.5 rather than 1.5^2) or calculating an incorrect area (<0.5 rather than >0.5).

(ii) This part was not as well attempted. Many candidates stated that T and P had to be independent, and some omitted a response for this part.

Question 6

(a) Most candidates were able to identify $\left(\frac{2}{3}\right)^{10}$ as the required probability but others incorrectly calculated $\left(\frac{1}{3}\right)^{10}$, or $1-\left(\frac{2}{3}\right)^{10}$, or summed until they got their sum of probabilities to be less than $\frac{1}{3}$. As highlighted in the Key messages, this was another case where some candidates did not gain full credit by giving an answer to two significant figures only.

(b) A large number of candidates struggled to set up the correct equation here. Errors included using a mix of both p and q. Candidates who set up the correct equation were usually successful in reaching the correct answer.

Question 7

(a) (i) This was well attempted, with most candidates using the fact that the area of the triangle was equal to 1. However, there was some confusion between the height of the triangle $\left(\frac{1}{2}\right)$ and the value of $k\left(\frac{1}{8}\right)$, with a number of candidates stating that k was $\frac{1}{2}$.

(ii) Again, this was well attempted. Most candidates attempted to integrate x multiplied by their $\mathrm{f}(x)$ using correct limits.

(b) This part was not well attempted, with very few candidates able to reach the correct value for a. Few candidates realised they could use a ratio method (using similar triangles), so the most common method used was to find $\mathrm{g}(w)$ in terms of a and integrate from 0 to 1 (or 1 to a), knowing that this was equal to 0.5. Finding $\mathrm{g}(w)$ in terms of a proved difficult for a large number of candidates.

MATHEMATICS

Paper 9709/63
Probability & Statistics 2 (63)

Key messages

Candidates should be aware that questions on probability density functions may require understanding and use of the pdf properties in different situations. Additionally, candidates are encouraged to make use of diagrams and sketches where appropriate.

General comment

There are several question parts for which the answers are required to be written in context, including in the conclusions to hypothesis tests. Candidates should be prepared to respond to questions in this manner.

Comments on specific questions

Question 1

Many candidates answered this question correctly, integrating $x\mathrm{f}(x)$ between the limits 0 and 1. Some candidates made mistakes when multiplying or when integrating the terms. A few candidates omitted the x and gained no marks.

Question 2

(a) Most candidates found the number for the third member from the list of random numbers. Fewer candidates found the appropriate fourth number. Some candidates suggested 121, but this was a repetition of the second number. Other candidates suggested 473, but this was greater than 264 and so was beyond the highest member's number and so should have been ignored.

(b) Candidates were required to state that these numbers had already been used and so were not random. Some candidates suggested only that they were not random or were biased, which was not sufficient.

Question 3

(a) Some very efficient answers to this question were seen with candidates dealing well with the variance of proportions and the structure of the question. Some candidates found difficulty dealing with $\frac{x}{100}$ and $\left(1-\frac{x}{100}\right)$, and dividing by 100. Other candidates found proportional values (0.36 and 0.64) but did not convert these into the final values for x. Most candidates used the correct z-value of 1.645.

(b) The probability of one of the confidence intervals not containing the true value of p was 0.1. Then the probability of neither of the confidence intervals containing the true value of p was 0.1^2. Some candidates tried squaring 0.9 or some other value. Other candidates tried to use probabilities extracted from a normal distribution.

Question 4

There were two common ways seen of finding the probability. One method involved working with the mass of steel over the 7 days and to find and use N(456.4, 90.72). The main alternative method involved working with the profit over the 7 days and to find and use N(22820, 226800). For either way, it was necessary to standardise and to select the correct probability area. Common errors included not finding the correct variance and working with ‘per day’ instead of ‘per week’. Candidates should be advised that a diagram could be helpful in choosing the correct area, as mentioned in the Key messages.

Question 5

(a) Many candidates found the required unbiased estimates of the population mean and variance. These candidates substituted accurately in the formulae and calculated correctly. A few candidates confused $\frac{1700^2}{50}$ with $\left(\frac{1700}{50}\right)^2$, depending on which of the two formulae for variance was being used.

(b) This significance test required the hypotheses, standardisation, a comparison and a conclusion. The standardisation required the correct use of the sample size of 50. The comparison could be carried out with the critical value for z (2.326) or with the critical probability value (0.01). An alternative method was to find the critical value for t and then compare. The conclusion needed to be stated in context, to not be definite and to have no contradictions.

(c) The question on the circumstances when it would not be necessary to use the Central Limit Theorem was only answered correctly by about a third of candidates.

Question 6

(a) The sample size and the probability led to $np = 2.5$ which indicated the use of a Poisson distribution $\text{Po}(2.5)$. The sum of the probabilities of X being from 0 to 3 was required. It was necessary to list these four individual terms as well as giving the final answer. Many candidates did this correctly.

(b) The value of $\text{E}(X)$ was found from np and the value of $\text{Var}(X)$ from npq. For the second mark, it was required that these two values were referred to in order to explain the appropriateness of the Poisson distribution. Other properties, such as $np < 5$, were not relevant here. Some candidates did not give the answer for the variance accurately enough.

Question 7

(a) This question required the use of the knowledge that the area under a pdf curve is 1. Here, candidates were expected to find the area under the given semicircular graph by using $\frac{\pi r^2}{2}$. Having found the area to be 1, a statement that this indicated that f could be a pdf was required. Some candidates omitted this statement. It was not necessary to state that $\text{f}(x) \geq 0$ for this question.

(b) The angle AOB could be shown to be $\frac{\pi}{4}$ directly by use of the cosine function, but not by other trigonometrical functions unless AB was found first. The further property of pdf functions, i.e., that the probability is given by the specific area under the curve, was then to be used. This area could be found by subtracting the area of the triangle AOB from the area of the sector. This involved careful use of the given lengths in their unusual formats. Answers in decimal form or in terms of π were accepted. Some candidates attempted to use integration to find the area and probability. Many of these candidates were unsuccessful as the integration required was challenging and the formula for the pdf had to be found. However, some marks could be gained for showing correct steps.

Question 8

(a) In this significance test there was a one-tail test and required $\text{P}(X \geq 6)$ using the Poisson distribution $\text{Po}(3.03)$. This tail was found from $1 - \text{P}(X \leq 5)$. This probability (0.0870) then needed to be compared to 0.05 for the 5 per cent significance level. Some candidates used 0.9130 < 0.95 correctly. The conclusion required the same conditions as earlier. Some candidates included an extra term $\mathrm{P}(X = 6)$ incorrectly. This work could be followed through to gain some marks.

(b) The Type I error required $\mathrm{P}(X > 6)$ which could be found from $1 - \mathrm{P}(X \le 6)$ or directly from **part (a)** with the one extra term. For some candidates, work in **part (a)** involving this could be given credit here. Some candidates incorrectly stated that the probability was 0.05.

(c) This statement needed to be expressed in context. Many candidates omitted some of the required parts.

(d) For a Type II error to take place the null hypothesis had to be false. Thus, a new value for the Poisson parameter was required. This had to be expressed in context relating to the mean number of people, the path and the 20 minutes time in the evening.

Grade thresholds – June 2023

Cambridge International AS & A Level Mathematics (9709)

Grade thresholds taken for Syllabus 9709 (Mathematics) in the June 2023 examination.

		Minimum raw mark required for grade:				
	Maximum raw mark available	A	B	C	D	E
Component 11	75	50	43	33	23	14
Component 12	75	49	40	30	20	10
Component 13	75	62	53	40	28	16
Component 21	50	33	28	23	19	14
Component 22	50	40	30	23	16	9
Component 23	50	40	30	23	16	9
Component 31	75	52	45	36	26	17
Component 32	75	52	45	36	27	17
Component 33	75	57	51	42	34	25
Component 34	75	53	45	36	26	16
Component 41	50	36	31	26	22	16
Component 42	50	38	30	24	18	13
Component 43	50	38	31	25	18	12
Component 51	50	35	30	24	19	13
Component 52	50	38	31	25	18	12
Component 53	50	40	34	27	20	13
Component 58	75	66	61	53	46	39
Component 59	75	64	59	49	39	30
Component 60	75	64	59	52	45	37
Component 61	50	38	34	28	23	16
Component 62	50	39	34	29	24	19
Component 63	50	38	31	26	20	15
Component 68	75	61	49	45	41	38
Component 69	75	60	52	48	44	39

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Grade thresholds continued

Cambridge AS & A Level Mathematics (9709)

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AX	250	11, 31, 41, 51	197	173	149	119	89	60
AY	250	12, 32, 42, 52	208	177	146	114	83	52
AZ	250	13, 33, 43, 53	223	197	169	134	100	66
CX	250	11, 31, 51, 61	198	175	152	121	90	60
CY	250	12, 32, 52, 62	206	178	150	119	88	58
CZ	250	13, 33, 53, 63	223	197	169	135	102	69
DO	250	34, 85	209	179	149	115	82	49
DX	250	31, 51, 84	203	177	151	121	92	63
DY	250	32, 52, 85	212	181	150	117	84	51
DZ	250	33, 53, 86	220	193	166	135	104	74
EX	250	31, 41, 87	205	180	155	125	95	66
EY	250	32, 42, 88	212	180	148	115	82	49
EZ	250	33, 43, 89	219	191	163	132	102	72
GX	250	31, 61, 87	206	182	158	127	96	66
GY	250	32, 62, 88	210	181	152	119	87	55
GZ	250	33, 63, 89	219	191	163	133	104	75
HO	250	34, 95	209	183	157	123	89	55
HY	250	32, 52, 95	212	185	158	124	90	57
HZ	250	33, 53, 96	208	182	156	126	97	68
IX	250	31, 41, 97	212	184	156	127	98	69
IY	250	32, 42, 98	213	186	159	125	91	57
IZ	250	33, 43, 99	215	188	161	131	101	71
KX	250	31, 61, 97	213	186	159	129	99	69
KY	250	32, 62, 98	211	187	163	129	96	63
KZ	250	33, 63, 99	215	188	161	132	103	74
P2	75	58	–	66	61	53	46	39
P3	75	59	–	64	59	49	39	30
P4	150	59, 60	138	128	118	101	84	67
P6	150	68, 80	135	121	102	90	79	68
P8	150	69, 81	135	120	105	93	81	69
S1	125	11, 21	–	83	71	56	42	28
S2	125	11, 41	–	86	74	59	44	30
S3	125	11, 51	–	85	73	57	42	27
S4	125	12, 22	–	89	70	53	36	19

Grade thresholds continued

Cambridge AS & A Level Mathematics (9709)

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
S5	125	12, 42	–	87	70	54	38	23
S6	125	12, 52	–	87	71	54	38	22
S7	125	13, 23	–	102	83	63	44	25
S8	125	13, 43	–	100	84	65	46	28
S9	125	13, 53	–	102	87	67	48	29

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3967186601

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC23 06_9709_11/2R

1 Solve the equation $4\sin\theta + \tan\theta = 0$ for $0° < \theta < 180°$. [3]

2 **(a)** Find the first three terms in the expansion, in ascending powers of x, of $(2 + 3x)^4$. [2]

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(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

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(c) Hence find the coefficient of x^2 in the expansion of $(2 + 3x)^4(1 - 2x)^5$. [2]

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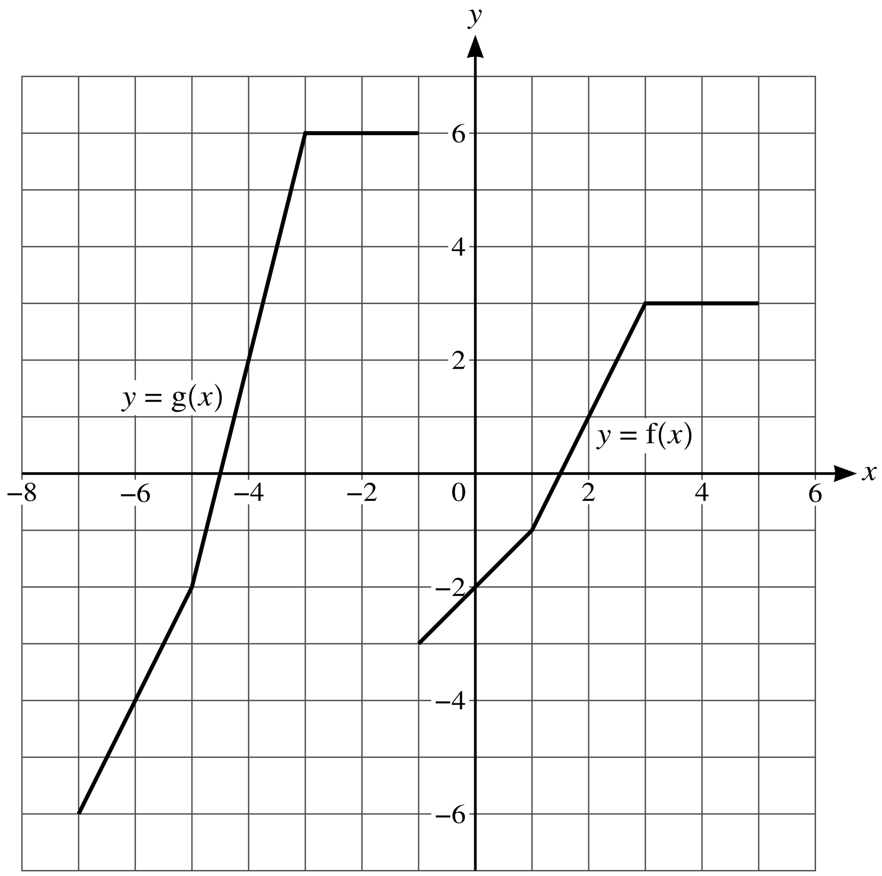

The diagram shows graphs with equations $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = \mathrm{f}(x)$ to $y = \mathrm{g}(x)$. [4]

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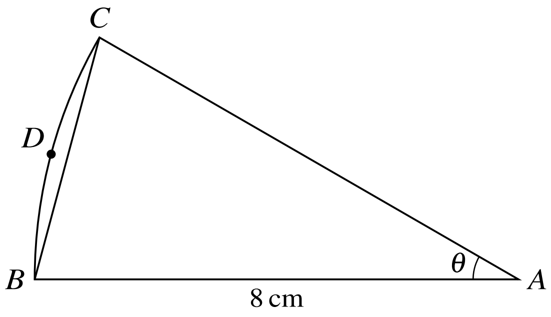

The diagram shows a sector ABC of a circle with centre A and radius 8 cm. The area of the sector is $\frac{16}{3}\pi\,\text{cm}^2$. The point D lies on the arc BC.

Find the perimeter of the segment BCD. [4]

5 The line with equation $y = kx - k$, where k is a positive constant, is a tangent to the curve with equation $y = -\frac{1}{2x}$.

Find, in either order, the value of k and the coordinates of the point where the tangent meets the curve. [5]

6 The first three terms of an arithmetic progression are $\frac{p^2}{6}$, $2p - 6$ and p.

(a) Given that the common difference of the progression is not zero, find the value of p. [3]

(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\frac{p^2}{6}$ and $2p - 6$. [2]

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7 A curve has equation $y = 2 + 3\sin\frac{1}{2}x$ for $0 \leqslant x \leqslant 4\pi$.

(a) State greatest and least values of y. [2]

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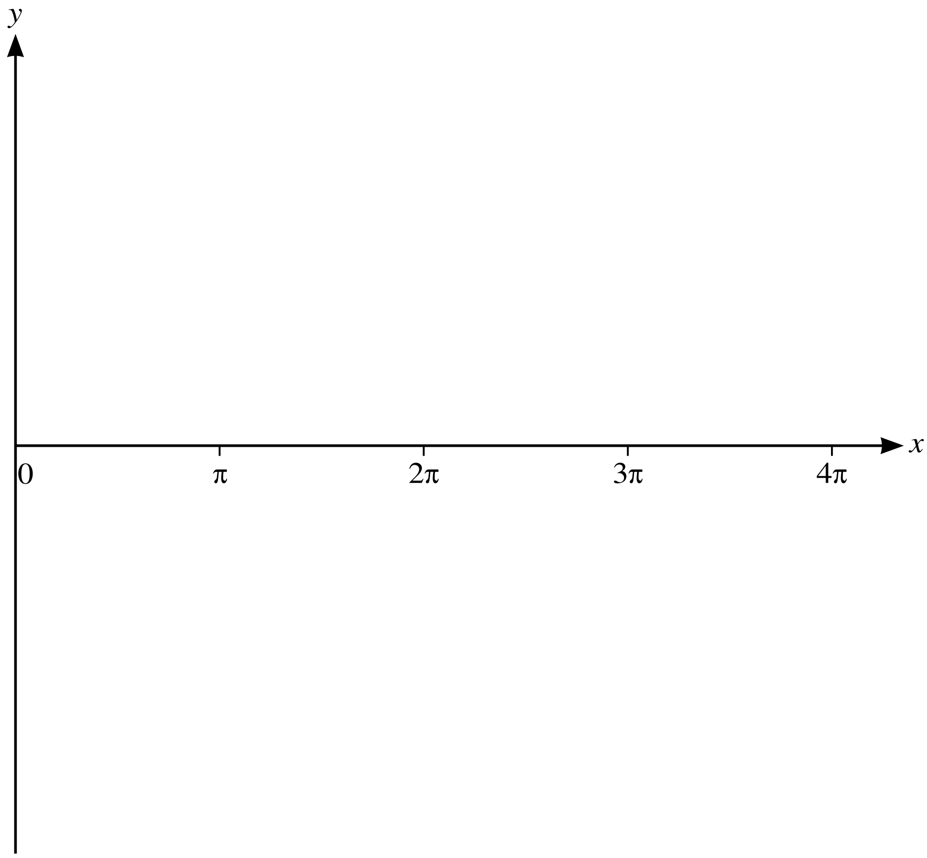
(b) Sketch the curve. [2]



(c) State the number of solutions of the equation

$$2 + 3\sin\tfrac{1}{2}x = 5 - 2x$$

for $0 \leqslant x \leqslant 4\pi$. [1]

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8 The functions f and g are defined as follows, where a and b are constants.

$$\mathrm{f}(x) = 1 + \frac{2a}{x-a} \text{ for } x > a$$

$$\mathrm{g}(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

(a) Given that $\mathrm{f}(7) = \frac{5}{2}$ and $\mathrm{gf}(5) = 4$, find the values of a and b. [4]

For the rest of this question, you should use the value of a which you found in **(a)**.

(b) Find the domain of f^{-1}. [1]

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(c) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

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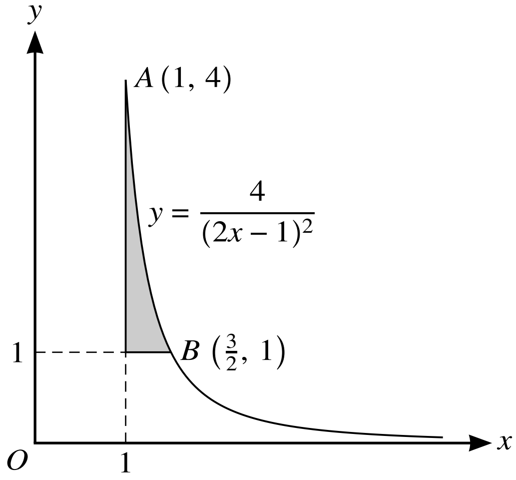
9 Water is poured into a tank at a constant rate of $500\,\text{cm}^3$ per second. The depth of water in the tank, t seconds after filling starts, is h cm. When the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by the formula $V = \frac{4}{3}(25 + h)^3 - \frac{62500}{3}$.

(a) Find the rate at which h is increasing at the instant when $h = 10$ cm. [3]

(b) At another instant, the rate at which h is increasing is 0.075 cm per second.

Find the value of V at this instant. [3]

10



The diagram shows part of the curve with equation $y = \dfrac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A\,(1, 4)$ and $B, \left(\frac{3}{2}, 1\right)$.

(a) Find the exact volume generated when the shaded region is rotated through 360° about the x-axis. [5]

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(b) A triangle is formed from the tangent to the curve at B, the normal to the curve at B and the x-axis.

Find the area of this triangle. [6]

11 The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a. [2]

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(b) Determine the nature of this stationary point. [2]

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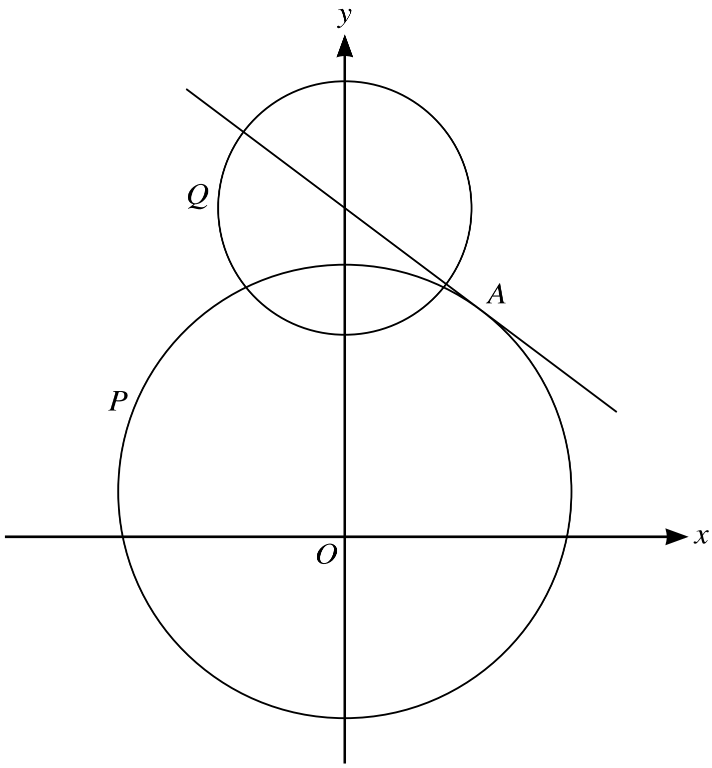
(c) Find the equation of the curve. [3]

(d) Find the coordinates of any other stationary points on the curve. [2]

12



The diagram shows a circle P with centre $(0, 2)$ and radius 10 and the tangent to the circle at the point A with coordinates $(6, 10)$. It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P. [1]

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(b) Find the equation of the tangent to the circle P at A. [2]

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(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11. [3]

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(d) Find the coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

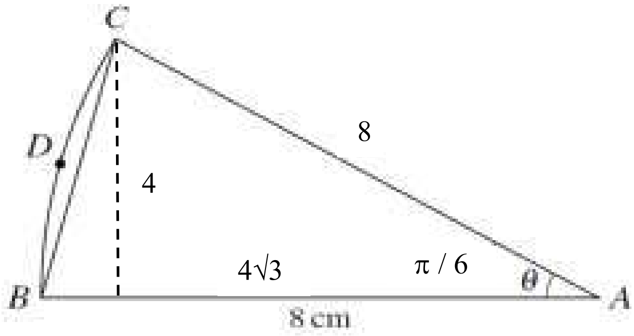
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$4\sin\theta + \tan\theta = 0 \Rightarrow 4\sin\theta + \frac{\sin\theta}{\cos\theta}[=0]$	**M1**	For use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$. BOD if θ missing.
	$\Rightarrow \sin\theta(4\cos\theta + 1)[=0 \Rightarrow \sin\theta = 0 \text{ or}]\ \cos\theta = -\frac{1}{4}$	**M1**	WWW Factorise, not divide by $\sin\theta$ or $\tan\theta$. May see $\tan\theta(4\cos\theta+1)[=0]$ or $\sin\theta(4+\sec\theta)[=0]$.
	$\theta = 104.5°$	**A1**	AWRT 1.82 rads A0. Ignore answers outside (0, 180°). If M1 M0, **SC B1** for $\theta = 104.5°$ max 2/3.
		3	

Question	Answer	Marks	Guidance
2(a)	$16 + 96x + 216x^2$	**B2, 1, 0**	ISW (higher powers of x). Terms may be in any order or presented as a list.
		2	
2(b)	$1 - 10x + 40x^2$	**B2, 1, 0**	ISW (higher powers of x). Terms may be in any order or presented as a list.
		2	
2(c)	$(16\times 40) - (10\times 96) + (1\times 216)$	**M1**	*Their* 3 products which would give the term in x^2 (FT *their* values). Look for $640 - 960 + 216$.
	-104	**A1**	Condone $-104x^2$.
		2	

Question	Answer	Marks	Guidance
3	{Stretch} {factor 2} {in y-direction}	**B2, 1, 0**	**B2** for fully correct, **B1** with two elements correct. {} indicates different elements.
	{Translation} $\begin{pmatrix} \{-6\} \\ \{0\} \end{pmatrix}$	**B2, 1, 0**	**B2** for fully correct, **B1** with two elements correct. {} indicates different elements.
		4	Transformations may be in either order.

Question	Answer	Marks	Guidance
4	$\frac{1}{2}\times 8^2 \times \theta = \frac{16\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$	**B1**	SOI OE e.g. $\frac{2\pi}{12}$, $0.524(3\text{s.f.})$ Use of degrees acceptable throughout provided conversion used in formulae for sector area and arc length.
	Arc length = $8\times their\frac{\pi}{6}$ [= 4.1887…]	**M1**	OE FT *their* θ. Look for $\frac{4\pi}{3}$.
	$[BC =]\ 2\times 8\sin\left(\frac{1}{2}\times their\frac{\pi}{6}\right)$ [= 4.1411…]	**M1**	Attempt to find BC or BC^2 (see alt. methods below) FT *their* θ. Look for $16\sin\frac{\pi}{12}$ or $4\sqrt{6}-4\sqrt{2}$.
	Perimeter = 8.33	**A1**	AWRT Must be combined into one term.

Question	Answer	Marks	Guidance
4	**Alternative methods for Question 4: 2nd M1 mark (use normal scheme for the other marks)**		
	ALT 1 $BC^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \cos\left(their \frac{\pi}{6}\right) [\Rightarrow BC = 4.14\ldots]$ **ALT 2** $BC^2 = \left(8 - 4\sqrt{3}\right)^2 + 4^2 [\Rightarrow BC = 4.14\ldots]$  **ALT 3** $\dfrac{BC}{\sin\left(\frac{\pi}{6}\right)} = \dfrac{8}{\sin\left(\frac{5\pi}{12}\right)} [\Rightarrow BC = 4.14\ldots]$		ALT 1 Substitute into correct cosine rule. FT *their* θ Look for $128 - 64\sqrt{3}$ ALT 2 Find lengths 4 and $4\sqrt{3}$ then use Pythagoras in the left hand triangle. ALT 3 Substitute into correct sine rule.
		4	

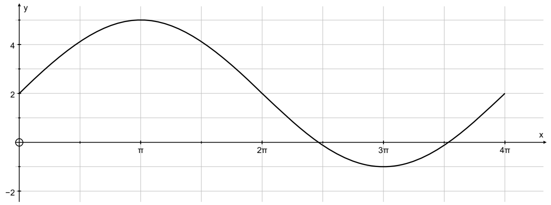
Question	Answer	Marks	Guidance
5	$kx-k=-\frac{1}{2x} \Rightarrow 2kx^2-2kx+1[=0]$ OR quadratic in y: $x=\frac{y+k}{k} \Rightarrow y=-\frac{1}{2\left(\frac{y+k}{k}\right)} \Rightarrow 2y^2+2ky+k=0$	*M1	OE e.g. $kx^2-kx+\frac{1}{2}[=0]$, $x^2-x+\frac{1}{2k}[=0]$ Equate line and curve to form 3-term quadratic (all terms on one side).
	$b^2-4ac[=0] \Rightarrow ([-]2k)^2-4(2k)(1)[=0]$ or $4k^2-8k\ [=0 \Rightarrow 4k(k-2)=0]$ OR using equation in y: $(2k)^2-4(2)(k)=0$	DM1	Use discriminant correctly with their a,b,c not in quadratic formula. DM0 if x still present. May see $k^2-4(k)\left(\frac{1}{2}\right)=0$ or $1-4\left(\frac{1}{2k}\right)=0$.
	$k=2$ only	A1	If DM0 then $k=2$, award A0 XP then B0 B0 Allow A1 even if divides by k to solve. If $k=0$ also present but uses $k=2$, award A1.
	$4x^2-4x+1=0\left[\Rightarrow (2x-1)^2=0\right] \Rightarrow x=\frac{1}{2}$	B1	
	$y=2\times\frac{1}{2}-2=-1$	B1	

Question	Answer	Marks	Guidance
5	**Alternative method for Q5**		
	$\frac{dy}{dx} = \frac{1}{2x^2}$ or $\frac{1}{2}x^{-2}$	***M1**	Differentiate $-\frac{1}{2x}$ M0 for $2x^{-2}$. No errors.
	$[y =]\frac{1}{2x^2}x - \frac{1}{2x^2} = -\frac{1}{2x}$ or $\frac{1}{x} = \frac{1}{2x^2}$ $\left[\Rightarrow 2x^2 - x = 0\right]$	**DM1**	Sub *their* $\frac{dy}{dx}$ into equation of line or set gradient = k to form equation in x.
	$x = \frac{1}{2}$ only	**A1**	If DM0 then $x = \frac{1}{2}$, award A0XP then B0 B0.
	$y = \left[2 \times \frac{1}{2} - 2\right] = -1$	**B1**	
	$k = 2$	**B1**	
		5	

Question	Answer	Marks	Guidance
6(a)	$2(2p-6)=p+\frac{p^2}{6} \Rightarrow \frac{p^2}{6}-3p+12[=0]$ OR $(2p-6)-\frac{p^2}{6}=p-(2p-6) \Rightarrow \frac{p^2}{6}-3p+12[=0]$ OR $\frac{1}{6}d^2+d[=0]$	*M1	Correct method leading to formation of a 3-term quadratic in p (all terms on one side) or 2-term quadratic in d. OE e.g. $p^2-18p+72[=0]$, $\frac{1}{2}p^2-9p+36[=0]$.
	$p^2-18p+72[=0] \Rightarrow (p-6)(p-12)[=0]$ **or** $\frac{18\pm\sqrt{(-18)^2-4(1)(72)}}{2}$ OR $d\left(\frac{1}{6}d+1\right)[=0] \Rightarrow d=-6$	DM1	Solve a 3-term quadratic in p by factorisation, formula or completing the square or solve a 2-term quadratic in d by factorisation.
	$p = 12$ only	A1	Since $p = 6$ gives $d = 0$. If *M1 DM0 then $p = 12$ only, award **SC B1**, max 2/3 marks. A0 XP if error in either factor and $p=12$ only. $p=12$ only by trial and improvement 3/3.
		3	

Question	Answer	Marks	Guidance
6(b)	For GP $r = \left[\dfrac{2p-6}{\frac{p^2}{6}}\right] = \dfrac{18}{24}\left[=\dfrac{3}{4}\right]$	**B1**	OE SOI.
	Sum to infinity = $\dfrac{24}{1-\frac{3}{4}}$ = 96	**B1 FT**	FT *their* value of p if used correctly to find r (B0 if 'p' used) provided $\lvert r\rvert < 1$. e.g. $p = 18 \Rightarrow [S_\infty =]\dfrac{54}{1-\frac{5}{9}} = 121.5$.
		2	

Question	Answer	Marks	Guidance
7(a)	[Greatest =] 5	**B1**	No inequality required.
	[Least =] –1	**B1**	No inequality required.
			Condone $(-1,5)$ or equivalent.
		2	

Question	Answer	Marks	Guidance
7(b)		B1	One complete cycle starting and finishing at $y = 2$. Maximum and minimum in correct quadrants. Shape and curvature approximately correct.
		B1 FT	Maximum and minimum (indicated on y-axis with numbers or lines, or labelled on graph). FT *their* greatest and least values. Award **B1** for 5 and -1 even if *their* values were incorrect in **(a)**.
		2	
7(c)	1	B1	WWW
		1	

Question	Answer	Marks	Guidance
8(a)	$1+\frac{2a}{7-a}=\frac{5}{2}\left[\Rightarrow \frac{2a}{7-a}=\frac{3}{2} \Rightarrow 7a=21\right]\Rightarrow a=\ldots$ OR $1+\frac{2a}{7-a}=\frac{5}{2}\left[\Rightarrow (7-a)+2a=\frac{5}{2}(7-a)\ [\Rightarrow 7a=21\right]\Rightarrow a=\ldots$	**M1**	OE Substitute $x=7$ then solve for a via legitimate mathematical steps. Condone sign errors only.
	$a=3$	**A1**	If M0, **SC B1** for $a=3$ with no working.
	$f(5)=1+\frac{2(their3)}{5-their3}=4\,[\Rightarrow 4b-2=4]\Rightarrow b=\ldots$ OR $gf(5)=b\left(1+\frac{2(their3)}{5-their3}\right)-2\ [\Rightarrow 4b-2=4]\Rightarrow b=\ldots$	**M1**	Evaluate f(5)**,** either separately or within gf then solve for b via legitimate mathematical steps. Condone sign errors only. FT *their* a value.
	$b=\frac{3}{2}$	**A1**	OE e.g. $\frac{6}{4}$, 1.5 .
		4	
8(b)	$x>1$	**B1**	Accept $(1,\infty)$ or {*: * > 1} where * is any variable. B0 for $f^{-1}(x)>1$ or $f(x)>1$ or $y>1$.
		1	

Question	Answer	Marks	Guidance
8(c)	EITHER $x-1=\dfrac{6}{y-3}$ $\left[\Rightarrow (y-3)(x-1)=6\right]$ OR $x=1+\dfrac{6}{y-3}$ $\Rightarrow x(y-3)=(y-3)+6$	*M1	OE $y-1=\dfrac{6}{x-3}$ $\Rightarrow (x-3)(y-1)=6$. OE $y=1+\dfrac{6}{x-3}$ $\Rightarrow y(x-3)=(x-3)+6$. Allow *M1 for use of *their* 3 from **(a)**.
	$y-3=\dfrac{6}{x-1}$ or $y(x-1)=3x+3$	DM1	OE $x-3=\dfrac{6}{y-1}$ or $x(y-1)=3y+3$. Allow DM1 for use of *their* 3 from **(a)**.
	$\left[f^{-1}(x)\right]=3+\dfrac{6}{x-1}$	A1	OE Correct answer e.g. $\dfrac{3x+3}{x-1}$ ISW. Must be in terms of x.
			*M1 DM1 possible for ‘ a ’ used, but A0 so max 2/3.
		3	

Question	Answer	Marks	Guidance
9(a)	$\dfrac{dV}{dh}=\dfrac{4}{3}\times 3(25+h)^2$ [= 4900 when h = 10]	**B1**	Correct expression for $\dfrac{dV}{dh}$.
	$\dfrac{dV}{dh}\times\dfrac{dh}{dt}=\dfrac{dV}{dt}\Rightarrow$ *their* "$4(25+10)^2$"$\times\dfrac{dh}{dt}=500\Rightarrow\dfrac{dh}{dt}=\left[\dfrac{500}{4900}\right]$	**M1**	Use chain rule correctly to find a numerical expression for $\dfrac{dh}{dt}$. Accept e.g. $\dfrac{500}{2500+2000+400}$.
	$\dfrac{dh}{dt}=0.102\left[\text{cms}^{-1}\right]$	**A1**	AWRT OE e.g. $\dfrac{5}{49}$ ISW.
		3	
9(b)	$\dfrac{dV}{dt}=\dfrac{dV}{dh}\times\dfrac{dh}{dt}\Rightarrow 500=$ *their* "$4(25+h)^2$"$\times 0.075$	***M1**	SOI Use chain rule correctly to form equation in h.
	$\left[(25+h)^2=\dfrac{5000}{3}\right]\Rightarrow h=[15.8\,248\ldots]$	**DM1**	Solve quadratic to find h. Exact value of h is $\sqrt{\dfrac{5000}{3}}-25$ or $\dfrac{50\sqrt{6}}{3}-25$ $h+25=40.82\ldots$
	$V=69900\text{ cm}^3$	**A1**	AWRT ISW Look for 698(88.5) .
		3	

Question	Answer	Marks	Guidance
10(a)	$[\pi]\int \frac{16}{(2x-1)^4}[dx]=[\pi]\int 16(2x-1)^{-4}[dx]=[\pi]\left(-\frac{16}{3\times 2\times(2x-1)^3}\right)$	*M1	Integrate $\boldsymbol{y}^2$ (power incr. by 1 or div by *their* new power). M0 if more than 1 error or $-\frac{16}{6}\boldsymbol{x}(2x-1)^{-3}$.
	$[\pi]\left(-\frac{16}{3\times 2\times(2x-1)^3}\right)$	A1	OE e.g. $\left(-\frac{8}{3}(2x-1)^{-3}\right)$.
	$[\pi]\left(-\frac{16}{6\times 8}+\frac{16}{6\times 1}\right)$ $\left[=[\pi]\frac{112}{48}=[\pi]\frac{7}{3}\right]$	DM1	Sub correct limits into *their* integral: $F\left(\frac{3}{2}\right)-F(1)$. Must see at least $\left(-\frac{1}{3}+\frac{8}{3}\right)$. Allow 1 sign error. Decimal: 2.33π or 7.33 .
	Volume of cylinder $\left[=\pi\times 1^2\times\frac{1}{2}\right]=\frac{1}{2}\pi$ OR $[\pi]\int_1^{1.5}1[dx]=\frac{1}{2}\pi$	B1	$\frac{1}{2}\pi$ or $\pm\pi\left(\frac{3}{2}-1\right)$ seen.
	Volume of revolution $\left[=\frac{7}{3}\pi-\frac{1}{2}\pi\right]=\frac{11}{6}\pi$	A1	A0 for 5.76 (not exact). If DM0 for insufficient substitution, or B0, **SC B1** for $\frac{11}{6}\pi$.
		5	

Question	Answer	Marks	Guidance
10(b)	$\left[\dfrac{dy}{dx}=\right]\left\{-8(2x-1)^{-3}\right\}\{\times 2\}$	**B2, 1, 0**	OE B1 for each correct element in $\{\}$.
	At B gradient $= -2$	**B1**	
	Eqn of tangent $y-1 = their\ "-2"\left(x-\dfrac{3}{2}\right)$ OR Eqn of normal $y-1 = their\ "\dfrac{1}{2}"\left(x-\dfrac{3}{2}\right)$	**M1**	SOI Following differentiation OE e.g. $y=-2x+4$ or $y=\dfrac{1}{2}x+\dfrac{1}{4}$. (Must have $m_N = -\dfrac{1}{m_T}$ for M1).
	Tangent crosses x-axis at 2 **or** normal crosses x-axis at $-\dfrac{1}{2}$	**A1**	SOI For at least one intercept correct or correct integration.
	Area $=\dfrac{5}{4}$	**A1**	From intercepts: $\dfrac{1}{2}\times\dfrac{5}{2}\times 1=\dfrac{5}{4}$ or $1+\dfrac{1}{4}=\dfrac{5}{4}$, from lengths: $\dfrac{1}{2}\times\sqrt{5}\times\dfrac{\sqrt{5}}{2}=\dfrac{5}{4}$ or by integration.
		6	

Question	Answer	Marks	Guidance
11(a)	$6a^2-30a+6a=0$ $[\Rightarrow 6a(a-4)=0]$	**B1**	Sub $x=a$ into $\dfrac{dy}{dx}=0$. May see $a^2-5a+a=0$.
	$a=4$ only	**B1**	
		2	

Question	Answer	Marks	Guidance
11(b)	$\frac{d^2y}{dx^2} = 12x - 30$ or correct values of $\frac{dy}{dx}$ either side of $x = 4$	**M1**	Differentiate $\frac{dy}{dx}$ (mult. by power or dec. power by 1) M0 if no values of $\frac{dy}{dx}$, only signs.
	At $x = 4, \frac{d^2y}{dx^2} > 0 \therefore$ minimum or $\frac{d^2y}{dx^2} = 18 \therefore$ minimum or concludes minimum from $\frac{dy}{dx}$ values	**A1**	WWW A0 XP if $a = 4$ obtained incorrectly in (a) Must see 'minimum'. If M0, **SC B1** for 'minimum' from $\frac{dy}{dx}$ sign diagram.
		2	
11(c)	$[y =]\ \frac{6}{3}x^3 - \frac{30}{2}x^2 + 6(\textit{their } a)x[+c]$	**B1 FT**	Expect $2x^3 - 15x^2 + 24x[+c]$. B1 poss. even if uses 'a' – no value in (a) – max 1/3.
	$-15 = 2(\textit{their } "4")^3 - 15(\textit{their } "4")^2 + 6(\textit{their } "4")^2 + c$	**M1**	Sub $x = \textit{their}"4"$, $y = -15$ into integral (must incl $+c$) Look for $-15 = 128 - 240 + 96 + c$ $[\Rightarrow c = 1]$.
	$y = 2x^3 - 15x^2 + 24x + 1$	**A1**	Coefficients must be correct and simplified. Need to see '$y =$' or '$f(x) =$' in the working.
		3	
11(d)	$\frac{dy}{dx} = 6x^2 - 30x + 6(\textit{their } "4")[= 0]$ If correct, $[6](x-1)(x-4)[= 0]$ or $\frac{30 \pm \sqrt{(-30)^2 - 4(6)(24)}}{12}$	**M1**	OE Forming a 3-term quadratic using the given $\frac{dy}{dx}$ and solving by factorisation, formula or completing the square. Check for working in **(b)**.
	Coordinates $(1, 12)$	**A1**	Allow $x = 1, y = 12$ (ignore $x = 4$ if present). If M0, award **SC B1** for $(1, 12)$.
		2	

Question	Answer	Marks	Guidance
12(a)	$x^2+(y-2)^2=100$	B1	OE e.g. $(x-0)^2+(y-2)^2=10^2$ ISW.
		1	
12(b)	Gradient of radius $=\left[\frac{10-2}{6-0}=\right]\frac{4}{3}$ or gradient of tangent $=\frac{-3}{4}$	M1	OE SOI Use coordinates to find gradient of radius or differentiate to find m_T e.g. $2x+2(y-2)\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3}{4}$ at (6, 10) $y=2+\sqrt{100-x^2}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}(100-x^2)^{-\frac{1}{2}}(-2x)=-\frac{3}{4}$.
	Equation of tangent is $y-10=-\frac{3}{4}(x-6)$ $\left[\Rightarrow y=-\frac{3}{4}x+\frac{29}{2}\right]$	A1	OE ISW Allow e.g. $\frac{58}{4}$.
		2	
12(c)	Coordinates of centre of circle Q are $\left(0, their\frac{29}{2}\right)$	M1	SOI From a linear equation in **(b)**.
	Equation of circle Q is $x^2+\left(y-their\frac{29}{2}\right)^2=\left(\frac{5\sqrt{5}}{2}\right)^2\left[=\frac{125}{4}\right]$	A1FT	OE e.g. $(x-0)^2+(y-14.5)^2=31.25$ ISW.
	$x^2+(11-2)^2=100\Rightarrow x^2=19$ and $x^2+\left(11-\frac{29}{2}\right)^2=\frac{125}{4}\Rightarrow x^2=19$ OR e.g. $\frac{125}{4}-\left(y-\frac{29}{2}\right)^2+(y-2)^2=100\Rightarrow 25y=275\Rightarrow y=11$	B1	OE e.g. $x=[\pm]\sqrt{19}$, $x^2-19=x^2-19$ Correct argument to verify both y-coords are 11 ISW.
		3	

Question	Answer	Marks	Guidance
12(d)	$x^2+\left(-\frac{3}{4}x+\frac{29}{2}-\frac{29}{2}\right)^2=\frac{125}{4}\ \left[\Rightarrow\frac{25}{16}x^2=\frac{125}{4}\Rightarrow x^2=20\right]$ or $y^2-29y+199[=0]$	**M1**	Substitute equation of *their* tangent into equation of *their* circle. May see $y=\sqrt{31.25-x^2}+14.5$.
	$x=\pm2\sqrt{5}$ or $y=\frac{29\mp3\sqrt{5}}{2}$	**A1**	OE e.g. $x=\pm\sqrt{20}$ For 2 x-values or 2 y-values or correct (x,y) pair.
	$y\left[=\left(-\frac{3}{4}\times\pm\sqrt{20}\right)+\frac{29}{2}\right]=\frac{29\mp3\sqrt{5}}{2}$	**A1**	OE e.g. $\frac{58}{4}+\frac{3\sqrt{20}}{4},\ \frac{58}{4}-\frac{3\sqrt{20}}{4}$ Correct (x,y) pairs.
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

* 2 3 3 0 9 7 3 6 0 5 *

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point (4, 5).

Find the equation of the curve. [3]

2 The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q. It is given that $p + q = 276$.

Find the possible values of the constant a. [4]

3 **(a)** Express $4x^2 - 24x + p$ in the form $a(x+b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. [2]

(b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

4 Solve the equation $8x^6 + 215x^3 - 27 = 0$. [3]

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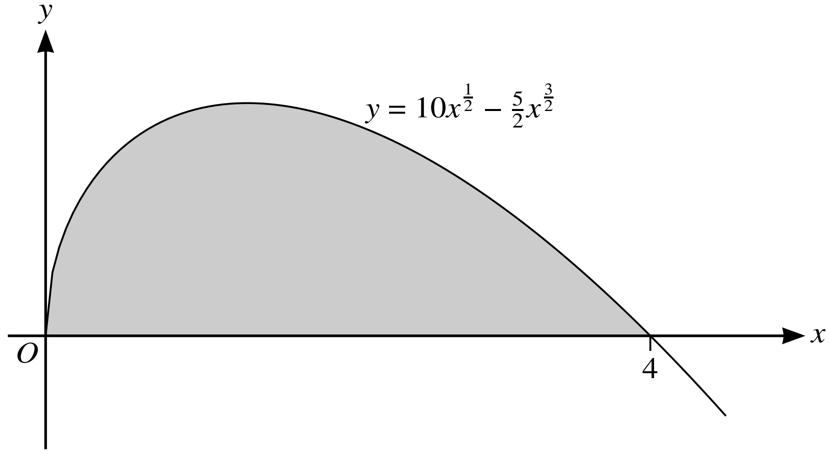

The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points $(0, 0)$ and $(4, 0)$.

Find the area of the shaded region. [4]

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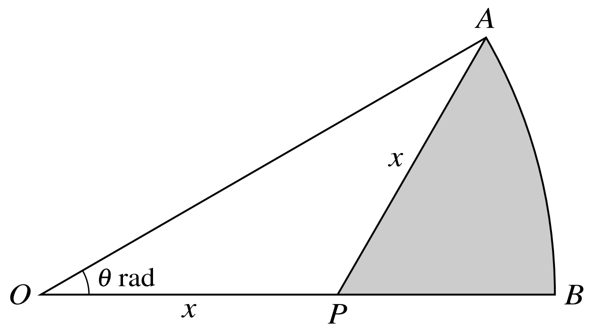

The diagram shows a sector OAB of a circle with centre O. Angle $AOB = \theta$ radians and $OP = AP = x$.

(a) Show that the arc length AB is $2x\theta\cos\theta$. [2]

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(b) Find the area of the shaded region APB in terms of x and θ. [4]

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7 **(a)** **(i)** By first expanding $(\cos\theta + \sin\theta)^2$, find the three solutions of the equation

$$(\cos\theta + \sin\theta)^2 = 1$$

for $0 \leqslant \theta \leqslant \pi$. [3]

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(ii) Hence verify that the only solutions of the equation $\cos\theta + \sin\theta = 1$ for $0 \leqslant \theta \leqslant \pi$ are 0 and $\frac{1}{2}\pi$. [2]

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(b) Prove the identity $\dfrac{\sin\theta}{\cos\theta+\sin\theta}+\dfrac{1-\cos\theta}{\cos\theta-\sin\theta}\equiv\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}$. [3]

(c) Using the results of **(a)(ii)** and **(b)**, solve the equation

$$\frac{\sin\theta}{\cos\theta+\sin\theta}+\frac{1-\cos\theta}{\cos\theta-\sin\theta}=2(\cos\theta+\sin\theta-1)$$

for $0\leqslant\theta\leqslant\pi$. [3]

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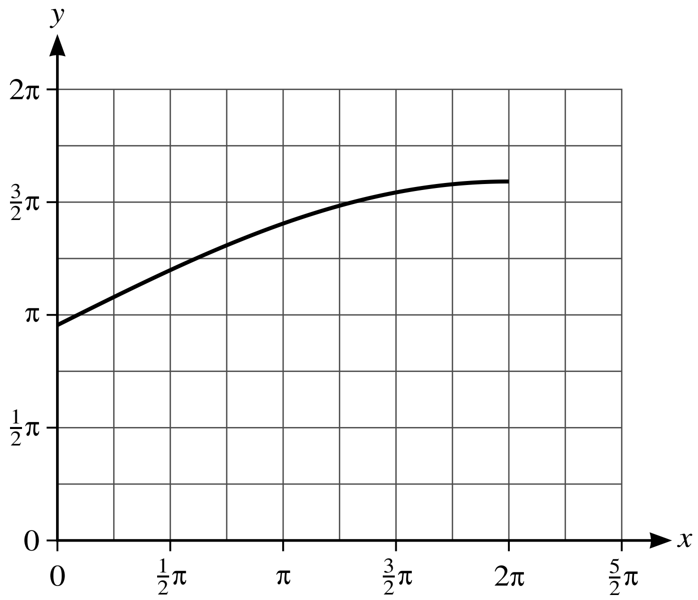

The diagram shows the graph of $y = \mathrm{f}(x)$ where the function f is defined by

$$\mathrm{f}(x) = 3 + 2\sin\tfrac{1}{4}x \text{ for } 0 \leqslant x \leqslant 2\pi.$$

(a) On the diagram above, sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

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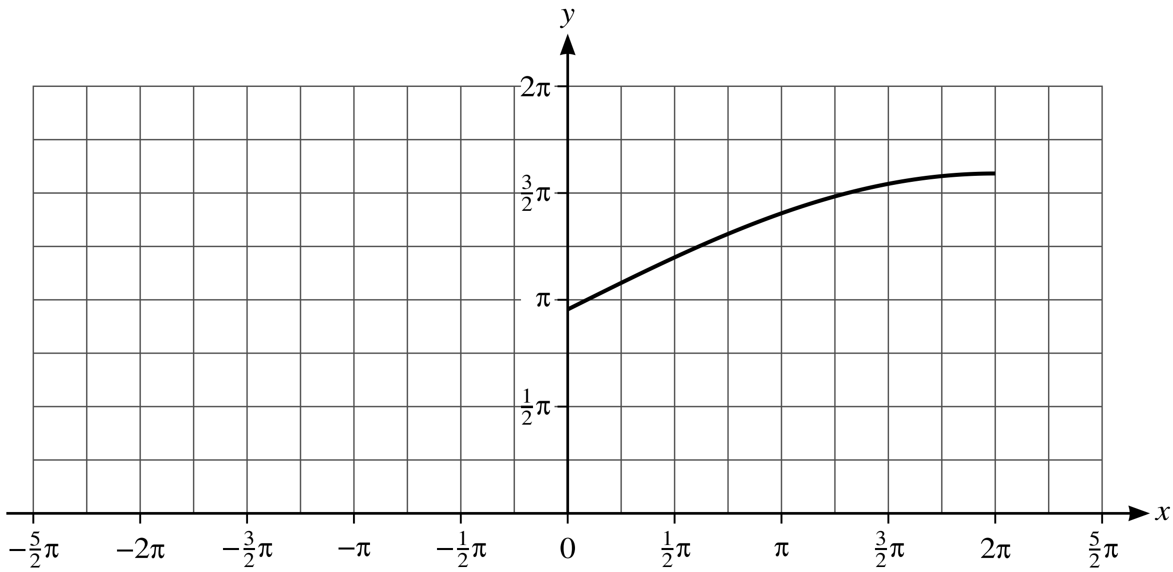
(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2\sin\frac{1}{4}x$ for $-2\pi \leqslant x \leqslant 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

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(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

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9 The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term. [4]

(b) Show that the nth term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the nth term of the other geometric progression. [4]

10 The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other possible value. [5]

(b) For $a = 4$, find the equation of the normal to the circle at P. [4]

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(c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in **(b)**. [4]

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11 The equation of a curve is

$$y = k\sqrt{4x+1} - x + 5,$$

where k is a positive constant.

(a) Find $\frac{dy}{dx}$. [2]

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(b) Find the x-coordinate of the stationary point in terms of k. [2]

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(c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x-axis. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$[y] = \frac{4}{-2}(x-3)^{-3+1}$ or $\frac{4}{-2(x-3)^2}[+c]$	**B1**	OE Allow $\frac{4}{-3+1}$ and $-3+1$ for the power.
	$5 = \frac{4}{-2}(4-3)^{-2} + c$ or $5 = \frac{4}{-2(4-3)^2} + c$ leading to $c =$	**M1**	Correct use of $(4, 5)$ to find c in an integrated expression (defined by the correct power and no extra x's or terms).
	$y = \frac{-2}{(x-3)^2} + 7$ or $y = -2(x-3)^{-2} + 7$	**A1**	OE $-\frac{4}{2}$ must be simplified to **–**2. Condone c = 7 as their final line as long as either y or f(x) = is seen elsewhere. Do not ISW if the result is of the form y = mx+c.
		3	

Question	Answer	Marks	Guidance
2	[Coefficient of $x^4 = p =$] $15a^2$	**B1**	May be seen in an expansion or with x^4.
	[Coefficient of $x^2 = q =$] $54a^2$	**B1**	May be seen in an expansion or with x^2.
	Equating *their* p + *their* q to 276 leading to an equation in a^2 only	**M1**	No x terms and no extra terms. If p and q are not identified then it needs to be clear from the expansion that the appropriate coefficients are being used. $69a^2 = 276$ implies the first 3 marks.
	$a = \pm 2$	**A1**	CAO
		4	

Question	Answer	Marks	Guidance
3(a)	$4(x-3)^2$ seen or $a=4$ and $b=-3$	**B1**	OE Award marks for the correct expression or their values a, b and c. Condone $4(x-3)+p-36=0$ and $4\left(\frac{p}{4}-9\right)$.
	$-36+p$ or $p-36$ seen or $c=p-36$	**B1**	
		2	
3(b)	$p-36>0$ leading to $p>36$ or $24^2-4\times4p\langle0\Rightarrow p\rangle36$ or $36<p$	**B1**	Allow $(36,\infty)$ or $36<p<\infty$. Consider final answer only.
		1	

Question	Answer	Marks	Guidance
4	$[8x^6+215x^3-27=0]$ leading to $(8x^3-1)(x^3+27)[=0]$ **OR** $\dfrac{-215\pm\sqrt{215^2-4.8.-27}}{2.8}$ or $\dfrac{-215\pm\sqrt{47089}}{2.8}$	**M1**	OE If a substitution is used then the correct coefficients must be retained. Condone substitution of $x=x^3$.
	$\frac{1}{8}$, -27	**A1**	Both correct values seen. **SC**: if M0 scored **SC B1** is available for sight of $\frac{1}{8}$ and -27 OE
	$\frac{1}{2}$ or $0.5, -3$	**A1**	**SC**: if M0SCB1 scored then **SCB1** is available for the correct answers and no others. Do not ISW if answers given as a range.
		3	

Question	Answer	Marks	Guidance
5	$\left[\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=\right]\left\{\frac{10}{\frac{3}{2}}x^{\frac{3}{2}}\right\}\left\{-\frac{5}{2\times\frac{5}{2}}x^{\frac{5}{2}}\right\}\ \left[=\frac{20}{3}x^{\frac{3}{2}}-x^{\frac{5}{2}}\right]$	**B1 B1**	B1 for contents of each { } then ISW.
	$=\left(their\,\frac{20}{3}\times 8-32\right)[-0]$	**M1**	Using limit(s) correctly in an integrated expression (defined by one correct power). Minimum acceptable working is their ($\frac{160}{3}-32$).
	[Area of shaded region =] $\frac{64}{3}$, $21\frac{1}{3}$ or 21.3[333…]	**A1**	Condone the presence of π for the first 3 marks. Condone using the limits the wrong way around for the M mark and if -21.3 is corrected to 21.3 allow the A mark. **SC**: if M0 scored **SCB1** is available for correct final answer If $\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=21.3$ and no integration seen B1 only.
		4	

Question	Answer	Marks	Guidance
6(a)	$\frac{1}{2}OA = x\cos\theta$ or $\frac{OA}{\sin(\pi-2\theta)} = \frac{x}{\sin\theta}$ or $OA^2 = x^2 + x^2 - 2x^2\cos(\pi - 2\theta)$ or $x^2 = r^2 + x^2 - 2rx\cos\theta$ or other valid method.	***B1**	Correct expression containing $\frac{1}{2}OA$, OA or OA^2 (allow *p*, *a* or *r* for *OA*) containing only terms with *x* and θ but not just $OA = 2x\cos\theta$. Do not condone $\sin\pi - 2\theta$ until missing brackets recovered or cos(180 −2θ) until it becomes $-cos2\theta$ etc.
	$OA = 2x\cos\theta$ leading to Arc length = $2x\theta\cos\theta$	**DB1**	AG Complete correct method showing all necessary working. Condone $2x\cos\theta\times\theta$.
		2	If B0 but www then **SCB1** for $OA = 2x\cos\theta$ leading to Arc length = $2x\theta\cos\theta$.
6(b)	Sector area = $\frac{1}{2}(2x\cos\theta)^2\times\theta$	**M1**	OE Using sector formula with a correct OA. Condone $cos\theta^2$ for $cos^2\theta$ and missing brackets.
	Triangle area = $\frac{1}{2}\times 2x\cos\theta x\sin\theta$ OR $\frac{1}{2}x^2\sin(\pi-2\theta)$	**M1**	Using a correct triangle formula for the correct triangle. Condone missing brackets and 180 for π.
	[Area *APB* =] *Their* sector area – *their* triangle area	**M1**	Both expressions must be areas involving terms with x^2 and θ only. Condone missing brackets and 180 for π for the triangle. Condone calling the sector a segment.
	[Area *APB* =] $\frac{1}{2}(2x\cos\theta)^2\times\theta - \frac{1}{2}x^2\sin(\pi-2\theta)$ $[= x^2(2\theta\cos^2\theta - \frac{1}{2}\sin 2\theta)$ or $x^2\cos\theta(2\theta\cos\theta - \sin\theta)$]	**A1**	OE A correct expression. Mark the first unsimplified result of subtraction and ISW any incorrect ‘simplifications’.
		4	

Question	Answer	Marks	Guidance
7(a)(i)	$\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 1$ leading to $2\sin\theta\cos\theta = 0$ or $\sin 2\theta = 0$	***B1**	Or arriving at $\cos\theta = 0$ or $\sin\theta = 0$ or $\tan\theta = 0$ after first expanding and www.
	$[\theta =]\ 0,\ \frac{\pi}{2},\pi$	**DB 2,1,0**	B2 for three correct answers only. B1 for two correct answers and one incorrect or 3 correct answers plus other values in the range. **SC DB1** for correct 3 answers in degrees and no others. Ignore extras outside of the range and allow decimal equivalents.
		3	Verifying 3 answers rather than expanding and solving 0/3.
7(a)(ii)	$\cos 0 + \sin 0 = [1+0=]1$ and $\cos\frac{\pi}{2} + \sin\frac{\pi}{2}[=0+1] = 1$	**B1**	Checking both correct values. Do not allow solving an equation. Condone use of 90 degrees.
	$\cos\pi + \sin\pi[=-1+0] = -1$ or $\neq 1$	**B1**	www
		2	

Question	Answer	Marks	Guidance
7(b)	$\dfrac{(\cos\theta-\sin\theta)\sin\theta+(\cos\theta+\sin\theta)(1-\cos\theta)}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}$	**M1**	Correct common denominator and correct products in the numerator and no missing terms. Correct factors in the denominator can be implied by $\cos^2\theta-\sin^2\theta$. Condone brackets missing if recovered.
	$=\dfrac{\cos\theta\sin\theta-\sin^2\theta+\cos\theta-\cos^2\theta+\sin\theta\ -\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$	**A1**	
	$=\dfrac{\sin\theta+\cos\theta-\cos^2\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta}=\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}$	**A1**	AG Clear evidence of using $sin^2\theta+cos^2\theta=1$ in either the numerator or denominator. Condone c, s and/or omission of θ. Working from both sides of the identity and correctly arriving at the same expression can score M1A1. A final statement is then required for the A1.
		3	

Question	Answer	Marks	Guidance
7(c)	$\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}=2(\cos\theta+\sin\theta-1)$ leading to $1=2(1-2\sin^2\theta)$	*M1	Replacing LHS with the expression from **(b)** and attempting to simplify i.e. condone omission of $(\cos\theta+\sin\theta-1)=0$ at this stage. M0 for $0=2(1-2\sin^2\theta)$
	$k\sin^2\theta=1$ or 3 leading to $\sin\theta=[\pm]\sqrt{\dfrac{1\text{ or }3}{k}}$ $\left[4\sin^2\theta=1 \text{ leading to } \sin\theta=\pm\dfrac{1}{2}\right]$	DM1	Dividing by k and taking the square root of a positive value < 1. This mark can be implied by the solutions $\dfrac{1}{6}\pi, \dfrac{5}{6}\pi$.
	Solutions $0, \dfrac{1}{6}\pi, \dfrac{1}{2}\pi, \dfrac{5}{6}\pi$	A1	Allow 0, 0.524, 1.57, 2.62 AWRT. If M0 **SCB1** for $(\cos\theta+\sin\theta-1)=0 \Rightarrow 0, \dfrac{1}{2}\pi$. If M0 **SCB1** for all four correct answers and no others. Ignore answers outside of the range. Answers in degrees A0.
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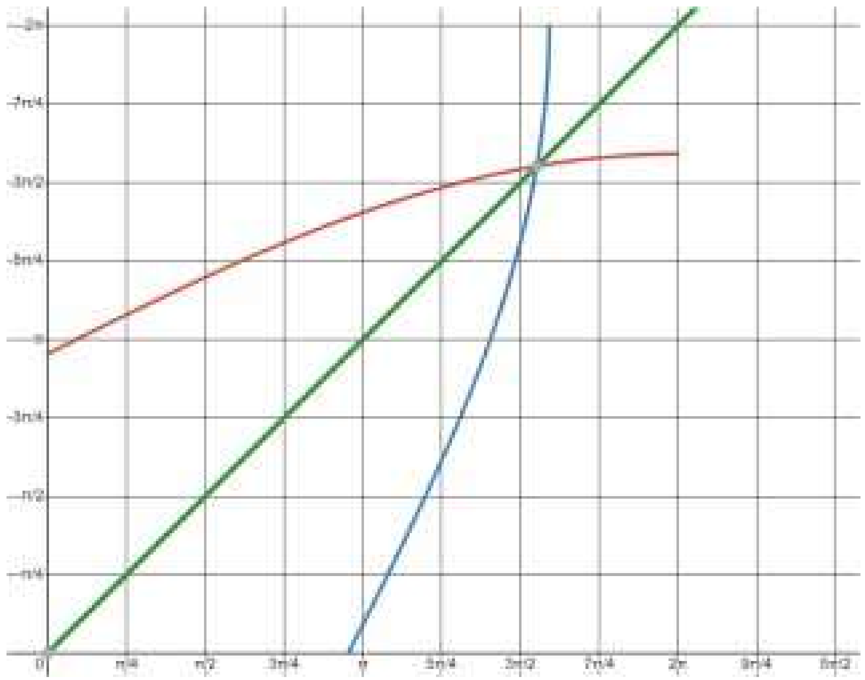
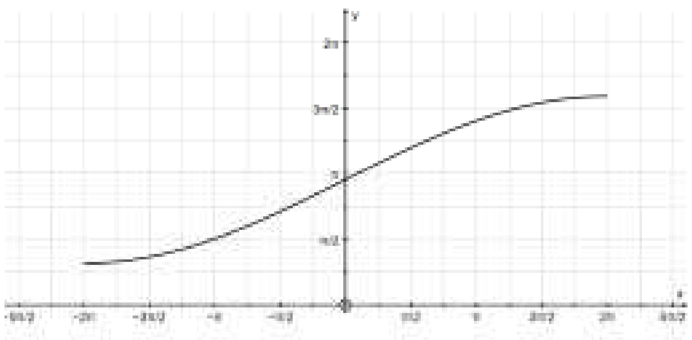
Question	Answer	Marks	Guidance
8(a)		*B1	The line $y = x$ correctly drawn. Can be implied by reasonably correct graph of $\mathrm{f}^{-1}(x)$.
		DB1	Fully correct (needs to reach $y = 2\pi$ and x-axis and cross the line $y = x$ in the correct squares).
		2	

Question	Answer	Marks	Guidance
8(b)	$y = 3 + 2\sin\frac{1}{4}x$ leading to $\sin\frac{1}{4}x = \frac{y \pm 3}{2}$	**M1**	Attempting to arrive at an expression for $\sin\frac{1}{4}x$; condone $\pm$ sign errors. Variables may be interchanged initially. M1 not implied by $x = \frac{y \pm 3}{2\sin\frac{1}{4}}$.
	$x = 4\sin^{-1}\left(\frac{y-3}{2}\right)$ leading to [$f^{-1}(x)$or y =] $4\sin^{-1}\left(\frac{x-3}{2}\right)$	**A1**	ISW Must clearly be $\sin^{-1}\left(\frac{x-3}{2}\right)$ NOT $\frac{\sin^{-1}(x-3)}{2}$. Allow $\left(\frac{3-x}{-2}\right)$ but not $\div\frac{1}{4}$.
		2	
8(c)		**B1**	Continuing given graph from y intercept to -2π. The correct shape needed between 0 and -2π, including starting to level off (gradient in the final two squares needs to be reducing) as -2π is approached. The y co-ordinate at -2π must be in the correct square.
	Yes it does have an inverse, because the graph is always increasing OR because it is one-one OR because it passes the horizontal line test OR it is not a many to one [function].	**B1 FT**	If there is no graph to the left of the y axis, no mark is available. FT an incorrect graph and if the answer is now 'No' provide an appropriate reason.
		2	

Question	Answer	Marks	Guidance
8(d)	**{ } indicates different elements throughout.**		
	{Stretch} {factor 4} {in x-direction}	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched. Allow any mention of x-axis, horizontally or y-axis invariant. Wavelength or period increased by a factor of 4 for B2 or by 4 for B1.
	{Stretch} {factor 2} {in y-direction}	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched. Allow any mention of y-axis, vertically or x-axis invariant. Allow y 'co-ordinates' doubled or amplitude doubled for B2.
	{Translation} $\begin{pmatrix} \{0\} \\ \{3\} \end{pmatrix}$	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. Allow shift. Any mention of y axis, y-direction or vertically implies $\{0\}$, so shift by 3 vertically is B2, but shift by a factor of 3 vertically or a translation of 3 'up' is B1.
		6	After scoring B2, B2 the final transformation can only be awarded B2 if the order is fully correct i.e. the translation must not be applied before the y stretch. If all correct except the order award B2B2B1.

Question	Answer	Marks	Guidance
9(a)	$\left[ar = 16, \dfrac{a}{1-r} = 100\right]$ leading to $a = \dfrac{16}{r}$ and $a = 100(1-r)$	**B1**	Rearranging two algebraic statements to give $a =$. These can be implied by a correct equation in one variable.
	$100(1-r)r = 16$ leading to $100r^2 - 100r + 16[=0]$	***M1**	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors only.
	$(5r-4)(5r-1) = 0$ OR $\dfrac{25 \pm \sqrt{25^2 - 4.25.4}}{2.25}$ leading to $r = \left[\dfrac{4}{5} \text{ or } \dfrac{1}{5}\right]$	**DM1**	Condone $(5r-4)(5r-1)$ following $100r^2 - 100r + 16$.
	$a = 20$, $a = 80$	**A1**	**SC**: if DM0 scored **SCB1** is available for sight of 20 and 80.
	Alternative Method for Question 9(a)		
	$\left[ar = 16, \dfrac{a}{1-r} = 100\right]$ leading to $r = \dfrac{16}{a}$ and $r = \dfrac{100-a}{100}$	**B1**	Rearranging two algebraic statements to give $r =$. These can be implied by a correct equation in one variable.
	$1600 = 100a - a^2$ leading to $a^2 - 100a + 1600[=0]$	***M1**	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors and 160 instead of 1600 only.
	$(a-20)(a-80) = 0$ OR $\dfrac{100 \pm \sqrt{100^2 - 4.1600}}{2}$	**DM1**	
	$a = 20$, $a = 80$	**A1**	**SC**: if DM0 scored **SCB1** is available for sight of 20 and 80.
		4	

Question	Answer	Marks	Guidance
9(b)	$r = \frac{4}{5}\ ,\ \frac{1}{5}$	**B1**	OE SOI
	$[u_n =]their\,20 \times their\left(\frac{4}{5}\right)^{n-1}$ $[v_n =]their\,80 \times their\left(\frac{1}{5}\right)^{n-1}$	**B1FT**	2 expressions for the nth term FT *their* values from part **(a)** if $\lvert r \rvert$ less than 1.
	Method 1 for final 2 marks		
	$20 \times \left(\frac{1}{5}\right)^{n-1} \times 4^{n-1}$	**M1**	Correctly separating the numerator and denominator of *their* $\left(\frac{4}{5}\right)^{n-1}$ or one correct step towards the solution eg $u_n = 80 \times \frac{4^{n-2}}{5^{n-1}}$.
	$u_n = \frac{1}{4} \times 80 \times \left(\frac{1}{5}\right)^{n-1} \times 4^{n-1} = 4^{n-2} \times 80 \times \left(\frac{1}{5}\right)^{n-1} = 4^{n-2} \times v_n$	**A1**	AG Given result clearly shown
	Method 2 for final 2 marks		
	$\frac{20 \times 0.8^{n-1}}{80 \times 0.2^{n-1}} = \frac{1}{4} \times 4^{n-1}$	**M1**	Dividing two nth terms of the correct format and simplifying their terms in r.
	$= 4^{-1} \times 4^{n-1} = 4^{n-2}$	**A1**	AG
		4	

Question	Answer	Marks	Guidance
10(a)	$(x-a)^2+\left(\frac{1}{2}x+6-3\right)^2=20$ or using $x=2y-12$	*M1	Obtaining an unsimplified equation in x or y only.
	$\frac{5}{4}x^2+(3-2a)x+a^2-11[=0]$	A1	OE e.g. $5x^2+4(3-2a)x+4a^2-44$ Rearranging to get a correct 3-term quadratic on one side. Condone terms not grouped together. $5y^2-y(54+4a)+133+a^2+24$.
	$(3-2a)^2-4\times\frac{5}{4}(a^2-11)[=0]$	DM1	OE Using b^2-4ac on *their* 3 term quadratic $[=0]$.
	Method 1 for final 2 marks		
	Using $a=4$: $(3-8)^2-5(5)=0$	A1	Clearly substituting $a=4$.
	$a=-16$	B1	Condone no method shown for this value.
	Method 2 for final 2 marks		
	$-a^2-12a+64=0 \Rightarrow (a-4)(a+16)=0 \Rightarrow a=4$	A1	AG Full method clearly shown.
	$a=-16$	B1	Condone no method shown for this value.
		5	If M0, **SCB1** available for substituting $a=4$, finding P(2, 7) and verifying that $CP^2=20$.

Question	Answer	Marks	Guidance
10(b)	Centre (4, 3) identified or used or the point P is (2, 7)	**B1**	
	∴ gradient of normal $=-2$	**B1**	SOI
	Forming normal equation using their gradient (not 0.5) and their centre or P	**M1**	Condone use of $(\pm 4, \pm 3)$.
	$\frac{y-3}{(x-4)}=-2$ or $y-7=-2(x-2)$	**A1**	OE Condone $\mathrm{f}(x)=$.
		4	

Question	Answer	Marks	Guidance
10(c)	**Method 1 for Question 10(c)**		
	Diameter: $y-3=\frac{1}{2}(x-4)$ $\left[\text{leading to } y=\frac{1}{2}x+1\right]$ Or $2(x-4)+2(y-3)\frac{dy}{dx}=0$ $\left[\text{leading to } y=\frac{1}{2}x+1\right]$	*M1	Using gradient $\frac{1}{2}$ with their centre. By implicit differentiation.
	$(x-4)^2+\left(\frac{1}{2}x+1-3\right)^2=20$ $\left[\frac{5}{4}x^2-10x=0\right]$	DM1	Obtaining an unsimplified equation in x or y only. $[y^2-6y+5=0]$.
	$x=0$ or 8, $y=1$ or 5 [(0, 1) and (8, 5)]	A1	Correct co-ordinates for both points. Condone no method shown for solution.
	Equations are $y-1=-2x$ and $y-5=-2(x-8)$	A1	$2x+y=1$ and $2x+y=21$.
	Method 2 for Question 10(c)		
	Coordinates of points at which tangents meet curve are (4+4, 3+2) = (8, 5) and (4 – 4, 3 – 2) = (0, 1)	*M1 A1	Vector approach using their centre and gradient = 0.5 . Condone answers only with no working.
	Equations are $y-5=-2(x-8)$ and $y-1=-2x$	DM1 A1	Forming equations of tangents using *their* (0, 1) and (8, 5).
	Method 3 for Question 10(c)		
	$(x-4)^2+(-2x+c-3)^2=20$ $\left[5x^2+(4-4c)x+(c-3)^2-4=0\right]$	*M1	Obtaining an unsimplified equation in x only using equation of circle with $y=-2x+c$.
	$(4-4c)^2-20\left((c-3)^2-4\right)[=0]$ [leading to $-4c^2-32c+120c+16-100=0$]	DM1	Using $b^2-4ac[=0]$.

Question	Answer	Marks	Guidance
10(c)	$4c^2 - 88c + 84[= 0]$ [leading to $c^2 - 22c + 21 = 0$]	**A1**	
	$c = 21$ and $c = 1$ or $y = -2x + 21$ **and** $y = -2x + 1$	**A1**	Condone no method shown for solution.
		4	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \left\{ k\frac{1}{2}(4x+1)^{-\frac{1}{2}} \right\}\{\times 4\}\{-1\}$	**B 2,1,0**	OE e.g. $2k(4x+1)^{-\frac{1}{2}} - 1$ B2 Three correct unsimplified **{ }** and no others. B1 Two correct **{ }** or three correct **{ }** and an additional term e.g. +5. B0 More than one error.
		2	
11(b)	$2k(4x+1)^{-\frac{1}{2}} - 1 = 0$ leading to $(4x+1)^{\frac{1}{2}} = 2k$ or $\frac{2k}{(4x+1)^{\frac{1}{2}}} = 1$	**M1**	OE Equating their $\frac{dy}{dx}$ of the form $ak(4x+1)^{-\frac{1}{2}} - 1$ where a = 2 or 0.5, to 0 and dealing with the negative power correctly including k not multiplied by $(4x+1)^{\frac{1}{2}}$.
	$x = \frac{4k^2 - 1}{4}$	**A1**	CAO OE simplified expression ISW.
		2	

Question	Answer	Marks	Guidance
11(c)	$2\times 10.5(4x+1)^{-\frac{1}{2}}-1=2$	**M1**	Putting k= 10.5 into their $\frac{dy}{dx}$ and equating to 2.
	$7=(4x+1)^{\frac{1}{2}}$ leading to $4x+1=49$ leading to $x=12$	**A1**	If M1 earned **SCB1** available for $x=\frac{33}{64}$ from $a=\frac{1}{2}$.
	$y=\left[10.5\sqrt{4x+1}-x+5=\right]66.5$ [leading to (12, 66.5)]	**A1**	
	$y-66.5=-\frac{1}{2}(x-12)$	**A1**	OE
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME										
CENTRE NUMBER						CANDIDATE NUMBER				

* 2 7 8 0 8 2 1 7 9 6 *

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

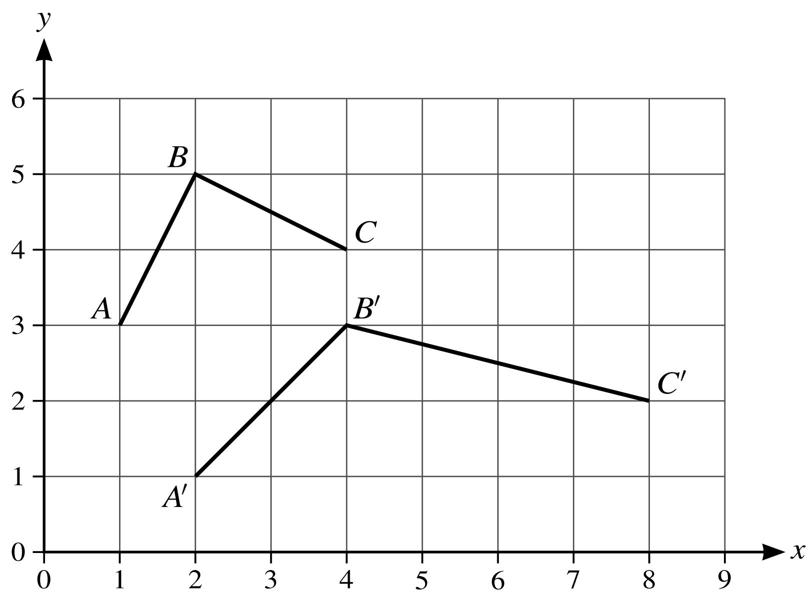
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

1



The diagram shows the graph of $y = \mathrm{f}(x)$, which consists of the two straight lines AB and BC. The lines $A'B'$ and $B'C'$ form the graph of $y = \mathrm{g}(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = \mathrm{f}(x)$.

State fully the two transformations. [4]

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2 The function f is defined for $x \in \mathbb{R}$ by $\mathrm{f}(x) = x^2 - 6x + c$, where c is a constant. It is given that $\mathrm{f}(x) > 2$ for all values of x.

Find the set of possible values of c. [4]

3 **(a)** Give the complete expansion of $\left(x + \dfrac{2}{x}\right)^5$. [2]

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(b) In the expansion of $(a + bx^2)\left(x + \dfrac{2}{x}\right)^5$, the coefficient of x is zero and the coefficient of $\dfrac{1}{x}$ is 80.

Find the values of the constants a and b. [4]

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4 **(a)** Show that the equation

$$3\tan^2 x - 3\sin^2 x - 4 = 0$$

may be expressed in the form $a\cos^4 x + b\cos^2 x + c = 0$, where a, b and c are constants to be found. [3]

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(b) Hence solve the equation $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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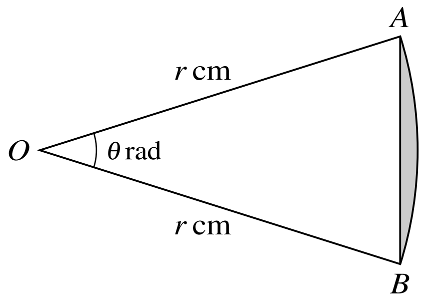
5 A circle has equation $(x-1)^2 + (y+4)^2 = 40$. A line with equation $y = x - 9$ intersects the circle at points A and B.

(a) Find the coordinates of the two points of intersection. [4]

(b) Find an equation of the circle with diameter AB. [3]

6



The diagram shows a sector OAB of a circle with centre O and radius r cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6 cm and that the area of the sector OAB is $76.8\,\text{cm}^2$.

(a) Find the area of the shaded region. [5]

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(b) Find the perimeter of the shaded region. [2]

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7 The function f is defined by $\mathrm{f}(x) = 2 - \dfrac{5}{x+2}$ for $x > -2$.

(a) State the range of f. [1]

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(b) Obtain an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

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The function g is defined by $g(x) = x + 3$ for $x > 0$.

(c) Obtain an expression for $fg(x)$ giving your answer in the form $\frac{ax + b}{cx + d}$, where a, b, c and d are integers. [3]

8 A progression has first term a and second term $\dfrac{a^2}{a+2}$, where a is a positive constant.

(a) For the case where the progression is geometric and the sum to infinity is 264, find the value of a. [5]

(b) For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480. [5]

9 A curve which passes through $(0, 3)$ has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = 1 - \dfrac{2}{(x-1)^3}$.

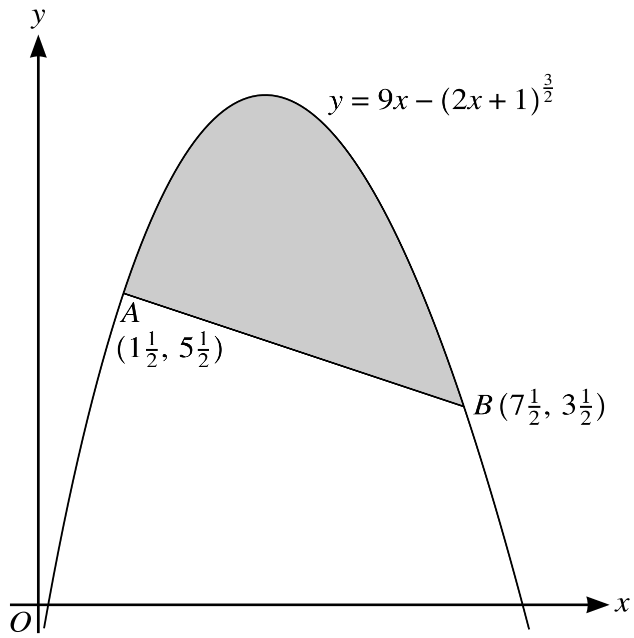
(a) Find the equation of the curve. [4]

The tangent to the curve at $(0,\ 3)$ intersects the curve again at one other point, P.

(b) Show that the x-coordinate of P satisfies the equation $(2x+1)(x-1)^2 - 1 = 0$. [4]

(c) Verify that $x = \frac{3}{2}$ satisfies this equation and hence find the y-coordinate of P. [2]

10



The diagram shows the points $A\left(1\frac{1}{2},\ 5\frac{1}{2}\right)$ and $B\left(7\frac{1}{2},\ 3\frac{1}{2}\right)$ lying on the curve with equation $y = 9x - (2x+1)^{\frac{3}{2}}$.

(a) Find the coordinates of the maximum point of the curve. [4]

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(b) Verify that the line AB is the normal to the curve at A. [3]

(c) Find the area of the shaded region. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **16** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	{Translation} $\begin{pmatrix} \{0\} \\ \{-2\} \end{pmatrix}$	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
	{Stretch} {[scale] factor 2} {parallel to x-axis}	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct.
		4	Transformations can be in either order.

Question	Answer	Marks	Guidance
2	$x^2-6x+c>2$ leading to $(x-3)^2-9+c>2$	**M1 A1**	M1 for completion of the square with an equation or in equality with the '2'.
	$c>11-(x-3)^2$ and $(x-3)^2\geq 0$	**M1**	SOI
	$c>11$	**A1**	
	Alternative Method 1		
	$\frac{dy}{dx}=2x-6=0$	**M1**	M1 for differentiating and setting $\frac{dy}{dx}=0$.
	$x=3$	**A1**	
	When $x=3$, $y=9-18+c$	**M1**	
	$[-9+c>2]$ $c>11$	**A1**	
	Alternative Method 2		
	$x^2-6x+c>2$ leading to $x^2-6x+c-2[>0]$ then use of 'b^2-4ac'	**M1**	
	$36-4(1)(c-2)<0$	**M1 A1**	OE Must be correct inequality for M1.
	$c>11$	**A1**	
		4	

Question	Answer	Marks	Guidance
3(a)	$x^5+10x^3+40x+\frac{80}{x}+\frac{80}{x^3}+\frac{32}{x^5}$ or $x^5+10x^3+40x+80x^{-1}+80x^{-3}+32x^{-5}$	**B2, 1, 0**	B2, all terms correct, B1 5 terms correct. Terms must be simplified. Lists of terms allowed.
		2	
3(b)	*their* $40\times a+\left(\textit{their}\text{ coefficient of }x^{-1}\right)\times b=0$	**M1**	Coefficients of a and b must be non-zero, allow x's so long as they are dealt with correctly.
	$\left(\textit{their}\text{ coefficient of }x^{-1}\right)\times a+\left(\textit{their}\text{ coefficient of }x^{-3}\right)\times b=80$	**M1**	Coefficients of a and b must be non-zero, allow x's as long as they are dealt with correctly.
	$a=2 \quad b=-1$	**A1 A1**	Dependent on both M marks, may be seen without working.
		4	

Question	Answer	Marks	Guidance
4(a)	$3\sin^2 x-3\sin^2 x\cos^2 x-4\cos^2 x\ [=0]$	**M1**	Replace $\tan^2 x$ with $\frac{\sin^2 x}{\cos^2 x}$ and multiply by $\cos^2 x$.
	$3\left(1-\cos^2 x\right)-3\left(1-\cos^2 x\right)\cos^2 x-4\cos^2 x\ [=0]$	**M1**	Replace $\sin^2 x$ by $1-\cos^2 x$ twice.
	$3\cos^4 x-10\cos^2 x+3=0$ or $-3\cos^4 x+10\cos^2 x-3=0$	**A1**	Or multiple of these equations.
		3	

Question	Answer	Marks	Guidance
4(b)	$(3\cos^2 x - 1)(\cos^2 x - 3)$ $[=0]$	**M1**	OE, using *their* equation in the given form. Allow unusual notation if meaning is clear.
	$\cos x = [\pm]\frac{1}{\sqrt{3}}$	**A1**	SOI Answer only **SC B1**.
	54.7°,	**A1**	
	125.3°	**A1 FT**	Only other answer and must be from correct factorisation for A1. FT for $180° -$ *their* first answer. Answers only **SC B1**, **SC B1 FT**.
		4	

Question	Answer	Marks	Guidance
5(a)	$(x-1)^2 + (x-9+4)^2 = 40$	**M1**	Substitute line into circle.
	$x^2 - 6x - 7$ $[=0]$ leading to $(x+1)(x-7)$ $[=0]$	**M1**	Simplify to 3-term quadratic and factorise OE.
	(–1, –10), (7, –2) or $x = -1$ and 7, $y = -10$ and –2	**A1 A1**	Answers only **SC B1**, **SC B1** but must see a correct quadratic equation.
		4	

Question	Answer	Marks	Guidance
5(b)	[C is mid-point =] $\left(\frac{their\ x_1 + their\ x_2}{2}, \frac{their\ y_1 + their\ y_2}{2}\right)$	M1	Expect (3, –6).
	Radius = $\sqrt{(their\ x - their\ 3)^2 + (their\ y - their\ (-6))^2}$ OR $their \sqrt{\left((7-(-1))^2 + (-2-(-10))^2\right)} / 2$	M1	Expect $\sqrt{32}$.
	$(x-3)^2 + (y+6)^2 = 32$	A1	OE
		3	

Question	Answer	Marks	Guidance
6(a)	$\frac{½r^2\theta}{r\theta} = \frac{76.8}{9.6}$ or $\frac{1}{2}\left(\frac{9.6^2}{\theta^2}\right)\theta = 76.8$	M1	Eliminate θ *or* r using correct formulae SOI.
	$r = 16$	A1	
	$\theta = 0.6$	A1	Accept 34.4°
	ΔOAB = ½ × *their* 16^2 × sin *their* 0.6	M1	Allow Segment = 76.8 –½ × *their* 16^2 × sin *their* 0.6. Expect 72.27 .
	[Area = 76.8 – 72.27 =] 4.53	A1	AWRT
		5	
6(b)	$AB = 2 \times 16 \times \sin 0.3$ OR $AB^2 = 16^2 + 16^2 - 2 \times 16^2 \cos 0.6$	M1	Any valid method with *their* r, θ. Expect *AB* = 9.46.
	Perimeter = 9.6 + 9.46 = 19.1	A1	AWRT
		2	

Question	Answer	Marks	Guidance
7(a)	$[y] < 2$ OR $[f(x)] < 2$	**B1**	OE e.g. $f < 2, (-\infty, 2), -\infty < f[x] < 2$. Do not accept $x < 2$ or $f(x) \leqslant 2$.
		1	
7(b)	$y = 2 - \dfrac{5}{x+2}$ leading to $y(x+2) = 2(x+2) - 5$ leading to $xy + 2y = 2x - 1$	**M1**	or $\dfrac{5}{x+2} = 2 - y$ (allow sign errors).
	$2y + 1 = 2x - xy$ leading to $2y + 1 = x(2 - y)$	**DM1**	or $\dfrac{5}{2-y} = x + 2$ (allow sign errors).
	$x = \dfrac{2y+1}{2-y} \rightarrow f^{-1}(x) = \dfrac{2x+1}{2-x}$	**A1**	OE or $y = \dfrac{5}{2-x} - 2$.
	Domain is $x < 2$	**B1 FT**	FT on the numerical part of *their* range from part **(a),** including $x \neq 2$ not penalized. No FT for $x \in \mathcal{R}, x = k, x \neq k$.
		4	
7(c)	$fg(x) = 2 - \dfrac{5}{x+3+2}$	**B1**	
	$= \dfrac{2(x+5)-5}{x+5}$ or $\dfrac{2(x+5)}{x+5} - \dfrac{5}{x+5}$	**M1**	Use of *their* common denominator.
	$= \dfrac{2x+5}{x+5}$	**A1**	
		3	

Question	Answer	Marks	Guidance
8(a)	$r = \dfrac{a}{a+2}$	**B1**	OE SOI
	$\dfrac{a}{1-\dfrac{a}{a+2}} = 264$	**M1**	Use of S∞ formula.
	$\dfrac{a(a+2)}{a+2-a} = 264$ leading to $\dfrac{a(a+2)}{2} = 264$ leading to $a^2 + 2a - 528\ [=0]$	**M1***	Process to a 3 term quadratic or a 3 term cubic. May contain terms on LHS and RHS.
	$(a-22)(a+24)\ [=0]$	**DM1**	Attempt to solve.
	$a = 22$ (only)	**A1**	22 without working **SC DB1** (dep on 2nd M1).
		5	
8(b)	$d = \dfrac{6^2}{6+2} - 6 = -\dfrac{3}{2}$	**B1**	
	$\dfrac{n}{2}\left\{12 + (n-1)\left(\dfrac{-3}{2}\right)\right\}[<] -480$	**M1***	Forming an inequation with *their* numerical *d*. May use an equality.
	$[3](n^2 - 9n - 640)\ [> 0]$	**A1**	OE May contain terms on LHS and RHS.
	$[n =]\ \dfrac{9 \pm \sqrt{81+2560}}{2}$	**DM1**	OE. Expect 30.19 . Working for solution must be shown.
	31 only	**A1**	Must come from a correct first inequality (or an equality). 31 no working **SC DB1** (dep on correct quadratic and correct inequality/equality).
		5	

Question	Answer	Marks	Guidance
9(a)	$\left[y=\right]\{x\}\ \{+(x-1)^{-2}\ \}\ \ [+c]$	B1 B1	May be unsimplified.
	Sub $x = 0$, $y = 3$ leading to $3 = 0 + 1 + c$	M1	Substitution into an integral, expect $c = 2$.
	$y=x+(x-1)^{-2}+2$ or $f(x)=x+(x-1)^{-2}+2$	A1	$\dfrac{-2}{(-2)(x-1)^2}$ or $\dfrac{-2(x-1)^{-2}}{-2}$ must be simplified.
		4	
9(b)	[Gradient of tangent =] $f'(0) = 3$	B1	
	Equation of tangent is $y-3=$ *their* gradient at $x=0(x-0)$	M1*	Expect $y=3x+3$, normal gets M0.
	Intersection given by $3x+3=x+(x-1)^{-2}+2$	DM1	FT *their* equation from part **(a)**.
	$2x+1=\dfrac{1}{(x-1)^2}\rightarrow(2x+1)(x-1)^2-1=0$ or solve equation before given form reached and show solution ($x=3/2$) satisfies given result	A1	WWW AG
		4	
9(c)	Substitute $x=\dfrac{3}{2}$ leading to $(2x+1)(x-1)^2-1$ leading to $4\times¼-1=0$. Hence $x=\dfrac{3}{2}$ If shown in **(b)** must be referenced here (in part **(c)**)	B1	Evaluation of each bracket must be shown. Allow $\left(\dfrac{1}{2}\right)^2$ for second bracket. Solution of $(2x+1)(x-1)^2-1=0$ is acceptable.
	When $x=\dfrac{3}{2}$ $y=7½$	B1	
		2	

Question	Answer	Marks	Guidance
10(a)	$\left[\frac{dy}{dx}=\right]\{9\}+\left\{-\frac{3}{2}(2x+1)^{1/2}\times 2\right\}$	**B1, B1**	Including '+c' makes the second term B0.
	$9-3(2x+1)^{1/2}=0$ leading to $2x+1=9$	**M1**	Set differential to zero and solve by squaring SOI. Beware $9^2-3^2(2x+1)=0$ M0A0. $2\boldsymbol{x}+1=\sqrt{3}\; or\; 2x+1=\pm 9$ get M0.
	Max point = (4, 9)	**A1**	WWW y = 9 must come from original equation.
		4	
10(b)	When x = 1½, shows substitution or $\frac{dy}{dx}=3$	**M1**	Substituting x = 1½ into their $\frac{dy}{dx}$.
	Gradient of AB is $\frac{5½-3½}{1½-7½}\left[=\frac{-1}{3}\right]$	**M1**	Substituting into a correct expression for m_{AB}.
	$-\frac{1}{3}\text{x}3=-1$. [Hence AB is the normal]	**A1**	
	Alternative method for Question 10(b)		
	When x = 1½ $\frac{dy}{dx}=3$,[perpendicular gradient is -1/3]	**M1**	
	Perpendicular through A has equation $y=\frac{-x}{3}+6$ which contains B(7.5,3.5) leading to AB is a normal to the curve at A	**M1** **A1**	
		3	

Question	Answer	Marks	Guidance
10(c)	$\left\{\frac{9x^2}{2}\right\}+\left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}\right\}$	B1 B1	Integrating y with respect to x.
	$\left\{\frac{9}{2}7.5^2-\frac{1}{5}(2\times 7.5+1)^{2.5}\right\}-\left\{\frac{9}{2}1.5^2-\frac{1}{5}(2\times 1.5+1)^{2.5}\right\}$ or $\left(\frac{9}{2}\times\frac{225}{4}-\frac{1024}{5}\right)-\left(\frac{81}{8}-\frac{32}{5}\right)$ or $\frac{1933}{40}-\frac{149}{40}$ or $48.325-3.725$	M1	OE Apply limits 1½ to 7½ to an integral. Working must be seen. Expect 44.6 .
	$\frac{1}{2}\left(5\frac{1}{2}+3\frac{1}{2}\right)\times 6$ or $\int_{\frac{3}{2}}^{\frac{15}{2}}(\frac{-1}{3}x+6)\mathrm{d}x=$ $\left(\frac{-1}{6}\times\left(\frac{15}{2}\right)^2+6\times\frac{15}{2}\right)-\left(\frac{-1}{6}\times\left(\frac{3}{2}\right)^2+6\times\frac{3}{2}\right)$ or $\frac{285}{8}-\frac{69}{8}$ $[=27]$	B1	SOI Area of trapezium. May be seen combined with the area under the curve integral.
	[Shaded area = 44.6 – 27 =] 17.6	A1	**SC B1** if no substitution of the limits seen.
		5	

Question	Answer	Marks	Guidance
10(c)	**Alternative method for Question 10(c)**		
	$A = \int_{\frac{3}{2}}^{\frac{15}{2}} ((9x - (2x+1)^{\frac{3}{2}}) - \left(\frac{-1}{3}x + 6\right))\mathrm{d}x = \int_{\frac{3}{2}}^{\frac{15}{2}} ((\frac{28}{3}x - (2x+1)^{\frac{3}{2}} - 6)\mathrm{d}x$	**M1**	Finding the equation of AB and subtracting from the equation of the curve.
	$= \left\{\frac{28}{3\times 2}x^2 - 6x\right\} + \left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}\right\}$	**A1 A1**	
	$\frac{127}{10} - \frac{-49}{10}$	**M1**	Apply limits 1½ to 7½ to an integral. Working must be seen.
	17.6	**A1**	**SC B1** if no substitution of limits seen.
		5	

Cambridge Assessment International Education

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MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

JC23 06_9709_21/RP

1 Use logarithms to solve the equation $12^x = 3^{2x+1}$. Give your answer correct to 3 significant figures. [4]

2 A curve has equation $y = \dfrac{2 + 3\ln x}{1 + 2x}$.

Find the equation of the tangent to the curve at the point $\left(1, \frac{2}{3}\right)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

3 It is given that $\int_0^a (3e^{2x} - 1)\,dx = 12$, where a is a positive constant.

(a) Show that $a = \frac{1}{2}\ln\left(9 + \frac{2}{3}a\right)$. [4]

(b) Use an iterative formula, based on the equation in **(a)**, to find the value of a correct to 4 significant figures. Use an initial value of 1 and give the result of each iteration to 6 significant figures. [3]

4 The polynomial $p(x)$ is defined by

$$p(x) = 2x^3 + 3x^2 + kx - 30,$$

where k is a constant. It is given that $(x - 3)$ is a factor of $p(x)$.

(a) Find the value of k. [2]

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(b) Hence find the quotient when $p(x)$ is divided by $(x - 3)$ and factorise $p(x)$ completely. [3]

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(c) It is given that a is one of the roots of the equation $p(x) = 0$.

Given also that the equation $|4y - 5| = a$ is satisfied by two real values of y, find these two values of y. [3]

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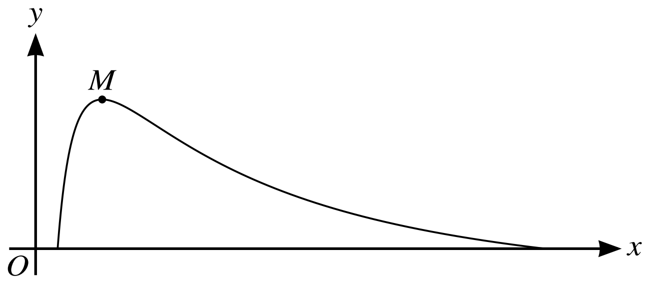

The diagram shows the curve with parametric equations

$$x = 4e^{2t}, \qquad y = 5e^{-t}\cos 2t,$$

for $-\frac{1}{4}\pi \leqslant t \leqslant \frac{1}{4}\pi$. The curve has a maximum point M.

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. [3]

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(b) Find the coordinates of M, giving each coordinate correct to 3 significant figures. [5]

6 Show that $\displaystyle\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \left(4\cos^2 2x + \frac{1}{\cos^2 x}\right) \mathrm{d}x = \tfrac{3}{4}\sqrt{3} + \tfrac{1}{6}\pi - 1.$ [7]

7 **(a)** Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(b) Solve the equation $7\cos\theta + 24\sin\theta = 18$ for $0° < \theta < 360°$. [4]

(c) As β varies, the greatest possible value of

$$\frac{150}{7\cos\frac{1}{2}\beta + 24\sin\frac{1}{2}\beta + 50}$$

is denoted by V.

Find the value of V and determine the smallest positive value of β (in degrees) for which the value of V occurs. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS Level

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Apply logarithms to both sides and apply power law correctly at least once	***M1**	OE with x not in a power.
	Obtain $x\ln 12 = (2x+1)\ln 3$	**A1**	
	Attempt solution of linear equation	**DM1**	
	Obtain 3.82	**A1**	Do not condone incorrect use of logarithms or greater accuracy.
		4	

Question	Answer	Marks	Guidance
2	Use quotient rule (or equivalent) to find first derivative	**M1**	
	Obtain $\dfrac{(1+2x)\frac{3}{x}-(2+3\ln x)2}{(1+2x)^2}$	**A1**	OE
	Substitute $x=1$ and obtain $\frac{5}{9}$	**A1**	
	Attempt equation of tangent through $(1,\ \frac{2}{3})$ with *their* numerical gradient	**M1**	Must have made an attempt at differentiation.
	Obtain $5x-9y+1=0$ or equivalent of required form	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	Integrate to obtain the form $k_1e^{2x}+k_2x$	*M1	Where $k_1k_2 \neq 0$.
	Obtain correct $\frac{3}{2}e^{2x}-x$	A1	
	Use limits correctly and attempt rearrangement at least as far as $e^{2a}=...$	DM1	For DM1, must be equated to 12 and simplified using a correct method. Do not condone verification.
	Confirm given result $a=\frac{1}{2}\ln(9+\frac{2}{3}a)$ with sufficient detail	A1	AG
		4	
3(b)	Use iteration process correctly at least once	M1	Need to see 1.13434 and 1.13895 .
	Obtain final answer 1.139	A1	Final answer needed to exactly 4 sf.
	Show sufficient iterations to 6 sf to justify answer or show a sign change in interval [1.1385, 1.1395]	A1	
		3	

Question	Answer	Marks	Guidance
4(a)	Substitute $x=3$, equate to zero and attempt solution	M1	Condone $-\dfrac{51}{3}$.
	Obtain $k=-17$	A1	
		2	

Question	Answer	Marks	Guidance
4(b)	Divide by $x-3$ at least as far as $2x^2+mx$	**M1**	
	Obtain $2x^2+9x+10$	**A1**	
	Obtain $(x-3)(2x+5)(x+2)$	**A1**	**SC** – no attempt at division (or equivalent) and only correct fully factorised form shown: award B1 only.
		3	
4(c)	Solve linear equation $4y-5=3$ to obtain $y=2$	**B1 FT**	following any positive root from *their* factorised $\mathrm{p}(x)$.
	Attempt solution of linear equation $4y-5=-3$ or equivalent	**M1**	with RHS of equation being – (*their* positive root).
	Obtain $y=\frac{1}{2}$	**A1**	
	Alternative method for Question 4(c)		
	State or imply $(4y-5)^2=3^2$	**B1 FT**	following any positive root from *their* factorised $\mathrm{p}(x)$.
	Attempt solution of 3-term quadratic equation $(4y-5)^2=3^2$	**M1**	with RHS of equation involving *their* positive root.
	Obtain $\frac{1}{2}$ and 2 and no other solutions	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	Use product rule to find $\dfrac{dy}{dt}$	**M1**	
	Obtain $\dfrac{dy}{dt} = -5e^{-t}\cos 2t - 10e^{-t}\sin 2t$	**A1**	or (unsimplified) equivalent (do not condone poor use of brackets.
	Obtain $\dfrac{dy}{dx} = \dfrac{-5e^{-t}\cos 2t - 10e^{-t}\sin 2t}{8e^{2t}}$	**A1**	OE following *their* expression for $\dfrac{dy}{dt}$.
		3	
5(b)	Equate $\dfrac{dy}{dx}$ to zero and simplify at least as far as $\tan 2t = ...$	**M1***	now condoning any error with $\dfrac{dx}{dt}$.
	Obtain $\tan 2t = -\frac{1}{2}$	**A1**	
	Obtain $t = -0.231...$	**A1**	allow $t = -0.232$.
	Substitute negative value of t in expressions for x and y	**DM1**	
	Obtain $x = 2.52$ and $y = 5.64$	**A1**	or greater accuracy.
		5	

Question	Answer	Marks	Guidance
6	Express $4\cos^2 2x$ in the form $k_1 \cos 4x + k_2$	**M1**	where $k_1 k_2 \neq 0$.
	Obtain correct $2\cos 4x + 2$	**A1**	Allow unsimplified.
	State or imply $\dfrac{1}{\cos^2 x} = \sec^2 x$	**B1**	Maybe implied by integration.
	Integrate to obtain $k_3 \sin 4x + k_4 x + \tan x$	***M1**	where $k_3 k_4 \neq 0$.
	Obtain correctly $\frac{1}{2}\sin 4x + 2x + \tan x$	**A1**	
	Use limits correctly with correct values of $\sin\frac{4}{3}\pi$ and $\tan\frac{1}{3}\pi$ indicated	**DM1**	
	Confirm given result $\frac{3}{4}\sqrt{3} + \frac{1}{6}\pi - 1$ with sufficient detail	**A1**	AG
		7	

Question	Answer	Marks	Guidance
7(a)	State $R = 25$	**B1**	
	Use appropriate trigonometry to find α	**M1**	Allow if found in radians .
	Obtain $\alpha = 73.74$	**A1**	or greater accuracy.
		3	

Question	Answer	Marks	Guidance
7(b)	Use correct method to find one value of θ	**M1**	
	Obtain 29.8 (or 117.7)	**A1**	or greater accuracy.
	Use correct method to find second value of θ between 0 and 360	**M1**	
	Obtain 117.7 (or 29.8)	**A1**	or greater accuracy; and no others between 0 and 360.
		4	
7(c)	State or imply expression is $\dfrac{150}{25\cos(\frac{1}{2}\beta - 73.74) + 50}$	**B1 FT**	following *their* R and α.
	Obtain $V = 6$	**B1**	
	Attempt complete method to find positive value from $\cos(\frac{1}{2}\beta - 73.74) = -1$	**M1**	for *their* α.
	Obtain 507.5	**A1**	or greater accuracy.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

2139890131

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

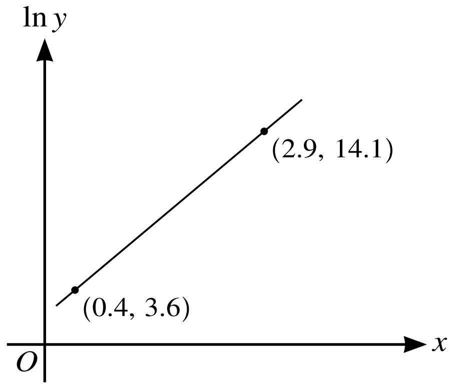
JC23 06_9709_22/2R

1 Solve the equation

$$\sec^2\theta + 5\tan^2\theta = 9 + 17\sec\theta$$

for $0° < \theta < 360°$. [5]

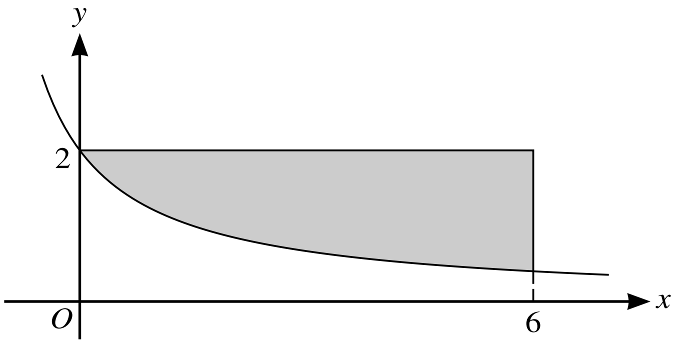
2



The variables x and y satisfy the equation $y = Ae^{(A-B)x}$, where A and B are constants. The graph of $\ln y$ against x is a straight line passing through the points (0.4, 3.6) and (2.9, 14.1), as shown in the diagram.

Find the values of A and B correct to 3 significant figures. [5]

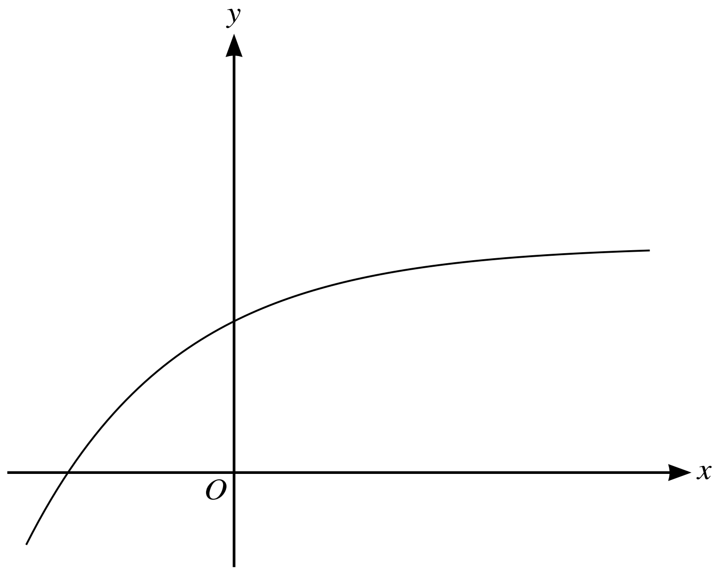
3



The diagram shows part of the curve $y = \dfrac{6}{2x+3}$. The shaded region is bounded by the curve and the lines $x = 6$ and $y = 2$.

Find the exact area of the shaded region, giving your answer in the form $a - \ln b$, where a and b are integers. [5]

4 (a)



The diagram shows the graph of $y = 3 - e^{-\frac{1}{2}x}$.

On the diagram, sketch the graph of $y = |5x - 4|$, and show that the equation $3 - e^{-\frac{1}{2}x} = |5x - 4|$ has exactly two real roots. [2]

It is given that the two roots of $3 - e^{-\frac{1}{2}x} = |5x - 4|$ are denoted by α and β, where $\alpha < \beta$.

(b) Show by calculation that α lies between 0.36 and 0.37. [2]

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(c) Use the iterative formula $x_{n+1} = \frac{1}{5}\left(7 - e^{-\frac{1}{2}x_n}\right)$ to find β correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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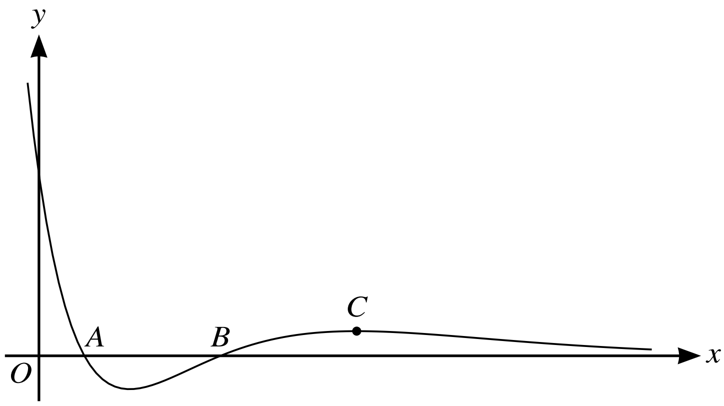

The diagram shows the curve with equation $y = e^{-\frac{1}{2}x}(x^2 - 5x + 4)$. The curve crosses the x-axis at the points A and B, and has a maximum at the point C.

(a) Find the exact gradient of the curve at B. [5]

(b) Find the exact coordinates of C. [4]

6 **(a)** Show that $4\sin\left(\theta + \frac{1}{3}\pi\right)\cos\left(\theta - \frac{1}{3}\pi\right) \equiv \sqrt{3} + 2\sin 2\theta$. [4]

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(b) Find the exact value of $4\sin\frac{17}{24}\pi\cos\frac{1}{24}\pi$. [2]

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(c) Find the exact value of $\int_0^{\frac{1}{8}\pi} 4\sin\left(2x+\frac{1}{3}\pi\right)\cos\left(2x-\frac{1}{3}\pi\right)\mathrm{d}x$. [4]

7 A curve has parametric equations

$$x = \frac{2t+3}{t+2}, \qquad y = t^2 + at + 1,$$

where a is a constant. It is given that, at the point P on the curve, the gradient is 1.

(a) Show that the value of t at P satisfies the equation

$$2t^3 + (a+8)t^2 + (4a+8)t + 4a - 1 = 0.$$ [4]

(b) It is given that $(t + 1)$ is a factor of

$$2t^3 + (a + 8)t^2 + (4a + 8)t + 4a - 1.$$

Find the value of a. [2]

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(c) Hence show that P is the only point on the curve at which the gradient is 1. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS Level

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Attempt to express equation in terms of $\sec\theta$ only	**M1**	or equivalent in terms of $\cos\theta$ only, using a correct identity, allow if '5' omitted.
	Obtain $6\sec^2\theta - 17\sec\theta - 14(=0)$	**A1**	or $14\cos^2\theta + 17\cos\theta - 6 = 0$.
	Attempt solution of 3-term quadratic equation to find one value of θ , from $\cos\theta = \ldots$	**M1**	
	Obtain 73.4	**A1**	or greater accuracy.
	Obtain 286.6	**A1**	or greater accuracy; and no others between 0 and 360.
		5	

Question	Answer	Marks	Guidance
2	State or imply equation $\ln y = \ln A + (A - B)x$	**B1**	Allow inclusion of ln e.
	Equate $A - B$ to gradient of line	**M1**	
	Obtain $A - B = 4.2$	**A1**	
	Substitute appropriate values to find value of $\ln A$	**M1**	
	Obtain $\ln A = 1.92$ and hence $A = 6.82$ and $B = 2.62$	**A1**	or greater accuracy.
	Alternative Method 1		
	State or imply equation $\ln y = \ln A + (A - B)x$	**B1**	Allow inclusion of ln e.
	Use of coordinates to obtain equation of line $\dfrac{\ln y - 3.6}{14.1 - 3.6} = \dfrac{x - 0.4}{2.9 - 0.4}$	**M1**	condone use of y in place of $\ln y$.
	Obtain gradient equal to 4.2	**A1**	
	Substitute appropriate values to find value of $\ln A$	**M1**	
	Obtain $\ln A = 1.92$ and hence $A = 6.82$ and $B = 2.62$	**A1**	or greater accuracy.
	Alternative Method 2		
	State or imply equation $\ln y = \ln A + (A - B)x$	**B1**	Allow inclusion of ln e.
	$3.6 = \ln A + 0.4(A - B)$	**M1**	For one correct equation.
	$14.1 = \ln A + 2.9(A - B)$	**A1**	For both correct equations.
	Obtain $A = 6.82$ and $B = 2.62$	**M1 A1**	For attempt at solution by elimination of $\ln A$ to obtain both values. **SC B1** for $y = 4.2x + 1.92$.
		5	

Question	Answer	Marks	Guidance
3	Integrate to obtain the form $k\ln(2x+3)$	*M1	Condone lack or misuse of brackets if recovered later.
	Obtain correct $3\ln(2x+3)$	A1	Allow unsimplified.
	Apply limits 0 and 6 correctly to obtain $k\ln 15 - k\ln 3$	*DM1	Allow unsimplified.
	Apply relevant logarithm properties correctly to obtain form $\ln b$	DM1	
	Obtain $12 - \ln 125$	A1	
		5	

Question	Answer	Marks	Guidance
4(a)	Draw (more or less) correct sketch with vertex on positive *x*-axis	*B1	crossing *y*-axis above given graph, may be implied by extrapolation.
	Indicate in some way the two roots	DB1	
		2	
4(b)	Consider sign of $3-\mathrm{e}^{-\frac{1}{2}x}+5x-4$ or of $3-\mathrm{e}^{-\frac{1}{2}x}-\lvert 5x-4\rvert$ for 0.36 and 0.37	M1	but not for sign of $3-\mathrm{e}^{-\frac{1}{2}x}-5x+4$. May be implied by $-0.035...$ and $0.018...$, or equivalents.
	Obtain $-0.035...$ and $0.018...$, or equivalents, and justify conclusion	A1	AG necessary detail needed.
		2	

Question	Answer	Marks	Guidance
4(c)	Use iteration process correctly at least once	**M1**	
	Obtain final answer 1.295	**A1**	answer required to exactly 4 sf.
	Show sufficient iterations to 6 sf to justify answer or show sign change in interval $[1.2945,\ 1.2955]$	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	Attempt use of product rule to find first derivative	***M1**	
	Obtain $-\frac{1}{2}e^{-\frac{1}{2}x}(x^2-5x+4)+e^{-\frac{1}{2}x}(2x-5)$	**A1**	OE
	Obtain $x=4$ for point B	**B1**	
	Substitute $x=4$ to find the value of the derivative	**DM1**	
	Obtain $3e^{-2}$	**A1**	or exact equivalent.
		5	
5(b)	Equate *their* first derivative to zero and simplify as far as quadratic equation	***M1**	allow if it appears in part **(a)**.
	Obtain at least $x^2-9x+14=0$	**A1**	OE
	Solve to find relevant x value and substitute to find the value of y	**DM1**	
	Obtain $x=7$ and $y=18e^{-\frac{7}{2}}$	**A1**	or exact equivalent.
		4	

Question	Answer	Marks	Guidance
6(a)	Obtain at least either $(\frac{1}{2}\sin\theta + \frac{1}{2}\sqrt{3}\cos\theta)$ or $(\frac{1}{2}\cos\theta + \frac{1}{2}\sqrt{3}\sin\theta)$	**B1**	Allow if implied by decimal values.
	Expand and simplify with correct use of $\sin^2\theta + \cos^2\theta = 1$	**M1**	
	Use $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$	**M1**	
	Confirm given result $\sqrt{3} + 2\sin 2\theta$	**A1**	AG necessary detail required.
		4	
6(b)	Identify value of θ is $\frac{3}{8}\pi$	***B1**	OE
	Obtain $\sqrt{3} + 2\sin\frac{3}{4}\pi$ and conclude $\sqrt{3} + \sqrt{2}$	**DB1**	or exact equivalent.
		2	
6(c)	Identify integrand as $\sqrt{3} + 2\sin 4x$	**B1**	
	Integrate to obtain form $k_1 x + k_2 \cos 4x$	**M1**	where $k_1 k_2 \neq 0$.
	Obtain correct $\sqrt{3}\,x - \frac{1}{2}\cos 4x$	**A1**	
	Obtain $\frac{1}{8}\pi\sqrt{3} + \frac{1}{2}$	**A1**	or exact equivalent.
		4	

Question	Answer	Marks	Guidance
7(a)	Attempt use of quotient rule (or equivalent) to find $\frac{dx}{dt}$	**M1***	
	Obtain $\frac{1}{(t+2)^2}$	**A1**	or (unsimplified) equivalent.
	Equate $\frac{dy}{dx}$ to 1	**DM1**	Must be using *their* $\frac{dy}{dx}$.
	Obtain $(2t+a)(t+2)^2 = 1$ and expand to confirm given result	**A1**	AG necessary detail required.
		4	
7(b)	Substitute $t = -1$, equate to zero and attempt solution for a	**M1**	Allow a complete method using algebraic long division or synthetic division.
	Obtain $a = 3$	**A1**	
		2	
7(c)	Divide their cubic by $t+1$ at least as far as the x term	**M1**	or equivalent (inspection, identity, …).
	Obtain $2t^2 + 9t + 11$	**A1**	
	Calculate discriminant, obtain -7 and conclude no further value of t	**A1**	OE
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

8595284852

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
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- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

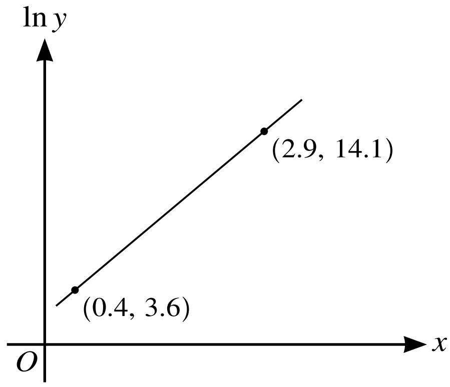
This document has **12** pages.

1 Solve the equation

$$\sec^2\theta + 5\tan^2\theta = 9 + 17\sec\theta$$

for $0° < \theta < 360°$. [5]

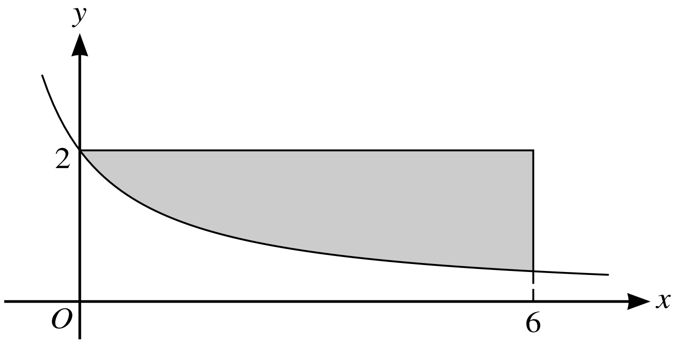
2



The variables x and y satisfy the equation $y = Ae^{(A-B)x}$, where A and B are constants. The graph of $\ln y$ against x is a straight line passing through the points (0.4, 3.6) and (2.9, 14.1), as shown in the diagram.

Find the values of A and B correct to 3 significant figures. [5]

3



The diagram shows part of the curve $y = \dfrac{6}{2x+3}$. The shaded region is bounded by the curve and the lines $x = 6$ and $y = 2$.

Find the exact area of the shaded region, giving your answer in the form $a - \ln b$, where a and b are integers. [5]

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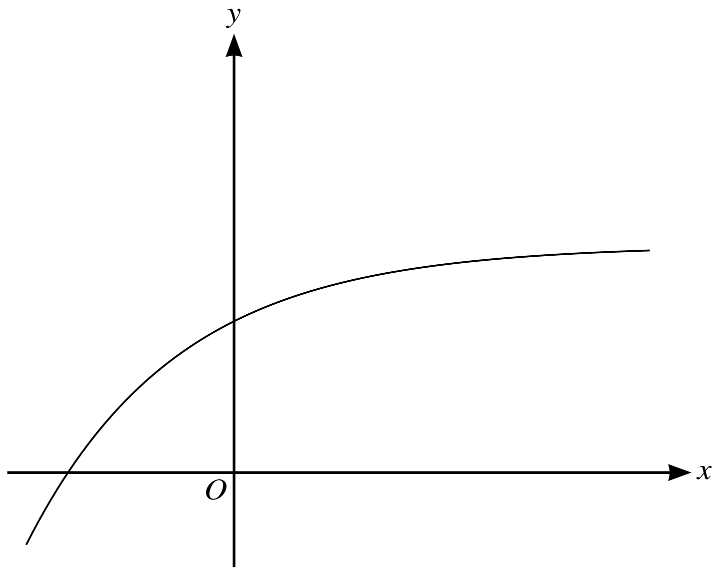
4 **(a)**



The diagram shows the graph of $y = 3 - e^{-\frac{1}{2}x}$.

On the diagram, sketch the graph of $y = |5x - 4|$, and show that the equation $3 - e^{-\frac{1}{2}x} = |5x - 4|$ has exactly two real roots. [2]

It is given that the two roots of $3 - e^{-\frac{1}{2}x} = |5x - 4|$ are denoted by α and β, where $\alpha < \beta$.

(b) Show by calculation that α lies between 0.36 and 0.37. [2]

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(c) Use the iterative formula $x_{n+1} = \frac{1}{5}\left(7 - e^{-\frac{1}{2}x_n}\right)$ to find β correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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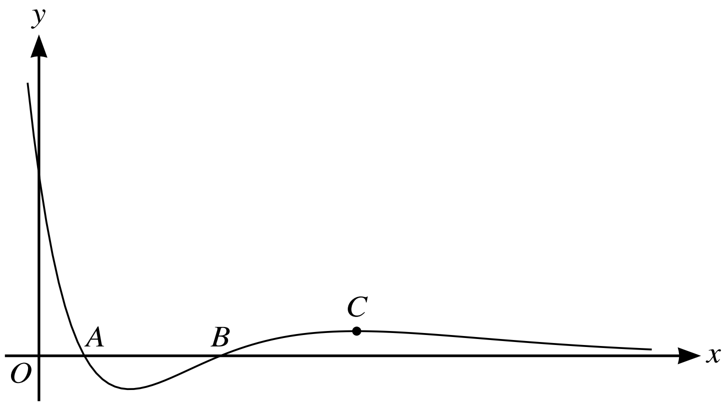

The diagram shows the curve with equation $y = e^{-\frac{1}{2}x}(x^2 - 5x + 4)$. The curve crosses the x-axis at the points A and B, and has a maximum at the point C.

(a) Find the exact gradient of the curve at B. [5]

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(b) Find the exact coordinates of C. [4]

6 **(a)** Show that $4\sin\left(\theta + \frac{1}{3}\pi\right)\cos\left(\theta - \frac{1}{3}\pi\right) \equiv \sqrt{3} + 2\sin 2\theta$. [4]

(b) Find the exact value of $4\sin\frac{17}{24}\pi\cos\frac{1}{24}\pi$. [2]

(c) Find the exact value of $\int_0^{\frac{1}{8}\pi} 4\sin\left(2x + \frac{1}{3}\pi\right)\cos\left(2x - \frac{1}{3}\pi\right)\,\mathrm{d}x$. [4]

7 A curve has parametric equations

$$x = \frac{2t+3}{t+2}, \qquad y = t^2 + at + 1,$$

where a is a constant. It is given that, at the point P on the curve, the gradient is 1.

(a) Show that the value of t at P satisfies the equation

$$2t^3 + (a+8)t^2 + (4a+8)t + 4a - 1 = 0.$$ [4]

(b) It is given that $(t + 1)$ is a factor of

$$2t^3 + (a + 8)t^2 + (4a + 8)t + 4a - 1.$$

Find the value of a. [2]

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(c) Hence show that P is the only point on the curve at which the gradient is 1. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS Level

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

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GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
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GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Attempt to express equation in terms of $\sec\theta$ only	**M1**	or equivalent in terms of $\cos\theta$ only, using a correct identity, allow if '5' omitted.
	Obtain $6\sec^2\theta - 17\sec\theta - 14(=0)$	**A1**	or $14\cos^2\theta + 17\cos\theta - 6 = 0$.
	Attempt solution of 3-term quadratic equation to find one value of θ, from $\cos\theta = ...$	**M1**	
	Obtain 73.4	**A1**	or greater accuracy.
	Obtain 286.6	**A1**	or greater accuracy; and no others between 0 and 360.
		5	

Question	Answer	Marks	Guidance
2	State or imply equation $\ln y = \ln A + (A - B)x$	**B1**	Allow inclusion of ln e.
	Equate $A-B$ to gradient of line	**M1**	
	Obtain $A-B=4.2$	**A1**	
	Substitute appropriate values to find value of $\ln A$	**M1**	
	Obtain $\ln A=1.92$ and hence $A=6.82$ and $B=2.62$	**A1**	or greater accuracy.
	Alternative Method 1		
	State or imply equation $\ln y = \ln A + (A - B)x$	**B1**	Allow inclusion of ln e.
	Use of coordinates to obtain equation of line $\frac{\ln y-3.6}{14.1-3.6}=\frac{x-0.4}{2.9-0.4}$	**M1**	condone use of y in place of $\ln y$.
	Obtain gradient equal to 4.2	**A1**	
	Substitute appropriate values to find value of $\ln A$	**M1**	
	Obtain $\ln A=1.92$ and hence $A=6.82$ and $B=2.62$	**A1**	or greater accuracy.
	Alternative Method 2		
	State or imply equation $\ln y = \ln A + (A - B)x$	**B1**	Allow inclusion of ln e.
	$3.6 = \ln A + 0.4(A - B)$	**M1**	For one correct equation.
	$14.1 = \ln A + 2.9(A - B)$	**A1**	For both correct equations.
	Obtain $A=6.82$ and $B=2.62$	**M1 A1**	For attempt at solution by elimination of $\ln A$ to obtain both values. **SC B1** for $y = 4.2x+1.92$.
		5	

Question	Answer	Marks	Guidance
3	Integrate to obtain the form $k\ln(2x+3)$	*M1	Condone lack or misuse of brackets if recovered later.
	Obtain correct $3\ln(2x+3)$	A1	Allow unsimplified.
	Apply limits 0 and 6 correctly to obtain $k\ln 15 - k\ln 3$	*DM1	Allow unsimplified.
	Apply relevant logarithm properties correctly to obtain form $\ln b$	DM1	
	Obtain $12 - \ln 125$	A1	
		5	

Question	Answer	Marks	Guidance
4(a)	Draw (more or less) correct sketch with vertex on positive *x*-axis	*B1	crossing *y*-axis above given graph, may be implied by extrapolation.
	Indicate in some way the two roots	DB1	
		2	
4(b)	Consider sign of $3 - e^{-\frac{1}{2}x} + 5x - 4$ or of $3 - e^{-\frac{1}{2}x} - \lvert 5x-4 \rvert$ for 0.36 and 0.37	M1	but not for sign of $3 - e^{-\frac{1}{2}x} - 5x + 4$. May be implied by $-0.035...$ and $0.018...$, or equivalents.
	Obtain $-0.035...$ and $0.018...$, or equivalents, and justify conclusion	A1	AG necessary detail needed.
		2	

Question	Answer	Marks	Guidance
4(c)	Use iteration process correctly at least once	**M1**	
	Obtain final answer 1.295	**A1**	answer required to exactly 4 sf.
	Show sufficient iterations to 6 sf to justify answer or show sign change in interval $[1.2945,\ 1.2955]$	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	Attempt use of product rule to find first derivative	***M1**	
	Obtain $-\frac{1}{2}e^{-\frac{1}{2}x}(x^2-5x+4)+e^{-\frac{1}{2}x}(2x-5)$	**A1**	OE
	Obtain $x=4$ for point *B*	**B1**	
	Substitute $x=4$ to find the value of the derivative	**DM1**	
	Obtain $3e^{-2}$	**A1**	or exact equivalent.
		5	
5(b)	Equate *their* first derivative to zero and simplify as far as quadratic equation	***M1**	allow if it appears in part **(a)**.
	Obtain at least $x^2-9x+14=0$	**A1**	OE
	Solve to find relevant *x* value and substitute to find the value of *y*	**DM1**	
	Obtain $x=7$ and $y=18e^{-\frac{7}{2}}$	**A1**	or exact equivalent.
		4	

Question	Answer	Marks	Guidance
6(a)	Obtain at least either $(\frac{1}{2}\sin\theta+\frac{1}{2}\sqrt{3}\cos\theta)$ or $(\frac{1}{2}\cos\theta+\frac{1}{2}\sqrt{3}\sin\theta)$	**B1**	Allow if implied by decimal values.
	Expand and simplify with correct use of $\sin^2\theta+\cos^2\theta=1$	**M1**	
	Use $\sin\theta\cos\theta=\frac{1}{2}\sin 2\theta$	**M1**	
	Confirm given result $\sqrt{3}+2\sin 2\theta$	**A1**	AG necessary detail required.
		4	
6(b)	Identify value of θ is $\frac{3}{8}\pi$	***B1**	OE
	Obtain $\sqrt{3}+2\sin\frac{3}{4}\pi$ and conclude $\sqrt{3}+\sqrt{2}$	**DB1**	or exact equivalent.
		2	
6(c)	Identify integrand as $\sqrt{3}+2\sin 4x$	**B1**	
	Integrate to obtain form $k_1x+k_2\cos 4x$	**M1**	where $k_1k_2\neq 0$.
	Obtain correct $\sqrt{3}x-\frac{1}{2}\cos 4x$	**A1**	
	Obtain $\frac{1}{8}\pi\sqrt{3}+\frac{1}{2}$	**A1**	or exact equivalent.
		4	

Question	Answer	Marks	Guidance
7(a)	Attempt use of quotient rule (or equivalent) to find $\frac{dx}{dt}$	**M1***	
	Obtain $\frac{1}{(t+2)^2}$	**A1**	or (unsimplified) equivalent.
	Equate $\frac{dy}{dx}$ to 1	**DM1**	Must be using *their* $\frac{dy}{dx}$.
	Obtain $(2t+a)(t+2)^2=1$ and expand to confirm given result	**A1**	AG necessary detail required.
		4	
7(b)	Substitute $t=-1$, equate to zero and attempt solution for a	**M1**	Allow a complete method using algebraic long division or synthetic division.
	Obtain $a=3$	**A1**	
		2	
7(c)	Divide their cubic by $t+1$ at least as far as the x term	**M1**	or equivalent (inspection, identity, …).
	Obtain $2t^2+9t+11$	**A1**	
	Calculate discriminant, obtain -7 and conclude no further value of t	**A1**	OE
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME										
CENTRE NUMBER						CANDIDATE NUMBER				

* 2 9 6 8 7 5 0 1 7 5 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC23 06_9709_31/2R

1 Solve the equation

$$3e^{2x} - 4e^{-2x} = 5.$$

Give the answer correct to 3 decimal places. [3]

2 **(a)** Sketch the graph of $y = |2x + 3|$. [1]

(b) Solve the inequality $3x + 8 > |2x + 3|$. [3]

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3 Find the coefficient of x^3 in the binomial expansion of $(3 + x)\sqrt{1 + 4x}$. [4]

4 **(a)** Show that the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ can be expressed in the form

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0.$$

[2]

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(b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ for $0° < \theta < 180°$. [4]

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5 The equation of a curve is $x^2y - ay^2 = 4a^3$, where a is a non-zero constant.

(a) Show that $\dfrac{dy}{dx} = \dfrac{2xy}{2ay - x^2}$. [4]

(b) Hence find the coordinates of the points where the tangent to the curve is parallel to the y-axis. [4]

6 Relative to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \qquad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}.$$

The quadrilateral $ABCD$ is a parallelogram.

(a) Find the position vector of D. [3]

(b) The angle between BA and BC is θ.

Find the exact value of $\cos\theta$. [3]

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(c) Hence find the area of $ABCD$, giving your answer in the form $p\sqrt{q}$, where p and q are integers. [4]

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7 The variables x and y satisfy the differential equation

$$\cos 2x \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 \tan 2x}{\sin^2 3y},$$

where $0 \leqslant x < \frac{1}{4}\pi$. It is given that $y = 0$ when $x = \frac{1}{6}\pi$.

Solve the differential equation to obtain the value of x when $y = \frac{1}{6}\pi$. Give your answer correct to 3 decimal places. [8]

8 Let $f(x) = \dfrac{3 - 3x^2}{(2x+1)(x+2)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

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(b) Hence find the exact value of $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $a + b\ln c$, where a, b and c are integers. [5]

9 The constant a is such that $\int_0^a x\mathrm{e}^{-2x}\,\mathrm{d}x = \frac{1}{8}$.

(a) Show that $a = \frac{1}{2}\ln(4a + 2)$. [5]

(b) Verify by calculation that a lies between 0.5 and 1. [2]

(c) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

10 The polynomial $x^3 + 5x^2 + 31x + 75$ is denoted by $\mathrm{p}(x)$.

(a) Show that $(x + 3)$ is a factor of $\mathrm{p}(x)$. [2]

(b) Show that $z = -1 + 2\sqrt{6}\mathrm{i}$ is a root of $\mathrm{p}(z) = 0$. [3]

(c) Hence find the complex numbers z which are roots of $\mathrm{p}(z^2) = 0$. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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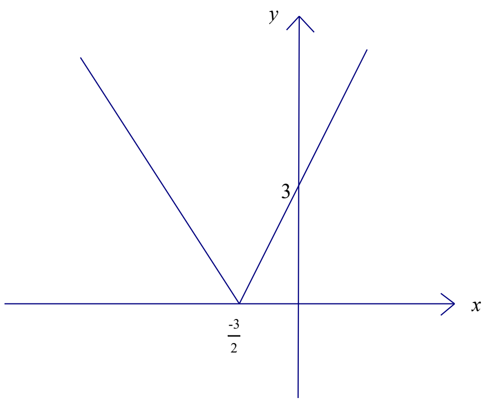
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$3\left(e^{2x}\right)^2 - 5\left(e^{2x}\right) - 4 = 0$	**B1**	OE Form 3 term quadratic in e^{2x}.
	$e^{2x} = \frac{5 \pm \sqrt{73}}{6}$, $x = \frac{1}{2}\ln\left(\frac{5+\sqrt{73}}{6}\right)$	**M1**	Use correct method to solve for *x*.
	$x = 0.407$	**A1**	Only
		3	

Question	Answer	Marks	Guidance
2(a)		**B1**	Show a recognizable sketch graph of $y = \lvert 2x+3 \rvert$. (Ignore any attempt to sketch $y = 3x + 8$). Straight lines. Vertex in approximately correct position on *x* axis. Symmetry.
		1	

Question	Answer	Marks	Guidance
2(b)	Find x-coordinate of intersection with $y = 3x + 8$	M1	
	Obtain $x = -\frac{11}{5}$	A1	
	State final answer $x > -\frac{11}{5}$ only	A1	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
	Alternative Method 1		
	Solve the linear inequality $3x + 8 > -(2x + 3)$, or corresponding linear equation	M1	
	Obtain critical value $x = -\frac{11}{5}$	A1	
	State final answer $x > -\frac{11}{5}$ only	A1	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
	Alternative Method 2		
	Solve the quadratic inequality $(3x+8)^2 > (2x+3)^2$, or corresponding quadratic equation	(M1)	$5x^2 + 36x + 55$.
	Obtain critical value $x = -\frac{11}{5}$	(A1)	Ignore -5 if seen.
	State final answer $x > -\frac{11}{5}$ only	(A1)	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
		3	

Question	Answer	Marks	Guidance
3	State unsimplified term in x^3, or its coefficient, in the expansion of $\left(1+4x\right)^{\frac{1}{2}}$	**B1**	$\frac{\frac{1}{2}\times\frac{-1}{2}\times\frac{-3}{2}}{6}\left(4x\right)^3\left(=4\right)$ Must expand binomial coefficient.
	State unsimplified term in x^2, or its coefficient, in the expansion of $\left(1+4x\right)^{\frac{1}{2}}$	**B1**	$\frac{\frac{1}{2}\times\frac{-1}{2}}{2}\left(4x\right)^2\left(=-2\right)$ Must expand binomial coefficient.
	Multiply by $\left(3+x\right)$ and combine terms in x^3, or their coefficients	**M1**	$\left(3\times4-1\times2\right)$ Allow if they expanded with x rather than $4x$.
	Obtain answer 10	**A1**	Accept $10x^3$
		4	

Question	Answer	Marks	Guidance
4(a)	Use correct double angle formulae	**M1**	e.g. $2\sin\theta\cos\theta+\cos^2\theta-\sin^2\theta=2\sin^2\theta$
	Obtain $\cos^2\theta+2\sin\theta\cos\theta-3\sin^2\theta=0$ from **full and correct** working	**A1**	AG Check conclusion is complete and matches the working.
		2	

Question	Answer	Marks	Guidance
4(b)	Factorise to obtain $(\cos\theta - \sin\theta)(\cos\theta + 3\sin\theta) = 0$	**B1**	OE
	Solve a quadratic in $\sin\theta$ and $\cos\theta$ to obtain a value for θ.	**M1**	$\tan\theta = 1$ *or* $\tan\theta = -\frac{1}{3}$.
	Obtain one correct value e.g. $45°$	**A1**	
	Obtain a second correct value e.g. $161.6°$ and no others in the interval	**A1**	Mark answers in radians (0.785 and 2.82) as a misread. Accept awrt 161.6 .
	Alternative Method 1		
	Obtain $3\tan^2\theta - 2\tan\theta - 1 = 0$	**B1**	
	Solve a 3 term quadratic in $\tan\theta$ to obtain a value for θ.	**M1**	$\tan\theta = 1$ *or* $\tan\theta = -\frac{1}{3}$.
	Obtain one correct value e.g. $45°$	**A1**	
	Obtain a second correct value e.g. $161.6°$ and no others in the interval	**A1**	Mark answers in radians (0.785 and 2.82) as a misread.
	Alternative Method 2		
	Obtain $(\cos\theta + \sin\theta)^2 = (2\sin\theta)^2$	**B1**	
	Solve to obtain a value for θ.	**M1**	$\tan\theta = 1$ *or* $\tan\theta = -\frac{1}{3}$.
	Obtain one correct value e.g. $45°$	**A1**	
	Obtain a second correct value e.g. $161.6°$ and no others in the interval	**A1**	Mark answers in radians (0.785 and 2.82) as a misread.
		4	

Question	Answer	Marks	Guidance
5(a)	State or imply $2xy + x^2 \frac{dy}{dx}$ as derivative of $x^2 y$	**B1**	Accept partial: $\frac{\partial}{\partial x} \to 2xy$.
	State or imply $2ay\frac{dy}{dx}$ as derivative of ay^2	**B1**	Accept partial: $\frac{\partial}{\partial y} \to x^2 - 2ay$.
	Equate attempted derivative to zero and solve for $\frac{dy}{dx}$	**M1**	
	Obtain answer $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$ from correct working	**A1**	AG
		4	
5(b)	State or imply $2ay - x^2 = 0$	***M1**	
	Substitute into equation of curve to obtain equation in x and a or in y and a	**DM1**	e.g. $2ay^2 - ay^2 = 4a^3$ or $\frac{x^4}{2a} - \frac{x^4}{4a} = 4a^3$.
	Obtain one correct point	**A1**	e.g. $(2a, 2a)$.
	Obtain second correct point and no others	**A1**	e.g. $(-2a, 2a)$.
		4	**SC**: Allow A1 A0 for $x = \pm 2a$ or for $y = 2a$.

Question	Answer	Marks	Guidance
6(a)	Obtain a vector for one side of the parallelogram	B1	e.g. $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix} -1 \\ -5 \\ -6 \end{pmatrix}$.
	Correct method to obtain $\pm\overrightarrow{OD}$	M1	e.g. $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$. MO if use $\overrightarrow{AB} = \overrightarrow{CD}$ or $\overrightarrow{BC} = \overrightarrow{DA}$.
	Obtain $\overrightarrow{OD} = \mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$	A1	Any equivalent form. Accept coordinates.
		3	
6(b)	Using the correct process, evaluate the scalar product $\overrightarrow{BA}.\overrightarrow{BC}$	M1	$(2+10-6)$ Scalar product of two relevant vectors. OE
	Using the correct process for the moduli, divide the scalar product by the product of the moduli.	M1	$\dfrac{2+10-6}{\sqrt{9}\times\sqrt{62}}$.
	Obtain answer $\dfrac{2}{\sqrt{62}}$	A1	ISW Or simplified equivalent i.e. $\dfrac{\sqrt{62}}{31}$.
		3	

Question	Answer	Marks	Guidance
6(c)	State or imply $\sin\theta = \sqrt{\frac{58}{62}}$	**B1 FT**	Follow *their* $\cos\theta$.
	Use correct method to find the area of *ABCD*	**M1**	**e.g.** $2 \times \frac{1}{2} BA \times BC \sin\theta$. Condone decimals.
	Correct unsimplified expression for the area	**A1 FT**	**e.g.** $2 \times \frac{1}{2} \times 3 \times \sqrt{62} \times \sin\theta$. Condone decimals. Follow *their* sides and angle.
	Obtain answer $3\sqrt{58}$	**A1**	Correct only.
		4	

Question	Answer	Marks	Guidance
7	Correct separation of variables	**B1**	$\int \sin^2 3y\,dy = \int 4\sec 2x \tan 2x\,dx$ or equivalent. Condone missing integral signs or dx and dy.
	Integrate to obtain $k\sec 2x$	**M1**	
	Obtain $2\sec 2x$	**A1**	
	Use double angle formula and integrate to obtain $py + q\sin 6y$	**M1**	Or two cycles of integration by parts.
	Obtain $\frac{1}{2}y - \frac{1}{12}\sin 6y$	**A1**	
	Use $y = 0$, $x = \frac{\pi}{6}$ in a solution containing terms $\lambda \sec 2x$ and $\mu \sin 6y$ to find the constant of integration	**M1**	
	Obtain $\frac{1}{2}y - \frac{1}{12}\sin 6y = 2\sec 2x - 4$	**A1**	Or equivalent seen or implied by $\frac{\pi}{2}\left(-\frac{1}{12}\sin \pi\right) = 2\sec 2x - 4$.
	Obtain $x = 0.541$	**A1**	From correct working (not by using the calculator to integrate).
		8	

Question	Answer	Marks	Guidance
8(a)	State or imply the form $\dfrac{A}{2x+1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}$	**B1**	Accept $\dfrac{A}{2x+1}+\dfrac{Dx+E}{(x+2)^2}$.
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A=1, B=-2, C=3$	**A1**	For alternative form: $A=1, D=-2, E=-1$.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
8(b)	Integrate and obtain one of $\frac{1}{2}\ln(2x+1), -2\ln(x+2), \dfrac{-3}{x+2}$	**B1 FT**	The follow through is on *their* A, B, C.
	Obtain a second term	**B1 FT**	If the alternative form is used, then either need to use integration by parts or split the fraction further.
	Obtain the third term	**B1 FT**	
	Substitute limits correctly in an integral with at least two terms of the form $\frac{1}{2}\ln(2x+1), -2\ln(x+2)$ and $\dfrac{-3}{x+2}$ and subtract in correct order	**M1**	The terms used need to have been obtained correctly. Must be exact values, not decimals.
	Obtain $1-\ln 3$	**A1**	
		5	

Question	Answer	Marks	Guidance
9(a)	Commence integration and reach $pxe^{-2x} + q\int e^{-2x}dx$	*M1	OE
	Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}dx$	A1	OE
	Complete integration and obtain $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$	A1	
	Use limits correctly and equate to $\frac{1}{8}$, having integrated twice	DM1	$-\frac{1}{2}a\,e^{-2a} - \frac{1}{4}e^{-2a} + \frac{1}{4} = \frac{1}{8}$.
	Obtain $a = \frac{1}{2}\ln(4a+2)$ correctly	A1	AG
		5	
9(b)	Calculate the values of a relevant expression or pair of expressions at $a = 0.5$ and $a = 1$	M1	
	Justify the given statement with correct calculated values	A1	e.g. $0.5 < 0.69\ldots$, $1 > 0.89\ldots$ $0.193 > 0$, $-1.105 < 0$ $0.066 < 0.125$, $0.148 > 0.125$ if put limits in the integral. Condone if they use calculator for the definite integral.
		2	
9(c)	Use the iterative process $a_{n+1} = \frac{1}{2}\ln(4a_n + 2)$ correctly at least once.	M1	
	Obtain final answer 0.84	A1	
	Show sufficient iterations to at least 4 d.p. to justify 0.84 to 2 d.p. or show that there is a sign change in (0.835, 0.845)	A1	e.g. 0.75, 0.8047, 0.8261, 0.8343, 0.8373, 0.8385 1, 0.8959, 0.8599, 0.8469, 0.8420, 0.8402 .
		3	

Question	Answer	Marks	Guidance
10(a)	Substitute $x = -3$ to obtain value of $\mathrm{p}(-3)$	M1	
	Obtain $\mathrm{p}(-3) = 0$ and hence given result	A1	
	Alternative method for Question 10(a)		
	Divide $\mathrm{p}(x)$ by $(x+3)$ to obtain quotient $x^2 \pm 2x + \ldots$	M1	
	Obtain quotient $x^2 + 2x + 25$, with zero remainder and hence given result	A1	
		2	

Question	Answer	Marks	Guidance
10(b)	Substitute $z=-1+2\sqrt{6}\mathrm{i}$ and attempt expansions of z^2 and z^3	**M1**	$z^2=-23-4\sqrt{6}\mathrm{i}$, $z^3=-1+6\sqrt{6}\mathrm{i}+72-48\sqrt{6}\mathrm{i}$.
	Use $\mathrm{i}^2=-1$	**M1**	Seen at least once.
	Obtain $\mathrm{p}(z)=0$ and hence given result	**A1**	**SC B1** if there is no evidence of working for the square or the cube. Total 1/3.
	Alternative Method 1		
	Use roots $z=-1+2\sqrt{6}\mathrm{i}$ to form quadratic factor	**M1**	$z^2+2z+25$.
	Divide $\mathrm{p}(z)$ by *their* quadratic factor	**M1**	
	Obtain zero remainder and hence given result.	**A1**	
	Alternative Method 2		
	Set *their* quadratic factor from **(a)** equal to zero	**M1**	
	Solve for z	**M1**	Need to see method here as answer is given.
	Obtain $z=-1+2\sqrt{6}\mathrm{i}$ (and $z=-1-2\sqrt{6}\mathrm{i}$)	**A1**	
	Alternative Method 3		
	Substitute $z=-1+2\sqrt{6}\mathrm{i}$ into *their* quadratic factor and attempt expansion of z^2	**M1**	
	Use $\mathrm{i}^2=-1$	**M1**	
	Obtain 0 and hence given result	**A1**	
		3	

Question	Answer	Marks	Guidance
10(c)	State $z_1 = \sqrt{3}\mathrm{i}$ and $z_2 = -\sqrt{3}\mathrm{i}$	**B1**	
	Expand $(x+\mathrm{i}y)^2 = -1+2\sqrt{6}\mathrm{i}$ and compare real and imaginary parts	**M1**	Allow for use of $z^2 = -1-2\sqrt{6}\mathrm{i}$.
	Obtain $x^2 - y^2 = -1$ and $xy = \sqrt{6}$	**A1**	
	Solve to obtain x and y	**M1**	
	Obtain $z_3 = \sqrt{2}+\sqrt{3}\mathrm{i}$ and $z_4 = -\sqrt{2}-\sqrt{3}\mathrm{i}$	**A1**	
	Use $z^2 = -1-2\sqrt{6}\mathrm{i}$ to obtain z_5 and z_6	**M1**	Allow for use of $z^2 = -1+2\sqrt{6}\mathrm{i}$.
	Obtain $z_5 = \sqrt{2}-\sqrt{3}\mathrm{i}$ and $z_6 = -\sqrt{2}+\sqrt{3}\mathrm{i}$	**A1**	
		7	

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CENTRE NUMBER						CANDIDATE NUMBER	

* 7 9 4 3 7 1 9 4 5 2 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Solve the inequality $|5x - 3| < 2|3x - 7|$. [4]

2 Solve the equation $\ln(2x^2 - 3) = 2\ln x - \ln 2$, giving your answer in an exact form. [3]

3 **(a)** On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z + 3 - 2\mathrm{i}| = 2$. [2]

(b) Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [2]

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4 Solve the equation $2\cos x - \cos\frac{1}{2}x = 1$ for $0 \leqslant x \leqslant 2\pi$. [5]

5 The complex number $2 + yi$ is denoted by a, where y is a real number and $y < 0$. It is given that $f(a) = a^3 - a^2 - 2a$.

(a) Find a simplified expression for $f(a)$ in terms of y. [3]

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(b) Given that $\mathrm{Re}\big(f(a)\big) = -20$, find $\arg a$. [3]

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6 The equation $\cot \frac{1}{2}x = 3x$ has one root in the interval $0 < x < \pi$, denoted by α.

(a) Show by calculation that α lies between 0.5 and 1. [2]

(b) Show that, if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{3}\left(x_n + 4\tan^{-1}\left(\frac{1}{3x_n}\right)\right)$$

converges, then it converges to α. [2]

(c) Use this iterative formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The equation of a curve is $3x^2 + 4xy + 3y^2 = 5$.

(a) Show that $\dfrac{dy}{dx} = -\dfrac{3x + 2y}{2x + 3y}$. [4]

(b) Hence find the exact coordinates of the two points on the curve at which the tangent is parallel to $y + 2x = 0$. [5]

8 **(a)** The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{4 + 9y^2}{e^{2x+1}}.$$

It is given that $y = 0$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x. [7]

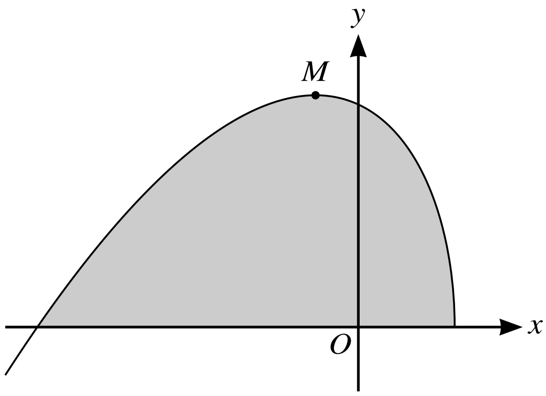
(b) State what happens to the value of y as x tends to infinity. Give your answer in an exact form. [1]

9 Let $f(x) = \dfrac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \frac{5}{2} - \ln 72$. [5]

10



The diagram shows the curve $y = (x + 5)\sqrt{3 - 2x}$ and its maximum point M.

(a) Find the exact coordinates of M. [5]

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(b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x-axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

11 The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$.

(a) Show that l does not intersect the line passing through A and B. [5]

(b) Find the position vector of the foot of the perpendicular from A to l. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

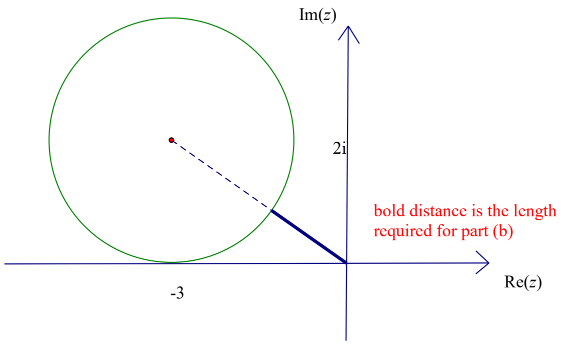
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(5x-3)^2 < 2^2(3x-7)^2$, or corresponding quadratic equation, or pair of linear equations $(5x-3) = \pm 2(3x-7)$	**B1**	$11x^2 - 138x + 187 > 0$.
	Solve a 3-term quadratic, or solve **two** linear equations for x	**M1**	If no working is shown, the M1 is implied by the correct roots for an incorrect quadratic.
	Obtain critical values $x = \frac{17}{11}$ and $x = 11$	**A1**	Accept 1.55 or better.
	State **final** answer $x < \frac{17}{11}$, $x > 11$	**A1**	Strict inequality required. In set notation, allow notation for open sets but not for closed sets e.g. accept $\left(-\infty, \frac{17}{11}\right) \cup (11, \infty)$ or $(-\infty, \frac{17}{11}[\cup]11, \infty)$ but not $(-\infty, \frac{17}{11}] \cup [11, \infty)$. Allow 'or' but not 'and'. Accept $\cup$. Final A0 for $\frac{17}{11} > x > 11$. Exact values expected but ISW if exact inequalities seen followed by decimal approx.
	Alternative Method for Question 1		
	Obtain critical value $x = 11$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x = \frac{17}{11}$ similarly	**B2**	Accept decimal value.
	State final answer $x < \frac{17}{11}$, $x > 11$	**B1**	Strict inequality required. See notes above.
		4	

Question	Answer	Marks	Guidance
2	Use law of the logarithm of a power, quotient or product	**M1**	Must be used correctly on a **correct** term. e.g. M1 for $2 \ln x = \ln x^2$ but M0 for $2 \ln x - \ln 2 = 2 \ln \frac{x}{2}$. M0 for $\ln(2x^2 - 3) = \ln 2x^2 - \ln 3$ $= \ln 2 + 2 \ln x - \ln 3$.
	Remove logarithms and obtain a correct equation in x	**A1**	e.g. $2x^2 - 3 = \frac{x^2}{2}$.
	Obtain final answer $x = \sqrt{2}$ only	**A1**	If $x = -\sqrt{2}$ is mentioned, it must be rejected.
		3	

Question	Answer	Marks	Guidance
3(a)		**B1**	Show a circle with centre $-3 + 2\text{i}$. Allow for a curved figure with 'centre' in roughly the correct position. Accept marks or numbers on axes, coordinates of centre shown. B0B1 available for axes the wrong way round (and M1 A1 in part **(b)**).
	Show a circle with radius 2	**B1 FT**	FT centre not at the origin. Allow 'near miss' on x axis. Different scales on axes require an ellipse for B1 B1. Scales on the axes and any label of the radius must be consistent for B1 B1. Correct circle shaded scores B1 B0.
		2	

Question	Answer	Marks	Guidance
3(b)	Carry out a correct method for finding the least value of $\|z\|$	M1	e.g. distance of centre from origin – radius or find point of intersection of circle and $3y = -2x$ and use Pythagoras. If they subtract the wrong way round M0. If their diagram is a reflection or a rotation of the correct diagram, M1 A1 is available (requires equivalent work). Any other circle M0.
	Obtain answer $\sqrt{13}-2$ or $\sqrt{17-4\sqrt{13}}$	A1	Or exact equivalent e.g. $\sqrt{17-\frac{26}{3}\sqrt{\frac{36}{13}}}$. Correct solution only. Allow A1 if exact answer seen and then decimal given.
		2	

Question	Answer	Marks	Guidance
4	Use correct double angle formula to obtain an equation in $\cos\left(\frac{x}{2}\right)$ only	*M1	e.g. $2\left(2\cos^2\left(\frac{x}{2}\right)-1\right)-\cos\left(\frac{x}{2}\right)=1$.
	Obtain a 3 term quadratic in $\cos\left(\frac{x}{2}\right)$,	A1	e.g. $4\cos^2\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)-3=0$. Allow $4\cos^2 u-\cos u-3=0$. Condone $\frac{x}{2}=x$.
	Obtain $\cos\left(\frac{x}{2}\right)=-\frac{3}{4}$ and $\cos\left(\frac{x}{2}\right)=1$	A1	Allow answer in u e.g. $(4\cos u+3)(\cos u-1)$ and condone $\frac{x}{2}=x$.
	Solve for the **original** x	DM1	Must see evidence of doubling, not halving.
	Obtain $x = 0$ and 4.84 and no others in the interval	A1	Ignore any answers outside interval. Accept AWRT 4.84. Accept 1.54π. Must be in radians. 277.2 indicates M1 but is A0.
	Alternative Method for Question 4		
	Use correct double angle formula to obtain an equation in $\cos x$ only	*M1	e.g. $2\cos x-1=\sqrt{\frac{\cos x+1}{2}}$.
	Obtain a 3 term quadratic in $\cos x$,	A1	e.g. $8\cos^2 x-9\cos x+1=0$.
	Obtain $\cos x=\frac{1}{8}$ and $\cos x = 1$	A1	
	Solve for x	DM1	
	Obtain answers $x = 0$ and 4.84 and no others in the interval	A1	Ignore any answers outside interval. Accept AWRT 4.84. Must be in radians. 277.2 is A0.
		5	

Question	Answer	Marks	Guidance
5(a)	Substitute $2 + y\mathrm{i}$ in $a^3 - a^2 - 2a$ and attempt expansions of a^2 and a^3	M1	$a^2 = 4 + 4y\mathrm{i} - y^2$ $a^3 = 8 + 12y\mathrm{i} - 6y^2 - y^3\mathrm{i}$. If using $a\left(a^2 - a - 2\right)$ must then expand fully. Must see working.
	Use $\mathrm{i}^2 = -1$	M1	Seen at least once (e.g. in squaring).
	Obtain **final** answer $-5y^2 + (6y - y^3)\mathrm{i}$	A1	Or simplified equivalent e.g. $6y\mathrm{i} - 5y^2 - y^3\mathrm{i}$. Do not ISW.
		3	No evidence of working for the square or the cube can score SC B1 for the correct answer.
5(b)	Equate *their* $-5y^2$ to -20 and solve for y	M1	Need to obtain a value for y. Available even if *their* y is not real.
	Obtain $y = -2$	A1	From correct work. Allow after incorrect f(a) if the real part was correct. Condone ± 2 with positive not rejected.
	Obtain **final** answer $\arg a = -\dfrac{\pi}{4}$	A1	Correct only (must have rejected y positive). OE e.g. $-\frac{\pi}{4} \pm 2n\pi$. Accept –0.785, 5.50 . Allow after incorrect f(a) if the real part was correct. Accept degrees. Do not ISW.
		3	

Question	Answer	Marks	Guidance
6(a)	Calculate the values of a relevant expression or pair of expressions at x = 0.5 and x = 1	**M1**	Need to evaluate at both points, but M1 still available if one value incorrect. Use of degrees is M0. Correct use of a smaller interval is M1. If using g(x) – f(x), there needs to be a clear indication of the comparison being made e.g. by listing values in a table. Embedded values 0.5 and 1 are not sufficient. 3.92 and 1.83 alone are not sufficient.
	Complete the argument correctly with conclusion about change of sign or change of inequalities and with correct calculated values. Can all be in symbols – an explanation in words is not required.	**A1**	e.g. 3.92 > 1.5, 1.83 < 3 or 2.42 > 0, –1.17 < 0.
		2	
6(b)	State $x=\frac{1}{3}\left(x+4\tan^{-1}\frac{1}{3x}\right)$	**M1**	Or rearrange $\cot\left(\frac{x}{2}\right)=3x$ as far as $2x=4\tan^{-1}\left(\frac{1}{3x}\right)$
	Rearrange to the given equation $\cot\left(\frac{x}{2}\right)=3x$ Need intermediate step between $\frac{x}{2}=\tan^{-1}\frac{1}{3x}$ and $\cot\left(\frac{x}{2}\right)=3x$	**A1**	Or continue rearrangement to $x=\frac{1}{3}\left(x+4\tan^{-1}\frac{1}{3x}\right)$ and state iterative formula of $x_{n+1}=\frac{1}{3}\left(x_n+4\tan^{-1}\frac{1}{3x_n}\right)$ **AG**
		2	

Question	Answer	Marks	Guidance
6(c)	Use the iterative process correctly at least once	**M1**	Obtain one value and substitute that back in to obtain a second value. Working in degrees is M0.
	Obtain final answer 0.79	**A1**	Must be to 2 d.p.
	Show sufficient iterations to at least 4 d.p. to justify 0.79 to 2 d.p. or show there is a sign change in the interval (0.785, 0.795)	**A1**	e.g. 1, 0.7623, 0.8037, 0.7921, 0.7951, 0.7943, 0.7945 or 0.5, 0.9506, 0.7665, 0.8024, 0.7924, 0.7950, 0.7944, 0.7945 or 0.75, 0.8076, 0.7911, 0.7954, 0.7943, 0.7946, 0.7945 . Condone truncation. Allow recovery. Condone minor differences in the final d.p.
		3	If they do the iteration in **(b)** but restate the conclusion here, no marks in **(b)** but could score 3/3 for **(c)**.

Question	Answer	Marks	Guidance
7(a)	State or imply $6y\frac{\mathrm{d}y}{\mathrm{d}x}$ as the derivative of $3y^2$	**B1**	Allow y' for $\frac{\mathrm{d}y}{\mathrm{d}x}$ throughout. Accept $\frac{\partial f}{\partial x} = 6x + 4y$.
	State or imply $4x\frac{\mathrm{d}y}{\mathrm{d}x} + 4y$ as the derivative of $4xy$	**B1**	Accept $\frac{\partial f}{\partial y} = 4x + 6y$.
	Equate derivative of LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	Allow an extra $\frac{\mathrm{d}y}{\mathrm{d}x}$ in front of their differentiated equation. Allow if '= 0' is implied but not seen. Allow $\frac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{\partial f}{\partial x}\Big/\dfrac{\partial f}{\partial y}$
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3x+2y}{2x+3y}$	**A1**	AG – must come from correct working. The position of the negative must be clear.
		4	

Question	Answer	Marks	Guidance
7(b)	Equate $\frac{dy}{dx}$ to –2 and solve for x in terms of y or for y in terms of x	*M1	Must be using the given derivative.
	Obtain $x = -4y$ or $y = -\frac{x}{4}$	A1	Seen or implied by correct later work.
	Substitute *their* $x = -4y$ or *their* $y = -\frac{x}{4}$ in curve equation	DM1	Allow unsimplified.
	Obtain $y = \pm\frac{1}{\sqrt{7}}$ or $x = \pm\frac{4}{\sqrt{7}}$	A1	Or exact equivalent. Or $x = \frac{4}{\sqrt{7}}$ and $y = -\frac{1}{\sqrt{7}}$ or exact equivalent.
	Obtain both pairs of values	A1	Or $x = -\frac{4}{\sqrt{7}}$ and $y = \frac{1}{\sqrt{7}}$ or exact equivalent. A1 A0 for incorrect final pairing.
		5	

Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	**B1**	$\int \frac{1}{4+9y^2}\,dy = \int e^{-(2x+1)}\,dx$. Condone missing integral signs **or** dx and dy missing.
	Obtain term $-\frac{1}{2}e^{-2x-1}$	**B1**	OE e.g. $-\frac{1}{2e}e^{-2x}$.
	Obtain term of the form $a\tan^{-1}\left(\frac{3y}{2}\right)$	**M1**	
	Obtain term $\frac{1}{6}\tan^{-1}\left(\frac{3y}{2}\right)$	**A1**	OE e.g. $\frac{1}{9}\times\frac{3}{2}\tan^{-1}\frac{3y}{2}$.
	Use $x = 1$, $y = 0$ to evaluate a constant or as limits in a solution containing or derived from terms of the form $a\tan^{-1}(by)$ and $ce^{\pm(2x+1)}$	**M1**	If they rearrange before evaluating the constant, the constant must be of the correct form.
	Obtain correct answer in any form	**A1**	e.g. $\frac{1}{6}\tan^{-1}\frac{3y}{2} = \frac{1}{2}e^{-3} - \frac{1}{2}e^{-(2x+1)}$.
	Obtain final answer $y = \frac{2}{3}\tan\left(3e^{-3} - 3e^{-2x-1}\right)$	**A1**	OE Allow with $3e^{-3} = 0.149....$
		7	
8(b)	State that y approaches $\frac{2}{3}\tan\left(3e^{-3}\right)$	**B1 FT**	Or exact equivalent. The FT is on correct work on a solution containing e^{-2x-1}. Condone $y = \ldots$ Accept correct answer stated with minimal wording. 0.10032… is not exact so B0.
		1	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{1+2x}+\dfrac{B}{2-x}+\dfrac{C}{(2-x)^2}$	**B1**	Alternative form: $\dfrac{A}{1+2x} + \dfrac{Dx+E}{(2-x)^2}$
	Use a correct method for finding a coefficient	**M1**	e.g. $A(2-x)^2 + B(1+2x)(2-x) + C(1+2x)$ $= 2x^2 + 17x - 17$ and compare coefficients or substitute for x. $A(2-x)^3 + B(1+2x)(2-x)^2 + C(1+2x)(2-x)$ $= 2x^2 + 17x - 17$ scores M0.
	Obtain one of $A = -4$, $B = -3$ and $C = 5$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Extra term in partial fractions, then B0 unless recover at end. Allow the marks for any constants found correctly. Missing terms in partial fractions, B0 but M1A1 is available for a correct method that obtains at least one correct constant (e.g. cover-up rule) Max 2/5. Ignore any substitution back into their original expression. If alternative form used: $A = -4$, $D = 3$ and $E = -1$.
		5	

Question	Answer	Marks	Guidance
9(b)	Integrate and obtain terms $-2\ln(1+2x)+3\ln(2-x)+\dfrac{5}{2-x}$	B1FT B1FT B1FT	OE The FT is on correct use of *their* A, B and C; or on A, D and E. If using the A, D, E form then B1 for the A term, but no further marks until partial fractions are used to split the second term or they use integration by parts to obtain $\dfrac{Dx+E}{2-x}-\int\dfrac{D}{2-x}\mathrm{d}x$ for the 2nd B1 and 3rd B1 for correct completion. B0FT, B0FT, B0FT if they place *their* A, B, C with incorrect denominators.
	Substitute limits correctly in an integral with two terms (obtained correctly) of the form $a\ln(1+2x)+b\ln(2-x)+\dfrac{c}{2-x}$, where $abc \neq 0$	M1	Condone minor slips in substitution. Exact substitution required.
	Obtain answer $\dfrac{5}{2}-\ln 72$ after full and correct working	A1	AG – evidence of some correct work to combine or simplify logs is required e.g. allow from $-\ln 9+\ln\dfrac{1}{8}$ or $-\ln 2^3-\ln 3^2$.
		5	

Question	Answer	Marks	Guidance
10(a)	Use the product rule correctly to obtain $p(x+5)(3-2x)^n + q(3-2x)^{\frac{1}{2}}$	***M1**	Allow with incorrect chain rule. BOD over sign errors unless an incorrect rule is quoted.
	Obtain correct derivative in any form	**A1**	e.g. $-(x+5)(3-2x)^{-\frac{1}{2}} + (3-2x)^{\frac{1}{2}}$.
	Equate derivative to zero and obtain a linear equation	**DM1**	Allow with surd factor e.g. $(3-2x)^{-\frac{1}{2}}\left(-(x+5)+(3-2x)\right)=0$.
	Obtain a correct linear equation.	**A1**	e.g. $-(x + 5) + 3 - 2x = 0$.
	Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$.	**A1**	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. Accept with x, y stated separately. ISW
	Alternative Method for Question 10(a)		
	Obtain y^2 and differentiate	***M1**	Ignore *their* left hand side i.e. *their* $\frac{d}{dx}y^2$.
	Obtain correct derivative in any form	**A1**	e.g. $-6x^2 - 34x - 20$.
	Equate derivative to zero and solve for x	**DM1**	
	Obtain $-\frac{2}{3}$	**A1**	Ignore –5 if seen.
	Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$ only	**A1**	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. ISW
		5	

Question	Answer	Marks	Guidance
10(b)	Use the given substitution and reach $a\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\ \mathrm{d}u$	*M1	OE Need to see -2 or -½ used. Condone if du missing or the integral sign is missing. Allow M1A0 for complete substitution into $\int x\sqrt{3-2x}\,\mathrm{d}x$ to obtain first term of the line below.
	Obtain correct integral $-\frac{1}{2}\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\ \mathrm{d}u$	A1	OE e.g. $-\frac{1}{2}\left[\int\frac{3-u}{2}\sqrt{u}\,\mathrm{d}u+5\int\sqrt{u}\,\mathrm{d}u\right]$. Ignore limits at this stage. Condone if du missing.
	$x=-5$ and $\frac{3}{2}$	B1	SOI e.g. by u = 13 and 0. In any order and at any stage.
	Use correct limits the right way round in an integral of the form $a\left(\frac{26}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right)$	DM1	
	Obtain answer $\frac{169}{15}\sqrt{13}$ or $a=\frac{169}{15}$	A1	or exact equivalents.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>11(a)</td><td>Carry out correct method for finding a vector equation for AB</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain [$\mathbf{r}$ =] $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$</td><td>A1</td><td>OE e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$.</td></tr>
<tr><td></td><td>Equate two pairs of components of general points on their AB and l
and evaluate λ or μ</td><td>M1</td><td>$\begin{pmatrix} 1+\lambda \\ 2-3\lambda \\ -2+3\lambda \end{pmatrix} = \begin{pmatrix} 1+2\mu \\ -1-3\mu \\ 3+4\mu \end{pmatrix}$.</td></tr>
<tr><td></td><td>Obtain correct answer for λ or μ, e.g. $\lambda = -1$, $\mu = -2$</td><td>A1</td><td>Correct value from two correct component equations.</td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and the lines fail to intersect
(≠ is sufficient justification e.g. $0 \neq -3$).</td><td>A1</td><td>Conclusion needs to follow correct values.
Hybrid versions are possible e.g. using j and k to get one parameter and then i to obtain the other.
or e.g. solving two pairs of simultaneous equations and showing that the results are not the same.
Alternatives:
<table>
<tr><td>A</td><td>λ</td><td>μ</td><td></td><td>B</td><td>λ</td><td>μ</td><td></td></tr>
<tr><td>ij</td><td>2</td><td>1</td><td>$4 \neq 7$</td><td>ij</td><td>1</td><td>1</td><td>$4 \neq 7$</td></tr>
<tr><td>ik</td><td>5</td><td>5/2</td><td>$-13 \neq -17/2$</td><td>ik</td><td>4</td><td>5/2</td><td>$-13 \neq -17/2$</td></tr>
<tr><td>jk</td><td>–1</td><td>–2</td><td>$0 \neq -3$</td><td>jk</td><td>–2</td><td>–2</td><td>$0 \neq -3$</td></tr>
</table></td></tr>
<tr><td></td><td></td><td>5</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
11(b)	Find $\overrightarrow{AP}$ for a general point P on l, e.g. $-3\mathbf{j}+5\mathbf{k}+\mu(2\mathbf{i}-3\mathbf{j}+4\mathbf{k})$	**B1**	Or equivalent e.g. $\overrightarrow{PA}=-2\mu\mathbf{i}+(3\mu+3)\mathbf{j}-(4\mu+5)\mathbf{k}$.
	Calculate scalar product of *their* $\overrightarrow{AP}$ and a direction vector for l and equate the result to zero	**M1**	e.g. $4\mu+(9+9\mu)+(20+16\mu)=0$. M0 if using $\overrightarrow{OP}$. M0 if using parallel line through A.
	Obtain $\mu=-1$	**A1**	
	Obtain answer $-\mathbf{i}+2\mathbf{j}-\mathbf{k}$	**A1**	Accept coordinates in place of position vector.
	Alternative Method for Question 11(b)		
	Find $\overrightarrow{AP}$ for a general point P on l, e.g. $-3\mathbf{j}+5\mathbf{k}+\mu(2\mathbf{i}-3\mathbf{j}+4\mathbf{k})$	**B1**	Or equivalent e.g. $\overrightarrow{PA}=-2\mu\mathbf{i}+(3\mu+3)\mathbf{j}-(4\mu+5)\mathbf{k}$.
	Use Pythagoras and differentiate with respect to μ to obtain value of μ corresponding to minimum distance. (No need to prove it is a minimum)	**M1**	$\frac{\mathrm{d}}{\mathrm{d}\mu}\left(4\mu^2+9(\mu+1)^2+(4\mu+5)^2\right)=0$.
	Obtain $\mu=-1$	**A1**	
	Obtain answer $-\mathbf{i}+2\mathbf{j}-\mathbf{k}$	**A1**	Accept coordinates in place of position vector.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5471210840

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

JC23 06_9709_33/2R

1 Solve the equation $\ln(x+5) = 5 + \ln x$. Give your answer correct to 3 decimal places. [4]

2 Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. [3]

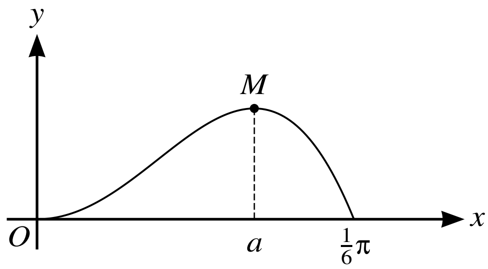
3 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z-3-\mathrm{i}| \leqslant 3$ and $|z| \geqslant |z-4\mathrm{i}|$. [4]

4 The parametric equations of a curve are

$$x = \frac{\cos\theta}{2 - \sin\theta}, \qquad y = \theta + 2\cos\theta.$$

Show that $\frac{dy}{dx} = (2 - \sin\theta)^2$. [5]

5



The diagram shows the part of the curve $y = x^2 \cos 3x$ for $0 \leqslant x \leqslant \frac{1}{6}\pi$, and its maximum point M, where $x = a$.

(a) Show that a satisfies the equation $a = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$. [3]

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(b) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(a)** Express $3\cos x + 2\cos(x - 60°)$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [4]

(b) Hence solve the equation

$$3\cos 2\theta + 2\cos(2\theta - 60°) = 2.5$$

for $0° < \theta < 180°$. [4]

7 **(a)** Use the substitution $u = \cos x$ to show that

$$\int_0^{\pi} \sin 2x \, e^{2\cos x} \, dx = \int_{-1}^{1} 2u e^{2u} \, du.$$ [4]

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(b) Hence find the exact value of $\int_0^{\pi} \sin 2x \, e^{2\cos x} \, dx$. [4]

8 The variables x and y satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 + 4}{x(y + 4)}$$

for $x > 0$. It is given that $x = 4$ when $y = 2\sqrt{3}$.

Solve the differential equation to obtain the value of x when $y = 2$. [8]

9 The lines l and m have equations

$$l: \quad \mathbf{r} = a\mathbf{i} + 3\mathbf{j} + b\mathbf{k} + \lambda(c\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}),$$
$$m: \quad \mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}).$$

Relative to the origin O, the position vector of the point P is $4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$.

(a) Given that l is perpendicular to m and that P lies on l, find the values of the constants a, b and c. [4]

(b) The perpendicular from P meets line m at Q. The point R lies on PQ extended, with $PQ : QR = 2 : 3$.

Find the position vector of R. [6]

10 Let $f(x) = \dfrac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

11 The complex number z is defined by $z = \dfrac{5a - 2\mathrm{i}}{3 + a\mathrm{i}}$, where a is an integer. It is given that $\arg z = -\frac{1}{4}\pi$.

(a) Find the value of a and hence express z in the form $x + \mathrm{i}y$, where x and y are real. [6]

(b) Express z^3 in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the simplified exact values of r and θ. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

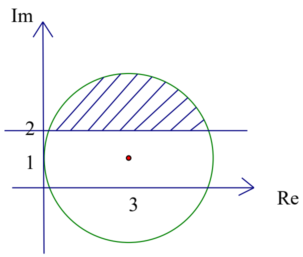
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use exponentials or law for the logarithm of a product, quotient or power	**M1***	$e^{\ln(5+x)} = e^{5+\ln x}$ insufficient. Need e.g. $\ln\left(\frac{x+5}{x}\right) = 5$ or $\ln(x+5) = \ln(e^5) + \ln x$ or $\ln(x + 5) = \ln(e^5x)$ or $x + 5 = e^{5 + \ln x}$ or $x + 5 = e^5 e^{\ln x}$ and others.
	Correctly remove logarithms	**DM1**	
	Obtain a correct equation in x	**A1**	e.g. $\frac{x+5}{x} = e^5$ (or 148.4…) or $x + 5 = xe^5$.
	Obtain 0.034	**A1**	CAO Final answer must be 3d.p.
		4	

Question	Answer	Marks	Guidance
2	Divide to obtain quotient $2x^2 \pm 2x + k \ (k \neq 0)$	**M1**	Obtain result in answer column, together with a linear polynomial or a constant as remainder. If correct: $\begin{array}{rl} & 2x^2 - 2x - 4 \\ \hline x^2+x+3 & 2x^4 \qquad\qquad\quad - 27 \\ & 2x^4+2x^3+6x^2 \\ \hline & -2x^3-6x^2 \\ & -2x^3-2x^2-6x \\ \hline & -4x^2+6x-27 \\ & -4x^2-4x-12 \\ \hline & 10x-15 \end{array}$
	Obtain [quotient] $2x^2 - 2x - 4$	**A1**	Allow unless quotient and remainder interchanged, then A0 A1.
	Obtain [remainder] $10x - 15$	**A1**	Allow $(x^2 + x + 3)(2x^2 - 2x - 4) + 10x - 15$.
	Alternative Method for Question 2		
	Expand $\left(x^2+x+3\right)\left(Ax^2+Bx+C\right)+\left(Dx+E\right)$ and reach $A = 2$, $B = \pm 2$, $C = k$	**M1**	Solve all 3 equations for A, B and C, allow sign errors in establishing equations and in solving. If correct, $A = 2$, $A + B = 0$, $3A + B + C = 0$, $3B + C + D = 0$, $3C + E = -27$. Obtain result in answer column, together with a linear polynomial or a constant as remainder.
	Obtain [quotient] $2x^2 - 2x - 4$	**A1**	Allow unless quotient and remainder interchanged, then A0 A1.
	Obtain [remainder] $10x - 15$	**A1**	Allow $(x^2 + x + 3)\ (2x^2 - 2x - 4) + 10x - 15$.
		3	

Question	Answer	Marks	Guidance
3	Show a circle with centre $3+\mathrm{i}$	**B1**	Must be some evidence of scale on both axes or centre stated as 3 + i or (3, 1).
	Show a circle with radius 3 and centre not at the origin	**B1**	Must be some evidence that radius = 3 or stated $r = 3$
	Show the line $y = 2$	**B1**	Line $y = 2$ can be represented by 2 or correct dashes.
	Shade the correct region	**B1**	Line and circle must be correct.
		**4**	Scales may be replaced by dashes on axes for all marks. Correct figure, with no scale on either axis then allow 1/3 and the B1 for correct shaded region Max 2/4. If B0 above for line but relatively correct position then B1 for correct shaded region Max 3/4. Re and Im axes interchanged but clearly labelled, allow **SCB1** for centre and radius of circle correct and **SCB1** for line and shading correct Max 2/4.

Question	Answer	Marks	Guidance
4	State $\dfrac{dy}{d\theta}=1-2\sin\theta$	**B1**	Ignore left side throughout dx/dt, dy/dt, dx, dy but must see $\dfrac{dy}{dx}$ for final A1.
	Use correct quotient rule, or product rule if rewrite x as $\cos\theta(2-\sin\theta)^{-1}$	**M1**	Incorrect formula seen M0 A0 otherwise BOD.
	Obtain $\dfrac{dx}{d\theta}=\dfrac{-(2-\sin\theta)\sin\theta+\cos^2\theta}{(2-\sin\theta)^2}$ o.e.	**A1**	$-\sin\theta(2-\sin\theta)^{-1}-\cos\theta(2-\sin\theta)^{-2}(-\cos\theta)$ or equivalent.
	Use $\dfrac{dy}{dx}=\dfrac{dy}{d\theta}\div\dfrac{dx}{d\theta}$	**M1**	$\left(\dfrac{dy}{dx}=(1-2\sin\theta)\div\dfrac{1-2\sin\theta}{(2-\sin\theta)^2}\right)$. Allow M1 even if errors in both derivatives.
	Obtain $\dfrac{dy}{dx}=(2-\sin\theta)^2$.	**A1**	AG – must see working in above cell to gain final A1. Allow $\cos^2\theta+\sin^2\theta=1$ to be implied. x instead of θ or missing θ more than twice on right side then A0 final mark.
		5	

Question	Answer	Marks	Guidance
5(a)	Use correct product rule	**M1**	$\frac{d}{dx}(x^2)\cos(3x) + x^2\frac{d}{dx}(\cos 3x)$.
	Obtain correct derivative in any form	**A1**	e.g. $2x\cos 3x - 3x^2 \sin 3x$.
	Equate derivative to zero and obtain $a = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$.	**A1**	AG Condone $a = \frac{1}{3}\tan^{-1}\frac{2}{3a}$. Must at least reach expression $2x = 3x^2 \tan(3x)$ or better <u>before</u> final answer to gain A1. Final answer must be in terms of a. Can work with x and switch to a at very end. Look for $\frac{2}{3}a$ or $\frac{2}{3}x$ in working not immediately corrected or as penultimate line A0.
		3	
5(b)	Use the iterative process $a_{n+1} = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a_n}\right)$ correctly at least twice during successive iterations in the numerous iterations	**M1**	Degrees 0/3.
	Obtain final answer 0.36	**A1**	Must be 2d.p.
	Show sufficient iterations to 4 or more d.p. to justify 0.36 to 2 d.p. or show there is a sign change in the interval $(0.355, 0.365)$	**A1**	Allow small errors in 4th d.p. Allow errors at start if self corrects later.
	0.5 0.4 0.3 0.2 0.1 π/6 π/12 0.3091 0.3435 0.3826 0.4264 0.4740 0.3017 0.3989 0.3789 0.3650 0.3499 0.3339 0.3176 0.3820 0.3439 0.3513 0.3566 0.3625 0.3688 0.3754 0.3502 0.3649 0.3619 0.3599 0.3576 0.3552 0.3526 0.3624 0.3567 0.3578 0.3604 0.3614 0.3576 0.3580	**3**	

Question	Answer	Marks	Guidance
6(a)	Expand $\cos(x-60°)$ correctly and evaluate $3\cos x + 2\cos(x - 60°)$ to obtain $4\cos x + \sqrt{3}\sin x$ or unsimplified coefficients	**B1**	Need to see $A\cos x + B\sin x$ with A and B correct A may be 4 or $3 + 2\cos 60$ and B may be $\sqrt{3}$ or $2\sin 60$.
	State $R = \sqrt{19}$ [$R\cos\alpha = 4$ $R\sin\alpha = \sqrt{3}$]	**B1 FT**	Follow through *their* 4 and $\sqrt{3}$. If coefficients are 3 and 2 then B0. $R = \sqrt{19}$ from $R = 4.36$ B0 but 4.36 seen after $\sqrt{19}$ ISW.
	Use correct trig formulae for their expansion to find α e.g. $\alpha = \tan^{-1}\frac{\sqrt{3}}{4}$ or $\cos^{-1}\frac{4}{\sqrt{19}}$ or $\sin^{-1}\frac{\sqrt{3}}{\sqrt{19}}$	**M1**	If $\sin\alpha = \sqrt{3}$ $\cos\alpha = 4$ seen then M0 A0. If $\underline{\tan\ \alpha} = 23.41°$ M0 A0 but can recover if $\alpha = 23.41°$ seen later. $\alpha = \tan^{-1}\frac{2}{3}$ M1 $(\alpha = 33.69°)$ but $\alpha = \tan^{-1}\frac{3}{2}$ M0
	Obtain $\alpha = 23.41°$	**A1**	Allow if x instead of α.
		4	

Question	Answer	Marks	Guidance
6(b)	$\cos^{-1}\left(\frac{2.5}{R}\right)$	**B1 FT**	SOI [55.0°]. Follow through *their* $\sqrt{19}$.
	Use a correct method to find a value of 2θ (not x) in the interval. Allow sign error in moving α to right side	**M1**	$2\theta = \cos^{-1}\left(\frac{2.5}{R}\right) + 23.41°$ or $2\theta = 360° - \cos^{-1}\left(\frac{2.5}{R}\right) + 23.41°$ with R substituted.
	Obtain one correct answer e.g. $39.2°$	**A1**	If working for **M1** not seen then **M1** implied by $39.2°$ or $164.2°$ Must be at least 1d.p.
	Obtain second correct answer e.g. $164.2°$ and no others in the interval	**A1**	Must be at least 1d.p. Ignore answers outside the given interval.
		4	

Question	Answer	Marks	Guidance
7(a)	$\dfrac{du}{dx} = -\sin x$	**B1**	SOI
	Use double angle formula and substitute for x and dx throughout the integral	**M1**	All x's must be removed, can be coefficient errors provided 2 seen in working.
	Obtain $\pm\int 2ue^{2u}\,du$	**A1**	Limits may be omitted, or left as 0 and π, during the change of variable stage.
	Justify new limits and obtain $\int_{-1}^{1} 2ue^{2u}\,du$ from correct working	**A1**	AG Must see $x = 0$, $u = 1$ and $x = \pi$, $u = -1$. Inequalities alone e.g. $0 \leqslant x \leqslant \pi$ and $1 \leqslant u \leqslant -1$ or $-1 \leqslant u \leqslant 1$ for limits are insufficient A0 If sign in expression and order of limits incorrect then A0. If negative sign is present in the integrand then this can be removed and limits introduced in correct order in a single step.
		4	

Question	Answer	Marks	Guidance
7(b)	Commence integration and reach $aue^{2u} + b\int e^{2u}\,du$, where $ab \neq 0$, $b < 0$	**M1***	Condone dx.
	Complete integration and obtain $ue^{2u} - \frac{1}{2}e^{2u}$	**A1**	OE Allow $(2u\frac{1}{2}e^{2u}) - \frac{1}{2}e^{2u}$.
	Use correct limits correctly in $c\,ue^{2u} + d\,e^{2u}$ having integrated twice or in $c\cos x\,e^{2\cos x} + d\,e^{2\cos x}$	**DM1**	1 and −1 for u, 0 and π for x e.g. $ce^2 + de^2 - (-ce^{-2} + de^{-2})$. Not decimals. Allow one sign error at most in going from $c\,ue^{2u} + d\,e^{2u}$ or $c\cos x\,e^{2\cos x} + d\,e^{2\cos x}$ to $ce^2 + de^2 - (-ce^{-2} + de^{-2})$. $[e^2 - ½\,e^2 - (-e^{-2} - ½\,e^{-2})]$ Complete reversal of sign by converting back to cos x and not making $x = 0$ upper limit is DM0 A0.
	Obtain $\frac{1}{2}e^2 + \frac{3}{2}e^{-2}$	**A1**	ISW Or equivalent 2-term expression e.g. $\frac{e^4+3}{2e^2}$ or $\frac{1}{2}\left(e^2 + \frac{3}{e^2}\right)$.
		4	

Question	Answer	Marks	Guidance
8	Separate the variables correctly	B1	$\dfrac{y+4}{y^2+4}\mathrm{d}y = \dfrac{1}{x}\mathrm{d}x$.
	Obtain $\ln x$	B1	
	Split the fraction and integrate to obtain $p\ln\left(y^2+4\right)$ or $q\tan^{-1}\frac{y}{2}$ correctly	*M1	Only following subdivision into $\dfrac{y}{y^2+4}+\dfrac{4}{y^2+4}$. If no subdivision seen then both terms $p\ln\left(y^2+4\right)$ and $q\tan^{-1}\dfrac{y}{2}$ must be present.
	Obtain $\frac{1}{2}\ln\left(y^2+4\right)$	A1	
	Obtain $2\tan^{-1}\frac{y}{2}$	A1	
	Use $\left(4, 2\sqrt{3}\right)$ in an expression containing at least 2 of $a\ln x, b\ln\left(y^2+4\right)$ and $c\tan^{-1}\frac{y}{2}$ to obtain constant of integration	DM1	Allow one sign or arithmetic error e.g. $\dfrac{2\pi}{3}$. May use $\left(4, 2\sqrt{3}\right)$ and $(x, 2)$ as limits to find x for the final 3 marks.
	Correct solution (any form) e.g. $\dfrac{1}{2}\ln\left(y^2+4\right)+2\tan^{-1}\dfrac{y}{2}=\ln x+\dfrac{2\pi}{3}$ or $\dfrac{1}{2}\ln\left(y^2+4\right)+2\tan^{-1}\dfrac{y}{2}=\ln x+2\tan^{-1}\sqrt{3}+\dfrac{1}{2}\ln 16-\ln 4$	A1	However solution not asked for so allow $\dfrac{1}{2}\ln 8+2\tan^{-1}1=\ln x+2\tan^{-1}\sqrt{3}+\dfrac{1}{2}\ln 16-\ln 4$.
	Obtain $\sqrt{8}\mathrm{e}^{-\frac{1}{6}\pi}$ or 1.68 or more accurate or $2\sqrt{2}\mathrm{e}^{-\frac{1}{6}\pi}$ or $\dfrac{\sqrt{8}}{\mathrm{e}^{\frac{1}{6}\pi}}$ or $\mathrm{e}^{0.516}$	A1	ISW Must remove ln so $x = \mathrm{e}^{(\ln 2\sqrt{2}-\pi/6)}$ A0.

Question	Answer	Marks	Guidance
8	**Alternative method for first *M1 A1 A1**		
	$p\left((y+4)\tan^{-1}\frac{y}{2}-\int\tan^{-1}\frac{y}{2}\,\mathrm{d}y\right)$	***M1**	Allow sign error.
	$(y+4)\frac{1}{2}\tan^{-1}\frac{y}{2}-\frac{y}{2}\tan^{-1}\frac{y}{2}+\int\frac{y}{y^2+4}\,\mathrm{d}y$	**A1**	
	Obtain $2\tan^{-1}\frac{y}{2}+\frac{1}{2}\ln\left(y^2+4\right)$	**A1**	
		8	

Question	Answer	Marks	Guidance
9(a)	Perform scalar product of direction vectors and set result equal to zero	**M1**	$2c+6+4=0$.
	Use P to find the value of λ	**M1**	$3-2\lambda=7 \Rightarrow \lambda=-2$ [$a+\lambda c=4$, $b+4\lambda=-2$]. Equation for line l may contain $-\lambda$ instead of $+\lambda$ leading to $\lambda=2$ all marks available.
	Obtain $c=-5$ or $b=6$	**A1**	
	$a=-6$, $b=6$ and $c=-5$ all correct	**A1**	
		4	**SC1**: Use P to find the value of λ M1 Substitute $\lambda=-2$ into point P, so $a-2c=4$, and put $\mu=-1$ and $\lambda=-1$ into l so $a-c=-1$, then solve to obtain $a=-6$, $b=6$ and $c=-5$. All 3 values correct A1. Max 2/4.

Question	Answer	Marks	Guidance
9(b)	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on m $= \pm((1+2\mu, 2-3\mu, 3+\mu)-(a+\lambda c, 3-2\lambda, b+4\lambda))$	B1	$\left[\overrightarrow{PQ} \text{ or } \overrightarrow{QP} = \pm\begin{pmatrix}-3+2\mu\\-5-3\mu\\5+\mu\end{pmatrix}\right]$ Could be *their* a, b, c and λ values provided M1 M1 gained in **(a)**. Allow expression in answer column.
	Equate the scalar product of $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) and a direction vector for m to zero and obtain an equation in μ	M1*	$\left(2(-3+2\mu)-3(-5-3\mu)+(5+\mu)\right)=0$. Allow $\overrightarrow{PQ} = \overrightarrow{OQ} + \overrightarrow{OP}$ sign problem.
	Solve and obtain $\mu=-1$	A1	$PQ^2 = (-3+2\mu)^2+(-5-3\mu)^2+(5+\mu)^2$. $[= 14(\mu+1)^2+45]$. Min when $\mu = -1$ or by differentiation.
	Obtain $\overrightarrow{OQ} = -\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ or $\overrightarrow{PQ} = -5\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ Must be labelled correctly	A1	The working may be in **(a)** provided at least this result is used in **(b)**.
	Carry out a method to find the position vector of R **Alternative method for DM1** $\overrightarrow{OR} = (4, 7, -2) + t(-5, -2, 4)$ $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$ Solve $\lvert\boldsymbol{QR}\rvert^2 = \frac{9}{4}\lvert\boldsymbol{PQ}\rvert^2$ or $\lvert\boldsymbol{QR}\rvert = \frac{3}{2}\lvert\boldsymbol{PQ}\rvert$ $t = 2.5$	DM1	e.g. Use $\overrightarrow{OR} = \overrightarrow{OP} + \frac{5}{2}\overrightarrow{PQ}$ or $\overrightarrow{OR} = \overrightarrow{OQ} + \frac{3}{2}\overrightarrow{PQ}$ or $\overrightarrow{OR} = \frac{5}{2}\overrightarrow{OQ} - \frac{3}{2}\overrightarrow{OP}$ or $2\overrightarrow{QR} = 2(\overrightarrow{OR} - \overrightarrow{OQ}) = 3\overrightarrow{PQ}$ where $\overrightarrow{OR} = (x, y, z)$. $\overrightarrow{PQ}$ used in all these approaches, may be incorrect, must be in the correct direction, i.e. not using $\overrightarrow{QP}$ for $\overrightarrow{PQ}$.

Question	Answer	Marks	Guidance
9(b)	Obtain $-\frac{17}{2}\boldsymbol{i}+2\boldsymbol{j}+8\boldsymbol{k}$ from correct working	**A1**	Accept coordinates. Don't accept $-\frac{17}{2}\mathbf{i}+\frac{4}{2}\mathbf{j}+\frac{16}{2}\mathbf{k}$.
		6	**SC2** Equate lines, attempt to find $\mu=-1$ or $\lambda=-1$ M1* $\overrightarrow{OQ}=-\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ A1. Attempt to find $\overrightarrow{OQ}$ using other parameter value DM1. $\overrightarrow{OQ}=-\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ therefore intersect A1. Then use main scheme for the final DM1 A1.
			First **DM1 A1** are available if they show the 3 coordinates are consistent for the 2 parameter values instead of attempting to find $\overrightarrow{OQ}$ using the other parameter value and then showing intersection

Question	Answer	Marks	Guidance
10(a)	State or imply the form $\dfrac{A}{1+2x}+\dfrac{B}{3-x}+\dfrac{C}{(3-x)^2}$	**B1**	Alternative form: $\dfrac{A}{1+2x}+\dfrac{Dx+E}{(3-x)^2}$.
	Use a correct method to find a constant	**M1**	Incorrect format for partial fractions: Allow M1 and a possible A1 if obtain one of these correct values. Max 2/5 Allow M1 even if multiply up by $(1 + 2x)(3 - x)^3$.
	Obtain one of $A=2$, $B=2$ and $C=-3$	**A1**	Alternative form: obtain one of $A=2$, $D=-2$ and $E=3$.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Do not need to substitute values back into original form.
		5	If $\dfrac{A}{1+2x}+\dfrac{B}{3-x}+\dfrac{Cx+D}{(3-x)^2}$ B0 but M1 A1 for *A*, A1 for *B* and A1 for *C* and *D*. If $C = 0$ then recovers B1 from above.

Question	Answer	Marks	Guidance
10(b)	Use a correct method to obtain the first two terms of one of the unsimplified expansions $(1+2x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-2}$ $(3-x)^{-1}$, $(3-x)^{-2}$	**M1**	$(1+2x)^{-1} = 1 + (-1)(2x) + \ldots$ $\left(1-\frac{1}{3}x\right)^{-1} = 1 + (-1)(-x/3) + \ldots$ $\left(1-\frac{1}{3}x\right)^{-2} = 1 + (-2)(-x/3) + \ldots$ $(3-x)^{-1} = 3^{-1} + (-1)3^{-2}(-x) \ldots$ $(3-x)^{-2} = 3^{-2} + (-2)3^{-3}(-x) + \ldots$
	Obtain the correct unsimplified expansions up to the term in x^2 for each partial fraction If correct, should be working with $\frac{2}{1+2x}+\frac{2}{3-x}-\frac{3}{(3-x)^2}$ or $\frac{2}{1+2x}+\frac{-2x+3}{(3-x)^2}$	**A1 FT** **A1 FT** **A1 FT**	Follow through on *their* A, B, C $A(1 + (-1)(2x) + ((-1)(-2)/2)(2x)^2 + \ldots)$ $\frac{B}{3}$ $(1 + (-1)(-x/3) + ((-1)(-2)/2)(-x/3)^2 + \ldots)$ $\frac{C}{3^2}$ $(1 + (-2)(-x/3) + ((-2)(-3)/2)(-x/3)^2 + \ldots)$. Must be *their* coefficients from **(a)** but may be unsimplified expansions for FT marks. If correct, expect to see $2(1 - 2x + (2x)^2)$ or $2-4x+8x^2$ $\frac{2}{3}\left(1 + \frac{x}{3} + \left(\frac{x}{3}\right)^2\right)$ or $\frac{2}{3}+\frac{2}{9}x+\frac{2}{27}x^2$ $-\frac{1}{3}\left(1 + \frac{2x}{3} + (3)\left(\frac{x}{3}\right)^2\right)$ or $-\frac{1}{3}-\frac{2}{9}x-\frac{x^2}{9}$.
	Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$	**A1**	Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW.

Question	Answer	Marks	Guidance
10(b)	**Alternative Method for Question 10(b)**		
	For the form $\dfrac{A}{1+2x}+\dfrac{Dx+E}{(3-x)^2}$	**M1***	For the first two terms of an expanded partial fraction, following their A, D, E.
	Obtain the correct unsimplified expansions up to the term in x^2 for each partial fraction	**A1FT**	$A(1+(-1)(2x)+((-1)(-2)/2)(2x)^2+\dots)+$
		A1FT	$(Dx+E)\dfrac{1}{3^2}(1+(-2)(-x/3)+((-2)(-3)/2)(-x/3)^2+)$ $2(1-2x+(2x)^2+\dots)$ $+\dfrac{-2x+3}{3^2}(1+\dfrac{2x}{3}+(3)\left(\dfrac{x}{3}\right)^2+\dots)$.
	Multiply out fully	**DM1**	Provided $DE\neq 0$. Ignore cubic terms and above. Allow error in one term but all terms must be present. If correct, expect to see $2-4x+8x^2-\dfrac{2}{9}x-\dfrac{4}{27}x^2+\dfrac{1}{3}+\dfrac{2}{9}x+\dfrac{1}{9}x^2$
	Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$	**A1**	Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW

Question	Answer	Marks	Guidance
10(b)	**Alternative Method for Question 10(b): Maclaurin's Series**		
	Correct derivatives for $A(1+2x)^{-1}$, $B(3-x)^{-1}$ and $C(3-x)^{-2}$ $(-1)(2)A(1+2x)^{-2}$, $(-1)(-1)B(3-x)^{-2}$ and $(-2)(-1)C(3-x)^{-3}$	**B1 FT**	
	One of following $(-2)(2)(-1)(2)A(1+2x)^{-3}$, $(-2)(-1)(-1)(-1)B(3-x)^{-3}$ and $(-3)(-1)(-2)(-1)C(3-x)^{-4}$	**B1 FT**	
	All correct	**B1 FT**	
	Substitute in $f(0)+xf'(0)+\frac{x^2}{2}f''(0)$	**M1**	
	Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$	**A1**	Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW
		5	

Question	Answer	Marks	Guidance
11(a)	Multiply numerator and denominator by $(3-a\mathrm{i})$	**M1**	Must perform complete multiplications but need not simplify i^2. Can have errors but no term duplicated or missing. $\dfrac{(5a-2\mathrm{i})(3-a\mathrm{i})}{9-a^2}=\dfrac{13a-i(5a^2+6)}{9-a^2}$ M0 M1 A0 No working so unsure if denominator multiplied by $3-a\mathrm{i}$ M1 M1 A0
	Use $\mathrm{i}^2=-1$ at least once and separate real and imaginary parts	**M1**	
	Obtain $\dfrac{13a-\mathrm{i}(5a^2+6)}{9+a^2}$ or $\dfrac{13a-5a^2\mathrm{i}-6\mathrm{i}}{9+a^2}$	**A1**	OE If $15a-2a=13a$ seen later award this A1.
	Use $\arg z$ to form equation in a $-\dfrac{5a^2+6}{13a}=\pm\tan\left(\pm\dfrac{\pi}{4}\right)$ or $-\dfrac{13a}{5a^2+6}=\pm\tan\left(\pm\dfrac{\pi}{4}\right)$ or $\tan^{-1}\left(-\dfrac{5a^2+6}{13a}\right)=\pm\dfrac{\pi}{4}$ or $\tan^{-1}\left(-\dfrac{13a}{5a^2+6}\right)=\pm\dfrac{\pi}{4}$	**M1**	Allow expression given in answer column or $5a^2+6=\pm 13a$ or use $-(x\pm x\mathrm{i})=(13a-\mathrm{i}(5a^2+6))/(9+a^2)$ and eliminate x so $5a^2+6=\pm 13a$ M1.
	Obtain $a=2$	**A1**	Need to reject $a=\dfrac{3}{5}$ or ignore it in future work. May not see second root, but if present, must be $\dfrac{3}{5}$.
	Obtain $z=2-2\mathrm{i}$ only	**A1**	Allow $z=-2\mathrm{i}+2$.

Question	Answer	Marks	Guidance
11(a)	**Alternative Method 1 for the first four marks**		
	$\arg z = \arg(5a - 2\text{i}) - \arg(3 + a\text{i})$	**M1**	
	$= \tan^{-1}\left(\frac{-2}{5a}\right) - \tan^{-1}\left(\frac{a}{3}\right)$ $= \tan^{-1}\left(\frac{-2}{5a} - \frac{a}{3}\right) / \left(1 + \left(\frac{-2}{5a}\right)\left(\frac{a}{3}\right)\right)$	**M1**	Allow one sign error in second M1.
	$= \tan^{-1}\left(-\frac{5a^2+6}{13a}\right)$ or $\tan^{-1}\left(-\frac{13a}{5a^2+6}\right)$	**A1**	
	$\pm\frac{\pi}{4} = \tan^{-1}\left(-\frac{5a^2+6}{13a}\right)$ or $\tan^{-1}\left(-\frac{13a}{5a^2+6}\right)$	**M1**	Equate *their* $\tan^{-1}\left(-\frac{5a^2+6}{13a}\right)$ to $\pm\frac{\pi}{4}$. Then as original scheme for final 2 marks.
	Alternative Method 2 for the first four marks		
	$(x + \text{i}y)(3 + a\text{i}) = 5a - 2\text{i}$ $3x - ay = 5a$ and $ax + 3y = -2$	**M1 A1**	
	$x = \pm y$ Find x or y in terms of a, e.g. $x = \frac{2}{3-a}$ or $x = \frac{5a}{3+a}$	**M1**	
	Substitute in other equation, for example $3\left(\frac{2}{3-a}\right) + a\left(\frac{2}{3-a}\right) = 5a$	**M1**	Then as original scheme for final 2 marks.
		6	

Question	Answer	Marks	Guidance
11(b)	State $\arg\left(z^3\right) = -\frac{3}{4}\pi$ or evaluate from $z = b - b\mathrm{i}$ or from $-2b^3(1 + \mathrm{i})$	**B1**	If 2 different values given award B0. Do not ISW.
	Complete method to obtain r from *their z*	**M1**	$\left\|z^3\right\| = \left(\sqrt{x^2 + y^2}\right)^3$. If z correct, may see $\left\|z^3\right\| = \left(\sqrt{2^2 + (-2)^2}\right)^3$ or $\left\|z^3\right\| = \sqrt{(-16)^2 + (-16)^2}$.
	$r = 16\sqrt{2}$	**A1**	CAO A1 if $z = 2 - 2\mathrm{i}$ obtained correctly. or $z =$ used with $a = 2$ found correctly, otherwise A0XP. May see arg and r given in a final answer i.e. $16\sqrt{2}e^{-\frac{3}{4}\pi\mathrm{i}}$. Allow this form for arg and r to collect full marks, even if i missing. Ignore answers outside the given interval. If 2 different values given award A0.
		3	

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

* 5 3 4 5 8 6 4 1 0 4 *

MATHEMATICS **9709/41**

Paper 4 Mechanics **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 Two particles P and Q, of masses m kg and 0.3 kg respectively, are at rest on a smooth horizontal plane. P is projected at a speed of $5\,\mathrm{m\,s^{-1}}$ directly towards Q. After P and Q collide, P moves with a speed of $2\,\mathrm{m\,s^{-1}}$ in the same direction as it was originally moving.

(a) Find, in terms of m, the speed of Q after the collision. [2]

After this collision, Q moves directly towards a third particle R, of mass 0.6 kg, which is at rest on the plane. Q is brought to rest in the collision with R, and R begins to move with a speed of $1.5\,\mathrm{m\,s^{-1}}$.

(b) Find the value of m. [2]

2 A particle P of mass $0.4\,\text{kg}$ is projected vertically upwards from horizontal ground with speed $10\,\text{m}\,\text{s}^{-1}$.

(a) Find the greatest height above the ground reached by P. [2]

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When P reaches the ground again, it bounces vertically upwards. At the first instant that it hits the ground, P loses $7.2\,\text{J}$ of energy.

(b) Find the time between the first and second instants at which P hits the ground. [4]

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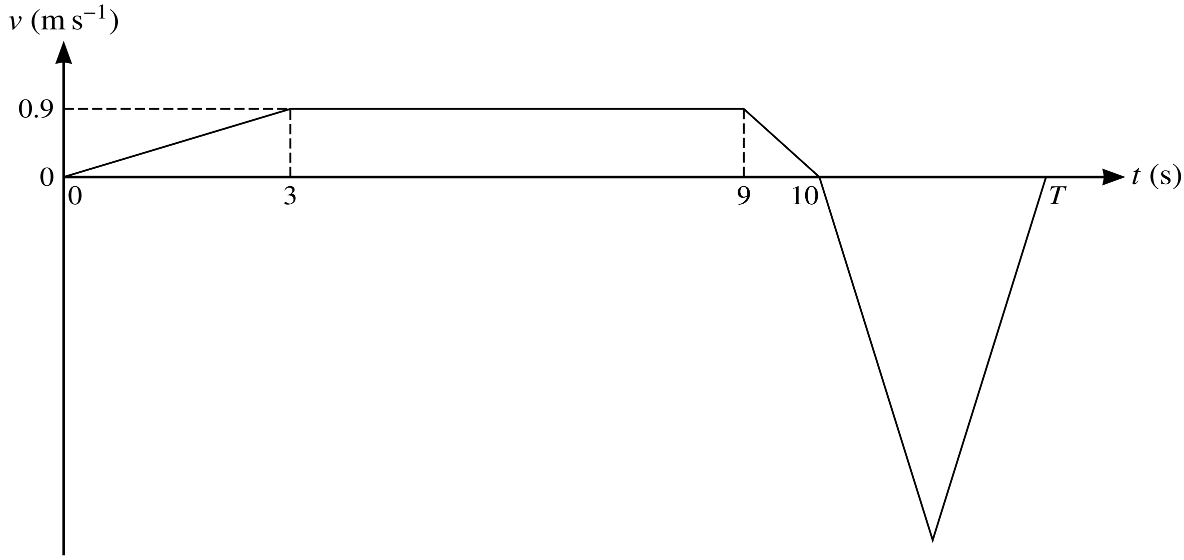
3 A particle moves in a straight line starting from rest. The displacement s m of the particle from a fixed point O on the line at time t s is given by

$$s = t^{\frac{5}{2}} - \frac{15}{4}t^{\frac{3}{2}} + 6.$$

Find the value of s when the particle is again at rest. [4]

4



The velocity of a particle at time t s after leaving a fixed point O is v m s^{-1}. The diagram shows a velocity-time graph which models the motion of the particle. The graph consists of 5 straight line segments. The particle accelerates to a speed of 0.9 m s^{-1} in a period of 3 s, then travels at constant speed for 6 s, and then comes instantaneously to rest 1 s later. The particle then moves back and returns to rest at O at time T s.

(a) Find the distance travelled by the particle in the first 10 s of its motion. [2]

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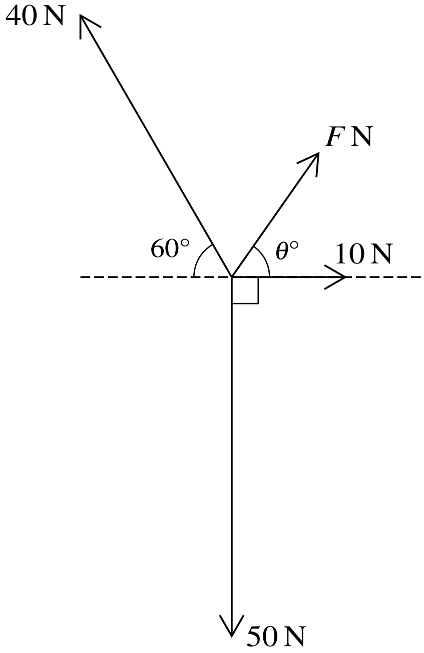
(b) Given that $T = 12$, find the minimum velocity of the particle. [2]

(c) Given instead that the greatest speed of the particle is $3\,\mathrm{m\,s^{-1}}$, find the value of T and hence find the average speed of the particle for the whole of the motion. [4]

5



Four coplanar forces act at a point. The magnitudes of the forces are F N, 10 N, 50 N and 40 N. The directions of the forces are as shown in the diagram.

(a) Given that the forces are in equilibrium, find the value of F and the value of θ. [6]

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(b) Given instead that $F = 10\sqrt{2}$ and $\theta = 45$, find the direction and the exact magnitude the resultant force. [3]

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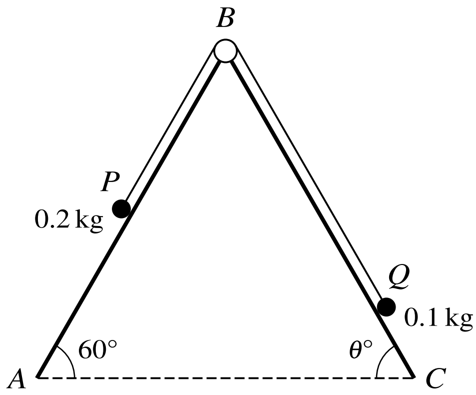

Two particles P and Q, of masses 0.2 kg and 0.1 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley at B which is attached to two inclined planes. Particle P lies on a smooth plane AB which is inclined at 60° to the horizontal. Particle Q lies on a plane BC which is inclined at an angle of $\theta°$ to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram).

(a) It is given that $\theta = 60$, the plane BC is rough and the coefficient of friction between Q and the plane BC is 0.7. The particles are released from rest.

Determine whether the particles move. [4]

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(b) It is given instead that the plane BC is smooth. The particles are released from rest and in the subsequent motion the tension in the string is $(\sqrt{3}-1)$ N.

Find the magnitude of the acceleration of P as it moves on the plane, and find the value of θ. [4]

7 A car of mass 1200 kg is travelling along a straight horizontal road. The power of the car's engine is constant and is equal to 16 kW. There is a constant resistance to motion of magnitude 500 N.

(a) Find the acceleration of the car at an instant when its speed is $20\,\mathrm{m\,s^{-1}}$. [3]

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(b) Assuming that the power and the resistance forces remain unchanged, find the steady speed at which the car can travel. [2]

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The car comes to the bottom of a straight hill of length 316 m, inclined at an angle to the horizontal of $\sin^{-1}\left(\frac{1}{60}\right)$. The power remains constant at 16 kW, but the magnitude of the resistance force is no longer constant and changes such that the work done against the resistance force in ascending the hill is 128 400 J. The time taken to ascend the hill is 15 s.

(c) Given that the car is travelling at a speed of $20\,\mathrm{m\,s^{-1}}$ at the bottom of the hill, find its speed at the top of the hill. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/41**

Paper 4 Mechanics **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **13** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$m \times 5 + 0 = m \times 2 + 0.3v$	**M1**	Attempt at conservation of momentum; 3 non-zero terms (with m appearing in two terms); allow sign errors.
	Speed $= 10m$ $(\mathrm{m\,s^{-1}})$	**A1**	M1A0 if using g in momentum terms. $v = -10m$ is A0.
		2	
1(b)	$0.3 \times 10m + 0 = 0 + 0.6 \times 1.5$ $[3m = 0.9]$	**M1**	Attempt at conservation of momentum between Q and R (so must be using correct masses of 0.3 and 0.6) to form a linear equation in m using their answer from **(a)**; 2 non-zero terms; allow sign errors.
	$m = 0.3$	**A1FT**	FT $\dfrac{3}{\textit{their} \text{ +ve coefficient of } m \text{ from } \textbf{(a)}}$ Condone including kg in answer.
		2	

Question	Answer	Marks	Guidance
2(a)	$0 = 10^2 + 2(-g)s \Rightarrow s = \ldots$	**M1**	For use of $v^2 = u^2 + 2as$ with $v = 0$, $u = \pm 10$ and $a = \pm g$ or ± 10 and solve for s (or any other complete SUVAT method). Or using an energy method: $0.4gh = \frac{1}{2}(0.4)(10)^2$ and solve for h (with two terms using correct given values – condone lack of masses in conservation of energy equation).
	Max. height = 5 (m)	**A1**	
		2	

Question	Answer	Marks	Guidance
2(b)	KE before impact $=\frac{1}{2}\times 0.4\times 10^2\,[=20]$	**B1FT**	Or loss of PE = $\pm\, 0.4\times g\times 5$ using their maximum height from **(a)**.
	$\frac{1}{2}\times 0.4\times v^2=20-7.2\,[v=8]$ or $0.4gh=20-7.2\,[h=3.2]$	***M1**	M1 for $\frac{1}{2}\times 0.4v^2=$ KE/PE before impact $-\,7.2$ (must be correct method of subtracting 7.2 (OE)). Or, for finding the maximum height after first impact. Need not solve for v (or h) for this mark.
	$-8=8+(-g)t\Rightarrow t=...$ or $0=8t+\frac{1}{2}\times -g\times t^2\Rightarrow t=...$	**DM1**	For use of a complete method to find t. Condone sign errors but $a=\pm g$. If calculating the time to the maximum height between the first and second impacts, then candidates must double this answer (OE). E.g., $0=8+(-g)T,\ t=2T=...$ or $3.2=0-0.5\times -g\times T^2\Rightarrow t=2T=...$.
	Time = 1.6(s)	**A1**	
		4	

Question	Answer	Marks	Guidance
3	For an attempt at differentiation.	*M1	Decrease power by 1 and a change in coefficient in at least one term. $v = \frac{s}{t}$ is M0.
	$(v =)\frac{5}{2}t^{\frac{3}{2}} - \frac{45}{8}t^{\frac{1}{2}}$	A1	Allow unsimplified, including indices (including a $+c$ is A0).
	$v = 0 \Rightarrow \frac{1}{8}t^{\frac{1}{2}}(20t - 25) = 0 \Rightarrow t = ...$	DM1	Attempting to find t by equating v to 0 and attempt to solve a linear equation for t (if correct $t = 2.25$). Must be of the form $t = \ldots$.
	$s = \frac{15}{16}[= 0.9375]$	A1	Condone 0.938 .
		4	

Question	Answer	Marks	Guidance
4(a)	Distance $= \frac{1}{2}(6 + 10) \times 0.9$	M1	Completely correct method for finding the total area underneath the velocity-time graph from $t = 0$ to 10 only. Can be done as two triangles and a rectangle e.g., $\frac{1}{2} \times 3 \times 0.9 + (9 - 3) \times 0.9 + \frac{1}{2} \times 1 \times 0.9$ (allow a slip in one value); need not see all three components added together.
	Distance = 7.2(m)	A1	
		2	

Question	Answer	Marks	Guidance
4(b)	$\frac{1}{2}(12-10)v_{\min} = -7.2$	**M1**	Setting $\frac{1}{2}(12-10)v_{\min}$ equal to $\pm$ their **(a)**.
	Minimum velocity = -7.2 (m s^{-1})	**A1**	Must be negative – allow those who solve $\frac{1}{2}(12-10)v_{\min} = 7.2$ and obtain $v_{\min} = 7.2$ and then change to -7.2 without justification. **SC B1** for assuming the triangle is isosceles.
		2	
4(c)	$\frac{1}{2}\times(T-10)\times 3 = 7.2$ or $\frac{1}{2}\times t\times 3 = 7.2$	***M1**	Correct method for finding T or t. Condone sign errors but must equate to their answer to **(a)**.
	$T = 14.8$	**A1**	OE (e.g. from $t + 10 = 4.8 + 10 = 14.8$).
	$\frac{14.4}{14.8}$	**DM1**	**M1** for $\frac{2(\textit{their}\ \textbf{(a)})}{\textit{their}\ T}$ or $\frac{2(\textit{their}\ \textbf{(a)})}{10+\textit{their}\ T}$.
	Average speed = $\frac{36}{37}$ (m s^{-1})	**A1**	OE 0.973 [For reference: 0.97297…]. **SC *B1** (for $T = 14.8$). DM1A1 for assuming the triangle is isosceles.
		4	

Question	Answer	Marks	Guidance
5(a)	Resolving either direction.	**M1**	3 terms; allow sign errors and allow sin/cos mix. Must be an equation with either = 0 or with an attempt to balance forces.
	Vertical: $F\sin\theta + 40\sin 60 - 50 = 0$	**A1**	$\left[F\sin\theta = 50 - 20\sqrt{3} = 15.358...\right]$
	Horizontal: $F\cos\theta + 10 - 40\cos 60 = 0$	**A1**	$\left[F\cos\theta = 10\right]$
	$\theta = \tan^{-1}(5 - 2\sqrt{3})$	**M1**	Attempt to solve for θ; one missing term in total $\theta = \tan^{-1}1.535898...$.
	$F = \sqrt{15.358...^2 + 10^2}$	**M1**	Attempt to solve for F: one missing term in total.
	$\theta = 56.9$, $F = 18.3$	**A1**	Both correct (18.327530…, 56.932462…).
		6	
5(b)	$(Y =) \pm(10\sqrt{2}\sin 45 + 40\sin 60 - 50)\ [= \pm(20\sqrt{3} - 40)]$	**B1**	Allow non-exact values for $\sqrt{2}$ etc. in correct expression.
	$(X =) \pm(10\sqrt{2}\cos 45 + 10 - 40\cos 60)\ [= 0]$	**B1**	Allow non-exact values for $\sqrt{2}$ etc. in correct expression. Could be implied by correct answer.
	Resultant force is $40 - 20\sqrt{3}$ (N) in the same direction as the 50(N) force.	**B1**	Allow vertically downwards, south, 180°, negative y-direction. Resultant force must be exact and positive (so $20\sqrt{3} - 40$ is B0).
		3	

Question	Answer	Marks	Guidance
6(a)	$R = 0.1g\cos 60 [= 0.5]$	**B1**	Correctly resolving perpendicular to the plane for *Q*.
	$F = 0.7 \times 0.1g\cos 60\ [= 0.35]$	**M1**	Use of $F = \mu R$ for *Q* where *R* is a component of weight but not mass; allow sin/cos mix.
	For whole system: LHS of Newton's second law: $0.2g\sin 60 - 0.1g\sin 60 - F\ \ [= 0.866... - F]$ Or separately for *P* and *Q*: $0.2g\sin 60 - T (= 0.2a)$ and $T - 0.1g\sin 60 - F (= 0.1a)$, and eliminate *T* to get $0.2g\sin 60 - 0.1g\sin 60 - F (= 0.3a)$	**M1**	Complete method to determine the resultant force for the whole system. Allow sign errors and sin/cos mix, but must include all required terms and be dimensionally correct. If considering either the while system or *P* and *Q* separately then ignore the RHS of their Newton's second law equations.
	As $\frac{\sqrt{3}}{2} - 0.35 > 0$ the particles do move.	**A1**	Correct indication (with no incorrect working) that the resultant force is positive (e.g. $0.8660... - 0.35 > 0$ or 0.516… (to at least 1 sf) which is positive) together with a correct conclusion. Candidates may calculate the acceleration which is $\frac{10\sqrt{3} - 7}{6} = 1.72008...$ and then say that the particles are moving.
		4	

Question	Answer	Marks	Guidance
6(b)	Attempt to use Newton's second law for *P*: $0.2g\sin 60-(\sqrt{3}-1)=0.2a$	**M1**	Allow sign errors, sin/cos mix. but must be dimensionally consistent.
	$a=5\,(\mathrm{m\,s^{-2}})$	**A1**	
	Newton's second law for system: $0.2g\sin 60-0.1g\sin\theta=0.3(5)$ or Newton's second law for *Q*: $(\sqrt{3}-1)-0.1g\sin\theta=0.1(5)$	**M1**	Attempt Newton's second law for *Q*, or for the whole system. Allow sign errors, sin/cos mix, but must be dimensionally consistent.
	$\theta=13.4$	**A1**	13.41784…
		4	

Question	Answer	Marks	Guidance
7(a)	Driving force $F=\dfrac{16000}{20}[=800]$	**B1**	OE e.g. $16000=20\times F$
	$F-500=1200a$	**M1**	Use of Newton's second law; allow sign errors but must be 3 terms. Allow *F* or any non-zero value for the driving force (allow 0.8 from using 16 rather than 16000) but not 16000, 16, 20 or 500 for *F*.
	$a=0.25\,(\mathrm{m\,s^{-2}})$	**A1**	
		3	

Question	Answer	Marks	Guidance
7(b)	$\frac{16000}{v} - 500 = 0$	**M1**	Allow sign errors but must be 2 terms. Condone $\frac{16}{v} - 500 = 0$ for M1.
	$v = 32$ (m s^{-1})	**A1**	
		2	
7(c)	Work done by engine = $16000 \times 15 [= 240000]$	**B1**	Or $16000 = \frac{\text{WD}}{15}$.
	KE change = $\pm\left(\frac{1}{2} \times 1200 \times v^2 - \frac{1}{2} \times 1200 \times 20^2\right)$	**B1**	$\pm(600v^2 - 240000)$
	PE change = $\pm\left(1200 \times g \times 316 \times \frac{1}{60}\right) [= \pm 63200]$	**B1**	Allow $1200 \times g \times 316 \times \sin 0.955$ or $1200 \times g \times 316 \times \sin 0.95$ or $1200 \times g \times \frac{79}{15}$ or $1200 \times g \times 5.266...$.
	Attempt at work-energy equation.	**M1**	Use of work-energy principle with 5 terms; dimensionally correct. Allow sign errors and sin/cos mix on PE term
	$16000 \times 15 - 128400 = \frac{1}{2} \times 1200 \times v^2 - \frac{1}{2} \times 1200 \times 20^2 + 1200g \times 316 \times \frac{1}{60}$ $(240000 - 128400 = 600v^2 - 240000 + 63200)$	**A1**	Allow a value in the interval [62870,63600] for the PE term from using non-exact values for the given angle (but not if from incorrect working).
	$v = 21.9$ (m s^{-1})	**A1**	21.924111…
		6	

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

5254654679

MATHEMATICS **9709/42**

Paper 4 Mechanics **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

1 A particle of mass 1.6 kg is dropped from a height of 9 m above horizontal ground. The speed of the particle at the instant before hitting the ground is $12\,\mathrm{m\,s^{-1}}$.

Find the work done against air resistance. [3]

2 Two particles A and B, of masses 3.2 kg and 2.4 kg respectively, lie on a smooth horizontal table. A moves towards B with a speed of $v\,\mathrm{m\,s^{-1}}$ and collides with B, which is moving towards A with a speed of $6\,\mathrm{m\,s^{-1}}$. In the collision the two particles come to rest.

(a) Find the value of v. [2]

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(b) Find the loss of kinetic energy of the system due to the collision. [2]

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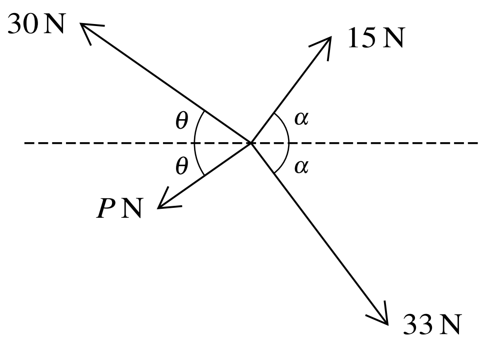

Coplanar forces of magnitudes 30 N, 15 N, 33 N and P N act at a point in the directions shown in the diagram, where $\tan\alpha = \frac{4}{3}$. The system is in equilibrium.

(a) Show that $\left(\dfrac{14.4}{30 - P}\right)^2 + \left(\dfrac{28.8}{P + 30}\right)^2 = 1.$ [4]

(b) Verify that $P = 6$ satisfies this equation and find the value of θ. [2]

4 An athlete of mass 84 kg is running along a straight road.

(a) Initially the road is horizontal and he runs at a constant speed of $3\,\mathrm{m\,s^{-1}}$. The athlete produces a constant power of 60 W.

Find the resistive force which acts on the athlete. [1]

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(b) The athlete then runs up a 150 m section of the road which is inclined at $0.8°$ to the horizontal. The speed of the athlete at the start of this section of road is $3\,\mathrm{m\,s^{-1}}$ and he now produces a constant driving force of 24 N. The total resistive force which acts on the athlete along this section of road has constant magnitude 13 N.

Use an energy method to find the speed of the athlete at the end of the 150 m section of road. [6]

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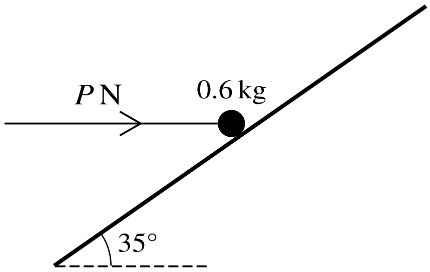

A particle of mass 0.6 kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and plane is 0.4.

Find the least possible value of P. [6]

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6 A particle P starts at rest and moves in a straight line from a point O. At time t s after leaving O, the velocity of P, $v\,\mathrm{m\,s^{-1}}$, is given by $v = bt + ct^{\frac{3}{2}}$, where b and c are constants. P has velocity $8\,\mathrm{m\,s^{-1}}$ when $t = 4$ and has velocity $13.5\,\mathrm{m\,s^{-1}}$ when $t = 9$.

(a) Show that $b = 3$ and $c = -0.5$. [1]

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(b) Find the acceleration of P when $t = 1$. [2]

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(c) Find the positive value of t when P is at instantaneous rest and find the distance of P from O at this instant. [5]

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(d) Find the speed of P at the instant it returns to O. [3]

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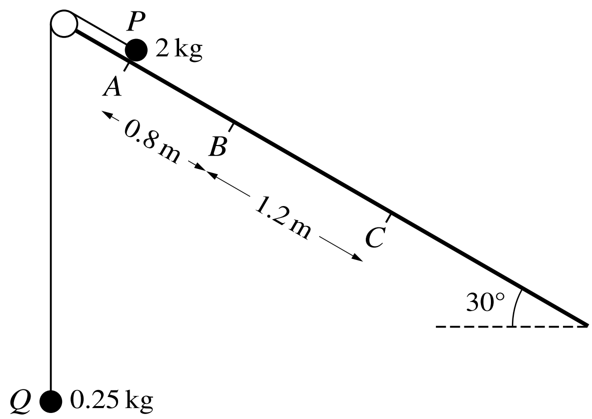

Two particles P and Q, of masses 2 kg and 0.25 kg respectively, are connected by a light inextensible string that passes over a fixed smooth pulley. Particle P is on an inclined plane at an angle of 30° to the horizontal. Particle Q hangs below the pulley. Three points A, B and C lie on a line of greatest slope of the plane with $AB = 0.8$ m and $BC = 1.2$ m (see diagram).

Particle P is released from rest at A with the string taut and slides down the plane. During the motion of P from A to C, Q does not reach the pulley. The part of the plane from A to B is rough, with coefficient of friction 0.3 between the plane and P. The part of the plane from B to C is smooth.

(a) **(i)** Find the acceleration of P between A and B. [4]

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(ii) Hence, find the speed of P at C. [5]

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(b) Find the time taken for P to travel from A to C. [4]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/42**

Paper 4 Mechanics **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$12^2 = 2\times9\times a$. **OR** $a = 8$.	**M1**	Use of suvat to get an equation in *a*.
	$1.6g - R = 1.6a$. [may see $R = 3.2$]	**M1**	Use Newton's second law with 3 terms, allow sign errors. Allow *their a* ≠ *g*. Allow *a* if it isn't subsequently replaced with *g*.
	WD $[= 3.2\times9] = 28.8$ J	**A1**	
	Alternative method for Question 1		
	(KE =) $\frac{1}{2}\times1.6\times12^2$ **OR** 115.2	**B1**	Allow for the expression for KE.
	(Loss of PE =) $\pm1.6g\times9$ **OR** ±144	**B1**	Allow for the expression for PE.
	WD $= 28.8$ J	**B1**	Allow if get –28.8 and then say 28.8 without explanation. Do not allow –28.8 as final answer to working, so if get 28.8 and state –28.8 then ISW.
		3	

Question	Answer	Marks	Guidance
2(a)	$\pm[3.2v + 2.4\times(-6)] = 0$	**M1**	Attempt at conservation of momentum; 2 non-zero terms; allow sign errors.
	$v = 4.5$	**A1**	M1A0 for use of *mgv*. $v = -4.5$ is A0.
		2	

Question	Answer	Marks	Guidance
2(b)	$\text{KE} = \pm\frac{1}{2}\times 3.2\times(\textit{their } 4.5)^2$ OR $\pm\frac{1}{2}\times 2.4\times 6^2$	**M1**	Attempt at either KE term, using *their v*. Do not allow $\frac{1}{2}\times 3.2\times(\textit{their } 4.5\pm 6)^2$, or $\frac{1}{2}\times 2.4\times(\textit{their } 4.5\pm 6)^2$, or $\frac{1}{2}\times(3.2+2.4)\times(\textit{their } 4.5\pm 6)^2$, or $\frac{1}{2}\times 3.2\times(\textit{their } 4.5-0)^2$, or $\frac{1}{2}\times 2.4\times(6-0)^2$.
	$\text{KE}_{loss} = 75.6$ J	**A1**	Allow –75.6. Note $\frac{1}{2}\times(3.2+2.4)\times 6^2$ or $\frac{1}{2}\times(3.2+2.4)\times(\textit{their } 4.5)^2$ is M1A0.
		2	

Question	Answer	Marks	Guidance
3(a)	Resolving either direction.	**M1**	Correct number of terms, allow sign errors, allow sin/cos mix. Do not allow with just $\sin\alpha$ and $\cos\alpha$.
	$(33+15)\times\frac{3}{5}=P\cos\theta+30\cos\theta$ OR $(33+15)\cos\left(\tan^{-1}\frac{4}{3}\right)=P\cos\theta+30\cos\theta$ OR $19.8+9=P\cos\theta+30\cos\theta$	**A1**	OE, but see note for final A1. Allow: $28.8=(P+30)\cos\theta$ $(33+15)\cos 53(.1)=P\cos\theta+30\cos\theta$ $19.81+9.01=P\cos\theta+30\cos\theta$ $19.86+9.03=P\cos\theta+30\cos\theta$.
	$15\times\frac{4}{5}+30\sin\theta=33\times\frac{4}{5}+P\sin\theta$ OR $15\sin\left(\tan^{-1}\frac{4}{3}\right)+30\sin\theta=33\sin\left(\tan^{-1}\frac{4}{3}\right)+P\sin\theta$ OR $12+30\sin\theta=26.4+P\sin\theta$	**A1**	OE, but see note for final A1. Allow: $14.4=(30-P)\sin\theta$ $15\sin 53(.1)+30\sin\theta=33\sin 53(.1)+P\sin\theta$ $12.00+30\sin\theta=26.39+P\sin\theta$ $11.98+30\sin\theta=26.35+P\sin\theta$.
	[Use $\cos^2\theta+\sin^2\theta=1$ with] $\cos\theta=\frac{28.8}{P+30}$ and $\sin\theta=\frac{14.4}{30-P}$ to get $\left(\frac{14.4}{30-P}\right)^2+\left(\frac{28.8}{P+30}\right)^2=1$	**A1**	AG**.** Must have evidence of where 28.8 and 14.4 come from. A0 for any error seen. A0 if use of inexact angles seen. Any inexact decimals seen for force components, i.e. if 14.4 and/or 28.8 have come from rounding to 3sf, scores M1A1A1A0 max 3/4. If exact values of $\sin\alpha$ and $\cos\alpha$ not shown (e.g. $28.8=(P+30)\cos\theta$ or $14.4=(30-P)\sin\theta$ from no working), this scores M1A1A1A0 max 3/4 marks.
		4	

Question	Answer	Marks	Guidance
3(b)	Sub $P=6$ into $\left(\frac{14.4}{30-P}\right)^2+\left(\frac{28.8}{P+30}\right)^2$ to get $\left[\left(\frac{14.4}{24}\right)^2+\left(\frac{28.8}{36}\right)^2\right]=\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2=0.36+0.64=1$	**B1**	Must see either $\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2=1$ or $0.36+0.64=1$ as minimum working.
	$\theta=36.9$	**B1**	AWRT 36.9 .
		2	

Question	Answer	Marks	Guidance
4(a)	$\left(\text{Resistive force}=DF=\frac{60}{3}=\right)20\text{ N}$	**B1**	
		1	

Question	Answer	Marks	Guidance
4(b)	PE $=\pm 84g\times 150\sin 0.8$	**B1**	$\pm 1759.23\ldots$
	KE change $=\pm\left(\frac{1}{2}\times 84v^2-\frac{1}{2}\times 84\times 3^2\right)$	**B1**	$\pm\left(\frac{1}{2}\times 84v^2-378\right)$
	Work Done $=\pm(24\times 150-13\times 150)$	**B1**	$\pm(3600-1950)=\pm 1650$
	Attempt at work-energy equation.	**M1**	5 terms, dimensionally correct, allow sign errors, sin/cos mix on PE term, PE must include $\sin 0.8$ or $\cos 0.8$.
	$84g\times 150\sin 0.8+\frac{1}{2}\times 84v^2-\frac{1}{2}\times 84\times 3^2=24\times 150-13\times 150$ $\left[1759.23\ldots+42v^2-378=3600-1950\to 42v^2=268.765\ldots\right]$	**A1**	
	$[v=]$ 2.53 $\mathrm{m\,s^{-1}}$	**A1**	AWRT 2.53; 2.5296…
		6	
	Special case for use of constant acceleration: Maximum 4 marks		
	Resolve parallel to slope and use Newton's second law	***M1**	Four terms, allow sign errors, allow sin/cos mix.
	$24-13-84g\sin 0.8=\pm 84a$	**A1**	For reference $a=\pm 0.008669\ldots$
	Use constant acceleration formula to get an equation in v or v^2	**DM1**	E.g. $v^2=3^2+2\times(\textit{their } a)\times 150$.
	$[v=]$ 2.53 $\mathrm{m\,s^{-1}}$	**A1**	AWRT 2.53; 2.5296…
		4	

Question	Answer	Marks	Guidance
5	Attempt at resolving parallel or perpendicular to the plane.	*M1	3 terms, allow sign errors, allow sin/cos mix, allow g missing. Forces that need resolving should be resolved, forces that do not need resolving should not be resolved.
	$R = P\sin 35 + 0.6g\cos 35 \left[R = (0.573\ldots)P + 4.914\ldots\right]$	A1	
	$F + P\cos 35 = 0.6g\sin 35 \left[F + (0.819\ldots)P = 3.441\ldots\right]$	A1	Their F.
	Use of $F = 0.4R$	*M1	Where R is initially a linear combination of a P component and a weight component (or a mass component).
	Solve for P.	DM1	From equations with the correct number of relevant resolved terms. $R = \dfrac{0.6g}{\cos 35 + 0.4\sin 35} = 5.7222$. Must get to $P = \ldots$, e.g. $P = \dfrac{0.6g\sin 35 - 0.4 \times 0.6g\cos 35}{\cos 35 + 0.4\sin 35}$ If no working seen, allow this mark if correct solution for their equations. If $F \leqslant 0.4R$ used, it should be used correctly. e.g. $0.6g\sin 35 - P\cos 35 \leqslant 0.4(P\sin 35 + 0.6g\cos 35)$.
	$P = 1.41$	A1	AWRT 1.41 . If $P \geqslant 1.41$ seen, must then state the least value explicitly for A1.

Question	Answer	Marks	Guidance
5	**Alternative for Question 5: Resolving vertically and horizontally**		
	Attempt at resolving vertically or horizontally.	*M1	3 terms, allow sign errors, allow sin/cos mix, allow g missing. Forces that need resolving should be resolved, forces that do not need resolving should not be resolved.
	$R\cos 35 + F\sin 35 = 0.6g$	A1	*Their* F or R.
	$P + F\cos 35 = R\sin 35$	A1	*Their* F or R.
	Use of $F = 0.4R$	*M1	To get 2 equations, one in R (or F) and the other in P and R (or P and F) from resolved equations with correct number of relevant terms. Allow g missing.
	Solve for P	DM1	From equations with the correct number of relevant resolved terms. May see $R = \frac{0.6g}{\cos 35 + 0.4\sin 35} = 5.7222$; Must get to $P = \ldots$, e.g. $P = \frac{0.6g\sin 35 - 0.4 \times 0.6g\cos 35}{\cos 35 + 0.4\sin 35}$. If no working seen, allow this mark if correct solution for *their* equations.
	$P = 1.41$	A1	AWRT 1.41 .
		6	

Question	Answer	Marks	Guidance
6(a)	$4b+4^{\frac{3}{2}}c=8\left[\to 4b+8c=8\right]$ and $9b+9^{\frac{3}{2}}c=13.5\left[\to 9b+27c=13.5\right]$ State $b=3$ and $c=-0.5$ OR $3\times4+(-0.5)\times4^{\frac{3}{2}}=8$ AND $3\times9+(-0.5)\times9^{\frac{3}{2}}=13.5$	**B1**	Must have 2 correct equations, which do not have to be simplified. Allow to just state the values of b and c. Allow substitution of $b=3$ and $c=-0.5$ in both equations to verify. No further calculation required. B0 if any incorrect work seen.
		1	
6(b)	$\left(a=\frac{dv}{dt}=\right)3-0.5\times\frac{3}{2}\times t^{\frac{1}{2}}$ OR $b+c\times\frac{3}{2}\times t^{\frac{1}{2}}$	**M1**	Attempt to differentiate, decrease power by 1 and a change in coefficient in at least one term (which must be the same term); allow unsimplified; $a=\frac{v}{t}$ is M0.
	acceleration = 2.25 m s^{-2}	**A1**	OE, e.g. $\frac{9}{4}$ or $2\frac{1}{4}$. **SC B1** for 2.25 if no differentiation seen.
		2	

Question	Answer	Marks	Guidance
6(c)	Equate v to 0 and attempt to solve for t	**M1**	Must get to $t = \ldots$ and must be positive.
	$t = 36$ ONLY	**A1**	WWW. Allow if $t = 0$ seen and not rejected.
	Attempt to integrate	**M1**	Increase power by 1 and a change in coefficient in at least one term (which must be the same term). $s = vt$ is M0.
	$s = \frac{3}{2}t^2 - 0.5 \times \frac{2}{5} \times t^{\frac{5}{2}} (+D) = \frac{3}{2}t^2 - \frac{1}{5}t^{\frac{5}{2}} (+D)$ OR $s = \frac{b}{2}t^2 + c \times \frac{2}{5} \times t^{\frac{5}{2}} (+D)$	**A1**	Allow unsimplified (including indices).
	Sub $t = 36$ (or use limits 36 and 0) to get distance = 388.8 m ONLY	**A1**	Allow 389 m. If no integration seen for the last 3 marks, allow **SC B1** for 388.8 m. Max M1A1B1 for 3/5 marks.
		5	
6(d)	$\frac{3}{2}t^2 - 0.5 \times \frac{2}{5} \times t^{\frac{5}{2}} = 0$	**M1**	Equate *their s* (that has come from an integration attempt) to 0 and attempt to solve for *t*. Must get to $t = \ldots$
	$t = 56.25$	**A1**	WWW. OE, e.g. $\frac{225}{4}$. Allow 2sf or better. Allow any correct unsimplified expression equivalent to 56.25 e.g. $\frac{15^2}{2^2}$ or 7.5^2.
	Speed = 42.2 m s^{-1} ONLY	**A1**	AWRT 42.2 . Speed = −42.2 is A0. Allow A1 if negative sign dropped without justification.
		3	

Question	Answer	Marks	Guidance
7(a)(i)	Particle P: $2g\sin 30 - F - T = 2a$ $[10 - F - T = 2a]$ Particle Q: $T - 0.25g = 0.25a$ System: $2g\sin 30 - F - 0.25g = (2+0.25)a$	**M1**	Newton's second law on either particle or for the system with correct masses; correct number of terms, allow sin/cos mix, allow sign errors. Allow with *their F*.
		A1	Both particle equations correct (with the same T) or system equation correct. Allow with *their F*. If *their a* direction is different to ours, allow if *their a* is consistently used e.g. $-2g\sin 30 + F + T = 2a'$ and $0.25g - T = 0.25a'$.
	$F = 0.3R = 0.3 \times 2g\cos 30\left[= 3\sqrt{3} = 5.1961......\right]$	**M1**	Use of $F = 0.3R$, where R is a component of weight.
	Acceleration from A to B = 1.02 m s^{-2}	**A1**	Solving for the acceleration from A to B. Allow $\dfrac{10-4\sqrt{3}}{3}$; AWRT 1.02 . May see $T = \dfrac{10-\sqrt{3}}{3} = 2.7559...$
		4	

Question	Answer	Marks	Guidance
7(a)(ii)	Use of suvat from A to B to get an equation in v^2 or v.	*M1	Using $u = 0$ and *their* $\lvert a \rvert$ from **(a)(i)** to get a positive v^2. E.g. $v^2 = 2 \times 0.8 \times (\textit{their}\ 1.02)$ or $v^2 = 2 \times (-0.8) \times (-\textit{their}\ 1.02)$.
	$v^2 = \left[\frac{80 - 32\sqrt{3}}{15}\right] = 1.64$ OR $v = 1.28$	A1	Not $v = 1.29$. Allow 2sf or better without wrong work, i.e. $v^2 = 1.6$ or $v = 1.3$.
	Find the acceleration from B to C: $2g\sin 30 - 0.25g = (2 + 0.25)a$	*M1	Resolving on both particles and eliminate T (if *their a* direction is different to ours, allow if *their a* is consistently used) OR for the system to get an equation in a only. Correct number of relevant terms, allow sin/cos mix, allow sign errors. For reference $a = \frac{10}{3}$ or 3.33. May see $T = \frac{10}{3}$.
	$v^2 = (\textit{their}\ 1.28)^2 + 2 \times \left(\textit{their}\ \frac{10}{3}\right) \times 1.2$	DM1	Use of suvat from B to C, allow *their* positive $a \neq g$ from **(a)(ii)** not *their a* from **(a)(i)** and *their* 1.28. Dependent on previous two marks.
	Velocity = 3.1(0) $\mathrm{m\,s^{-1}}$	A1	AWRT 3.1(0) to 3sf.

Question	Answer	Marks	Guidance
7(a)(ii)	**Alternative Method for Question 7(a)(ii): using suvat in first stage and energy in second stage**		
	Use of suvat from A to B to get an equation in v^2 or v.	***M1**	Using *their* positive a from **(a)(i)** E.g. $v^2 = 2 \times 0.8 \times (their\ 1.02)$.
	$v^2 = \left[\dfrac{80 - 32\sqrt{3}}{15}\right] = 1.64$ OR $v = 1.28$	**A1**	Allow 2sf or better, i.e. $v^2 = 1.6$ or $v = 1.3$.
	Change in PE $= \pm(2g \times 1.2\sin 30 - 0.25g \times 1.2)$ OR Change in KE $= \pm\left[\frac{1}{2}(2+0.25)v^2 - \frac{1}{2}(2+0.25)(their\ 1.28)^2\right]$	**B1**	
	$\frac{1}{2}(2+0.25)v^2 - \frac{1}{2}(2+0.25)(their\ 1.28)^2 = 2g \times 1.2\sin 30 - 0.25g \times 1.2$	**DM1**	Use of work-energy 6 terms; dimensionally correct. Allow sign errors. Allow sin/cos mix on PE. Dependent on previous M.
	Velocity = 3.1(0) m s^{-1}	**A1**	AWRT 3.1(0) to 3sf.

Question	Answer	Marks	Guidance
7(a)(ii)	**Alternative Method for Question 7(a)(ii): using energy for complete motion**		
	Change in PE $= \pm(2g \times 2\sin 30 - 0.25g \times 2)$	**B1**	
	Work done against friction $= 0.3 \times 2g\cos 30 \times 0.8$	**B1**	
	Change in KE $= \frac{1}{2}(2+0.25)v^2$	**B1**	
	$\frac{1}{2}(2+0.25)v^2 + 0.3 \times 2g\cos 30 \times 0.8 = (2g \times 2\sin 30 - 0.25g \times 2)$	**M1**	Use of work-energy 5 terms; dimensionally correct. Must be considering both particles. Allow sign errors. Allow sin/cos mix on PE and/or WD against friction.
	Velocity = 3.1(0) $\mathrm{m\,s^{-1}}$	**A1**	AWRT 3.1(0) to 3sf.

Question	Answer	Marks	Guidance
7(a)(ii)	**Alternative Method for Question 7(a)(ii): using energy in two stages**		
	$\frac{1}{2}(2+0.25)v^2+0.3\times 2g\cos 30\times 0.8 = 2g\times 0.8\sin 30 - 0.25g\times 0.8$ OR $\frac{1}{2}\times 2\times v^2+(\textit{their}\ 2.7559\ldots)\times 0.8+0.3\times 2g\cos 30\times 0.8 = 2g\times 0.8\sin 30$ OR $\frac{1}{2}\times 0.25\times v^2+0.25g\times 0.8=(\textit{their}\ 2.7559\ldots)\times 0.8$	*M1	Use of work-energy 5 terms; dimensionally correct. Allow sign errors. Allow sin/cos mix on PE. OR Use of work-energy 4 terms; dimensionally correct. Allow sign errors. Allow sin/cos mix on PE. OR Use of work-energy 3 terms; dimensionally correct. Allow sign errors.
	$v^2=\left[\frac{80-32\sqrt{3}}{15}\right]=1.64$ OR $v=1.28$	A1	Allow 2sf or better, i.e. $v^2=1.6$ or $v=1.3$.
	Change in PE $=\pm(2g\times 1.2\sin 30-0.25g\times 1.2)$ OR Change in KE $=\pm\left[\frac{1}{2}(2+0.25)v^2-\frac{1}{2}(2+0.25)1.28^2\right]$	B1	
	$\frac{1}{2}(2+0.25)v^2-\frac{1}{2}(2+0.25)1.28^2=2g\times 1.2\sin 30-0.25g\times 1.2$	DM1	Use of work-energy 6 terms; dimensionally correct. Allow sign errors. Allow sin/cos mix on PE. Dependent on previous 2 marks.
	Velocity = 3.1(0) m s^{-1}	A1	AWRT 3.1(0) to 3sf.
		5	

Question	Answer	Marks	Guidance
7(b)	$0.8 = 0 + \frac{1}{2} \times$ (*their* positive answer to **(a)(i)**) $\times t_1^2$ and solve for t_1 OR $0.8 = \frac{1}{2}$(0 + *their* positive 1.28 from **(a)(ii)**) t_1 and solve for t_1 OR (*their* positive 1.28 from **(a)(ii)**) = (*their* positive answer to **(a)(i)**) t_1 and solve for t_1	**M1**	Use of suvat from A to B to find t_1, using $s = 0.8$ and *their* positive $a \neq g$ from **(a)(i)**. Must get to $t_1 = \ldots$ OR using their positive 1.28 . OR using their positive $a \neq g$ from **(a)(i)** and positive 1.28 .
	$1.2 =$ (*their* 1.28 from **(a)(ii)**) $t_2 + \frac{1}{2} \times \left(their\ \frac{10}{3}\right) \times t_2^2$ and solve for t_2 OR $1.2 = \frac{1}{2}$ ((*their* 1.28 from **(a)(ii)**) + (*their* answer to **(a)(ii)**)) t_2 and solve for t_2 OR (*their* answer to **(a)(ii)**) = (*their* 1.28 from **(a)(ii)**) $+ \left(their\ \frac{10}{3}\right) t_2$ and solve for t_2	**M1**	Use of suvat from B to C to find t_2, using $s = 1.2$ and *their* positive $a \neq g$ from **(a)(ii)** (not *their* a from **(a)(i)**) and *their* 1.28 which would lead to a positive t_2 value. Must get to $t_2 = \ldots$ OR using *their* 1.28 and/or *their* answer to **(a)(ii)** which would lead to a positive t_2 value. OR using *their* 1.28 and/or *their* answer to **(a)(ii)** and *their* $a \neq g$ from **(a)(ii)** (not *their* a from **(a)(i)**) which would lead to a positive t_2 value.
	$t_1 = 1.25$ or $t_2 = 0.547$	**A1**	These can be seen as an expression. Allow $t_1 = \frac{0.8}{0.64}$ $t_2 = \frac{1.2}{2.19}$ OE. Allow 2sf or better, e.g. $t_1 = 1.3$ or $t_1 = 0.55$.
	Total time = 1.8(0) s	**A1**	WWW. AWRT 1.8(0) to 3sf.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4965349912

MATHEMATICS **9709/43**

Paper 4 Mechanics **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

1 Two particles P and Q, of masses 0.1 kg and 0.4 kg respectively, are free to move on a smooth horizontal plane. Particle P is projected with speed $4\,\mathrm{m\,s^{-1}}$ towards Q which is stationary. After P and Q collide, the speeds of P and Q are equal.

Find the two possible values of the speed of P after the collision. [3]

2 A car of mass 1500 kg is towing a trailer of mass m kg along a straight horizontal road. The car and the trailer are connected by a tow-bar which is horizontal, light and rigid. There is a resistance force of F N on the car and a resistance force of 200 N on the trailer. The driving force of the car's engine is 3200 N, the acceleration of the car is $1.25\,\mathrm{m\,s^{-2}}$ and the tension in the tow-bar is 300 N.

Find the value of m and the value of F. [4]

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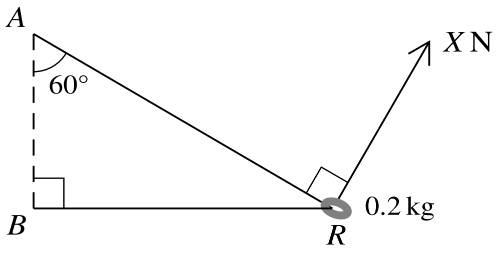

A smooth ring R of mass 0.2 kg is threaded on a light string ARB. The ends of the string are attached to fixed points A and B with A vertically above B. The string is taut and angle $ABR = 90°$. The angle between the part AR of the string and the vertical is 60°. The ring is held in equilibrium by a force of magnitude X N, acting on the ring in a direction perpendicular to AR (see diagram).

Calculate the tension in the string and the value of X. [5]

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4 A lorry of mass 15 000 kg moves on a straight horizontal road in the direction from A to B. It passes A and B with speeds $20\,\mathrm{m\,s^{-1}}$ and $25\,\mathrm{m\,s^{-1}}$ respectively. The power of the lorry's engine is constant and there is a constant resistance to motion of magnitude 6000 N. The acceleration of the lorry at B is 0.5 times the acceleration of the lorry at A.

(a) Show that the power of the lorry's engine is 200 kW, and hence find the acceleration of the lorry when it is travelling at $20\,\mathrm{m\,s^{-1}}$. [5]

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The lorry begins to ascend a straight hill inclined at $1°$ to the horizontal. It is given that the power of the lorry's engine and the resistance force do not change.

(b) Find the steady speed up the hill that the lorry could maintain. [2]

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5 A particle starts from rest from a point O and moves in a straight line. The acceleration of the particle at time t s after leaving O is $a\,\mathrm{m\,s^{-2}}$, where $a = kt^{\frac{1}{2}}$ for $0 \leqslant t \leqslant 9$ and where k is a constant. The velocity of the particle at $t = 9$ is $1.8\,\mathrm{m\,s^{-1}}$.

(a) Show that $k = 0.1$. [3]

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For $t > 9$, the velocity $v\,\mathrm{m\,s^{-1}}$ of the particle is given by $v = 0.2(t-9)^2 + 1.8$.

(b) Show that the distance travelled in the first 9 seconds is one tenth of the distance travelled between $t = 9$ and $t = 18$. [4]

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(c) Find the greatest acceleration of the particle during the first 10 seconds of its motion. [3]

6 An elevator is pulled vertically upwards by a cable. The elevator accelerates at $0.4\,\mathrm{m\,s^{-2}}$ for 5 s, then travels at constant speed for 25 s. The elevator then decelerates at $0.2\,\mathrm{m\,s^{-2}}$ until it comes to rest.

(a) Find the greatest speed of the elevator and hence draw a velocity-time graph for the motion of the elevator. [3]

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(b) Find the total distance travelled by the elevator. [2]

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The mass of the elevator is 1200 kg and there is a crate of mass m kg resting on the floor of the elevator.

(c) Given that the tension in the cable when the elevator is decelerating is 12 250 N, find the value of m. [3]

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(d) Find the greatest magnitude of the force exerted on the crate by the floor of the elevator, and state its direction. [3]

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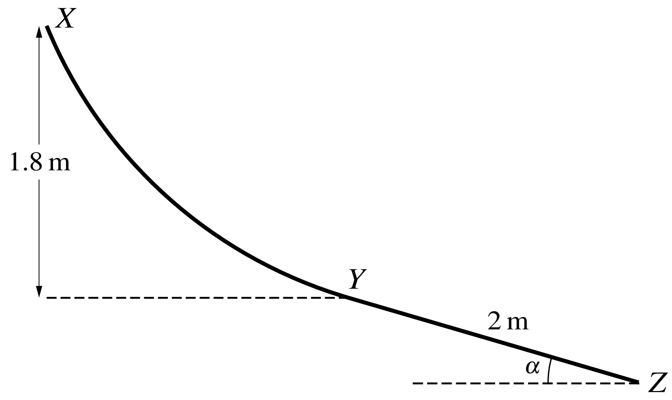

The diagram shows the vertical cross-section XYZ of a rough slide. The section YZ is a straight line of length 2 m inclined at an angle of α to the horizontal, where $\sin\alpha = 0.28$. The section YZ is tangential to the curved section XY at Y, and X is 1.8 m above the level of Y. A child of mass 25 kg slides down the slide, starting from rest at X. The work done by the child against the resistance force in moving from X to Y is 50 J.

(a) Find the speed of the child at Y. [4]

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It is given that the child comes to rest at Z.

(b) Use an energy method to find the coefficient of friction between the child and YZ, giving your answer as a fraction in its simplest form. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

MATHEMATICS **9709/43**

Paper 4 Mechanics **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **17** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	For attempt at use of conservation of momentum in one case	M1	$0.1\times4+0=0.4v+0.1v$ or $0.1\times4+0=0.4v+0.1(-v)$ OE. Must have correct number of terms. Allow sign errors.
	Speed = $0.8\,[\mathrm{m\,s^{-1}}]$ or $\frac{4}{5}$	A1	Must be positive. Allow Max M1A1A0 if g included with the masses.
	Speed = $\frac{4}{3}$ $[\mathrm{m\,s^{-1}}]$ Allow 1.33	A1	Must be positive.
		3	

Question	Answer	Marks	Guidance
2	Attempt to use Newton's second law	M1	Must have correct number of terms. Allow sign errors. Must use 300 and 1.25, not T and a.
	Trailer $300-200=m\times1.25$ or Car $3200-F-300=1500\times1.25$ System $3200-F-200=(1500+m)\times1.25$	A1	Any 2 equations. Third equation could be with *their m* substituted if found already.
	Solve for m or F	M1	Must get to 'm =' or 'F ='. Must have correct number of terms. Allow sign errors. Can be implied by correct answers.
	m = 80 and F = 1025	A1	
		4	

Question	Answer	Marks	Guidance
3	For attempt to resolve in one direction	**M1**	Must use 0.2 substituted for m if just awarding M1 for vertical equation. Must have correct number of relevant terms (forces must have components as required). Allow sin/cos mix. Allow sign errors. Allow g missing.
	$X\sin 60 + T\sin 30 - 0.2g = 0$	**A1**	OE. Correct vertical.
	$X\cos 60 - T - T\cos 30 = 0$	**A1**	OE. Correct horizontal. If the two Ts are different, they can get max M1A1A0M0A0, unless they subsequently state that the two Ts are equal.
	For attempt to solve for tension or X	**M1**	Must have correct number of relevant terms in both equations. Must get to '$T =$' or '$X =$'. Allow g missing. Can be implied by correct answers. If no working shown their values must follow from their equations.
	$X = 2$, tension in string = 0.536 [N]	**A1**	Allow exact value of tension = $4 - 2\sqrt{3}$. Allow awrt 2.00 for X.

Question	Answer	Marks	Guidance
3	**Alternative method for Question 3: Resolving parallel and perpendicular to the X N force**		
	For attempt to resolve in one direction, with 0.2 substituted for m	**M1**	Must have correct number of relevant terms (forces must have components as required). Allow sin/cos mix. Allow sign errors. Allow g missing.
	$X - 0.2g\cos 30 - T\cos 60 = 0$	**A1**	OE. Correct parallel to X.
	$T + T\cos 30 - 0.2g\cos 60 = 0$	**A1**	OE. Correct perp to X. If the two Ts are different, they can get max M1A1A0M0A0 unless they subsequently state that the two Ts are equal.
	For attempt to solve for the tension or for X	**M1**	Must have correct number of relevant terms in both equations. Must get to '$T =$' or '$X =$'. Allow g missing. Can be implied by correct answers. If no working shown their values must follow from their equations.
	$X = 2$, Tension in string = 0.536[N] [0.53589…]	**A1**	Allow exact value of tension = $4 - 2\sqrt{3}$. Allow awrt 2.00 for X.
		5	

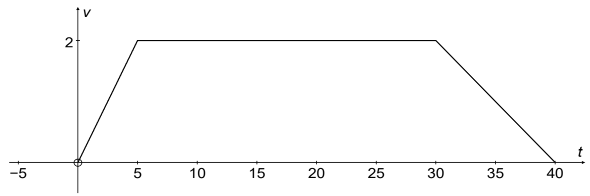
Question	Answer	Marks	Guidance
4(a)	For use of $P = Fv$	**B1**	$P = 20F$ or $P = 25F$ OE (e.g. $F = \frac{P}{20}$ or $F = \frac{P}{25}$). But not with wrong F substituted (e.g. 6000).
	Attempt to use Newton's second law in at least one case	**M1**	Must have 3 terms. Allow sign errors. Allow F.
	$\frac{P}{20} - 6000 = 15000a$ and $\frac{P}{25} - 6000 = 15000\left(\frac{1}{2}a\right)$	**A1**	OE for both. Allow $2a'$ and a'. Must be the same P for both.
	For solving simultaneously	**M1**	Dependent on 2 equations of the correct form with the correct number of relevant terms. Must get to '$P =$' or '$a =$', but $P =$ 200 kW or 200 000 W with no attempt at a gets M0. Must be the same P for both.
	Power [= 200 000W] = 200 kW, $a = \frac{4}{15}$ [ms^{-2}]	**A1**	AG. OE awrt 0.267. Do not allow 200 000 [W] as final answer. Must show some working when they find P.
	Alternative Method for Question 4(a): Using two expressions for *P*		
	For use of $P = Fv$	**B1**	$P = 20F$ or $P = 25F$ OE (e.g. $F = \frac{P}{20}$ or $F = \frac{P}{25}$). But not with wrong F substituted (e.g. 6000).
	For one expression for P in terms of a only	**M1**	Allow sign errors. Need 2 term expression.
	$(15000a + 6000) \times 20 = (15000 \times 0.5a + 6000) \times 25$	**A1**	Correct equation.
	For solving for a	**M1**	Must get to '$a =$'.
	Power [= 200 000W] = 200 kW, $a = \frac{4}{15}$ [ms^{-2}]	**A1**	AG. OE awrt 0.267. Do not allow 200 000 [W] as final answer. Must show some working when they find P.

Question	Answer	Marks	Guidance
4(a)	**Alternative Method for Question 4(a): Using the given value of P = 200 kW**		
	For use of $P = Fv$	**B1**	e.g. 200 000 = 20F or 200 000 = 25F OE. e.g. $F = \frac{200000}{20}[=10000]$ or $F = \frac{200000}{25}[=8000]$.
	Attempt to use Newton's second law in at least one case	**M1**	Must have 3 terms. Allow sign errors. Allow with F. Allow 200 in place of 200 000.
	$\frac{200000}{20} - 6000 = 15000a$ and $\frac{200000}{25} - 6000 = 15000\left(\frac{1}{2}a\right)$	**A1**	For both. Allow 2a' and a' here.
	For solving for a in both cases.	**M1**	
	For showing that both equations lead to $a = \frac{4}{15}$ [m s^{-2}]	**A1**	awrt 0.267.
		5	
4(b)	For attempt at resolving up hill $\frac{200000}{v} - 6000 - 15000g\sin 1 = 0$	**M1**	Or $\frac{200000}{v} - 6000 - 2618 = 0$. May see $\frac{200000}{8618}$. Must have correct number of terms. Allow sin/cos mix. Allow sign errors. Allow g missing, but not a different acceleration. Do not allow F.
	Steady speed = 23.2 [m s^{-1}]	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	For attempt at integration	**M1**	The power of t must increase by 1 with a change of coefficient. Do not penalise missing c. Use of $v = at$ scores M0.
	$v = \frac{2}{3}kt^{\frac{3}{2}}[+c]$	**A1**	Allow unsimplified e.g. $v = \frac{1}{1.5}kt^{\frac{1}{2}+1}[+c]$.
	$1.8 = \frac{2}{3}k \times 9^{\frac{3}{2}} \quad \Rightarrow k = \left[\frac{3}{2} \times 1.8 \div 27 = \right] 0.1$	**B1**	AG. Must show values substituted OE (e.g. $1.8 = 18k$).
		3	

Question	Answer	Marks	Guidance
5(b)	For attempt at integration of either $\int\left(\frac{2}{3}kt^{\frac{3}{2}}\right)\mathrm{d}t$ or $\int\left(0.2(t-9)^2+1.8\right)\mathrm{d}t$ or $\int\left(0.2t^2-3.6t+18\right)\mathrm{d}t$	M1	The power of t or $(t-9)$ must increase by 1 with a change of coefficient in at least one term. Use of $s=vt$ is M0.
	$\left[\frac{4}{150}t^{\frac{5}{2}}\right]_0^9$ and $\left[\left(\frac{0.2}{3}(t-9)^3+1.8t\right)\right]_9^{18}$ or $\left[\left(\frac{0.2}{3}t^3-\frac{3.6}{2}t^2+18t\right)\right]_9^{18}$	A1	Allow unsimplified. No need for limits. Could include '+c' with either or both.
	$=\frac{4}{150}\times 9^{\frac{5}{2}}$ $\left[=6.48 \text{ or } \frac{162}{25}\right]$ or $=\frac{0.2}{3}(18-9)^3+1.8\times 18-1.8\times 9=\left[64.8 \text{ or } \frac{324}{5}\right]$ or $\left(\frac{0.2}{3}\times 18^3-\frac{3.6}{2}\times 18^2+18\times 18\right)-\left(\frac{0.2}{3}\times 9^3-\frac{3.6}{2}\times 9^2+18\times 9\right)$ $\left[=\frac{648}{5}-\frac{324}{5}=\frac{324}{5}\right]$	M1	Correct use of limits 0, 9 or limits 9, 18. Can be implied by either answer following integration "+c" method is as follows: $s=\frac{0.2}{3}(t-9)^3+1.8t-\frac{243}{25}$, or $s=\frac{0.2}{3}t^3-\frac{3.6}{2}t^2+18t-\frac{1458}{25}$, then substitute t = 18 to get $\frac{1782}{25}$, subtract $\frac{162}{25}$, so distance = $\frac{324}{5}$.
	$6.48=\frac{1}{10}\times 64.8$ or $64.8=10\times 6.48$ or $\frac{324}{5}=10\times\frac{162}{25}$	A1	AG OE**.** Check working as can get answer from wrong working NFWW (not from wrong working).

Question	Answer	Marks	Guidance
5(b)	**Alternative Method for Question 5(b): Special Case for those who use a calculator to integrate. Award max 2/4**		
	Either $\int_0^9 \left(\frac{2}{3}kt^{\frac{3}{2}}\right)dt = 6.48$ or $\frac{162}{25}$ Or $\int_9^{18} \left(0.2(t-9)^2 + 1.8\right)dt = 64.8$ or $\frac{324}{5}$	**SC B1**	
	$6.48 = \frac{1}{10} \times 64.8$ or $64.8 = 10 \times 6.48$ or $\frac{324}{5} = 10 \times \frac{162}{25}$	**SC B1**	OE.
		4	
5(c)	For differentiation Should get $a = 0.4(t-9)$ or $a = 0.4t - 3.6$	**M1**	The power of t or $(t-9)$ must decrease by 1 with a change of coefficient. M0 for $a = \frac{v}{t}$.
	0.4 [ms^{-2}] [at $t = 10$]	**A1**	**SC B1** for 0.4 with no differentiation seen.
	0.3 seen (from the first phase) and state that 0.4 is final answer	**B1**	No working needed. If M1A0 or M0A0 scored, then **SC B1** for 0.3 without mention of the maximum acceleration.
		3	

Question	Answer	Marks	Guidance
6(a)	Greatest speed = $2[\text{m s}^{-1}]$ $[0.4 \times 5]$	**B1**	This can be seen on the graph and not stated explicitly.
	Trapezium shape	**B1**	Sitting on t-axis, starting at origin.
		**B1**	All correct including height of 2 and t-values of 5, 30, 40 on the horizontal axis. Labels not needed. Does not need to be to scale.
		3	
6(b)	Distance = $\frac{1}{2}(25+5+25+\mathit{their}\,10)\times \mathit{their}\,2$ or $\frac{1}{2}\times 5\times \mathit{their}\,2+25\times \mathit{their}\,2+\frac{1}{2}\times \mathit{their}\,10\times \mathit{their}\,2$	**M1**	Allow M1 for finding total area under their trapezium or appropriate 'suvat' in each phase. If presented as 3 areas, they do not need to be added for M1. Allow one wrong value but must represent all 3 phases of motion.
	Distance = $65[\text{m}]$	**A1**	
		2	
6(c)	Attempt at Newton's second law	**M1**	Must have correct number of terms (5). Allow sign errors. Allow g missing. Use of $a = g$ is M0A0A0 but condone use of $a = 0.4$ (from wrong phase).
	$12250 - 1200g - mg = (1200 + m)\times(-0.2)$ Or $1200g + mg - 12250 = (1200 + m)\times 0.2$	**A1**	Correct equation. Note that taking $a = 0.2$ and omitting mg gets M0A0A0.
	$m = 50$	**A1**	
		3	

Question	Answer	Marks	Guidance
6(d)	Realise that this is when accelerating and attempt Newton's second law for the crate only	**M1**	Must have correct number of terms (3). Allow sign errors. Allow g missing. Must use $a = \pm 0.4$, M0A0A0 otherwise.
	$R - 50g = 50 \times 0.4$ or $50g - R = 50 \times (-0.4)$	**A1FT**	Correct equation using their 50.
	Force $R = 520$ [N], **upwards**	**A1**	Must include 'upwards' OE.
		3	

Question	Answer	Marks	Guidance
7(a)	PE lost = $mgh = 25g \times 1.8 [= 450]$	**B1**	
	For work energy equation	**M1**	Must have correct number of terms. Allow sign errors. Dimensionally correct. Must use 25, not m. Candidates who try to use constant acceleration can only score B1.
	$25g \times 1.8 - 50 = \frac{1}{2} \times 25v^2$	**A1**	OE. Must be correct.
	$v = 4\sqrt{2}$ [$\mathrm{m\,s^{-1}}$] or 5.66 [5.6568…]	**A1**	Allow $\sqrt{32}$.
		4	

Question	Answer	Marks	Guidance
7(b)	PE gained/lost = $\pm 25g \times 2 \times 0.28 [= \pm 140]$ or KE gained/lost = $\pm\frac{1}{2}\times 25\left(\textit{their}\, 4\sqrt{2}\right)^2$ [KE = ± 400]	**B1FT**	For either. FT from *their* v for KE. Must have α substituted for PE. Allow $25g \times 2\sin 16.26°$ or $25g \times 2\sin 16.3°$.
	For work energy equation	***M1**	Must have correct number of terms. Allow sign errors. Dimensionally correct. Allow sin/cos mix Do not allow with WD instead of $F \times 2$. Must have substituted α and v.
	$F \times 2 = 25g \times 2 \times 0.28 + \frac{1}{2}\times 25\left(4\sqrt{2}\right)^2$ [$\Rightarrow F = 270$]	**A1FT**	FT *their* v^2 or v.
	$R = 25g \times 0.96$ [= 240]	**B1**	Allow $25g\cos 16.26°$ or $25g\cos 16.3°$.
	Use of $F = \mu R$ to form an equation in μ only	**DM1**	Must be from 3 term F, dimensionally correct and single term R. Allow sin/cos mix but must be different components of weight. F and R must be numerical expressions.
	$\mu = \frac{9}{8}$	**A1**	CAO. Allow $1\frac{1}{8}$, but no other answer.
	Alternative method 1 for first 3 marks: Using energy from the initial position (use existing scheme for final 3 marks).		
	PE lost = $\pm 25g \times (1.8 + 2\times 0.28) [= \pm 590]$	**B1**	Allow $25g \times (1.8 + 2\sin 16.26°)$ or $25g \times (1.8 + 2\sin 16.3°)$.
	For work energy equation	***M1**	Must have correct number of terms. Allow sign errors. Dimensionally correct. Allow sin/cos mix. Do not allow with WD instead of $F \times 2$. Must have substituted α.
	$F \times 2 = 25g \times (1.8 + 2\times 0.28) - 50$ [$\Rightarrow F = 270$]	**A1**	

Question	Answer	Marks	Guidance
7(b)	**Special Case: Use of constant acceleration. Award max 4/6**		
	$0=\left(4\sqrt{2}\right)^2+2(\pm a)\times 2$		Use of $v^2=u^2+2as$.
	$a=\pm 8$	**SC B1FT**	FT *their* v^2 or v. Note: 8.01 or 8.0089 from use of 5.66.
	$R=25g\times 0.96$	**SC B1**	Allow $25g\cos 16.26°$ or $25g\cos 16.3°$.
	Use of $F=\mu R$ and attempt at N2L If correct should get $25g\sin 16.3° - \mu\times 25g\cos 16.3° = 25\times(-8)$ $[\Rightarrow 70-240\mu=-200]$	**SC M1**	To form an equation in μ only. Using *their a.* Allow sign errors. Allow sin/cos mix but must be different components of weight. *F* and *R* must be numerical expressions. Must have substituted α.
	$\mu=\frac{9}{8}$	**SC A1**	CAO. Allow $1\frac{1}{8}$, but no other answer.
		6	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6035393307

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

JC23 06_9709_51/2R

1 A summary of 50 values of x gives

$$\Sigma(x - q) = 700, \qquad \Sigma(x - q)^2 = 14\,235,$$

where q is a constant.

(a) Find the standard deviation of these values of x. [2]

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(b) Given that $\Sigma x = 2865$, find the value of q. [2]

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2 **(a)** Find the number of ways in which a committee of 6 people can be chosen from 6 men and 8 women if it must include 3 men and 3 women. [2]

A different committee of 6 people is to be chosen from 6 men and 8 women. Three of the 6 men are brothers.

(b) Find the number of ways in which this committee can be chosen if there are no restrictions on the numbers of men and women, but it must include no more than two of the brothers. [3]

3 **(a)** Find the number of different arrangements of the 8 letters in the word COCOONED. [1]

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(b) Find the number of different arrangements of the 8 letters in the word COCOONED in which the first letter is O and the last letter is N. [2]

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(c) Find the probability that a randomly chosen arrangement of the 8 letters in the word COCOONED has all three Os together given that the two Cs are next to each other. [3]

4 A mathematical puzzle is given to a large number of students. The times taken to complete the puzzle are normally distributed with mean 14.6 minutes and standard deviation 5.2 minutes.

(a) In a random sample of 250 of the students, how many would you expect to have taken more than 20 minutes to complete the puzzle? [4]

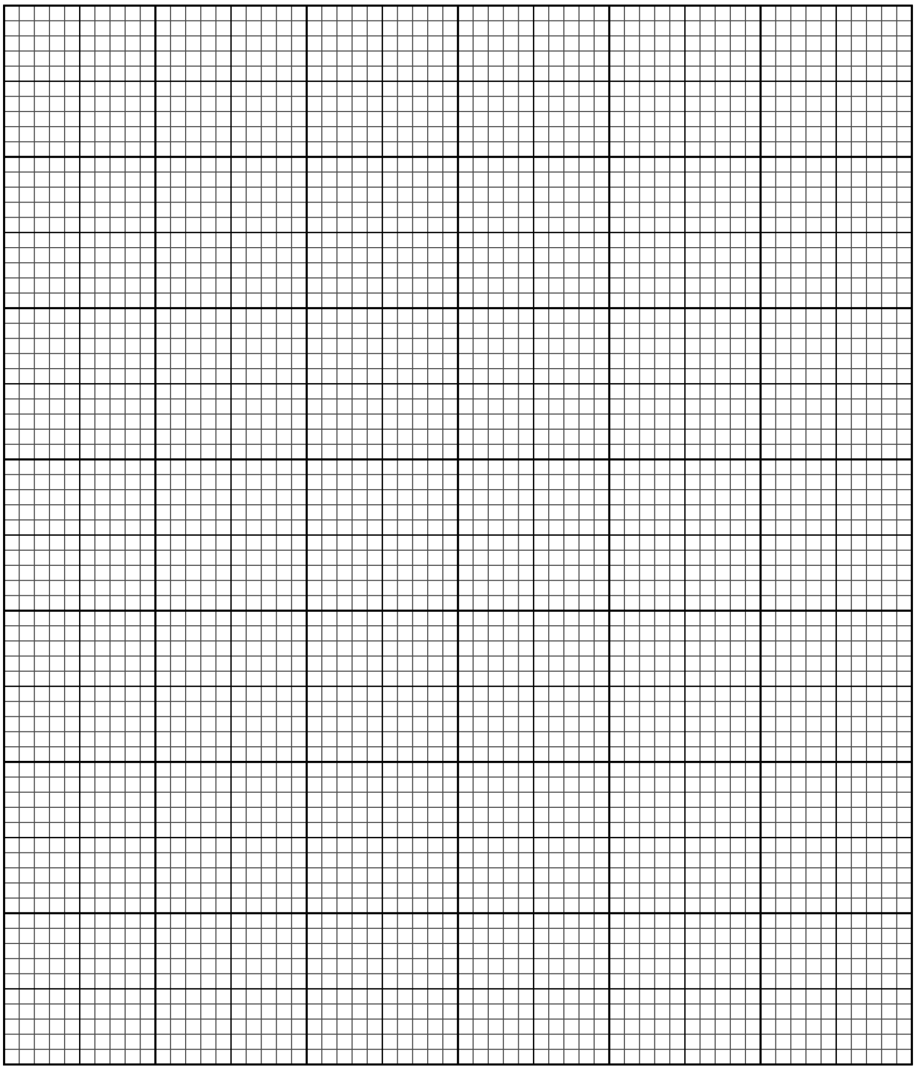
All the students are given a second puzzle to complete. Their times, in minutes, are normally distributed with mean μ and standard deviation σ. It is found that 20% of the students have times less than 14.5 minutes and 67% of the students have times greater than 18.5 minutes.

(b) Find the value of μ and the value of σ. [5]

5 The populations of 150 villages in the UK, to the nearest hundred, are summarised in the table.

Population	100 – 800	900 – 1200	1300 – 2000	2100 – 3200	3300 – 4800
Number of villages	8	12	50	48	32

(a) Draw a histogram to represent this information. [4]



(b) Write down the class interval which contains the median for this information. [1]

(c) Find the greatest possible value of the interquartile range for the populations of the 150 villages. [2]

6 Eli has four fair 4-sided dice with sides labelled 1, 2, 3, 4. He throws all four dice at the same time. The random variable X denotes the number of 2s obtained.

(a) Show that $P(X = 3) = \frac{3}{64}$. [2]

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(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	$\frac{81}{256}$			$\frac{3}{64}$	$\frac{1}{256}$

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(c) Find $E(X)$. [2]

Eli throws the four dice at the same time on 96 occasions.

(d) Use an approximation to find the probability that he obtains at least two 2s on fewer than 20 of these occasions. [5]

7 A children's wildlife magazine is published every Monday. For the next 12 weeks it will include a model animal as a free gift. There are five different models: tiger, leopard, rhinoceros, elephant and buffalo, each with the same probability of being included in the magazine.

Sahim buys one copy of the magazine every Monday.

(a) Find the probability that the first time that the free gift is an elephant is before the 6th Monday. [2]

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(b) Find the probability that Sahim will get more than two leopards in the 12 magazines. [3]

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(c) Find the probability that after 5 weeks Sahim has exactly one of each animal. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **15** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Var = $\left[\frac{\Sigma(x-q)^2}{50} - \left(\frac{\Sigma(x-q)}{50}\right)^2 =\right]\frac{14235}{50} - \left(\frac{700}{50}\right)^2$ $[= 284.7 - 196 = 88.7]$	M1	$\frac{14235}{a} - \left(\frac{700}{a}\right)^2$; where a = 49, 50, 51.
	[sd = $\sqrt{88.7}$ =] 9.42	A1	9.4180677 rounded to at least 3SF.
		2	
1(b)	$\sum x - 50q = 700$ $[2865 - 50q = 700]$	M1	Forming equation with Σx, $50q$ and 700.
	$q = 43.3, 43\frac{3}{10}$	A1	If M0 scored, **SC B1** for 43.3 WWW.
		2	

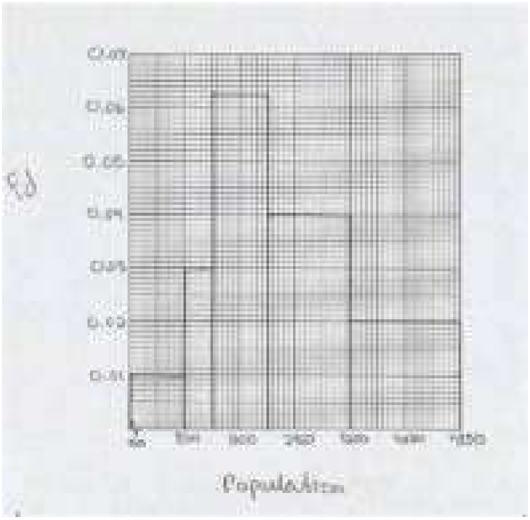
Question	Answer	Marks	Guidance
2(a)	${}^6C_3 \times {}^8C_3$	M1	${}^6C_3 \times b$ or $c \times {}^8C_3$ seen. b, c integers $\geqslant 1$ (1 may be implied).
	1120	A1	
		2	
2(b)	**Method 1**		
	0 brothers $[{}^3C_0] \times {}^{11}C_6$ 462 1 brother ${}^3C_1 \times {}^{11}C_5$ 1386 2 brothers ${}^3C_2 \times {}^{11}C_4$ 990	B1	${}^3C_x \times {}^{11}C_{6-x}$, with $x = 1$ or 2 seen.
		M1	Add values of 3 correct scenarios, (may be identified by the appropriate calculations) no incorrect/repeated scenarios, condone use of permutations.
	2838	A1	Only dependent on the M mark. **SC B1** for the correct calculation or 2838 seen WWW.
	Method 2		
	${}^{14}C_6 - {}^{11}C_3$ 3003 – 165	B1	${}^{14}C_6 - d$, where d a positive integer.
		M1	$e - {}^{11}C_3$, where e is a positive integer >165.
	= 2838	A1	
		3	

Question	Answer	Marks	Guidance
3(a)	$\left[\dfrac{8!}{2!3!}=\right]$ 3360	**B1**	
		1	
3(b)	$\dfrac{6!}{2!2!}$	**M1**	$\dfrac{6!}{2!f!}$; $f = 1, 2, 3$.
	180	**A1**	
		2	
3(c)	$\left[P(OOO \mid CC)=\dfrac{P(OOO\cap CC)}{P(CC)}=\right]$ $\dfrac{5!}{\frac{7!}{3!}}$	**M1**	$\dfrac{5!}{g}$ g a positive integer, $g \neq 3360, 1$. Condone numerator of $\dfrac{5!}{3360g}$.
		M1	$\dfrac{h}{\frac{7!}{3!}}$ or $\dfrac{h}{\frac{8!}{3!}}$, where h is a positive integer. Condone division by 3360 in denominator.
	$=\dfrac{120}{840}, \dfrac{1}{7}, 0.143$	**A1**	0.1428571… to at least 3SF. If M0 scored **SC B1** for $\dfrac{1}{7}$ WWW.
		3	

Question	Answer	Marks	Guidance
4(a)	$\text{P}(Z > \frac{20-14.6}{5.2}) = P(Z > 1.03846)$	**M1**	Use of ± standardisation formula with 20, 14.6 and 5.2 not σ^2, not $\sqrt{\sigma}$, no continuity correction.
	1 – 0.8504	**M1**	Calculating the appropriate probability area (leading to their final answer).
	0.150	**A1**	0.1496, $0.149 < p \leqslant 0.15[0]$. Only dependent on the 2nd M mark so M0M1A1 possible. **SC B1** for $0.149 < p \leqslant 0.15[0]$ if M0M0A0 awarded.
	[250 × *their* 0.1496 =] 37, 38	**B1 FT**	Strict FT *their* at least 4-figure probability seen anywhere (give BOD if they go on to use 0.150). Final answer must be positive integer, no approximation or rounding stated.
		4	
4(b)	$z_1 = \frac{14.5-\mu}{\sigma} = -0.842$ $z_2 = \frac{18.5-\mu}{\sigma} = -0.44$	**B1**	$-0.843 < z_1 < -0.841$ or $0.841 < z_1 < 0.843$.
		B1	$-0.441 < z_2 < -0.439$ or $0.439 < z_2 < 0.441$.
		M1	Use of the ±standardisation formula once with μ, σ and a z-value (not 0.20, 0.80, 0.67, 0.23, 0.5793, 0.7881, 0.7486, 0.591 or 1 - z i.e. 0.158 etc.). Condone continuity correction ±0.05, not $\sigma^2, \sqrt{\sigma}$.
	Solve, obtaining values for μ and σ. $\mu = 22.9, \quad \sigma = 9.95$	**M1**	Solve using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ.
		A1	AWRT 22.9, 9.95 .
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">5(a)</td><td rowspan="4">

cw	800	400	800	1200	1600
fd	0.01	0.03	0.0625	0.04	0.02



</td><td>M1</td><td>At least 4 frequency densities calculated (F/cw, e.g. $\frac{8}{800}\left(\text{condone}\frac{8}{n}, 799 \leqslant n \leqslant 801\right)$) Accept unsimplified, may be read from graph using their scale.</td></tr>
<tr><td>A1</td><td>All heights correct on graph.</td></tr>
<tr><td>B1</td><td>Bar ends at 50, 850, 1250, 2050, 3250, 4850 read at the axis with a horizontal linear scale with at least 3 values indicated.
$50 \leqslant$ horizontal scale $\leqslant 4850$.</td></tr>
<tr><td>B1</td><td>Axes labelled frequency density (fd) and population (pop) OE, or in a title. Linear vertical scale, with at least 3 values indicated. Vertical axis must cover at least the range $0 \leqslant$ vertical axis $\leqslant 0.0625$.
Axes may be reversed.</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="2">5(b)</td><td>2100 – 3200</td><td>B1</td><td>Accept 2050 – 3250 OE. Condone ‘4th interval’.</td></tr>
<tr><td></td><td>1</td><td></td></tr>
<tr><td rowspan="3">5(c)</td><td>3249 – 1250</td><td>M1</td><td>$2050 \leqslant \text{UQ} \leqslant 3250 - 1250 \leqslant \text{LQ} \leqslant 2050$.</td></tr>
<tr><td>1999</td><td>A1</td><td>Condone 3250 – 1250 = 2000.</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
6(a)	[P($X = 3$) =] $\frac{3}{4}\times\left(\frac{1}{4}\right)^3\times 4$	**M1**	$\frac{3}{4}\times\left(\frac{1}{4}\right)^3\times q$; q a positive integer (1 may be implied).
	$=\frac{3}{64}$	**A1**	AG.
		2	
6(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P($X = x$)</td><td>$\frac{81}{256}$</td><td>$\frac{27}{64}$</td><td>$\frac{27}{128}$</td><td>$\frac{3}{64}$</td><td>$\frac{1}{256}$</td></tr></table>	**B1**	Either P(1) = $\frac{27}{64}$,0.421875 or P(2) = $\frac{27}{128}$,0.2109375 correct to at least 3SF. Condone not in table.
		B1 FT	Both values in table. FT P(1) + P(2) = $\frac{81}{128}$,0.6328125 .
		2	
6(c)	[E(X) =] $[0\times\frac{81}{256}]+1\times \textit{their}\frac{27}{64}+2\times \textit{their}\frac{27}{128}+3\times\frac{12}{256}+4\times\frac{1}{256}$	**M1**	Correct method from *their* probability distribution table with at least 4 terms, $0 < \textit{their}\ \mathrm{P}(x) < 1$, accept partially evaluated. $= 0+\frac{27}{64}+\frac{54}{128}+\frac{36}{256}+\frac{4}{256}$
	= 1	**A1**	
		2	

Question	Answer	Marks	Guidance
6(d)	Mean $= 96 \times \frac{67}{256} = 25.125$ Var $= 96 \times \frac{67}{256} \times \frac{189}{256} = 18.549$	**B1**	25.125, $25\frac{1}{8}$ and 18.5493… to at least 3SF seen, allow unsimplified ($4.3068 \leqslant \sigma \leqslant 4.307$ implies correct variance).
	$\mathrm{P}(X < 20) = \mathrm{P}(Z < \frac{19.5 - 25.125}{\sqrt{18.549}})$	**M1**	Substituting *their* μ and σ into ± standardisation formula (any number for 19.5). Condone σ^2 and $\sqrt{\sigma}$.
		M1	Using continuity correction 19.5 or 20.5 in *their* standardisation formula. Note: $\frac{\pm 5.625}{\sqrt{18.549}}$ seen gains M2 BOD.
	[= P($Z < -1.306$) = 1 – Φ (1.306)=] 1 – 0.9042 =	**M1**	Appropriate area Φ, from final process. Must be a probability.
	0.0958	**A1**	$0.0957 \leqslant p \leqslant 0.0958$. **SC B1** for $0.0957 \leqslant p \leqslant 0.0958$ if B1M0M0M1 scored.
		5	

Question	Answer	Marks	Guidance
7(a)	**Method 1**		
	[P(X < 6) = P($X \leqslant 5$) =] $1-0.8^5$	**M1**	$1 - 0.8^r$, $r = 5, 6$.
	= 0.672	**A1**	
	Method 2		
	[P(X < 6) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) =] $\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}+\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}$	**M1**	Condone an extra term $(\frac{4}{5})^5\times\frac{1}{5}$. First, last and one of the 3 middle terms implies M1.
	= 0.672	**A1**	
		2	

Question	Answer	Marks	Guidance
7(b)	**Method 1**		
	$[1 - P(0, 1, 2)]$ $= 1 - ({}^{12}C_0\,(0.8)^{12} + {}^{12}C_1\,(0.2)(0.8)^{11} + {}^{12}C_2\,(0.2)^2\,(0.8)^{10})$ $[= 1 - (0.06872 + 0.20615 + 0.28347)]$	**M1**	One term ${}^{12}C_x\,(p)^x\,(1-p)^{12-x}$, $0 < p < 1$, $x \neq 0, 1, 2$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer. Correct unsimplified expression or better.
	= 0.442	**B1**	$0.411 < p \leqslant 0.442$ WWW.
	Method 2		
	$[P(3,4,5,6,7,8,9,10,11,12) =]$ ${}^{12}C_3\,(0.2)^3\,(0.8)^9 + {}^{12}C_4\,(0.2)^4\,(0.8)^8 + \ldots + {}^{12}C_{11}\,(0.2)^{11}\,(0.8)^1 + {}^{12}C_{12}\,(0.2)^{12}$ $[= 0.23622 + 0.13288 + \ldots + 1.966 \times 10^{-7} + 4.096 \times 10^{-9}]$	**M1**	One term ${}^{12}C_x\,(p)^x\,(1-p)^{12-x}$, $0 < p < 1$, $x \neq 0, 1, 2$.
		A1	Correct expression, accept unsimplified, leading to final answer. Accept first, last and 8 of the middle terms.
	=0.442	**B1**	$0.411 < p \leqslant 0.442$.
		3	

Question	Answer	Marks	Guidance
7(c)	$(0.2)^5 \times 5!$	**M1**	$(0.2)^5 \times s$, s a positive integer. 1 may be implied.
		M1	$t \times 5!$ where $0 < t < 1$.
	$= 0.0384,\ \dfrac{24}{625}$	**A1**	
	Alternative Method for Question 7(c)		
	$\dfrac{{}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times [{}^1C_1]}{({}^5C_1)^5}$	**M1**	$({}^5C_1)^5$ or 5^5 as denominator.
		M1	${}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times [{}^1C_1]$ or 5! as numerator.
	$= 0.0384,\ \dfrac{24}{625}$	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4124937800

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 The random variable X takes the values -2, 2 and 3. It is given that

$$P(X = x) = k(x^2 - 1),$$

where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

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(b) Find $E(X)$ and $Var(X)$. [3]

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2 A sports event is taking place for 4 days, beginning on Sunday. The probability that it will rain on Sunday is 0.4. On any subsequent day, the probability that it will rain is 0.7 if it rained on the previous day and 0.2 if it did not rain on the previous day.

(a) Find the probability that it does **not** rain on any of the 4 days of the event. [1]

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(b) Find the probability that the first day on which it rains during the event is Tuesday. [2]

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(c) Find the probability that it rains on exactly one of the 4 days of the event. [3]

3 The following back-to-back stem-and-leaf diagram represents the monthly salaries, in dollars, of 27 employees at each of two companies, A and B.

Company A		Company B
5 4 1 1 0	25	4 4 5 6 6 7
9 9 8 7 2 1 0	26	0 1 3 5 5 7 9 9
8 6 4 2 1 0	27	1 3 4 6 6 8 8
6 5 4 2 0	28	0 1 2 2 2
9 8 5	29	
1	30	9

Key: 1 | 27 | 6 means \$2710 for company A and \$2760 for company B

(a) Find the median and the interquartile range of the monthly salaries of employees in company A. [3]

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The lower quartile, median and upper quartile for company B are \$2600, \$2690 and \$2780 respectively.

(b) Draw two box-and-whisker plots in a single diagram to represent the information for the salaries of employees at companies A and B. [3]

(c) Comment on whether the mean would be a more appropriate measure than the median for comparing the given information for the two companies. [1]

4 A fair 5-sided spinner has sides labelled 1, 2, 3, 4, 5. The spinner is spun repeatedly until a 2 is obtained on the side on which the spinner lands. The random variable X denotes the number of spins required.

(a) Find $P(X = 4)$. [1]

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(b) Find $P(X < 6)$. [2]

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Two fair 5-sided spinners, each with sides labelled 1, 2, 3, 4, 5, are spun at the same time. If the numbers obtained are equal, the score is 0. Otherwise, the score is the higher number minus the lower number.

(c) Find the probability that the score is greater than 0 given that the score is **not** equal to 2. [3]

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The two spinners are spun at the same time repeatedly .

(d) For 9 randomly chosen spins of the two spinners, find the probability that the score is greater than 2 on at least 3 occasions. [3]

5 The lengths of Western bluebirds are normally distributed with mean 16.5 cm and standard deviation 0.6 cm.

A random sample of 150 of these birds is selected.

(a) How many of these 150 birds would you expect to have length between 15.4 cm and 16.8 cm? [4]

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The lengths of Eastern bluebirds are normally distributed with mean 18.4 cm and standard deviation σ cm. It is known that 72% of Eastern bluebirds have length greater than 17.1 cm.

(b) Find the value of σ. [3]

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A random sample of 120 Eastern bluebirds is chosen.

(c) Use an approximation to find the probability that fewer than 80 of these 120 bluebirds have length greater than 17.1 cm. [5]

6 In a group of 25 people there are 6 swimmers, 8 cyclists and 11 runners. Each person competes in only one of these sports. A team of 7 people is selected from these 25 people to take part in a competition.

(a) Find the number of different ways in which the team of 7 can be selected if it consists of exactly 1 swimmer, at least 4 cyclists and at most 2 runners. [4]

For another competition, a team of 9 people consists of 2 swimmers, 3 cyclists and 4 runners. The team members stand in a line for a photograph.

(b) How many different arrangements are there of the 9 people if the swimmers stand together, the cyclists stand together and the runners stand together? [2]

(c) How many different arrangements are there of the 9 people if none of the cyclists stand next to each other? [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

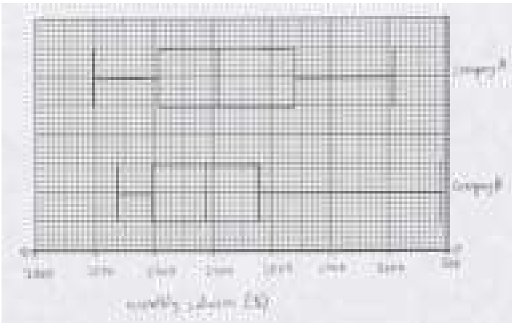
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">1(a)</td><td>$\left[3k+3k+8k=1, so\right] k=\frac{1}{14}$</td><td>B1</td><td></td></tr>
<tr><td rowspan="2"><table><tr><td>x</td><td>–2</td><td>2</td><td>3</td></tr><tr><td>P(x)</td><td>$\frac{3}{14}, 0.214$</td><td>$\frac{3}{14}, 0.214$</td><td>$\frac{8}{14}, 0.571$</td></tr></table></td><td>B1 FT</td><td>Table with correct values of x, and at least one correct probability linked with outcome. FT their k.
Condone any additional X values if probability stated as 0.</td></tr>
<tr><td>B1 FT</td><td>The outcomes in the table must be –2, 2 and 3.
2 further correct probabilities in table
or 3 correct probabilities not in table linked to outcomes,
or 3 correct FT probabilities in table using their k,
or 3 incorrect probabilities summing to 1 in table if k not stated.</td></tr>
<tr><td></td><td></td><td>If k not calculated, SC B1 for the below.<table><tr><td>x</td><td>–2</td><td>2</td><td>3</td></tr><tr><td>P(x)</td><td>$3k$</td><td>$3k$</td><td>$8k$</td></tr></table></td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
1(b)	$\left[E(X) = -2\times\frac{3}{14} + 2\times\frac{3}{14} + 3\times\frac{8}{14} = \right]$ $-\frac{6}{14} + \frac{6}{14} + \frac{24}{14}$	**M1**	Accept unsimplified expression. May be calculated in variance. FT *their* table with 3 probabilities summing to $0.999 \leqslant$ *total* $\leqslant 1$ $(0 < p < 1)$ or in terms of k.
	$\left[\mathrm{Var}(X) = (-2)^2\times\frac{3}{14} + 2^2\times\frac{3}{14} + 3^2\times\frac{8}{14} - (\textit{their}\ E(X))^2 = \right]$ $4\times\frac{3}{14} + 4\times\frac{3}{14} + 9\times\frac{8}{14} - \left(\textit{their}\ \frac{12}{7}\right)^2$ $\left[\frac{12+12+72}{14} - \left(\textit{their}\ \frac{12}{7}\right)^2\right]$	**M1**	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table with 3 or more probabilities $(0 < p < 1)$ which need not sum to 1, or in terms of k with an expression no more evaluated than shown.
	$E(X) = \frac{12}{7}$, 1.71, $1\frac{5}{7}$ $\mathrm{Var}(X) = \frac{192}{49}$, 3.92, $3\frac{45}{49}$	**A1**	Answers for $E(X)$ and $\mathrm{Var}(X)$ must be identified. $E(X)$ may be identified by correct use in Variance (condone E, V, μ, σ^2, etc.). If A0 earned, **SC B1** for identified correct final answers.
		3	

Question	Answer	Marks	Guidance
2(a)	[P(no rain) = $0.6\times(0.8)^3$ =] 0.3072, $\frac{192}{625}$	**B1**	Exact value required
		1	

Question	Answer	Marks	Guidance
2(b)	$0.6 \times 0.8 \times 0.2$	**M1**	$a \times b \times c$ where a, b = 0.6, 0.8, c = 0.2, 0.4, 0.7 . Condone including Wednesday with both 0.3 and 0.7 used.
	$= 0.096[0],\ \frac{12}{125}$	**A1**	
		2	
2(c)	$\text{P}(\text{RDDD}) = 0.4 \times 0.3 \times 0.8 \times 0.8 = 0.0768,\ \frac{48}{625}$ $\text{P}(\text{DRDD}) = 0.6 \times 0.2 \times 0.3 \times 0.8 = 0.0288,\ \frac{18}{625}$ $\text{P}(\text{DDRD}) = 0.6 \times 0.8 \times 0.2 \times 0.3 = 0.0288,\ \frac{18}{625}$ $\text{P}(\text{DDDR}) = 0.6 \times 0.8 \times 0.8 \times 0.2 = 0.0768,\ \frac{48}{625}$	**B1**	Correct probability for one clearly identified outcome evaluated accept unsimplified. A correct unsimplified expression is not sufficient.
		M1	Add 4 probability values, $0 < p < 1$, for appropriate identified scenarios. Accept unsimplified. Ways of identifying scenarios for this mark**:** Stating the days. All the unsimplified probability calculations exactly as stated in the mark scheme. Identifying the correct branches on a tree diagram and linking with the values. No repeated scenarios. No incorrect scenarios.
	$0.2112,\ \frac{132}{625}$	**A1**	Accept 0.211 If 0/3 scored **SC B1** for $0.2112,\ \frac{132}{625}$.
		3	

Question	Answer	Marks	Guidance
3(a)	Median = 2710	**B1**	Must be identified, condone Q2. Ignore units throughout.
	2840 – 2610	**M1**	2820 ⩽ UQ ⩽ 2850 – 2600 ⩽ LQ ⩽ 2620.
	230	**A1**	www If M0 scored **SC B1** for 230 www. If key ignored consistently: B0 Median = 271 **SC M1** 282 ⩽ UQ ⩽ 285 – 260 ⩽ LQ ⩽ 262 **SC A1** 23.
		3	
3(b)	Box-and-whisker plot on provided grid.  B: 2540 2600 2690 2780 3090 A: 2500 *2610* *2710* *2840* 3010	**B1**	All 5 key values for *B* plotted accurately in standard format using a linear scale with 3 identified values. Labelled *B*. Scale at least 1 cm = $100.
		B1FT	All 5 key values for *A*, **FT** from **(a)**, plotted accurately in standard format using a linear scale with 3 identified values. Labelled *A*. Scale at least 1cm = $100
		B1	Whiskers not through box for both, not drawn at corners of boxes, single linear scale for the diagram and labelled ‘salaries’ (oe) and $.
		3	

Question	Answer	Marks	Guidance
3(c)	Examples: Mean less appropriate than median because of extreme value for company *B* [at \$3090]. No, extreme value in company B. No, \$3090 is an anomaly.	**B1**	Must refer to company B, may be implied by appropriate use of \$3090. Must include an indication that the mean is not appropriate. No contradictory statements can be present, e.g. acceptable comment with 'but mean could be used for company A'. Condone reference to \$309.
		1	

Question	Answer	Marks	Guidance
4(a)	$[\mathrm{P}(X=4) = (0.8)^3(0.2) =]\ 0.1024, \frac{64}{625}$	**B1**	Condone 0.102 .
		1	

Question	Answer	Marks	Guidance
4(b)	$[\mathrm{P}(X < 6) =] 1 - 0.8^5$	**M1**	$1 - 0.8^d$, $d = 5, 6$.
	$= 0.672, \dfrac{2101}{3125}$	**A1**	0.67232 to at least 3SF. If M0 awarded, **SC B1** for $\dfrac{2101}{3125}$ or 0.67232 only.
	Alternative Method for Question 4(b)		
	$[\mathrm{P}(X < 6) =]\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$	**M1**	If answer correct, condone omission of 2 from 3 middle terms. Allow M1 for $\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)$
	$= 0.672, \dfrac{2101}{3125}$	**A1**	0.67232 to at least 3SF. If M0 awarded, **SC B1** for $\dfrac{2101}{3125}$ or 0.67232 only.
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="9">4(c)</td><td rowspan="3">$\left[\mathrm{P}(X>0 \mid X\neq 2)=\dfrac{\mathrm{P}(X>0\cap X\neq 2)}{\mathrm{P}(X\neq 2)}=\right]$
$=\dfrac{14}{25}\Big/\dfrac{19}{25}$
$=\dfrac{14}{19}$, 0.737</td><td>M1</td><td>[P($X>0\cap X\neq 2$) =] $\dfrac{14}{25}$, 0.56[0] seen as numerator or denominator of conditional probability fraction.</td></tr>
<tr><td>M1</td><td>[P($X\neq 2$) =] $\dfrac{19}{25}$, 0.76[0] seen as denominator of conditional probability fraction.</td></tr>
<tr><td>A1</td><td>Final answer = $\dfrac{14}{19}$, 0.7368421… to at least 3SF.
If A0, SC B1 for correct final answer www.</td></tr>
<tr><td colspan="3">Alternative Method for Question 4(c)</td></tr>
<tr><td rowspan="3">$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 & 1 & 2 \\ 4 & 3 & 2 & 1 & 0 & 1 \\ 5 & 4 & 3 & 2 & 1 & 0 \end{array}$
$\left[\mathrm{P}(X>0 \mid X\neq 2)=\dfrac{\text{Number of outcome}(X>0\cap X\neq 2)}{\text{Number of outcomes } X\neq 2}=\right]$
$\dfrac{14}{19}$, 0.737</td><td>M1</td><td>$\left[\text{Number of outcome}(X>0\cap X\neq 2)=\right]14$ seen as numerator or denominator of conditional probability fraction.</td></tr>
<tr><td>M1</td><td>$\left[\text{Number of outcome}(\mathrm{X}\neq 2)=\right]19$ seen as denominator of conditional probability fraction.</td></tr>
<tr><td>A1</td><td>Final answer = $\dfrac{14}{19}$, 0.7368421… to at least 3SF.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(d)	[P(X >2) = 1 – P(0, 1, 2) with $p = \frac{6}{25}$]	M1	One term ${}^9C_x\,(p)^x(1-p)^{9-x}$, $0 < p < 1$, $0 < x < 9$.
	$1-({}^9C_0\left(\frac{19}{25}\right)^9 + {}^9C_1\left(\frac{6}{25}\right)^1\left(\frac{19}{25}\right)^8 + {}^9C_2\left(\frac{6}{25}\right)^2\left(\frac{19}{25}\right)^7)$ [1 – (0.08459 + 0.2404 + 0.3037)]	A1	$1-({}^9C_0\,(1-p)^9 + {}^9C_1\,(p)^1(1-p)^8$ $+\,{}^9C_2\,(p)^2(1-p)^7)$, $0 < p < 1$. Correct expression from *their p*, accept unsimplified, no terms omitted leading to final answer. Condone omission of last bracket only.
	0.371	B1	$0.371 \leqslant p < 0.3715$.
	Alternative Method for Question 4(d)		
	[P(X>2) = P(3,4,5,6,7,8,9) with $p = \frac{6}{25}$]	M1	One term ${}^9C_x\,(p)^x(1-p)^{9-x}$, $0 < p < 1$, $0 < x < 9$.
	${}^9C_3\left(\frac{6}{25}\right)^3\left(\frac{19}{25}\right)^6 + {}^9C_4\left(\frac{6}{25}\right)^4\left(\frac{19}{25}\right)^5 + \ldots + {}^9C_8\left(\frac{6}{25}\right)^8\left(\frac{19}{25}\right)^1 + {}^9C_9\left(\frac{6}{25}\right)^9$ [$0.2238 + 0.1060 + \ldots + 7.529 \times 10^{-5} + 2.642 \times 10^{-6}$]	A1	${}^9C_3\,(p)^3(1-p)^6 + {}^9C_4\,(p)^4(1-p)^5 + \ldots + {}^9C_8$ $(p)^8(1-p)^1 + {}^9C_9\,(p)^9$, $0 < p < 1$. Correct expression from *their p*, accept unsimplified, no terms omitted leading to final answer.
	0.371	B1	$0.371 \leqslant p < 0.3715$.
		3	

Question	Answer	Marks	Guidance
5(a)	[P(15.4 < X < 16.8) =] $P(\frac{15.4-16.5}{0.6} < Z < \frac{16.8-16.5}{0.6})$ [= P(−1.833 < Z < 0.5)]	**M1**	Use of ± standardisation formula once with 16.5, 0.6 and either 15.4 or 16.8 substituted.
	[= $\Phi(0.5)+\Phi(1.833)-1$ =] 0.6915 + 0.9666 – 1	**M1**	Calculating the appropriate probability area (leading to their final answer, expect > 0.5). 0.6915 – (1 – 0.9666) or (0.6915 – 0.5) + (0.9666 – 0.5) OE are alternatives.
	= 0.658	**A1**	$0.658 \le p < 0.6585$. If A0 scored, **SC B1** for $0.658 \leqslant p < 0.6585$.
	[Expected number =] 0.6581 × 150 = 98, 99	**B1 FT**	FT *their* 4SF (or better) probability from a normal calculation. Must be a positive single integer answer. No approximation notation.
		4	
5(b)	$\left[P\left(Z > \frac{17.1-18.4}{\sigma}\right) = 0.72\right]$	**B1**	$0.5825 < z \leqslant 0.583$ or $-0.583 \leqslant z < -0.5825$ seen.
	$\frac{17.1-18.4}{\sigma} = -0.583$	**M1**	Use of the ± standardisation formula with 17.1, 18.4, σ and a z-value (not 0.28, 0.72, 0.4175, 0.2358, 0.7642, 0.6103, 0.3897, …). Condone continuity correct ± 0.05, not $\sigma^2, \sqrt{\sigma}$.
	$\sigma = 2.23$	**A1**	AWRT
		3	

Question	Answer	Marks	Guidance
5(c)	[Mean = 120 × 0.72 =] 86.4 [Var = 120 × 0.72 × 0.28 =] 24.192	B1	86.4, $84\frac{2}{5}$ and $24\frac{24}{125}$, 24.192 to at least 3SF seen, allow unsimplified. May be seen in standardisation formula. ($4.918 \leqslant \sigma \leqslant 4.919$ implies correct variance) Incorrect notation is penalised.
	$P(X < 80) = P(Z < \frac{79.5 - 86.4}{\sqrt{24.192}})$	M1	Substituting *their* mean (not 18.4) and *their positive* 4.9185 into ± standardisation formula (any number for 79.5), condone *their* 4.918^2 and $\sqrt{}$*their* 4.918 .
		M1	Using continuity correction 79.5 or 80.5 in *their* standardisation formula.
	$[P(Z < -1.4029) = 1 - \Phi(1.403)]$ 1 − 0.9196	M1	Appropriate area Φ, from final process, must be a probability. Expect final answer < 0.5 . Note: correct final answer implies this M1.
	0.0804	A1	$0.0803 \leqslant p \leqslant 0.0804$
		5	

Question	Answer	Marks	Guidance
6(a)	S + 4C + 2R ${}^6C_1 \times {}^8C_4 \times {}^{11}C_2$ [$= 6 \times 70 \times 55$] = 23 100 S + 5C + 1R ${}^6C_1 \times {}^8C_5 \times {}^{11}C_1$ [$= 6 \times 56 \times 11$] = 3696 S + 6C [+ 0R] ${}^6C_1 \times {}^8C_6$ [$\times {}^{11}C_0$][$= 6 \times 28$] = 168	**M1**	${}^6C_e \times {}^8C_f \times {}^{11}C_g$, with $e + f + g = 7$ seen.
		B1	Correct outcome/value for 1 identified scenario, accept unsimplified, www.
		M1	Add values of 3 correct scenarios. No incorrect scenarios, no repeated scenarios. Condone ${}^6C_e \times {}^8C_f \times {}^{11}C_g$, with $e + f + g = 7$ to identify S, C, R.
	[Total =] 26964	**A1**	cao
		4	
6(b)	$2! \times 3! \times 4! \times 6$	**M1**	$2! \times 3! \times 4! \times k$, k an integer > 0. 1 can be implied.
	=1728	**A1**	If A0 scored **SC B1** for 1728 www.
		2	

Question	Answer	Marks	Guidance
6(c)	**Method 1**		
	$6! \times 7 \times 6 \times 5$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times 7 \times n \times r$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$; $1 \leqslant n, r \leqslant 6$, $n \neq r$.
		M1	$\frac{m!}{a! \times b!} \times 7 \times 6 \times 5$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
	Method 2		
	$6! \times {}^7P_3$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times {}^7P_q$, or $\frac{m!}{a! \times b!} \times {}^7C_q \times q!$; $6 \le m \le 9$; $a = 1, 2$; $b = 1, 4$; $1 \le q \le 6$.
		M1	$\frac{m!}{a! \times b!} \times {}^7P_3$, or $\frac{m!}{a! \times b!} \times {}^7C_3 \times 3!$; $6 \le m \le 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.

Question	Answer	Marks	Guidance
6(c)	**Method 3**		
	$6! \times 35 \times 3!$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times 35 \times q!$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$; $1 \leqslant q \leqslant 3$.
		M1	$\frac{m!}{a! \times b!} \times 35 \times 6$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
	Method 4		
	$9! - 7!3! - {}^3P_2 \times 6! \times 7 \times 6$ Or $9! - 7!3! - 3! \times 7! \times 6$ [= 362 880 – 30 240 – 181 440]	**M1**	$9! - 7!r! - q$, r an integer > 1, q an integer $\leqslant 0$. 0 and 1 may be implied.
		M1	$\frac{s!}{a! \times b! \times c!} - 7!3! - q$; $s = 8, 9$; $a = 1, 2$; $b = 1, 3$; $c = 1, 4$; q an integer $\geqslant 0$. 0 and 1 may be implied.
		M1	$\frac{s!}{a! \times b! \times c!} - 7!3! - {}^3P_2 \times 6! \times 6 \times 7$, $6 \leqslant s \leqslant 9$, **or** $\frac{s!}{a! \times b! \times c!} - 7!3! - 3! \times 7! \times 6$, $6 \leqslant s \leqslant 9$. $a = 1, 2$ $b = 1, 3$ $c = 1, 4$. 1 may be implied.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME												
CENTRE NUMBER							CANDIDATE NUMBER					

0013731051

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

1 Two fair coins are thrown at the same time repeatedly until a pair of heads is obtained. The number of throws taken is denoted by the random variable X.

(a) State the value of $E(X)$. [1]

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(b) Find the probability that exactly 5 throws are required to obtain a pair of heads. [1]

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(c) Find the probability that fewer than 7 throws are required to obtain a pair of heads. [2]

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2 Anil is a candidate in an election. He received 40% of the votes. A random sample of 120 voters is chosen.

Use an approximation to find the probability that, of the 120 voters, between 36 and 54 inclusive voted for Anil. [5]

3 The random variable X takes the values 1, 2, 3, 4. It is given that $P(X = x) = kx(x + a)$, where k and a are constants.

(a) Given that $P(X = 4) = 3P(X = 2)$, find the value of a and the value of k. [4]

(b) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [1]

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(c) Given that $E(X) = 3.2$, find $Var(X)$. [2]

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4 The times taken, in minutes, to complete a cycle race by 19 cyclists from each of two clubs, the Cheetahs and the Panthers, are represented in the following back-to-back stem-and-leaf diagram.

Cheetahs		Panthers
9 8	7	4
8 7 3 2 0	8	6 8
9 8 7	9	1 7 8 9 9
6 5 3 3 1	10	2 3 4 4 5 6
9 8 2	11	1 2 8
4	12	0 6

Key: 7 | 9 | 1 means 97 minutes for Cheetahs and 91 minutes for Panthers

(a) Find the median and the interquartile range of the times of the Cheetahs. [3]

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The median and interquartile range for the Panthers are 103 minutes and 14 minutes respectively.

(b) Make two comparisons between the times taken by the Cheetahs and the times taken by the Panthers. [2]

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Another cyclist, Kenny, from the Cheetahs also took part in the race. The mean time taken by the 20 cyclists from the Cheetahs was 99 minutes.

(c) Find the time taken by Kenny to complete the race. [3]

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5 Jasmine throws two ordinary fair 6-sided dice at the same time and notes the numbers on the uppermost faces. The events A and B are defined as follows.

A: The sum of the two numbers is less than 6.

B: The difference between the two numbers is at most 2.

(a) Determine whether or not the events A and B are independent. [4]

(b) Find $\mathrm{P}(B \mid A')$. [3]

6 The mass of grapes sold per day by a large shop can be modelled by a normal distribution with mean 28 kg. On 10% of days less than 16 kg of grapes are sold.

(a) Find the standard deviation of the mass of grapes sold per day. [3]

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The mass of grapes sold on any day is independent of the mass sold on any other day.

(b) 12 days are chosen at random.

Find the probability that less than 16 kg of grapes are sold on more than 2 of these 12 days. [3]

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(c) In a random sample of 365 days, on how many days would you expect the mass of grapes sold to be within 1.3 standard deviations of the mean? [4]

7 **(a)** Find the number of different arrangements of the 10 letters in the word CASABLANCA in which the two Cs are **not** together. [3]

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(b) Find the number of different arrangements of the 10 letters in the word CASABLANCA which have an A at the beginning, an A at the end and exactly 3 letters between the 2 Cs. [3]

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Five letters are selected from the 10 letters in the word CASABLANCA.

(c) Find the number of different selections in which the five letters include at least two As and at most one C. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **16** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\left[\mathrm{P}(\mathrm{HH}) = \frac{1}{4}\right]$ [E(X) =] 4	**B1**	
		1	
1(b)	$\left[\mathrm{P}(X=5) = \left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) =\right]$ 0.0791	**B1**	$\frac{81}{1024}$
		1	
1(c)	[P(X < 7) =] $1-\left(\frac{3}{4}\right)^6$ or $\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\frac{3}{4}^2\times\frac{1}{4}+\ldots+\frac{3}{4}^5\times\frac{1}{4}$	**M1**	$1 - p^n$, $0 < p < 1$, $n = 6, 7$ or $p + p(1-p) + p(1-p)^2 + \ldots + p(1-p)^n$, where $n = 4, 5$.
	$= \frac{3367}{4096}$, 0.822	**A1**	Accept 0.82202148… to at least 3SF.
		2	

Question	Answer	Marks	Guidance
2	Mean $=120\times 0.4=48$ Var $=120\times 0.4\times 0.6=28.8$	**B1**	48 and $28\frac{4}{5}$, 28.8 seen, allow unsimplified. ($5.366 \leqslant \sigma \leqslant 5.367$ or $\frac{12\sqrt{5}}{5}$ implies correct variance).
	$\text{P}(36 \leqslant X \leqslant 54) = \text{P}\left(\frac{35.5-48}{\sqrt{28.8}} < Z < \frac{54.5-48}{\sqrt{28.8}}\right)$	**M1**	Substituting *their* μ and σ into one ±standardisation formula (any number for 35.5 or 54.5), condone σ^2 and $\sqrt{\sigma}$.
		M1	Using continuity correction 35.5, 36.5 or 53.5, 54.5 once in *their* standardisation formula. Note: $\frac{\pm 12.5}{\sqrt{28.8}}$ or $\frac{\pm 6.5}{\sqrt{28.8}}$ seen gains M2 BOD.
	[= $\text{P}(-2.3292 < Z < 1.211)$ =] $0.8871 + 0.9900 - 1$	**M1**	Appropriate area Φ, from final process. Must be a probability. Expect final answer > 0.5 . Note: correct final answer implies this M1.
	$= 0.877$	**A1**	$0.877 \leq p < 0.8772$.
		5	

Question	Answer	Marks	Guidance
3(a)	[P(X = 4) = 3P(X = 2)] $4k(4+a)=3\times 2k(2+a)$ $16k + 4ak = 12k + 6ak$	**M1**	Using P(X = 4) = 3P(X = 2) to form an equation in a and k.
	$a = 2$	**A1**	If M0 scored, **SC B1** for a = 2 www.
	$3k+8k+15k+24k=1$	**M1**	Using sum of probabilities = 1 to form an equation in k: $k(1 + a) + 2k(2 + a) + 3k(3 + a) + 4k(4 + a) = 1$.
	$k=\frac{1}{50}$	**A1**	If M0 scored, **SC B1** for $k=\frac{1}{50}$ www.
		4	
3(b)	<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>$\frac{3}{50}$, 0.06</td><td>$\frac{8}{50}$, 0.16</td><td>$\frac{15}{50}$, 0.3</td><td>$\frac{24}{50}$, 0.48</td></tr></table>	**B1 FT**	<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>$k(1 + a)$</td><td>$2k(2 + a)$</td><td>$3k(3 + a)$</td><td>$4k(4 + a)$</td></tr></table>$0 < p < 1$ for all outcomes, must be numerical.
		1	
3(c)	Var(X) = $\frac{3}{50}\times 1+\frac{8}{50}\times 2^2+\frac{15}{50}\times 3^2+\frac{24}{50}\times 4^2-3.2^2$	**M1**	Correct formula for variance method from their probability distribution table, $0 \leqslant$ *their* P(x) $\leqslant 1$. Accept $\frac{3+32+135+384}{50}-\frac{256}{25}$.
	[= 11.08 – 3.2^2 =] 0.84[0], $\frac{21}{25}$	**A1**	If M0 score **SC B1** for 0.84 www.
		2	

Question	Answer	Marks	Guidance
4(a)	Median = 99 [minutes]	**B1**	
	[IQR =] 106 – 83	**M1**	105 ⩽ UQ ⩽ 112 – 82 ⩽ LQ ⩽ 87.
	23 [minutes]	**A1**	www. If M0 scored **SC B1** for 23 www.
		3	
4(b)	The times for the Cheetahs are faster than the times for the Panthers	**B1**	Correct statement comparing central tendency in context.
	The times for the Cheetahs are more spread than the times for the Panthers	**B1**	Correct statement comparing range/IQR in context.
		2	
4(c)	[Total time including Kenny = 99 × 20 =]1980	**B1**	Accept unsimplified.
	[Kenny's time =] 1980 – 1862	**M1**	For *their* 1980 – *their* 1862.
	= 118 [minutes]	**A1**	Accept 1 hour 58 mins.
	Alternative Method for Question 4(c)		
	$\frac{1862 + \textit{their}\text{ Kenny's time}}{20} = 99$ [Kenny's time = $99 \times 20 - 1862$]	**B1**	$\frac{1862 + \textit{their}\text{ Kenny's time}}{20} = 99$ seen.
		M1	For *their* 99 ×20 – *their* 1862.
	= 118 [minutes]	**A1**	Accept 1 hour 58 mins.
		3	

Question	Answer	Marks	Guidance
5(a)	$P(A) = \frac{10}{36}$ $P(B) = \frac{24}{36}$	**B1**	Accept $P(A) = \frac{10}{36}, \frac{5}{18}, 0.278$ and $P(B) = \frac{24}{36}, \frac{2}{3}, 0.667$.
	$P(A \cap B) = \frac{8}{36}$	**B1**	
	$\frac{10}{36} \times \frac{24}{36}$	**M1**	*Their* P(A) × *their* P(B) seen numerically, $0 \leqslant$ their P(A), P(B) $\leqslant 1$.
	$= \frac{5}{27}, 0.185 \left[\neq \frac{8}{36}\right]$ Events are not independent	**A1 FT**	Multiplication evaluated correctly and compared with intersection that is not a product of multiplication, conclusion stated, notation P(A), P(B) and P($A \cap B$) used.
		4	

Question	Answer	Marks	Guidance
5(b)	$\left[\mathrm{P}(B\mid A')=\frac{\mathrm{P}(B\cap A')}{\mathrm{P}(A')}=\right]$ $\frac{16}{36}\Big/\left(1-\frac{10}{36}\right)$	**M1**	$\left[\mathrm{P}(B\cap A')=\right]\frac{16}{36}, 0.4444$ or *their* P(B) – *their* P($A \cap B$) seen as numerator or denominator of conditional probability fraction.
		M1	$\left[\mathrm{P}(A')=\right]\left(1-\frac{10}{36}\right), \frac{26}{36}, 0.7222$ or 1 – *their* P(A) seen as denominator of conditional probability fraction.
	$=\frac{8}{13}$	**A1**	Final answer $\frac{16}{26}, \frac{8}{13}, 0.6153846$ to at least 3SF.
	Alternative Method for Question 5(b): Direct from outcome tables		
	$\left[\mathrm{P}(B\mid A')=\frac{\text{Number of outcomes}(B\cap A')}{\text{Number of outcomes}(A')}=\right]$ $\frac{16}{26}$	**M1**	$\left[\text{Number of outcomes}(B\cap A')=\right]16$ seen as numerator or denominator of conditional probability fraction.
		M1	$\left[\text{Number of outcomes}(A')=\right]26$ seen as denominator of conditional probability fraction.
		A1	Final answer $\frac{16}{26}, \frac{8}{13}, 0.6153846$ to at least 3SF.
		3	

Question	Answer	Marks	Guidance
6(a)	$\left[\mathrm{P}(X<16)=\mathrm{P}\left(Z<\frac{16-28}{\sigma}\right)=0.1\right]$ $\frac{16-28}{\sigma}=-1.282$	**B1**	±1.282 seen, cao – critical value.
		M1	Use of the ±standardisation formula with 16, 28, σ and a z-value (not 0.1, 0.9, 0.282, 0.5398, 0.8159) equated to a z-value. Condone continuity correct ±0.5, not $\sigma^2, \sqrt{\sigma}$. Condone $\pm\frac{12}{\sigma}=-1.282$.
	$\sigma = 9.36$	**A1**	
		3	
6(b)	[1 − P(0, 1, 2) =] 1 – ($^{12}C_0(0.1)^0$ $(0.9)^{12}$ + $^{12}C_1$ $(0.1)^1$ $(0.9)^{11}$ + $^{12}C_2$ $(0.1)^2$ $(0.9)^{10}$) [1 – (0.2824 + 0.3766 + 0.2301)]	**M1**	One term $^{12}C_x\,(p)^x(1-p)^{12-x}$, $0<p<1$. $x \neq 0,1,2$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.111	**B1**	0.1108699… rounded to at least 3SF.
	Alternative Method for Question 6(b)		
	P(3,4,5,6,7,8,9,10,11,12) = $^{12}C_3$ $(0.1)^3$ $(0.9)^9$ + $^{12}C_4$ $(0.1)^4$ $(0.9)^8$+ … + $^{12}C_{11}$ $(0.1)^{11}$ $(0.9)^1$ + $^{12}C_{12}$ $(0.1)^{12}$ $(0.9)^0$ [0.08523 + 0.02131 + … + 1.08×10^{-10} + 1×10^{-12}]	**M1**	One term $^{12}C_x\,(p)^x(1-p)^{12-x}$, $0<p<1$. $x \neq 0,1,2$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.111	**B1**	0.1108699… rounded to at least 3SF.
		3	

Question	Answer	Marks	Guidance
6(c)	[P($-1.3 < Z < 1.3$) = 2 Φ(1.3) – 1] = 2 × 0.9032 – 1	**B1**	Identifying at least one of −1.3 or 1.3 as the appropriate *z*-values.
		M1	Calculating the appropriate probability area from 2 symmetrical *z*-values (leading to their final answer, expect > 0.5).
	= 0.806, $\frac{504}{625}$	**A1**	0.8064, $0.806 \leqslant p < 0.8065$.
	[In 365 days 0.8064×365] = 294 or 295	**B1 FT**	Strict FT *their* at least 4-figure probability (not *z*-value). Final answer must be positive integer, no approximation or rounding stated.
		4	

Question	Answer	Marks	Guidance
7(a)	**Method 1: Total number of arrangements – number of arrangements with Cs together**		
	$\frac{10!}{2!4!} - \frac{9!}{4!}$ [75600-15120]	**M1**	$\frac{10!}{a!b!} - c,\ a \neq b$, a = 1, 2, b = 1, 4, with c being a positive integer.
		M1	$d - \frac{e!}{4!}$, e = 8, 9, 10, with d being a positive integer.
	= 60480	**A1**	Exact value only. **SC B1** for final answer 60480 www.
	Method 2: Arrangements ^ ^ C ^ C ^ ^ ^ ^ ^		
	$\frac{8!}{4!} \times \frac{9 \times 8}{2}$	**M1**	$\frac{8!}{4!} \times f$ seen, with f being a positive integer.
		M1	$g \times \frac{9 \times 8}{h}$, with g being a positive integer, h = 1, 2. $g \times {}^9C_2$ and $g \times {}^9P_2$ are acceptable.
	= 60480	**A1**	Exact value only. **SC B1** for final answer 60480 www.
		3	

Question	Answer	Marks	Guidance
7(b)	AC^^^C^^^A $\frac{6!}{2!} \times 4$	M1	$\frac{6!}{2!} \times s$, with s being a positive integer.
		M1	$\frac{t!}{r!} \times 4$, $r = 1, 2, 3$ and $t = 8, 7, 6$.
	1440	A1	
	Alternative Method for Question 7(b)		
	$\frac{4 \times {}^6P_3 \times 3!}{2!}$	M1	$\frac{{}^6P_3}{2!} \times k$, with k being a positive integer.
		M1	$4 \times 3! \times \frac{{}^6P_m}{n!}$, $m = 2, 3$ and $n = 1, 2, 3$.
	1440	A1	
		3	

Question	Answer	Marks	Guidance
7(c)	Scenarios AA _ _ _ $^5C_3 = 10$ AAA _ _ $^5C_2 = 10$ AAAA _ $^5C_1 = 5$	**B1**	Correct number of ways for identified scenarios of 2 or 3 As, accept unsimplified, www.
		M1	Add 3 values for 2, 3 and 4 As, no additional, incorrect or repeated scenarios. Accept unsimplified.
	25	**A1**	
	Alternative Method 2 for Question 7(c)		
	Scenarios: AAC _ _ $^4C_2 = 6$ AA _ _ _ $^4C_3 = 4$ AAAC _ $^4C_1 = 4$ AAA _ _ $^4C_2 = 6$ AAAAC 1 AAAA _ 4	**B1**	Correct total number of ways for identified scenarios of 2 or 3 As, accept unsimplified, www (e.g., both values for AAC^^ and AA^^^ shown would be fine for 2As).
		M1	Add 6 values of appropriate scenarios only, no additional, incorrect or repeated scenarios. Accept unsimplified.
	25	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME		
CENTRE NUMBER		CANDIDATE NUMBER

* 0 2 5 9 9 1 0 9 4 4 *

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

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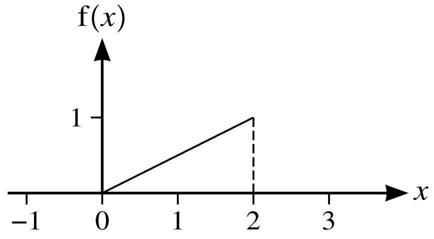
1 In a certain country, 20 540 adults out of a population of 6 012 300 have a degree in medicine.

(a) Use an approximating distribution to calculate the probability that, in a random sample of 1000 adults in this country, there will be fewer than 4 adults who have a degree in medicine. [4]

(b) Justify the approximating distribution used in part **(a)**. [2]

2 **(a)**



The graph of the function f is a straight line segment from (0, 0) to (2, 1).

Show that f could be a probability density function. [2]

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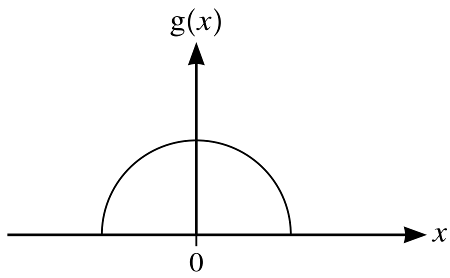
(b)



The graph of the function g is a semicircle, centre (0, 0), entirely above the x-axis.

Given that g is a probability density function, find the radius of the semicircle. [2]

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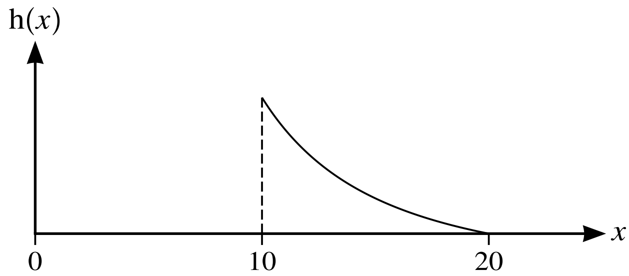
(c)



The time, X minutes, taken by a large number of students to complete a test has probability density function h, as shown in the diagram.

(i) Without calculation, use the diagram to explain how you can tell that the median time is less than 15 minutes. [1]

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It is now given that

$$\mathrm{h}(x) = \begin{cases} \dfrac{40}{x^2} - \dfrac{1}{10} & 10 \leqslant x \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(ii) Find the mean time. [3]

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3 In the past, the annual amount of wheat produced per farm by a large number of similar sized farms in a certain region had mean 24.0 tonnes and standard deviation 5.2 tonnes. Last summer a new fertiliser was used by all the farms, and it was expected that the mean amount of wheat produced per farm would be greater than 24.0 tonnes. In order to test whether this was true, a scientist recorded the amounts of wheat produced by a random sample of 50 farms last summer. He found that the value of the sample mean was 25.8 tonnes.

Stating a necessary assumption, carry out the test at the 1% significance level. [6]

4 A certain train journey takes place every day throughout the year. The time taken, in minutes, for the journey is normally distributed with variance 11.2.

(a) The mean time for a random sample of n of these journeys was found. A 94% confidence interval for the population mean time was calculated and was found to have a width of 1.4076 minutes, correct to 4 decimal places.

Find the value of n. [3]

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(b) A passenger noted the times for 50 randomly chosen journeys in January, February and March.

Give a reason why this sample is unsuitable for use in finding a confidence interval for the population mean time. [1]

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(c) A researcher took 4 random samples and a 94% confidence interval for the population mean was found from each sample.

Find the probability that exactly 3 of these confidence intervals contain the true value of the population mean. [2]

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5 Large packets of rice are packed in cartons, each containing 20 randomly chosen packets. The masses of these packets are normally distributed with mean 1010 g and standard deviation 3.4 g. The masses of the cartons, when empty, are independently normally distributed with mean 50 g and standard deviation 2.0 g.

(a) Find the variance of the masses of full cartons. [2]

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Small packets of rice are packed in boxes. The total masses of full boxes are normally distributed with mean 6730 g and standard deviation 15.0 g. The masses of the boxes and cartons are distributed independently of each other.

(b) Find the probability that the mass of a randomly chosen full carton is more than three times the mass of a randomly chosen full box. [5]

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6 A sample of 5 randomly selected values of a variable X is as follows:

$$1 \qquad 2 \qquad 6 \qquad 1 \qquad a$$

where $a > 0$.

Given that an unbiased estimate of the variance of X calculated from this sample is $\frac{11}{2}$, find the value of a. [3]

7 The number of accidents per week at a certain factory has a Poisson distribution. In the past the mean has been 1.9 accidents per week. Last year, the manager gave all his employees a new booklet on safety. He decides to test, at the 5% significance level, whether the mean number of accidents has been reduced. He notes the number of accidents during 4 randomly chosen weeks this year.

(a) State suitable null and alternative hypotheses for the test. [1]

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(b) Find the critical region for the test and state the probability of a Type I error. [6]

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(c) State what is meant by a Type I error in this context. [1]

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(d) During the 4 randomly chosen weeks there are a total of 3 accidents.

State the conclusion that the manager should reach. Give a reason for your answer. [2]

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(e) Assuming that the mean remains 1.9 accidents per week, use a suitable approximation to calculate the probability that there will be more than 100 accidents during a 52-week period. [4]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	20540/6012300 = 0.0034163	**B1**	
	[1000 × 0.0034163 = 3.4163]		
	Po(3.4163)	**B1**	Could be implied by expression seen.
	$e^{-their\text{ '}3.4163\text{'}}(1 + 3.4163 + \frac{3.4163^2}{2!} + \frac{3.4163^3}{3!})$ OR $e^{-their\text{ '}3.4163\text{'}}(1 + 3.4163 + 5.8356 + 6.6453)$ or 0.03283 + 0.1122 +0.1916 + 0.21819)	**M1**	Allow any λ. Allow with one end error. Must see expression.
	= 0.555 (3sf)	**A1**	CAO **SC** No working: B1 B1 (Po must be stated) B1 correct answer (max 3/4). **SC** Binomial: B1 B0 B1 correct answer (max 2/4).
		4	
1(b)	$n = 1000 > 50$	**B1**	Must show comparison with 50.
	$np = 3.4163 < 5$	**B1**	Must show comparison with 5.
		2	**SC B1**: $n > 50$ (or n large), $np < 5$. **SC B1**: n large, p small.

Question	Answer	Marks	Guidance
2(a)	$\frac{1}{2} \times 2 \times 1$ or $\int_0^2 \frac{1}{2}x\,dx = 1$, which is the correct area under a pdf.	**B1**	Calculation and result.
	$f(x) \geqslant 0$	**B1**	Condone $f(x) > 0$ or 'Line is above x-axis' OE.
		2	

Question	Answer	Marks	Guidance
2(b)	$\frac{1}{2}\pi r^2 = 1$	**M1**	Area of semi-circle equated to 1 OE. Missing factor of ½ gets M1A0.
	$r = \sqrt{\frac{2}{\pi}}$ or 0.798 (3sf)	**A1**	
		2	
2(c)(i)	Area to the left of 15 is greater than 0.5	**B1**	OE, e.g. 'The distribution of X is skewed to the right / positively skewed, suggesting the median will be less than the mid-point of the interval.' or 'The distribution of X is skewed to the right / positively skewed' or 'It is a decreasing function suggesting the median will be less than the mid-point of the interval'.
		1	
2(c)(ii)	$\int_{10}^{20} (\frac{40}{x} - \frac{x}{10})\mathrm{d}x$	**M1**	Integration of $x\mathrm{h}(x)$ attempted. Ignore limits.
	$\left[40\ln x - \frac{x^2}{20}\right]_{10}^{20}$	**A1**	Correct integration and limits (can be implied by final answer).
	$= 40\ln 2 - 15$ or 12.7 (3sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
3	Assume SD still = 5.2	**B1**	OE i.e. 'Assume the SD remains unchanged'.
	H_0: $\mu = 24.0$ H_1: $\mu > 24.0$	**B1**	Or population mean; not just mean.
	$\frac{25.8-24.0}{\frac{5.2}{\sqrt{50}}}$	**M1**	For standardising (could be implied). Must have $\sqrt{50}$.
	$= 2.448$	**A1**	Or $P(\overline{X} > 25.8) = 0.0071$.
	'2.448' > 2.326	**M1**	Or $0.0071 < 0.01$. For valid comparison.
	[Reject H_0] There is evidence that (mean) amount of wheat is greater.	**A1FT**	OE. FT their z_{calc}. In context, not definite, eg not 'Mean amount of wheat is greater' No contradictions CV method: CV= 25.71 M1A1 25.71<25.8 M1 A1FT or CV=24.09 M1 A1 24.09>24 M1 A1FT.
		6	

Question	Answer	Marks	Guidance
4(a)	$z \times \sqrt{\frac{11.2}{n}} = 1.4076 \div 2$	**M1**	Any z, but must be a z.
	$z = 1.881$ or 1.882	**B1**	
	$[n = \left(\frac{1.881}{0.7038}\right)^2 \times 11.2]$ $n = 80$	**A1**	Must be a whole number.
		3	

Question	Answer	Marks	Guidance
4(b)	Jan, Feb and March not typical of whole year.	**B1**	Or, e.g., weather is different at different times of year.
		1	
4(c)	$0.94^3 \times 0.06 \times 4$	**M1**	
	$= 0.199$ (3 sf)	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	$2.0^2 + 20 \times 3.4^2$	**M1**	
	$= 235.2$	**A1**	
		2	
5(b)	$E(C - 3B) = 50 + 20\times1010 - 3\times6730$ or 60	**B1**	
	$Var(C - 3B) = \text{‘}235.2\text{’} + 9\times15^2$ or 2260.2	**M1**	FT *their* values from **(a)**.
	[C – 3B ~ N(‘60’, ‘2260.2’)] $= \dfrac{0-60}{\sqrt{2260.2}}$ [= –1.262]	**M1**	Standardising with their values (could be implied).
	$1 - \Phi(\text{‘}-1.262\text{’}) = \Phi(\text{‘}1.262\text{’})$	**M1**	Probability area consistent with their values.
	$= 0.897$ (3 sf)	**A1**	
		5	

Question	Answer	Marks	Guidance
6	$\frac{5}{4}\left(\frac{1+2^2+6^2+1+a^2}{5}-\left(\frac{1+2+6+1+a}{5}\right)^2\right)=\frac{11}{2}$ or $\frac{1}{4}\left((42+a^2)-\frac{(10+a^2)}{5}\right)=\frac{11}{2}$	**M1***	OE attempted or e.g., $\frac{42+a^2}{5}-\left(\frac{10+a}{5}\right)^2=\frac{22}{5}$. Allow use of biased i.e., without $\frac{5}{4}$.
	$4a^2 - 20a + 0 = 0$ or $a^2 - 5a + 0 = 0$	**DM1**	Two- or three-term quadratic equation in a, with at least two terms correct.
	$a = 5$	**A1**	Ignore $a = 0$, if seen.
		3	

Question	Answer	Marks	Guidance
7(a)	H_0: $\lambda = 7.6$ [or 1.9] H_1: $\lambda < 7.6$ [or 1.9]	**B1**	Or Population mean = 7.6 or μ (not just 'mean'). Or Population mean < 7.6 or μ.
		1	

Question	Answer	Marks	Guidance
7(b)	Mean = 7.6	**B1**	Seen.
	$P(X \leqslant 2) = e^{-7.6}\left(1 + 7.6 + \frac{7.6^2}{2}\right)$ [= 0.0188 or 0.0187]	**M1**	OE.
	$P(X \leqslant 3) = e^{-7.6}\left(1 + 7.6 + \frac{7.6^2}{2} + \frac{7.6^3}{3!}\right)$ [= 0.0554 or 0.0553]	**M1**	OE. Expression must be seen in at least one probability calculation.
	0.0188 or 0.0187 and 0.0554 or 0.0553	**A1**	A1 for both values.
	Critical region is $X \leqslant 2$	**A1**	Dep on both M marks. **SC** No Poisson expression seen in either prob scores B1 for 0.0188 or 0.0187 and B1 for 0.0554 or 0.0553 and B1 for CR.
	P(Type I error) = $P(X \leqslant 2)$ = 0.0188 or 0.0187 (3 sf)	**B1FT**	FT *their* $P(X \leqslant 2)$ or *their* CR.
		6	
7(c)	Concluding that the (mean) no. of accidents has reduced when it has not.	**B1**	OE. Must be in context. Accept: 'It is believed that the booklet has helped to improve safety when actually it has not'.
		1	
7(d)	3 not in critical region.	**M1**	FT their CR or $P(X \leq 3) = 0.0554 > 0.05$.
	No evidence mean number of accidents has decreased.	**A1FT**	In context. Cannot be a definite statement, e.g., 'mean number accidents has not decreased'.
		2	

Question	Answer	Marks	Guidance
7(e)	N(98.8, 98.8)	**B1**	May be implied.
	$\frac{100.5-98.8}{\sqrt{98.8}}$ [= 0.171]	**M1**	For standardising (could be implied by correct answer). Allow with wrong or no continuity correction.
	1 – Φ('0.171')	**M1**	For probability area consistent with their working.
	= 0.432 (3 sf)	**A1**	
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0584750042

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

JC23 06_9709_62/2R

1 In a survey of 200 randomly chosen students from a certain college, 23% of the students said that they owned a car.

Calculate an approximate 93% confidence interval for the proportion of students from the college who own a car. [3]

2 **(a)** The random variable W has a Poisson distribution.

State the relationship between $\mathrm{E}(W)$ and $\mathrm{Var}(W)$. [1]

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(b) The random variable X has the distribution $\mathrm{B}(n, p)$. Jyothi wishes to use a Poisson distribution as an approximate distribution for X.

Use the formulae for $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ to explain why it is necessary for p to be close to 0 for this to be a reasonable approximation. [1]

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(c) Given that Y has the distribution $\mathrm{B}(20\,000, 0.000\,07)$, use a Poisson distribution to calculate an estimate of $\mathrm{P}(Y > 2)$. [3]

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3 The masses, in kilograms, of newborn babies in country A are represented by the random variable X, with mean μ and variance σ^2. The masses of a random sample of 500 newborn babies in this country were found and the results are summarised below.

$$n = 500 \qquad \Sigma x = 1625 \qquad \Sigma x^2 = 5663.5$$

(a) Calculate unbiased estimates of μ and σ^2. [3]

A researcher wishes to test whether the mean mass of newborn babies in a neighbouring country, B, is different from that in country A. He chooses a random sample of 60 newborn babies in country B and finds that their sample mean mass is 2.95 kg.

Assume that your unbiased estimates in part **(a)** are the correct values for μ and σ^2. Assume also that the variance of the masses of newborn babies in country B is the same as in country A.

(b) Carry out the test at the 1% significance level. [5]

4 The number, X, of books received at a charity shop has a constant mean of 5.1 per day.

(a) State, in context, one condition for X to be modelled by a Poisson distribution. [1]

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Assume now that X can be modelled by a Poisson distribution.

(b) Find the probability that exactly 10 books are received in a 3-day period. [2]

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(c) Use a suitable approximating distribution to find the probability that more than 180 books are received in a 30-day period. [4]

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The number of DVDs received at the same shop is modelled by an independent Poisson distribution with mean 2.5 per day.

(d) Find the probability that the total number of books and DVDs that are received at the shop in 1 day is more than 3. [3]

5 (a) Two random variables X and Y have the independent distributions $N(7, 3)$ and $N(6, 2)$ respectively. A random value of each variable is taken.

Find the probability that the two values differ by more than 2. [5]

(b) Each candidate's overall score in a science test is calculated as follows. The mark for theory is denoted by T, the mark for practical is denoted by P, and the overall score is given by $T + 1.5P$. The variables T and P are assumed to be independent with distributions $N(62, 158)$ and $N(42, 108)$ respectively. You should assume that no continuity corrections are needed when using these distributions.

(i) A pass is awarded to candidates whose overall score is at least 90.

Find the proportion of candidates who pass. [5]

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(ii) Comment on the assumption that the variables T and P are independent. [1]

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6 When a child completes an online exercise called a Mathlit, they might be awarded a medal. The publishers claim that the probability that a randomly chosen child who completes a Mathlit will be awarded a medal is $\frac{1}{3}$. Asha wishes to test this claim. She decides that if she is awarded no medals while completing 10 Mathlits, she will conclude that the true probability is less than $\frac{1}{3}$.

(a) Use a binomial distribution to find the probability of a Type I error. [2]

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The true probability of being awarded a medal is denoted by p.

(b) Given that the probability of a Type II error is 0.8926, find the value of p. [3]

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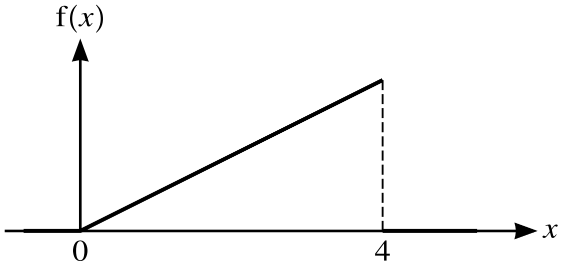
7 **(a)**



The diagram shows the graph of the probability density function, f, of a random variable X which takes values between 0 and 4 only. Between these two values the graph is a straight line.

(i) Show that $f(x) = kx$ for $0 \leqslant x \leqslant 4$, where k is a constant to be determined. [2]

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(ii) Hence, or otherwise, find $E(X)$. [3]

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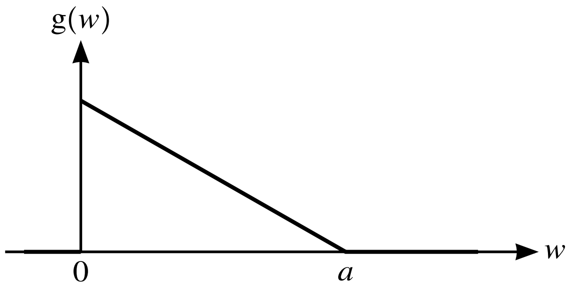
(b)



The diagram shows the graph of the probability density function, g, of a random variable W which takes values between 0 and a only, where $a > 0$. Between these two values the graph is a straight line.

Given that the median of W is 1, find the value of a. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$0.23 \pm z \times \sqrt{\frac{0.23\times(1-0.23)}{200}}$	**M1**	Expression of correct form. Any z, but $z = 0.8328$ scores B0M0.
	$z = 1.811$ or 1.812	**B1**	
	0.176 to 0.284 (3 sf)	**A1**	Must be an interval.
		3	

Question	Answer	Marks	Guidance
2(a)	E(W) = Var(W).	**B1**	Allow 'they are the same' OE. Must be = not ≈ (and not both = and ≈). Condone E(W) =λ and Var(W)= λ.
		1	
2(b)	$np \approx np(1-p)$, hence $1-p$ must be close to 1	**B1**	OE. Must see formulae and $q = 1 - p$ must be seen or implied and conclusion made.
		1	

Question	Answer	Marks	Guidance
2(c)	$\lambda = 1.4$	**B1**	Seen.
	$1 - e^{-1.4}(1 + 1.4 + \frac{1.4^2}{2})$ or $1 - e^{-1.4}(1 + 1.4 + 0.98)$ or 1-(0.2466+0.3452+0.2417)	**M1**	Allow any λ; allow one end error. Expression must be seen (accept correct sigma notation).
	= 0.167 (3 sf) or 0.166	**A1**	Use of Binomial scores SCB1 for 0.167 or 0.166 . No working: 0.167 [or 0.166] **SC B1**. Note: λ=1.4 and 0.167 with no working seen scores **SC B1B1**. Use of Normal scores B0M0.
		3	

Question	Answer	Marks	Guidance
3(a)	Est (μ) = 3.25 = 13/4 or 1625/500	**B1**	
	Est(σ^2) = $\frac{500}{499}(\frac{5663.5}{500} - "3.25"^2)$ or $\frac{1}{499}\left(5663.5 - \frac{1625^2}{500}\right)$	**M1**	Expression of correct form.
	= 0.766 (3 sf) or 1529/1996	**A1**	Biased variance of 0.7645 scores M0A0.
		3	

Question	Answer	Marks	Guidance
3(b)	H_0: Pop mean (or μ) = '3.25' H_1: Pop mean (or μ) ≠ '3.25'	**B1FT**	Not just 'mean'. FT their 3.25 .
	$\dfrac{2.95-"3.25"}{\sqrt{"0.766"\div 60}}$	**M1**	Standardising with their values. Must have √60.
	= –2.655	**A1**	Or $P(\bar{X} < 2.95) = 0.0039$ or 0.00396 or 0.00397 . **SC** FT their biased est(σ^2), i.e. 0.7645 to give $z = 2.658$ A1.
	'2.655' > 2.576 or '–2.655' < –2.576	**M1**	For valid comparison, e.g. 0.0039 or 0.00396 or 0.00397 < 0.005, or 0.0078 < 0.01, or 0.00792 < 0.01 .
	[Reject H_0] There is evidence that (mean) mass in (country B) is different (from country A).	**A1FT**	OE. Must be in context and not definite, e.g., not 'Mean mass is not different', No contradictions. Context needs either 'mass' or 'countries' OE.
			SC, Use of one-tail test. '2.655' > 2.326 or 0.0039 < 0.01 M1A0 (Max B0M1A1M1A0 3/5).
			Accept critical value method. Either: Xcrit=2.959 M1A1 2.95<2.959 M1A1FT with correct conclusion, or Xcrit=3.241 M1A1 3.25>3,241 M1A1FT with correct conclusion.
		5	

Question	Answer	Marks	Guidance
4(a)	Books received independently or singly or randomly.	**B1**	OE. Must be in context. If more than one condition given, ignore extras.
		1	

Question	Answer	Marks	Guidance
4(b)	$e^{-15.3} \times \frac{15.3^{10}}{10!}$	**M1**	Allow incorrect λ.
	= 0.0439 (3sf)	**A1**	**SC** No working shown but correct answer seen scores B1.
		2	
4(c)	N(153, 153)	**B1**	Seen or implied.
	$\frac{180.5-153}{\sqrt{153}}$ [= 2.223]	**M1**	For standardising with their values (can be implied). Allow with wrong or missing continuity correction.
	1 −ϕ('2.223')	**M1**	For correct probability area consistent with their values.
	= 0.0131 (3sf)	**A1**	
		4	
4(d)	(λ =) 5.1 + 2.5 [= 7.6]	**B1**	Give at early stage (seen or implied).
	$1 - e^{-7.6}(1+7.6+\frac{7.6^2}{2}+\frac{7.6^3}{3!}) = 1 - e^{-7.6}(1+7.6+28.88+73.16)$ = 1 – (0.0005005+0.003803+0.01445+0.03661)	**M1**	Allow incorrect λ. Allow one end error. Must see an expression (accept correct sigma notation).
	= 0.945 (3sf)	**A1**	**SC** No working, 0.945 B1(could be implied) **SC B1**.
		3	

Question	Answer	Marks	Guidance
5(a)	$E(X-Y)=1$ $Var(X-Y)=5$	B1	Seen or implied, OE e.g. $X-Y-2$.
	$\frac{2-1}{\sqrt{5}}$ [= 0.447] $\frac{-2-1}{\sqrt{5}}$ [= –1.342]	M1	Standardising with their values must come from a combination.
	$1-\Phi('0.447')$ $\Phi('-1.342')=1-\Phi('1.342)$	M1	Correct probability area consistent with their values.
	= 0.327 or 0.328 = 0.0898 or 0.0899	A1	Seen or implied.
	Probability that difference is more than 2 = 0.417 (3 sf) or 0.418	A1	
		5	
5(b)(i)	$E(X)=62+1.5(42)$ [= 125]	B1	OE.
	$Var(X)=158+1.5^2\times 108$ [= 401]	B1	Correct expression OE.
	$\frac{90-"125"}{\sqrt{"401"}}$ [= – 1.748]	M1	Correct standardisation using their $E(X)$ and $Var(X)$. Must both be from a combination attempt. Ignore any attempted continuity correction.
	Φ('1.748')	M1	Correct probability area consistent with their stated values.
	= 0.960 or 96.0% (3 sf)	A1	Allow 0.96 or 96%.
		5	
5(b)(ii)	Unlikely. A candidate who does well in Theory is likely to do well in Practical.	B1	Need both. Accept 'unlikely', 'not independent', 'dependent', 'not realistic', or similar; and accept 'both testing knowledge from the same syllabus', 'theory and practical share same content' or similar statement.
		1	

Question	Answer	Marks	Guidance
6(a)	$(1 - \frac{1}{3})^{10}$	**M1**	
	$= 0.0173$ (3 sf)	**A1**	No working scores **SC B1**.
		2	
6(b)	$1 - (1 - p)^{10} = 0.8926$	**M1**	Accept $1 - q^{10} = 0.8926$. Equation must be in p or in q but not both.
	$1 - p = 0.1074^{0.1}$ [= 0.800]	**M1**	For valid attempt to solve their (binomial) equation in p^{10} or q^{10}.
	$p = 0.200$ (3 sf) or 0.2	**A1**	
		3	

Question	Answer	Marks	Guidance
7(a)(i)	$\frac{1}{2} \times 4 \times a = 1$	**M1**	For use of area = 1 or let $f(x) = kx$ and attempt $\int_0^4 kx \, dx = 1$.
	$[a = \frac{1}{2}]$ $f(x) = \frac{1}{8}x$	**A1**	$k\left[\frac{x^2}{2}\right]_0^4 = 1;\ 8k = 1;\ k = \frac{1}{8}$. $f(x) = \frac{1}{8}x$ or $k = \frac{1}{8}$.
		2	

Question	Answer	Marks	Guidance
7(a)(ii)	$\int_0^4 x \times \frac{1}{8}x \, dx$	**M1**	Attempt to integrate $x \times$ *their* $f(x)$. Ignore limits accept in terms of k.
	$\left[\frac{x^3}{24}\right]_0^4$	**A1ft**	Their integral and correct limits accept in terms of k.
	$= \frac{8}{3}$ or 2.67 (3 sf)	**A1**	Note: Final answer of $64k/3$ scores 2/3.
		3	
7(b)	$\frac{a-1}{a} = \frac{1}{\sqrt{2}}$	**M1**	Or attempt $\int_0^1 g(w)dw = \frac{1}{2}$ i.e. $\int_0^1 (\frac{2}{a} - \frac{2}{a^2}w)dw = \frac{1}{2}$, or integral from 1 to a. $g(w)$ must be linear of form $g(w) = mw$ $(+c)$. Or area attempt: attempt to calculate heights using their linear equation ($h_1 = 2/a$ and $h_2 = -2/a^2 + 2/a$) and use in either area trapezium = 0.5, or area trapezium =area small triangle or area small triangle = 0.5 . Area trapezium $= 1/2 \times 1\ (2/a + -2/a^2 + 2/a)$ Area triangle $= 1/2(a-1)(-2/a^2 + 2/a))$ Note: alternative expression for $h_1 = (a-2)/(a-1)$.
	$a\sqrt{2} - \sqrt{2} = a$	**A1**	Or $a^2 - 4a + 2 = 0$. Any correct equation in a, a not in denominator.
	$a = 2 + \sqrt{2} = 3.41$	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER					
CANDIDATE NUMBER					

* 1 3 9 5 6 2 8 4 7 2 *

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

JC23 06_9709_63/RP

1 A random variable X has probability density function f, where

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X)$. [3]

2 A club has 264 members, numbered from 1 to 264. Donash wants to choose a random sample of members for a survey. In order to choose the members for the sample he uses his calculator to generate random digits. His first 20 random digits are as follows.

10612 11801 21473 22759

(a) The numbers of the first two members in the sample are 106 and 121.

Write down the numbers of the next two members in the sample. [2]

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(b) To obtain the numbers for members after the 4th member, Donash starts with the second random digit, 0, and obtains the numbers 061 and 211.

Explain why this method will not produce a random sample. [1]

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3 In a random sample of 100 students at Luciana's college, x students said that they liked exams. Luciana used this result to find an approximate 90% confidence interval for the proportion, p, of all students at her college who liked exams. Her confidence interval had width 0.157 92.

(a) Find the two possible values of x. [4]

Suzma independently took another random sample and found another approximate 90% confidence interval for p.

(b) Find the probability that neither of the two confidence intervals contains the true value of p. [1]

4 The mass, in tonnes, of steel produced per day at a factory is normally distributed with mean 65.2 and standard deviation 3.6. It can be assumed that the mass of steel produced each day is independent of other days. The factory makes \$50 profit on each tonne of steel produced.

Find the probability that the total profit made in a randomly chosen 7-day week is less than \$22 000. [6]

5 Last year the mean time for pizza deliveries from Pete's Pizza Pit was 32.4 minutes. This year the time, t minutes, for pizza deliveries from Pete's Pizza Pit was recorded for a random sample of 50 deliveries. The results were as follows.

$$n = 50 \qquad \Sigma t = 1700 \qquad \Sigma t^2 = 59\,050$$

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Test, at the 2% significance level, whether the mean delivery time has changed since last year. [5]

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(c) Under what circumstances would it **not** be necessary to use the Central Limit Theorem in answering **(b)**? [1]

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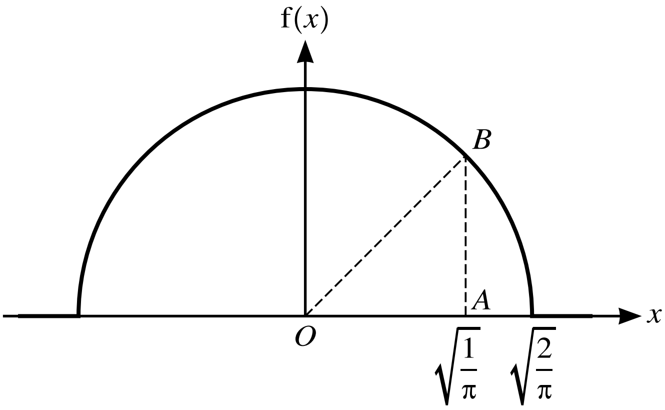
6 It is known that 1 in 5000 people in Atalia have a certain condition. A random sample of 12 500 people from Atalia is chosen for a medical trial. The number having the condition is denoted by X.

(a) Use an appropriate approximating distribution to find $P(X \leqslant 3)$. [3]

(b) Find the values of $E(X)$ and $Var(X)$, and explain how your answers suggest that the approximating distribution used in **(a)** is likely to be appropriate. [2]

7



A random variable X has probability density function f, where the graph of $y = \mathrm{f}(x)$ is a semicircle with centre (0, 0) and radius $\sqrt{\frac{2}{\pi}}$, entirely above the x-axis. Elsewhere $\mathrm{f}(x) = 0$ (see diagram).

(a) Verify that f can be a probability density function. [2]

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A and B are the points where the line $x = \sqrt{\frac{1}{\pi}}$ meets the x-axis and the semicircle respectively.

(b) Show that angle AOB is $\frac{1}{4}\pi$ radians and hence find $\mathrm{P}\left(X > \sqrt{\frac{1}{\pi}}\right)$. [6]

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8 A new light was installed on a certain footpath. A town councillor decided to use a hypothesis test to investigate whether the number of people using the path in the evening had increased.

Before the light was installed, the mean number of people using the path during any 20-minute period during the evening was 1.01.

After the light was installed, the total number, n, of people using the path during 3 randomly chosen 20-minute periods during the evening was noted.

(a) Given that the value of n was 6, use a Poisson distribution to carry out the test at the 5% significance level. [6]

(b) Later a similar test, at the 5% significance level, was carried out using another 3 randomly chosen 20-minute periods during the evening.

Find the probability of a Type I error. [2]

(c) State what is meant by a Type I error in this context. [1]

(d) State, in context, what further information would be needed in order to find the probability of a Type II error. Do not carry out any further calculation. [2]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\frac{3}{2}\int_0^1 (x - x^3)\mathrm{d}x$	**M1**	Attempt to integrate $x\mathrm{f}(x)$; ignore limits.
	$= \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$	**A1**	Correct integration and limits.
	$= \frac{3}{8}$	**A1**	
		3	

Question	Answer	Marks	Guidance
2(a)	180, 227	**B1**	One correct. Ignore incorrect numbers.
		B1	Both correct and no extra numbers seen. (Allow other correct use of list of digits).
		2	
2(b)	These numbers are not independent of the previous numbers OR Only a finite number of digits used	**B1**	Already used these numbers, so therefore not random. Does not include numbers not in the list, therefore not random (not random or biased needs a reason).
		1	

Question	Answer	Marks	Guidance
3(a)	$z = 1.645$	**B1**	
	$z \times \dfrac{\sqrt{\frac{x}{100} \times (1 - \frac{x}{100})}}{100} = 0.07896$	**M1**	OE. Equation of correct form. Accept $p = x/100$. Any z. Allow missing factor of 2.
	$[x(100 - x) = 100^3 \times 0.07896^2 \div 1.645^2]$ $x^2 - 100x + 2304 = 0$	**A1**	Any correct (likely scalar multiple) three-term quadratic equation in x or p with simplified coefficients. Accept $p^2 - p + 0.2304 = 0$ or $p(1-p) = 0.2304$.
	$x = 36$ or 64	**A1**	
		4	
3(b)	$0.1^2 = 0.01$	**B1**	Accept either.
		1	

Question	Answer	Marks	Guidance
4	**Method 1: Based on mass**		
	Mean = $7 \times 65.2 = 456.4$	**B1**	
	Var = 7×3.6^2 [= 90.72]	**M1**	
	22 000/50 = 440 used in standardising equation	**M1**	
	$\dfrac{'440' - '456.4'}{\sqrt{'90.72'}}$ [= –1.722] no mixed methods	**M1**	For standardising with their values. No mixed methods.
	ϕ(–'1.722') = 1 – ϕ('1.722')	**M1**	For correct probability area consistent with their values.
	= 0.0425 or 0.0426	**A1**	Note: accept alt method using per day. $N(65.2, \frac{3.6^2}{7})$. No mixed methods.
	Method 2: Based on profit		
	Mean = $7 \times 65.2 \times 50 = 22820$	**B1**	
	Var = 7×3.6^2	**M1**	
	Var = $50^2 \times$ '90.72' [= 226800]	**M1**	
	$\dfrac{22000 - '22820'}{\sqrt{'226800'}}$ [= –1.722] no mixed methods	**M1**	For standardising with their values. No mixed methods.
	ϕ(–'1.722') = 1 – ϕ('1.722')	**M1**	For correct probability area consistent with their values.
	= 0.0425 or 0.0426	**A1**	
		6	

Question	Answer	Marks	Guidance
5(a)	$\bar{x} = 1700/50 = 34$	**B1**	
	Est(σ^2) = $\frac{50}{49}\left(\frac{59050}{50} - 34^2\right)$ or $\frac{1}{49}\left(59050 - \frac{1700^2}{50}\right)$	**M1**	Est(σ^2) = $\frac{59050}{50} - 34^2$ biased scores M0.
	$= 25.5$ (3 sf) or $\frac{1250}{49}$	**A1**	= 25 scores A0.
		3	
5(b)	H_0: Population mean time = 32.4 H_1: Population mean time ≠ 32.4	**B1**	Not just 'mean' but allow just 'μ'.
	$\dfrac{34 - 32.4}{\frac{\sqrt{'25.5'}}{\sqrt{50}}}$	**M1**	Must have $\sqrt{50}$ and not 50. FT *their* mean and var. Can be implied.
	$= 2.24$ (3 sf)	**A1**	or $P(\overline{T} > 34) = 0.0125$. **SC** use of biased var (25) $z = 2.26$ or $p = 0.0119$, allow M1A1.
	'2.24' < 2.326	**M1**	Or 0.0125 > 0.01 for a valid comparison.
	[Not reject H_0] Insufficient evidence that (mean) time has changed	**A1FT**	In context, not definite, e.g. not 'Time not changed'. No contradictions. Note: accept CV method $x_{cri} = 34.06$ for M1A1. Compares 34 < 34.06 for M1, conclusion for A1. Condone $x = 32.34$ M1A1: compares 32.4 > 32.34 for M1, conclusion for A1.
		5	**SC** for using a one-tail method. Award max 3/5 (B0 M1 A1 M1 A0).

Question	Answer	Marks	Guidance
5(c)	Distribution of times in the population is normal	**B1**	Accept answers with no context here. Accept underlying distribution for population.
		1	

Question	Answer	Marks	Guidance
6(a)	$X \sim \text{Po}(2.5)$	**B1**	SOI.
	$e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!})$	**M1**	Any λ. Allow one end error.
	$= 0.758$ (3 sf)	**A1**	**SC** use of binomial B1 for 0.758. **SC** when no working is shown, $X \sim \text{Po}(2.5)$ seen scores B1, 0.758 seen also scores B1.
		3	
6(b)	$\text{E}(X) = \frac{5}{2}$ or 2.5 , $\text{Var}(X) = \frac{4999}{2000}$ or 2.4995	***B1**	Just an answer of 2.5 for the variance is not sufficient. However, 2.4995 is sufficient.
	These are almost equal	**DB1**	Condone 'equal'.
		2	

Question	Answer	Marks	Guidance
7(a)	$\frac{1}{2}\pi\left(\sqrt{\frac{2}{\pi}}\right)^2$	**M1**	
	= 1, which is the area under a PDF [and $f(x) \geq 0$]	**A1**	Result and statement are both needed.
		2	
7(b)	$\cos^{-1}\left(\frac{\sqrt{\frac{1}{\pi}}}{\sqrt{\frac{2}{\pi}}}\right) = \frac{\pi}{4}$	**B1**	AG. Accept alternative approaches, e.g. using Pythagoras, tangent, or isosceles right-angle triangles. Answer should be convincingly obtained and all correct.
	Area of sector = $\frac{1}{4}$	**B1**	
	Area of triangle $AOB = \frac{1}{2}OA \times OB = \frac{1}{2} \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{2}{\pi} - \frac{1}{\pi}}$ or Area of triangle $AOB = \frac{1}{2}OA \times OB \times \sin(AOB) = \frac{1}{2} \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{2}{\pi}} \sin\frac{\pi}{4}$	**M1**	Accept alternative approaches. Note: AB = $\sqrt{0.7979^2 - 0.5642^2}$ [= 0.5642] Allow values to 3sf.
	$\frac{1}{2\pi}$ or 0.1592	**A1**	
	'$\frac{1}{4}$' – '$\frac{1}{2\pi}$' or '0.25' – '0.1592'	**M1**	Attempt area of sector – area of triangle *AOB*.
	$= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf)	**A1**	

Question	Answer	Marks	Guidance
7(b)	**Alternative Method for Question Q7(b): Using integration**		
	Find equation of curve $x^2 + y^2 = \frac{2}{\pi}$	**M1**	
	$y = \sqrt{\frac{2}{\pi} - x^2}$	**A1**	
	Attempt to integrate (any limits)	**M1**	
	Use of correct limits $\sqrt{\frac{1}{\pi}}$ to $\sqrt{\frac{2}{\pi}}$	**B1**	
	Correct integration with correct limits	**A1**	
	$= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf)	**A1**	Correct final answer.
		6	

Question	Answer	Marks	Guidance
8(a)	H_0: Pop mean no. people = 3.03 or 1.01 (per 20 min) H_1: Pop mean no. people > 3.03 or 1.01 (per 20 min)	**B1**	These must not just be 'mean', but allow just 'λ' or 'μ'.
	Use of Po(3.03)	**M1**	
	$= 1 - e^{-3.03}(1 + 3.03 + \frac{3.03^2}{2} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!})$ $= 1 - e^{-3.03}(1 + 3.03 + 4.5905 + 4.6364 + 3.5120 + 2.128)$ $= 1 - (0.04832 + 0.1464 + 0.2218 + 0.2240 + 0.1697 + 0.1028)$	**M1**	Allow incorrect λ. Allow one end error. Must see Poisson expression used.
	= 0.0870 (3sf) [0.0869727]	**A1**	Allow 0.087 .
	0.0870 > 0.05	**M1**	For a valid comparison.
	(Do not reject H_0) Insufficient evidence to believe (mean) number of people has increased	**A1FT**	Conclusion stated must be in context, not definite and include no contradictions (e.g. not 'mean number people has not increased').
		6	If only $P(x = 6)$ award max 2/6 (single term not valid). **SC** No working B1 B2 M1 A1. Award maximum 5/6.

Question	Answer	Marks	Guidance
8(b)	$\text{"0.0869727"} - e^{-3.03} \times \dfrac{3.03^6}{6!}$ or $0.869727 - e^{-3.03}(1.0748)$ or $0.869727 - 0.05193$ or $1 - e^{-3.03}(1 + 3.03 + \frac{3.03^2}{2} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!} + \frac{3.03^6}{6!})$	**M1**	OE. Must see Poisson expression (may be in part (a)).
	0.0350 or 0.0351	**A1**	Accept 0.035. **SC** no working seen, award B1 for 0.0350, 0.0351 or 0.035.
		2	
8(c)	Concluding that the (mean) number of people (using the path per 20 mins in the evening) has increased when it has not	**B1**	OE. Conclusion must be in context.
		1	
8(d)	A value for the true mean	**B1**	Allow without context for this mark.
	Number of people using the path per 20 mins in the evening.	**B1**	Condone equivalent comment on three randomly chosen 20-minute periods.
		2	

Cambridge Assessment International Education

Grade thresholds – November 2023

Cambridge International AS & A Level Mathematics (9709)

Grade thresholds taken for Syllabus 9709 (Mathematics) in the November 2023 examination.

		Minimum raw mark required for grade:				
	Maximum raw mark available	A	B	C	D	E
Component 11	75	55	44	34	24	14
Component 12	75	55	46	35	24	13
Component 13	75	62	56	45	34	23
Component 21	50	39	35	27	20	13
Component 22	50	39	35	27	20	13
Component 23	50	39	35	27	20	13
Component 31	75	54	45	39	32	24
Component 32	75	54	45	39	32	24
Component 33	75	66	59	51	42	32
Component 41	50	38	34	26	19	12
Component 42	50	39	35	27	19	12
Component 43	50	45	40	33	27	21
Component 51	50	38	34	27	20	14
Component 52	50	40	36	28	21	14
Component 53	50	46	42	36	30	25
Component 58	75	66	61	53	46	39
Component 59	75	64	59	49	39	30
Component 61	50	41	36	29	23	17
Component 62	50	35	30	23	16	10
Component 63	50	41	36	29	23	17

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Grade thresholds continued

Cambridge AS & A Level Mathematics (9709)

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AX	250	11, 31, 41, 51	213	185	157	126	95	64
AY	250	12, 32, 42, 52	214	188	162	129	96	63
AZ	250	13, 33, 43, 53	234	219	197	165	133	101
CX	250	11, 31, 51, 61	217	188	159	129	99	69
CY	250	12, 32, 52, 62	211	184	157	125	93	61
CZ	250	13, 33, 53, 63	232	215	193	161	129	97
DX	250	31, 51, 84	203	178	153	124	96	68
DY	250	32, 52, 85	211	181	151	121	91	61
DZ	250	33, 53, 86	231	212	185	151	118	85
EX	250	31, 41, 87	202	177	152	122	92	63
EY	250	32, 42, 88	209	180	151	120	89	58
EZ	250	33, 43, 89	231	213	186	151	116	82
GX	250	31, 61, 87	206	180	154	125	96	68
GY	250	32, 62, 88	206	176	146	116	86	56
GZ	250	33, 63, 89	229	209	182	147	112	78
HY	250	32, 52, 95	215	189	163	130	98	66
HZ	250	33, 53, 96	222	197	172	143	115	87
IY	250	32, 42, 98	214	189	164	130	96	63
IZ	250	33, 43, 99	227	204	178	147	117	87
KX	250	31, 61, 97	220	191	161	133	105	77
KY	250	32, 62, 98	211	185	159	126	93	61
KZ	250	33, 63, 99	225	200	174	143	113	83
OV	250	32, 52, 75	219	191	163	132	102	72
P2	75	58	–	66	61	53	46	39
P3	75	59	–	64	59	49	39	30
PV	250	32, 42, 78	217	189	161	131	101	72
RV	250	32, 62, 78	214	185	156	127	98	70
S1	125	11, 21	–	94	79	61	44	27
S2	125	11, 41	–	93	78	60	43	26
S3	125	11, 51	–	93	78	61	44	28
S4	125	12, 22	–	94	81	62	44	26
S5	125	12, 42	–	94	81	62	43	25
S6	125	12, 52	–	95	82	63	45	27
S7	125	13, 23	–	101	91	72	54	36

Grade thresholds continued

Cambridge AS & A Level Mathematics (9709)

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
S8	125	13, 43	–	107	96	78	61	44
S9	125	13, 53	–	108	98	81	64	48

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5268134660

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Expand $(1+3x)^6$ in ascending powers of x up to, and including, the term in x^2. [2]

(b) Hence find the coefficient of x^2 in the expansion of $(1-7x+x^2)(1+3x)^6$. [2]

2 A line has equation $y = 2cx + 3$ and a curve has equation $y = cx^2 + 3x - c$, where c is a constant.

Showing all necessary working, determine which of the following statements is correct.

A The line and curve intersect only for a particular set of values of c.

B The line and curve intersect for all values of c.

C The line and curve do not intersect for any values of c. [4]

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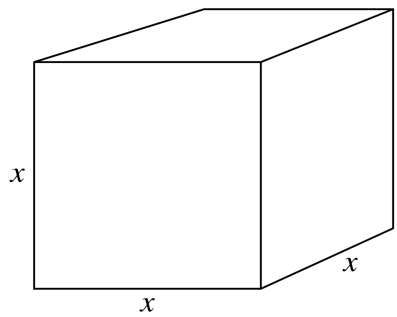

The diagram shows a cubical closed container made of a thin elastic material which is filled with water and frozen. During the freezing process the length, x cm, of each edge of the container increases at the constant rate of 0.01 cm per minute. The volume of the container at time t minutes is V cm^3.

Find the rate of increase of V when $x = 20$. [3]

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4 The transformation R denotes a reflection in the x-axis and the transformation T denotes a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

(a) Find the equation, $y = \mathrm{g}(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations R followed by T. [2]

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(b) Find the equation, $y = \mathrm{h}(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations T followed by R. [2]

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(c) State fully the transformation that maps the curve $y = \mathrm{g}(x)$ onto the curve $y = \mathrm{h}(x)$. [2]

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5 **(a)** Show that the equation

$$4\sin x + \frac{5}{\tan x} + \frac{2}{\sin x} = 0$$

may be expressed in the form $a\cos^2 x + b\cos x + c = 0$, where a, b and c are integers to be found. [3]

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(b) Hence solve the equation $4\sin x + \dfrac{5}{\tan x} + \dfrac{2}{\sin x} = 0$ for $0° \leqslant x \leqslant 360°$. [3]

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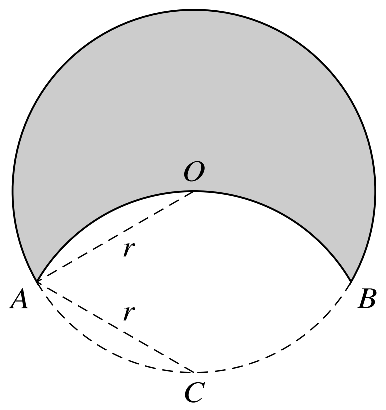

The diagram shows a motif formed by the major arc AB of a circle with radius r and centre O, and the minor arc AOB of a circle, also with radius r but with centre C. The point C lies on the circle with centre O.

(a) Given that angle $ACB = k\pi$ radians, state the value of the fraction k. [1]

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(b) State the perimeter of the shaded motif in terms of π and r. [1]

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(c) Find the area of the shaded motif, giving your answer in terms of π, r and $\sqrt{3}$. [5]

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7 The sum of the first two terms of a geometric progression is 15 and the sum to infinity is $\frac{125}{7}$. The common ratio of the progression is negative.

Find the third term of the progression. [7]

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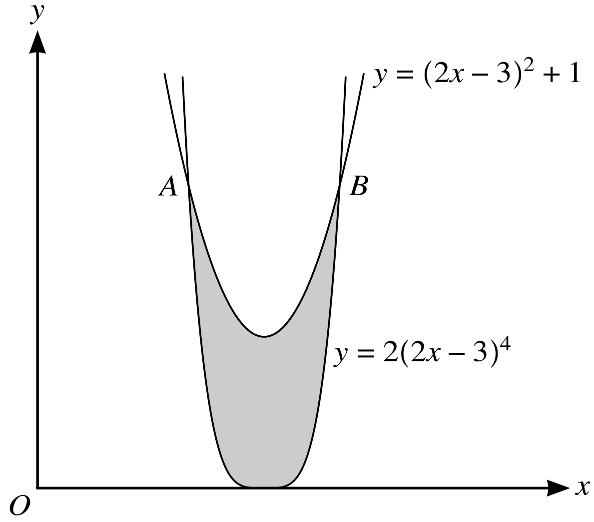

The diagram shows the curves with equations $y = 2(2x - 3)^4$ and $y = (2x - 3)^2 + 1$ meeting at points A and B.

(a) By using the substitution $u = 2x - 3$ find, by calculation, the coordinates of A and B. [4]

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(b) Find the exact area of the shaded region. [5]

9 **(a)** Express $4x^2 - 12x + 13$ in the form $(2x + a)^2 + b$, where a and b are constants. [2]

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The function f is defined by $f(x) = 4x^2 - 12x + 13$ for $p < x < q$, where p and q are constants. The function g is defined by $g(x) = 3x + 1$ for $x < 8$.

(b) Given that it is possible to form the composite function gf, find the least possible value of p and the greatest possible value of q. [3]

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(c) Find an expression for $\mathrm{gf}(x)$. [1]

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The function h is defined by $\mathrm{h}(x) = 4x^2 - 12x + 13$ for $x < 0$.

(d) Find an expression for $\mathrm{h}^{-1}(x)$. [3]

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10 A curve has a stationary point at $(2, -10)$ and is such that $\frac{d^2y}{dx^2} = 6x$.

(a) Find $\frac{dy}{dx}$. [3]

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(b) Find the equation of the curve. [3]

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(c) Find the coordinates of the other stationary point and determine its nature. [3]

(d) Find the equation of the tangent to the curve at the point where the curve crosses the y-axis. [2]

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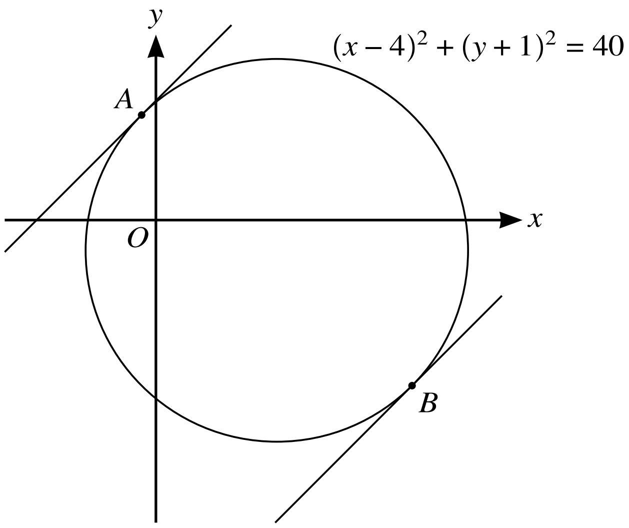

The diagram shows the circle with equation $(x-4)^2 + (y+1)^2 = 40$. Parallel tangents, each with gradient 1, touch the circle at points A and B.

(a) Find the equation of the line AB, giving the answer in the form $y = mx + c$. [3]

(b) Find the coordinates of A, giving each coordinate in surd form. [4]

(c) Find the equation of the tangent at A, giving the answer in the form $y = mx + c$, where c is in surd form. [2]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$1+18x+135x^2$	**B2, 1, 0**	Accept $1,\ 18x,\ 135x^2$ listed horizontally or vertically or $1x^0+18x+135x^2$.
		2	
1(b)	Coefficient of x^2 is $135-7\times18+1=10$	**M1 A1**	3 products, allow $10x^2$. If full expansion given, like terms must be collected for M1.
		2	

Question	Answer	Marks	Guidance
2	$cx^2+3x-c=2cx+3$ leading to $cx^2+(3-2c)x-(c+3)\ \ [=0]$	**M1**	Forming a 3-term quadratic, all terms on one side.
	$b^2-4ac=(3-2c)^2+4c(c+3)$	**M1**	2nd M1 for b^2-4ac correct for *their* $\underline{a}$, b, c i.e. no sign errors.
	$=8c^2+9$	**A1**	
	> 0 [for all values of c] leading to B [Intersects for all values of c]	**A1**	WWW
		4	

Question	Answer	Marks	Guidance
3	$\frac{dV}{dx} = 3x^2$	**B1**	SOI
	$\frac{dV}{dt}\left[= \frac{dV}{dx} \times \frac{dx}{dt}\right] = 3 \times 20^2 \times 0.01$	**M1**	Correct use of chain rule with $x = 20$ substituted into $\frac{dV}{dx}$.
	12	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	$\{-(x-3)^2\}\ \{-1\}$	**B1 B1**	OE. Must be a quadratic e.g. $3x - 1$ B0 B0. **SC B1** for correct use of generalised function notation.
		2	
4(b)	$\{-(x-3)^2\}\ \{+1\}$	**B1 B1**	OE. Must be a quadratic. **SC B1** for correct use of generalised function notation.
		2	
4(c)	$\{\text{Translation}\}\ \begin{pmatrix}\{0\}\\\{2\}\end{pmatrix}$	**B2, 1, 0**	FT from **(a)** and **(b)** if a translation parallel to the y axis. B2 for fully correct, B1 with two elements correct. {} indicates different elements.
		2	

Question	Answer	Marks	Guidance
5(a)	$4\sin^2 x + 5\cos x + 2 \quad [=0]$	*M1	Multiply by $\sin x$ (or writing as a single fraction) and using $\tan x = \frac{\sin x}{\cos x}$.
	$4(1-\cos^2 x) + 5\cos x + 2 \quad [=0]$	DM1	Correctly obtaining a quadratic in $\cos x$ (allow sign errors).
	$4\cos^2 x - 5\cos x - 6 = 0$	A1	Condone missing x. Must be = 0 unless 0 appears on RHS earlier.
		3	
5(b)	$(4\cos x + 3)(\cos x - 2) \ [=0]$	M1	Or use of formula or completing square.
	138.6º, 221.4º	A1 B1 FT	FT on 360º – 1st solution from quadratic in $\cos x$. Use of radians (2.42) A0 but allow B1 FT for 2π: 1st solution if use of radians is clear. **SC** If M0 scored **SC B1 B1** for correct final answer(s). If extra incorrect solutions in the range 0 → 360º are given award A1 B0.
		3	

Question	Answer	Marks	Guidance
6(a)	$k = \frac{2}{3}$	B1	Allow ACB = $\frac{2\pi}{3}$.
		1	
6(b)	Perimeter of shaded area $= 2\pi r$	B1	
		1	

Question	Answer	Marks	Guidance
6(c)	Major sector $OAB = \frac{1}{2}r^2 \times \frac{4\pi}{3}$	*M1	Expect $\frac{2}{3}\pi r^2$. Finds area of any relevant sector or triangle. Can be embedded in segment formula.
	One or both segments $=[2]\times\left(\frac{1}{2}r^2 \times \frac{\pi}{3} - \frac{1}{2}r^2 \sin\frac{\pi}{3}\right)$	*M1	
	$= [2]\left(r^2\frac{\pi}{6} - r^2\frac{\sqrt{3}}{4}\right)$	A1	
	Shaded area $=\frac{2}{3}\pi r^2 - 2\left(\frac{1}{6}\pi r^2 - \frac{r^2\sqrt{3}}{4}\right)$	DM1	
	$= \frac{\pi r^2}{3} + \frac{r^2\sqrt{3}}{2}$	A1	

Question	Answer	Marks	Guidance
6(c)	**Alternative method for Question 6(c)**		
	Sector $CAOB = [2] \times \frac{1}{2} r^2 their \frac{1}{3}\pi$	***M1**	Expect [2] × $\frac{1}{6}\pi r^2$. Can be embedded in segment formula.
	One or both segments $= [2] \times \left(\frac{1}{2} r^2 \times \frac{\pi}{3} - \frac{1}{2} r^2 \sin\frac{\pi}{3}\right)$	***M1**	
	$= [2]\left(r^2 \frac{\pi}{6} - r^2 \frac{\sqrt{3}}{4}\right)$	**A1**	
	Shaded area = $\pi r^2 - \left\{\frac{1}{3}\pi r^2 + 2\left(r^2 \frac{\pi}{6} - r^2 \frac{\sqrt{3}}{4}\right)\right\}$	**DM1**	
	$= \frac{\pi r^2}{3} + \frac{r^2\sqrt{3}}{2}$	**A1**	

Question	Answer	Marks	Guidance
6(c)	**Alternative method for Question 6(c)**		
	Area of rhombus AOBC $=[2]\times\frac{1}{2}r^2\sin\frac{\pi}{3}$	**M1**	Expect [2] $\times$ $\frac{\sqrt{3}}{4}$. Can be embedded in segment formula.
	One or both segments $=[2]\times\left(\frac{1}{2}r^2\times\frac{\pi}{3}-\frac{1}{2}r^2\sin\frac{\pi}{3}\right)$	**M1**	
	$=[2]\left(r^2\frac{\pi}{6}-r^2\frac{\sqrt{3}}{4}\right)$	**A1**	
	Shaded area = $\pi r^2-\left\{\frac{\sqrt{3}}{2}r^2-4\left(r^2\frac{\pi}{6}-r^2\frac{\sqrt{3}}{4}\right)\right\}$	**DM1**	
	$=\frac{\pi r^2}{3}+\frac{r^2\sqrt{3}}{2}$	**A1**	
		5	

Question	Answer	Marks	Guidance
7	$a(1+r)=15$	**B1**	Accept $\frac{a(1-r^2)}{1-r}$ = 15 for first B1.
	$\frac{a}{1-r}=\frac{125}{7}$	**B1**	
	$\frac{125}{7}(1-r)(1+r)=15$	**M1**	Eliminate *a*.
	$1-r^2=\frac{105}{125}$	**M1**	
	$r^2=\frac{4}{25}$ leading to $r=-\frac{2}{5}$	**A1**	Condone $\frac{2}{5}$ or $\pm\frac{2}{5}$.
	$a=\frac{125}{7}\times\frac{7}{5}=25$	**A1**	Ignore 2nd answer.
	3rd term $=25\times\frac{4}{25}=4$	**A1**	CAO

Question	Answer	Marks	Guidance
7	**Alternative method for Question 7**		
	$a(1+r)=15$	**B1**	
	$\frac{a}{1-r}=\frac{125}{7}$	**B1**	
	$7(15-15r) = (125 - 125r)(1 - r^2)$	**M1**	
	$125r^3 - 125r^2 - 20r + 20 = 0$	**M1**	
	$r = \frac{-2}{5}$ $[1, \frac{2}{5}]$	**A1**	Condone extra 'answer' of r = 1.
	a = 25	**A1**	Ignore 2nd answer.
	3rd term = 4	**A1**	CAO

Question	Answer	Marks	Guidance
7	**Alternative method for Question 7**		
	$a(1+r)=15$	**B1**	
	$\frac{a}{1-r}=\frac{125}{7}$	**B1**	
	$\frac{a}{1-\left(\frac{15}{a}-1\right)}=\frac{125}{7}$	**M1**	Eliminate *r*.
	$7a^2 - 250a + 1875$ [= 0]	**M1**	
	a = 25, $\left[\frac{75}{7}\right]$	**A1**	Condone extra 'answer' of $r = \left[\frac{75}{7}\right]$.
	r = $\frac{-2}{5}$	**A1**	Ignore 2nd answer.
	3rd term $=25\times\frac{4}{25}=4$	**A1**	CAO
		7	

Question	Answer	Marks	Guidance
8(a)	$u = 2x - 3$ leading to $2u^4 = u^2 + 1$ leading to $2u^4 - u^2 - 1[=0]$	**B1**	
	$(2u^2 + 1)(u^2 - 1)[=0]$	**M1**	Factors or formula or completing square must be shown.
	$u = \pm 1$ leading to $2x - 3 = \pm 1$ leading to x = 1 or 2	**A1**	
	(1, 2), (2, 2)	**A1**	**Special case:** If B1 M0 scored then **SC B2** can be awarded for correct coordinates or **SC B1** for correct x values only.
			Special case $2(2x - 3)^4 = (2x - 3)^2 + 1$ $32x^4 - 192x^3 + 428x^2 - 420x + 152 = 0$ x = 1, 2 finding both from a correct quartic **SC B1** (1, 2), (2, 2) **SC DB1** **Special case:** Trial and improvement without quartic. Both x values correct B1, both coordinates correct B2.
		4	

Question	Answer	Marks	Guidance
8(b)	$\left\{\dfrac{(2x-3)^3}{3\times 2}+x\right\}[-]\left\{\dfrac{2(2x-3)^5}{5\times 2}\right\}$	**B1 B1**	Integrate the 2 functions.
	$\left(\dfrac{1}{6}+2\right)-\left(-\dfrac{1}{6}+1\right)-\left\{\dfrac{1}{5}-\left(-\dfrac{1}{5}\right)\right\}$	**M1**	Apply *their* limits 1 → 2 (must be shown) to an integral. Some evidence of substitution. Minimum $(\frac{13}{6}-\frac{5}{6})-(\frac{1}{5}+\frac{1}{5})$ or equivalent. Allow 1 sign error for 1st M1.
	$\dfrac{4}{3}-\dfrac{2}{5}$	**M1**	Subtract (at some point) the 2 areas. Must subtract **areas** and not just integrals.
	$\dfrac{14}{15}$	**A1**	**Special case:** If M0 for substitution of limits can award **SC B1** for correct answer. Condone $-\dfrac{14}{15}$ if corrected.
			If subtraction is the wrong way round award B1 B1 M1 M1 A0. $\int y^2\,dx$ or $\int x\,dy$ scores 0 /5. $\pi\int y\,dx$ used. Award B1 B1 M1 M1 A0.

Question	Answer	Marks	Guidance
8(b)	**Alternative method for Question 8(b)**		
	u = 2x – 3 $\int\left(u^2+1-2u^4\right)\mathrm{d}u$ $\left\{\frac{1}{2}\right\}\left(\left\{\frac{1}{3}u^3+u\right\}-\left\{\frac{2}{5}u^5\right\}\right)$	**B2,1,0**	
	$\frac{1}{2}\left(\left(\frac{1}{3}+1-\frac{2}{5}\right)-\left(\frac{-1}{3}-1+\frac{2}{5}\right)\right)$	**M1**	Applies limits –1 → 1.
		M1	Subtract (at some point) the 2 areas.
	$\frac{1}{2}\left(\frac{14}{15}+\frac{14}{15}\right)$ $\frac{14}{15}$	**A1**	
		5	

Question	Answer	Marks	Guidance
9(a)	$\left(2x-3\right)^2+4$	**B1 B1**	Or a = –3, b = 4.
		2	

Question	Answer	Marks	Guidance
9(b)	$their\left(2x-3\right)^2+4[<]8$ OR $4x^2-12x+13[<]8$	*M1	Linking quadratic with 8.
	$\left(2x-3\right)^2<4$ leading to $-2<2x-3<2$ OR $4x^2-12x+5<0$ leading to $\left(2x-1\right)\left(2x-5\right)<0$	DM1	Simplify to 3-term quadratic and solve. Condone no method shown.
	$\frac{1}{2}<x<2\frac{1}{2}$ leading to [LEAST] $p=\frac{1}{2}$, [GREATEST] $q=2\frac{1}{2}$	A1	
		3	
9(c)	$\mathrm{gf}(x)=12x^2-36x+40$	B1	OE $\mathrm{gf}(x)=3\left(2x-3\right)^2+13$.
		1	
9(d)	$y=\left(2x-3\right)^2+4$ leading to $\left(2x-3\right)^2=y-4$ leading to $2x-3=[\pm]\sqrt{y-4}$	*M1	
	$2x=3[\pm]\sqrt{y-4}$ leading to $x=\frac{3}{2}[\pm]\frac{\sqrt{y-4}}{2}$	DM1	
	$\mathrm{h}^{-1}(x)=\frac{3}{2}-\frac{\sqrt{x-4}}{2}$	A1	
		3	

Question	Answer	Marks	Guidance
10(a)	$\frac{dy}{dx} = 3x^2 \; [+c]$	**B1**	
	$3 \times 2^2 + c = 0$	**M1**	Substitute $x = 2$ and $\frac{dy}{dx} = 0$ into an integral (c must be present).
	$\frac{dy}{dx} = 3x^2 \; -12$	**A1**	
		3	
10(b)	$y = x^3 - 12x \; [+k]$	**B1 FT**	FT on *their* non-zero c (dependent on c being found at some stage).
	$-10 = 2^3 - 12 \times 2 + k$	**M1**	Substitute $x = 2$, $y = -10$ (k present).
	$y = x^3 - 12x + 6$	**A1**	Must be $y =$ (unless $y = x^3 - 12x + k$ stated earlier).
		3	

Question	Answer	Marks	Guidance
10(c)	$3x^2 - 12 = 0$ [leading to $x = -2$]	**M1**	Set *their* two term $\frac{dy}{dx} = 0$. Expect $x = -2$. Ignore x = 2 given in addition.
	$y = (-2)^3 - 12 \times (-2) + 6 = 22$ leading to (–2, 22)	**A1**	
	When $x = -2$, $\frac{d^2y}{dx^2} < 0$ (or –12) hence Maximum	**A1**	Can be from correct conclusion from $\frac{dy}{dx}$ **sign** diagram if $\frac{dy}{dx}$ calculated correctly. Do not allow concave downward for final A1. Can be awarded if the only error is incorrect or missing *y*-coordinate.
		3	
10(d)	At $x = 0$, $\frac{dy}{dx} = -12$, $y = 6$	**M1**	Both required. FT on *their* $\frac{dy}{dx}$ and *y*.
	$y - 6 = -12x$	**A1**	OE
		2	

Question	Answer	Marks	Guidance
11(a)	Gradient of *AB* = –1	**B1**	SOI
	Centre of circle = (4, –1)	**B1**	SOI
	Equation of *AB* is $y + 1 = -1(x - 4)$ leading to $y = -x + 3$	**B1 FT**	FT *their* centre with gradient –1.
		3	

Question	Answer	Marks	Guidance
11(b)	$(x-4)^2+(-x+3+1)^2=40$	***M1**	Substitute *their AB* into circle equation.
	$2(x-4)^2=40$ OR [2] (x^2-8x-4) leading to $\frac{8\pm\sqrt{64+16}}{2}$ or $\frac{16\pm\sqrt{256+64}}{4}$	**DM1**	Forming and solving 3-term quadratic.
	$x=4[\pm]\sqrt{20}$	**A1**	OE. No fractions.
	$\left(4-\sqrt{20},-1+\sqrt{20}\right)$	**A1**	OE **Special case:** If M1 M0 scored then **SCB2** can be awarded for correct coordinates or **SCB1** for correct x values only. Ignore other coordinate
		4	
11(c)	$y-their\left(-1+\sqrt{20}\right)=1\left\{x-their\left(4-\sqrt{20}\right)\right\}$	**M1**	OE
	$y=x-5+2\sqrt{20}$ or $y=x-5+\sqrt{80}$ or $y=x-5+4\sqrt{5}$	**A1**	
		2	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

* 0 2 9 6 9 1 0 4 1 5 *

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

JC23 11_9709_12/3R

1 The coefficient of x^3 in the expansion of $(3 + 2ax)^5$ is six times the coefficient of x^2 in the expansion of $(2 + ax)^6$.

Find the value of the constant a. [4]

2 Find the exact solution of the equation

$$\tfrac{1}{6}\pi + \tan^{-1}(4x) = -\cos^{-1}\left(\tfrac{1}{2}\sqrt{3}\right).$$ [2]

3 The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{2}x + \frac{72}{x^4}$. The curve passes through the point $P(2, 8)$.

(a) Find the equation of the normal to the curve at P. [2]

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(b) Find the equation of the curve. [4]

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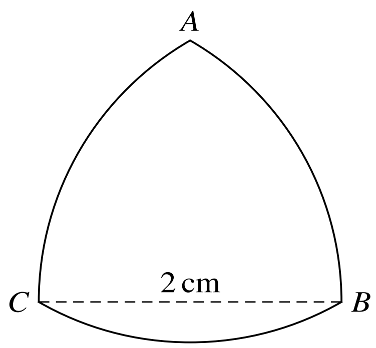

The diagram shows the shape of a coin. The three arcs AB, BC and CA are parts of circles with centres C, A and B respectively. ABC is an equilateral triangle with sides of length 2 cm.

(a) Find the perimeter of the coin. [2]

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(b) Find the area of the face ABC of the coin, giving the answer in terms of π and $\sqrt{3}$. [4]

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5 The first, second and third terms of a geometric progression are $\sin\theta$, $\cos\theta$ and $2 - \sin\theta$ respectively, where θ radians is an acute angle.

(a) Find the value of θ. [3]

(b) Using this value of θ, find the sum of the first 10 terms of the progression. Give the answer in the form $\dfrac{b}{\sqrt{c}-1}$, where b and c are integers to be found. [3]

6 The equation of a curve is $y = x^2 - 8x + 5$.

(a) Find the coordinates of the minimum point of the curve. [2]

The curve is stretched by a factor of 2 parallel to the y-axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

(b) Find the coordinates of the minimum point of the transformed curve. [2]

(c) Find the equation of the transformed curve. Give the answer in the form $y = ax^2 + bx + c$, where a, b and c are integers to be found. [4]

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7 **(a)** Verify the identity $(2x - 1)(4x^2 + 2x - 1) \equiv 8x^3 - 4x + 1$. [1]

(b) Prove the identity $\dfrac{\tan^2\theta + 1}{\tan^2\theta - 1} \equiv \dfrac{1}{1 - 2\cos^2\theta}$. [3]

(c) Using the results of **(a)** and **(b)**, solve the equation

$$\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = 4\cos\theta,$$

for $0° \leqslant \theta \leqslant 180°$. [5]

8 Functions f and g are defined by

$$\mathrm{f}(x) = (x + a)^2 - a \text{ for } x \leqslant -a,$$
$$\mathrm{g}(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

(a) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

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(b) **(i)** State the domain of the function f^{-1}. [1]

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(ii) State the range of the function f^{-1}. [1]

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(c) Given that $a = \frac{7}{2}$, solve the equation $gf(x) = 0$. [3]

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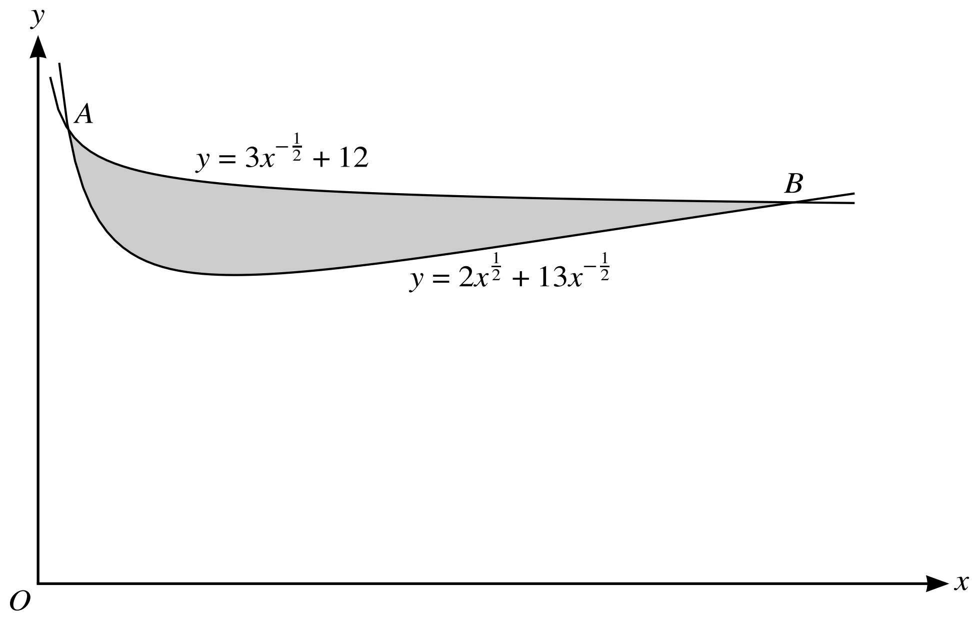

The diagram shows curves with equations $y = 2x^{\frac{1}{2}} + 13x^{-\frac{1}{2}}$ and $y = 3x^{-\frac{1}{2}} + 12$. The curves intersect at points A and B.

(a) Find the coordinates of A and B. [4]

(b) Hence find the area of the shaded region. [5]

10 The equation of a curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (4x-3)^{\frac{5}{3}} - \frac{20}{3}x$.

(a) Find the x-coordinates of the stationary points of the curve and determine their nature. [6]

(b) State the set of values for which the function f is increasing. [1]

11 The coordinates of points A, B and C are $(6,\ 4)$, $(p,\ 7)$ and $(14,\ 18)$ respectively, where p is a constant. The line AB is perpendicular to the line BC.

(a) Given that $p < 10$, find the value of p. [4]

A circle passes through the points A, B and C.

(b) Find the equation of the circle. [3]

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(c) Find the equation of the tangent to the circle at C, giving the answer in the form $dx + ey + f = 0$, where d, e and f are integers. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **23** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2. Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5. Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	[Coefficient of x^3 from $(3+2ax)^5 =$] $10\times9\times8a^3$ $\left[=720a^3\right]$	**B1**	May be seen in an expansion or with x^3.
	[Coefficient of x^2 from $(2+ax)^6 =$] $15\times16\times a^2$ $\left[=240a^2\right]$	**B1**	May be seen in an expansion or with x^2.
	$their\left(10\times9\times8a^3\right)=6\times their\left(15\times16\times a^2\right)$ $\left[\Rightarrow 720a^3=1440a^2\right]$	**M1**	OE Equating *their* coefficient of x^3 and 6 × *their* coefficient of x^2.
	$\boldsymbol{a}$ **= 2**	**A1**	Condone extra solution a = 0.
		4	

Question	Answer	Marks	Guidance
2	[$\tan^{-1}4x=$]$\left(their-\frac{\pi}{6}\right)\pm\frac{\pi}{6}$ $\left[\tan^{-1}4x=\pm\frac{\pi}{3},\pm1.047\text{ or }0\right]$	**M1**	OE Evaluating $\left(-\cos^{-1}\frac{\sqrt{3}}{2}\right)$ in rad and adding or subtracting $\frac{\pi}{6}$. Allow working with **both** angles in degrees.
	$\left[4x=-\sqrt{3},\ x=\right]-\frac{\sqrt{3}}{4}$	**A1**	**Note:** answer of $-0.43\text{ or }\frac{\sqrt{3}}{4}$ implies M1
		2	

Question	Answer	Marks	Guidance
3(a)	[Gradient of normal =] $\dfrac{-1}{\text{Their}\,\frac{11}{2}}$ $\left[\dfrac{-1}{\frac{11}{2}}=-\dfrac{2}{11}\right]$	**M1**	Tangent gradient must come from x = 2 substituted into the given expression.
	$\dfrac{y-8}{x-2}=-\dfrac{2}{11}$ or $11y+2x=92$ or $y=-\dfrac{2x}{11}+\dfrac{92}{11}$	**A1**	OE
		2	
3(b)	$[y=]\left\{\dfrac{1}{2}x^2\div 2\right\}\left\{+\dfrac{72}{x^3}\div -3\right\}$ $[+c]$ $\left[\dfrac{x^2}{4}-\dfrac{24}{x^3}+c\right]$	**B1, B1**	One mark for each correct unsimplified { }.
	$8=\dfrac{1}{4}\times 4-\dfrac{24}{8}+c$	**M1**	Substitution of x = 2, y = 8 into *their* integrated expression, defined by at least one correct power. Two terms and + c needed.
	$y=\left(\dfrac{1}{4}\text{ or }0.25\right)x^2-\dfrac{24}{x^3}+10$	**A1**	Both coefficients must be simplified but allow x^{-3}. Condone c = 10 as line as long as either y or f(x) = is seen elsewhere.
		4	

Question	Answer	Marks	Guidance
4(a)	[Arc length =] $2\times\dfrac{\pi}{3}$ or $\dfrac{60}{360}\times 2\pi\times 2$	**B1**	Finding one correct arc length – may be implied by correct final answer.
	[Perimeter =] 2π or 6.28	**B1**	AWRT
		2	

Question	Answer	Marks	Guidance
4(b)	[Area of one sector =] $\frac{1}{2}\times 2^2\times\frac{\pi}{3}$ or $\frac{60}{360}\times\pi\times 2^2\left[=\frac{2\pi}{3}\text{ or }2.09\right]$	**B1**	SOI AWRT
	[Area of triangle =]$\frac{1}{2}\times 2^2\times\sin\left(\frac{\pi}{3}\right)$ or other valid method $\left[=\sqrt{3}\text{ or }1.73\right]$	**B1**	AWRT Allow use of 60°
	$\left[\text{Area of coin} = 3\text{ segments} + \text{triangle} \Rightarrow\right]$ $3\left(\frac{2\pi}{3}-\sqrt{3}\right)+\sqrt{3}$ [= 2.82]	**M1**	OE Or 3 sectors – 2 triangles $\left(3\times\frac{2\pi}{3}-2\times\sqrt{3}\right)$ or Sector + 2 segments $\left(\frac{2\pi}{3}+2\left(\frac{2\pi}{3}-\sqrt{3}\right)\right)$
	$2\pi-2\sqrt{3}$ or $2(\pi-\sqrt{3})$	**A1**	Must be one of these simplified versions but equivalent decimal answers can score B1B1M1
		4	

Question	Answer	Marks	Guidance
5(a)	$\frac{\cos\theta}{\sin\theta}=\frac{2-\sin\theta}{\cos\theta}$ leading to $\cos^2\theta[\sin\theta]=\sin\theta(2-\sin\theta)[\sin\theta]$	***M1**	OE. Forming a correct equation in θ only using the terms of the GP and an attempt to clear fractions.
	$\cos^2\theta+\sin^2\theta=2\sin\theta$ leading to $\sin\theta=\left[\frac{1}{2}\right]$	**DM1**	Correct use of $\cos^2\theta+\sin^2\theta=1$ and attempt to solve for $\sin\theta$.
	$[\theta =]\frac{\pi}{6}$ or 0.524	**A1**	AWRT A0 for $\theta=30°$. Condone inclusion of $\frac{5\pi}{6}$ and/or
		3	

Question	Answer	Marks	Guidance
5(b)	$a=\frac{1}{2}\quad r=\sqrt{3}$	**B1**	OE SOI Trigonometric values need to have been evaluated but allow decimal equivalents (0.5 and 1.73 AWRT)
	$S_{10}=\sin\left(their\,\frac{\pi}{6}\right)\left(\frac{1-\left(their\sqrt{3}\right)^{10}}{1-\left(their\sqrt{3}\right)}\right)$	**M1**	Use of a correct formula for S_{10}, with *their* value of θ. Their $\sqrt{3}$ needs to come from $\frac{\cos(their\theta)}{\sin(their\theta)}$ or $\frac{2-sin(their\theta)}{cos(their\theta)}$ OE
	$[S_{10}=]\frac{121}{\sqrt{3}-1}$	**A1**	$\frac{-121}{1-\sqrt{3}}$ or 165 AWRT scores B1M1A0**.**
		3	

Question	Answer	Marks	Guidance
6(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 - 8x + 5\right) = 0$ $\left[2x - 8 = 0\right]$	**M1**	Correct differentiation of x^2 and equating their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to 0.
	Alternative method 1 for first mark of Question 6(a)		
	$y = \left(x-4\right)^2 - 11$	**M1**	Attempt to complete the square as far as $y = \left(x-4\right)^2 \pm k$.
	Alternative method 2 for first mark of Question 6(a)		
	$x = \frac{-b}{2a} = \frac{\pm 8}{2}$	**M1**	
	$x = 4, y = -11$	**A1**	Answers from $x = \frac{8 \pm \sqrt{64-20}}{2}$ leading to $x = 4 \pm \sqrt{11}$ scores M0A0
		2	
6(b)	$x = \left(\textit{their } x \text{ value from } a\right) + 4$ [=8]	**B1 FT**	Can be from finding the equation of the transformed curve, differentiating and putting $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.
	$y = \left\{\left(\textit{their } y \text{ value from } a\right) \times 2\right\} + 1$ [– 21]	**B1 FT**	Can be from putting x = 8 in the equation of the transformed curve.
		2	If B0B0 scored, **SC B1** for sight of $\left(4, -22\right)$.

Question	Answer	Marks	Guidance
6(c)	$2\left(x^2-8x+5\right)$ or $2\{\left(x-4\right)^2-11\}$	B1	Can be implied if both transformations done together: $2\left(\left(x-4\right)^2-8\left(x-4\right)+5\right)+1$ OE.
	$\left(\left(x-4\right)^2-8\left(x-4\right)+5\right)+1$ or $\left\{\left(x-4-4\right)^2-their11\right\}+1$	M1	For the x translation, each x becomes $\left(x-4\right)$.
		M1	For the y translation of +1.
	$y=2x^2-32x+107$ or $a=2, b=-32, c=107$	A1	Evidence to support *their* answer may be in **(b)** but answer must be seen in **(c).**
		4	

Question	Answer	Marks	Guidance
7(a)	$\left(2x-1\right)\left(4x^2+2x-1\right)=8x^3+4x^2-2x-4x^2-2x+1=8x^3-4x+1$	B1	AG Six correct terms leading to the correct answer.
		1	

Question	Answer	Marks	Guidance
7(b)	Starting with the LHS $\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}$ $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	For use of $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ in the numerator and denominator.
	$=\dfrac{1}{1-\cos^2\theta-\cos^2\theta}$ need to see clear evidence of this step	DM1	For use of $\sin^2\theta+\cos^2\theta=1$ twice, in a correct expression, resulting in an expression in $\cos^2\theta$.
	$=\dfrac{1}{1-2\cos^2\theta}$	A1	AG
	Alternative method 1 for Question 7(b)		
	Starting with the RHS $\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta+\cos^2\theta-2\cos^2\theta}$ $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	For use of $\sin^2\theta+\cos^2\theta=1$ twice.
	$=\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}$ need to see clear evidence of this step	DM1	Dividing throughout by $\cos^2\theta$.
	$=\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$	A1	AG

Question	Answer	Marks	Guidance
7(b)	**Alternative method 2 for Question 7(b)**		
	Starting with the LHS $\dfrac{\sec^2\theta}{\sec^2\theta - 2}$	***M1**	For use of 1+ $\tan^2\theta = \sec^2\theta$ twice.
	Clear statement $\Rightarrow \dfrac{1}{1-2\cos^2\theta}$	**DM1**	AG For multiplying throughout by $\cos^2\theta$ to give the RHS.
		A1	
		3	

Question	Answer	Marks	Guidance
7(c)	$\dfrac{1}{1-2\cos^2\theta}=4\cos\theta$ leading to $1=\ 4\cos\theta\left(1-2\cos^2\theta\right)$ $\left[8\cos^3\theta-4\cos\theta+1=0\right]$	**B1**	Replace LHS with RHS from **(b)** and clear fractions.
	$\left(2\cos\theta-1\right)\left(4\cos^2\theta+2\cos\theta-1\right)\left[=0\right]$	***B1**	Use of the expression from **(a)** with $x=\cos\theta$.
	$\left[x\text{ or }\cos\theta=\right]\dfrac{1}{2}\text{ and }\dfrac{-2\pm\sqrt{4+16}}{8}$ OR $0.31, -0.81$ AWRT	**DB1**	OE For all three values.
	$\left[\theta=\right]60°, 72°, 144°$	**B2,1,0**	B2 for three correct answers only, B1 for two correct answers and no others (but allow $36°$ instead of $144°$) in the given range or 3 correct answers plus other values in the given range. Ignore answers outside of the given range. Accept AWRT 72.0, 144.0 . **SC B1** for all 3 correct answers in radians and no others: $\dfrac{\pi}{3}, \dfrac{2\pi}{5}\text{ and }\dfrac{4\pi}{5}$.
		5	

Question	Answer	Marks	Guidance
8(a)	$y=(x+a)^2-a$ leading to $(x+a)^2=y\pm a$	*M1	x and y may be interchanged initially. Allow ± errors for these method marks.
	$x=[\pm]\sqrt{y\pm a}\pm a$	DM1	
	Alternative method for first 2 marks of Question 8(a)		
	$x=(y+a)^2-a$ leading to $y^2+2ay+a^2-a-x[=0]$	*M1	Allow ± errors for this method mark.
	$y=\dfrac{-2a\pm\sqrt{4a^2-4(a^2-a-x)}}{2}$	DM1	
	[y or $f^{-1}(x)$ =]$-\sqrt{x+a}-a$	A1	OE Must choose negative root.
		3	
8(b)(i)	$x\geqslant -a$	B1	Ignore infinity limit if included.
		1	
8(b)(ii)	y or $f^{-1}[(x)]\leqslant -a$	B1	Ignore negative infinity limit if included.
		1	

Question	Answer	Marks	Guidance
8(c)	$\left[\mathrm{gf}\left(\frac{7}{2}\right)=\right]2\left(\left(x+\frac{7}{2}\right)^2-\frac{7}{2}\right)-1$ or $2x^2+4\left(\frac{7}{2}\right)x+2\left(\frac{7}{2}\right)^2-2\left(\frac{7}{2}\right)-1[=0]$	**B1**	OE Alternatively, $\left[\mathrm{gf}(x)=0\Rightarrow\right]\mathrm{f}(x)=\frac{1}{2}$.
	$[x=]-\frac{7}{2}\pm 2$ or $\frac{-14\pm\sqrt{14^2-4\times 2\times\frac{33}{2}}}{4}$ $\left[\frac{-14\pm\sqrt{64}}{4}\right]$ or factorising	**M1**	OE Solving their three term quadratic equation as far as two solutions or correctly selecting the negative root only. Alternatively, $\pm\sqrt{\frac{1}{2}+\frac{7}{2}}-\frac{7}{2}$.
	$[x=]-\frac{11}{2}$	**A1**	If B1M0 scored then award **SCB1** for the correct final answer.
		3	

Question	Answer	Marks	Guidance
9(a)	$2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}=3x^{-\frac{1}{2}}+12$ all $\times x^{\frac{1}{2}} \Rightarrow x-6x^{\frac{1}{2}}+5=0$	*M1	OE Equating the two expressions in x and then multiplying each term by $x^{\frac{1}{2}}$ or by their substitution for $x^{\frac{1}{2}}$. Coefficients need to be retained but condone +/– sign errors. Allow $x^{\frac{1}{2}}$ replaced by x.
	$\left(x^{\frac{1}{2}}-1\right)\left(x^{\frac{1}{2}}-5\right)$ [= 0] or [x=] $\dfrac{6\pm\sqrt{36-4\times1\times5}}{2}$	DM1	OE Solving their three-term quadratic.
	Alternative method for first 2 marks of Question 9(a)		
	$2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}=3x^{-\frac{1}{2}}+12$ all $\times x^{\frac{1}{2}}$ leading to $2x+10=12x^{\frac{1}{2}}$	*M1	Equating the two expressions in x and isolating their term in $x^{\frac{1}{2}}$.
	$(2x+10)^2=144x$ leading to [4]$\left(x^2-26x+25\right)$[= 0] leading to [4]$(x-25)(x-1)$ [= 0] or [x=] $\dfrac{26\pm\sqrt{676-4\times1\times25}}{2}$	DM1	OE Squaring both sides, rearranging and solving a three-term quadratic.
	$x=1$ and 25 , $y=15$ and $12\frac{3}{5}$	A1, A1	A1 for both x-values and A1 for both y values. If M1DM0 scored then **SCB1B1** is available for final answers.
		4	Answers without working score 0/4

Question	Answer	Marks	Guidance
9(b)	Area = $\int\left(3x^{-\frac{1}{2}}+12\right)-\left(2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}\right)[dx]$ $\left[=-2x^{\frac{1}{2}}+12-10x^{-\frac{1}{2}}\right]$	**M1**	Attempt to integrate, defined by at least one correct fractional power, and subtract – condone the wrong way round.
	$=\left\{-\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}\right\}+12x\left\{-\frac{10x^{\frac{1}{2}}}{\frac{1}{2}}\right\}$	**B1 B1**	B1 for either { }. B1 for completely correct integration of their expression following through +/– sign errors from the subtraction.
	$\left(-\frac{4}{3}(\textit{their}\ 25)^{\frac{3}{2}}+12(\textit{their}\ 25)-20(\textit{their}\ 25)^{\frac{1}{2}}\right)-$ $\left(-\frac{4}{3}(\textit{their}\ 1)^{\frac{3}{2}}+12(\textit{their}\ 1)-20(\textit{their}\ 1)^{\frac{1}{2}}\right)$	**M1**	OE Substitution of *their* positive limits from part **(a)** in *their* integrated expression, defined by at least one correct fractional power, and subtraction.

Question	Answer	Marks	Guidance
9(b)	**Alternative method for first 4 marks of Question 9(b)**		
	Area = $\int\left(3x^{-\frac{1}{2}}+12\right)[\mathrm{dx}]-\int\left(2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}\right)[\mathrm{d}x]$	**M1**	Attempt to integrate, defined by at least one correct fractional power, and subtract – condone the wrong way round.
	$=\left\{\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}+12x\right\}[-]\left\{\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{13x^{\frac{1}{2}}}{\frac{1}{2}}\right\}$	**B1 B1**	OE One mark for each correct expression.
	$\left(\left(6(\mathit{their}\,25)^{\frac{1}{2}}+12(\mathit{their}\,25)\right)-\left(6(\mathit{their}\,1)^{\frac{1}{2}}+12(\mathit{their}\,1)\right)\right)$ [−] $\left(\left(\frac{4}{3}(\mathit{their}\,25)^{\frac{3}{2}}+26(\mathit{their}\,25)^{\frac{1}{2}}\right)-\left(\frac{4}{3}(\mathit{their}\,1)^{\frac{3}{2}}+26(\mathit{their}\,1)^{\frac{1}{2}}\right)\right)$	**M1**	OE Substitution of *their* positive limits from part **(a)** in both of their integrated expressions, defined by at least one correct fractional power, and subtraction.
	[Area =] $\frac{128}{3}, 42\frac{2}{3}$, 42.7	**A1**	AWRT If M1B1B1M0 then **SC B1** available for correct final answer. Condone negative answer if corrected.
		5	Condone the presence of π for the first 4 marks but use of $\int \boldsymbol{y}^2$ scores 0/5

Question	Answer	Marks	Guidance
10(a)	$\frac{dy}{dx}=\left\{\frac{5}{3}(4x-3)^{\frac{2}{3}}\right\}\{\times 4\}\left\{-\frac{20}{3}\right\}$	B2,1,0	B2 Three correct unsimplified { } and no others. B1 Two correct { }or three correct { } and an additional term e.g. + *c*. B0 More than one error.
	$\left[\frac{20}{3}(4x-3)^{\frac{2}{3}}-\frac{20}{3}=0\right]$ leading to $(4x-3)^2=k,\ k>0$ leading to $4x-3=\pm m$	M1	Equating *their* $\frac{dy}{dx}$ to 0 and using a valid method to arrive at 2 answers.
	$[4x-3=\pm 1]\ \ [x=]\frac{1}{2},1$	A1	
	$\frac{d^2y}{dx^2}=\frac{40}{9}(4x-3)^{-\frac{1}{3}}\times 4$	B1	OE
	$\left[x=\frac{1}{2}\right]\ \frac{d^2y}{dx^2}=\left(\frac{160}{9}\right)(4x-3)^{-\frac{1}{3}}<0$ or $-\frac{160}{9}$ or -17.8 so max $[x=1]\ \frac{d^2y}{dx^2}=\left(\frac{160}{9}\right)(4x-3)^{-\frac{1}{3}}>0$ or $\frac{160}{9}$ or 17.8 so min	B1 B1	If $\frac{d^2y}{dx^2}$ evaluated the answers for both must be correct OR Clear use of change in sign of $\frac{dy}{dx}$ correctly for both B1's. If B1M1A0B0B0 scored then **SCB1** can be awarded for: $\frac{dy}{dx}=\left\{\frac{5}{3}(4x-3)^{\frac{2}{3}}\right\}-\left\{\frac{20}{3}\right\}$ leading to $(4x-3)^2=64$ leading to $x=-\frac{5}{4},\frac{11}{4}$. $\frac{d^2y}{dx^2}=\frac{10}{9}(4x-3)^{-\frac{1}{3}}$, $x=-\frac{5}{4}$, $\frac{d^2y}{dx^2}<0$ so max, $x=\frac{11}{4}$, $\frac{d^2y}{dx^2}>0$ so min.
		6	

Question	Answer	Marks	Guidance
10(b)	$x < \frac{1}{2},\ x > 1$	B1	Allow $\leqslant$ and/or $\geqslant$. FT only from special case $x < -\frac{5}{4},\ x > \frac{11}{4}$ Condone: $1 < x < \frac{1}{2}$.
		1	

Question	Answer	Marks	Guidance
11(a)	$\left(their\,\frac{7-4}{p-6}\right)\times\left(their\,\frac{18-7}{14-p}\right)=-1$ OR Scalar product leading to $(14-p)(6-p)-33=0$	*M1	Their gradients must both come from $\frac{\text{Difference in the } y\text{s}}{\text{Difference in the } x\text{s}}$.
	$p^2-20p+84=33$ leading to $p^2-20p+51[=0]$ or $p^2-20p=-51$	A1	Clearing of fractions and collecting terms to arrive at the three-term quadratic. Allow integer multiples.
	Alternative method for first 2 marks of Question 11(a)		
	$(p-6)^2+(7-4)^2+(14-p)^2+(18-7)^2=(14-6)^2+(18-4)^2$ OR E.g. $(10-p)^2+4^2=4^2+7^2$	*M1	For correct use of Pythagoras with *A*,*B* and *C*. OR For correct use of Pythagoras with the centre, B and one of the other two points.
	$2p^2-40p+102[=0]$	A1	OE Collecting terms to arrive at the three-term quadratic.
	$[2](p-3)(p-17)$ or $\frac{20\pm\sqrt{20^2-4\times 51}}{2}$	DM1	OE Solving their three-term quadratic.
	$p=3$	A1	If M1A1DM0 scored then **SC B1** is available for final answer.
		4	

Question	Answer	Marks	Guidance
11(b)	[Midpoint or Centre is] (10, 11)	**B1**	SOI by final answer.
	$\frac{1}{2}\sqrt{(14-6)^2+(18-4)^2}$ or $(18-their11)^2+(14-their10)^2$ or $(their11-4)^2+(their10-6)^2$ $\left[r^2=65 \text{ or } r=\sqrt{65}\right]$	**M1**	Finding half of the length of *AC* or using their centre, which cannot be A, B or C, to find r^2 or r. Note: $r=65$ is M0.
	$(x-10)^2+(y-11)^2=65$ or $x^2+y^2-20x-22y+156=0$	**A1**	$(x-6)(x-14)+(y-4)(y-18)=0$ scores 3/3.
		3	
11(c)	$\frac{18-their11}{14-their10}$ or $\frac{their11-4}{their10-6}$ or $\frac{18-4}{14-6}$ $\left[=\frac{7}{4}\right]$	***M1**	Gradient of *their* centre, which cannot be A, B or C, from part **(b)**, to A or C or the gradient of AC but working needed if incorrect centre. OR by clearly differentiating and substitution of (14,18).
	$y-18=-\frac{1}{their\frac{7}{4}}(x-14)$	**DM1**	OE Using (14,18) and $-\frac{1}{their\frac{7}{4}}$ to form the equation of a straight line.
	$4x+7y-182=0$	**A1**	All terms on one side in any order. Allow multiples of this format by an integer only.
		3	

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2108969358

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

JC23 11_9709_13/3R

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1 A curve is such that its gradient at a point (x, y) is given by $\frac{dy}{dx} = x - 3x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, 1)$.

Find the equation of the curve. [4]

2 The circle with equation $(x-3)^2 + (y-5)^2 = 40$ intersects the y-axis at points A and B.

(a) Find the y-coordinates of A and B, expressing your answers in terms of surds. [2]

(b) Find the equation of the circle which has AB as its diameter. [2]

3 **(a)** Show that the equation

$$5\cos\theta - \sin\theta\tan\theta + 1 = 0$$

may be expressed in the form $a\cos^2\theta + b\cos\theta + c = 0$, where a, b and c are constants to be found. [3]

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(b) Hence solve the equation $5\cos\theta - \sin\theta\tan\theta + 1 = 0$ for $0 < \theta < 2\pi$. [4]

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4 **(a)** Expand the following in ascending powers of x up to and including the term in x^2.

(i) $(1+2x)^5$. [1]

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(ii) $(1-ax)^6$, where a is a constant. [2]

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In the expansion of $(1+2x)^5(1-ax)^6$, the coefficient of x^2 is -5.

(b) Find the possible values of a. [4]

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5 The first, second and third terms of a geometric progression are $2p + 6$, $5p$ and $8p + 2$ respectively.

(a) Find the possible values of the constant p. [3]

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(b) One of the values of p found in **(a)** is a negative fraction.

Use this value of p to find the sum to infinity of this progression. [4]

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6 A line has equation $y = 6x - c$ and a curve has equation $y = cx^2 + 2x - 3$, where c is a constant. The line is a tangent to the curve at point P.

Find the possible values of c and the corresponding coordinates of P. [7]

7 The function f is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.

(a) State the range of f. [1]

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(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

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The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$. [2]

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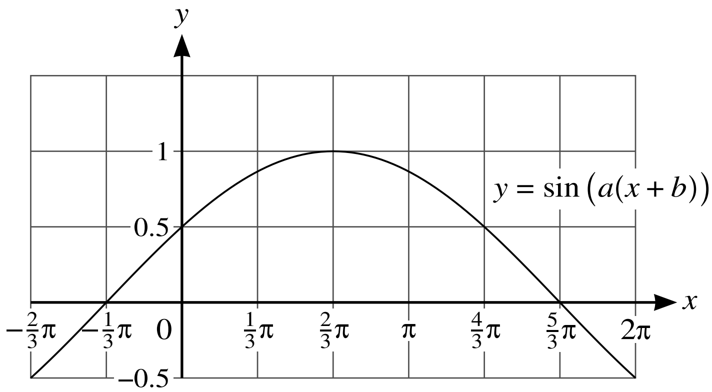

The diagram shows part of the graph of $y = \sin\big(a(x+b)\big)$, where a and b are positive constants.

(a) State the value of a and one possible value of b. [2]

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Another curve, with equation $y = \mathrm{f}(x)$, has a single stationary point at the point (p, q), where p and q are constants. This curve is transformed to a curve with equation

$$y = -3\mathrm{f}\big(\tfrac{1}{4}(x+8)\big).$$

(b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q. [3]

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9 A curve has equation $y = 2x^{\frac{1}{2}} - 1$.

(a) Find the equation of the normal to the curve at the point $A\,(4,\ 3)$, giving your answer in the form $y = mx + c$. [3]

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A point is moving along the curve $y = 2x^{\frac{1}{2}} - 1$ in such a way that at A the rate of increase of the x-coordinate is $3\,\text{cm}\,\text{s}^{-1}$.

(b) Find the rate of increase of the y-coordinate at A. [2]

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At A the moving point suddenly changes direction and speed, and moves down the normal in such a way that the rate of decrease of the y-coordinate is constant at $5\,\text{cm}\,\text{s}^{-1}$.

(c) As the point moves down the normal, find the rate of change of its x-coordinate. [3]

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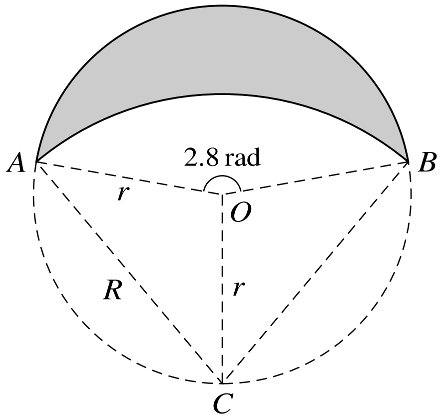

The diagram shows points A, B and C lying on a circle with centre O and radius r. Angle AOB is 2.8 radians. The shaded region is bounded by two arcs. The upper arc is part of the circle with centre O and radius r. The lower arc is part of a circle with centre C and radius R.

(a) State the size of angle ACO in radians. [1]

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(b) Find R in terms of r. [1]

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(c) Find the area of the shaded region in terms of r. [7]

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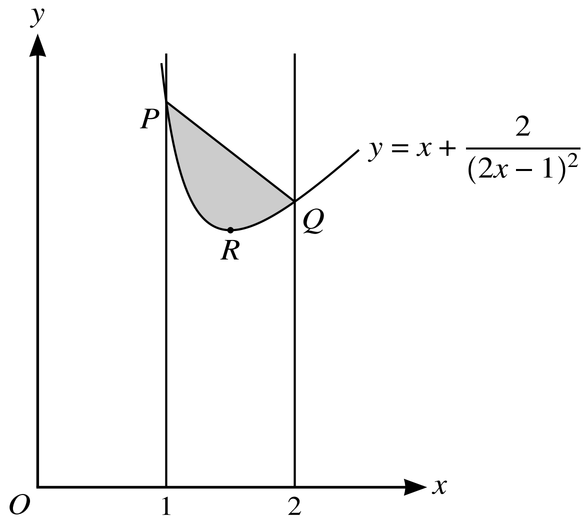

The diagram shows part of the curve with equation $y = x + \dfrac{2}{(2x-1)^2}$. The lines $x = 1$ and $x = 2$ intersect the curve at P and Q respectively and R is the stationary point on the curve.

(a) Verify that the x-coordinate of R is $\frac{3}{2}$ and find the y-coordinate of R. [4]

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(b) Find the exact value of the area of the shaded region. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$[y]=\left\{\frac{x^2}{2}\right\}\ \left\{\frac{-3x^{\frac{1}{2}}}{\frac{1}{2}}\right\}[+c]$	**B1 B1**	Any unsimplified correct form, ISW for extra terms, allow lists.
	$1 = 8 - 12 + c$	**M1**	Substitute (into an integrated expression) $x = 4$, $y = 1$. c must be present. Expect $c = 5$.
	$y=\frac{1}{2}x^2-6x^{\frac{1}{2}}+5$, allow f($x$) =	**A1**	Condone $c = 5$ as the final line so long as '$y =$' present.
		4	

Question	Answer	Marks	Guidance
2(a)	$(0-3)^2+(y-5)^2=40$	**M1**	OE. Substitute $x = 0$, may use $y^2-10y-6=0$.
	$y=5\pm\sqrt{31}$	**A1**	OE. Must be surd form.
		2	
2(b)	$\left\{x^2+(y-5)^2\right\}=\{31\}$ Allow $(x-0)^2$	**B1FT B1FT**	B1 FT for *their* 5 and B1 FT for *their* 31. Don't allow surd form.
		2	

Question	Answer	Marks	Guidance
3(a)	$5\cos^2\theta - \sin^2\theta + \cos\theta\ [=0]$	**M1**	Multiply by $\cos\theta$ and replace $\tan\theta$ by $\dfrac{\sin\theta}{\cos\theta}$.
	$5\cos^2\theta - \left(1-\cos^2\theta\right) + \cos\theta\ [=0]$	**M1**	
	$6\cos^2\theta + \cos\theta - 1 = 0$	**A1**	Missing '= 0' can be condoned if '= 0' appears earlier.
		3	
3(b)	$(3\cos\theta - 1)(2\cos\theta + 1) = 0$	**M1**	Must have 3 term quadratic, expect $\cos\theta = \frac{1}{3}, -\frac{1}{2}$. Factors (OE) must be shown.
	$\theta = \{1.23\}$; {2.09 or $\frac{2\pi}{3}$ } ; {5.05 and 4.19 (allow $\frac{4\pi}{3}$) }	**A1 A1** **A1 FT**	For A1 FT is for <u>both</u> 2π – 1st solutions.
		4	

Question	Answer	Marks	Guidance
4(a)(i)	$1 + 10x + 40x^2$	**B1**	Ignore any additional terms (ISW). Allow x^0 or 1 x^0 for the first term, allow lists.
		1	
4(a)(ii)	$\{1\}\{-6ax\}\{+15a^2x^2\}$	**B2, 1, 0**	Ignore any additional terms (ISW). Allow x^0 or 1 x^0 for the first term, allow lists.
		2	

Question	Answer	Marks	Guidance
4(b)	$15a^2 - 60a + 40 = -5$	**M1 A1**	Correct 3 products from *their* expansions for M1. Condone inclusion of x^2 for M1.
	$[15](a-1)(a-3)[=0]$ OE	**DM1**	OE. Rearranging and solving a quadratic.
	$a = 1$ and 3	**A1**	**Special case:** If M1 A1 DM0 scored then **SC B1** can be awarded for correct answers.
		4	

Question	Answer	Marks	Guidance
5(a)	$\frac{5p}{2p+6} = \frac{8p+2}{5p}$	**M1**	OE. Setting up a valid relationship in terms of *p*.
	$9p^2 - 52p - 12$ [= 0]	**DM1**	OE. Simplifying to a 3 term quadratic equation, only condone sign errors.
	$[(9p+2)(p-6) = 0]$ leading to $p = \frac{-2}{9}$ and 6	**A1**	
		3	
5(b)	$a = 2\left(-\frac{2}{9}\right) + 6\left[= \frac{50}{9}\right]$	***M1**	FT *their* $-\frac{2}{9}$, allow any negative non-integer.
	$r = -\frac{10}{9} \div \frac{50}{9}\left[= -\frac{1}{5}\right]$	***M1**	Ft *their* $-\frac{2}{9}$, allow any negative non-integer.
	$S_\infty = \frac{50}{9} \div \left(1 - -\frac{1}{5}\right) = \frac{125}{27}$	**DM1 A1**	Can only get DM1 if $\|r\| < 1$. Accept AWRT 4.63 .
		4	

Question	Answer	Marks	Guidance
6	$cx^2+2x-3=6x-c$ leading to $cx^2-4x+(c-3)$ $[=0]$	B1	3-term quadratic.
	$16-4c(c-3)=0$	*M1	Apply $b^2-4ac=0$ ('$=0$' may be implied in subsequent work). *Their* coefficients must be substituted correctly
	$4c^2-12c-16[=0]$ leading to $[4](c-4)(c+1)$ $[=0]$ leading to $c=4$ and -1	A1	Dependent on factorisation oe.
	When c = 4, $4x^2-4x+1[=0]$ $\left[(2x-1)^2=0\right]$	DM1	OE. Substituting *their c* = 4 into *their* quadratic equation.
	$x=\frac{1}{2}$, $y=-1$	A1	Both required.
	When $c=-1$, $x^2+4x+4[=0]$ $\left[(x+2)^2=0\right]$	DM1	OE. Substituting *their c* = -1 into *their* quadratic equation.
	$x=-2$, $y=-11$	A1	Both required.
	Alternative method for Question 6		
	$\frac{dy}{dx}=2cx+2$	B1	
	$2cx+2=6$	M1	Equating *the*ir curve gradient and 6.
	$c=\frac{2}{x}$	A1	SOI
	$2x^2+3x-2$ $[=0]$	DM1	Substitute $c=\frac{2}{x}$ into $cx^2+2x-3=6x-c$. Simplify to 3-term quadratic.

Question	Answer	Marks	Guidance
6	$(2x-1)(x+2)[=0] \rightarrow x=\frac{1}{2}$ or -2	**A1**	Dependent on factorisation. Both required.
	$c=4$ and -1	**A1**	Both required, if DM0 given **SC B1** for both.
	$y=-1$ and -11	**A1**	Both required, if DM0 given **SC B1** for both. **SC** one correct (x, y). A1 only
		7	

Question	Answer	Marks	Guidance
7(a)	Range is [y] > 1	**B1**	Allow f, f(x), $(1,\infty)$, etc.
		1	
7(b)	$y=\frac{3}{x-2}+1$ leading to $y-1=\frac{3}{x-2}$ leading to $(x-2)(y-1)=3$	**M1**	Clearing the fraction.
	$x-2=\frac{3}{y-1}$	**DM1**	Reaching a stage which only requires one further operation.
	$x=\frac{3}{y-1}+2$ leading to $f^{-1}(x)=\frac{3}{x-1}+2$	**A1**	OE. Slightly longer routes lead to $f^{-1}(x)=\frac{2x+1}{x-1}$.
	[Domain is] $x>1$	**B1FT**	Must use x FT *their* **(a)**, must be a range.
		4	

Question	Answer	Marks	Guidance
7(c)	$\text{gf}(x) = 2\left(\dfrac{3}{x-2}+1\right)-2$ or $2\left(\dfrac{x+1}{x-2}\right)-2$	**M1**	Substitute f(x) into g(x).
	$\dfrac{6}{x-2}$	**A1**	
		2	

Question	Answer	Marks	Guidance
8(a)	$a = \dfrac{1}{2}$	**B1**	
	$b = \dfrac{\pi}{3}$	**B1**	$b = \dfrac{\pi}{3} + 4n\pi, n \geqslant 0$.
		2	
8(b)	x-coordinate = {$4p$} {–8}	**B1 B1**	OE, e.g. $4(p-2)$.
	y-coordinate = $-3q$	**B1**	
		3	

Question	Answer	Marks	Guidance
9(a)	$\frac{dy}{dx} = x^{-\frac{1}{2}} \rightarrow m = \frac{1}{2}$ at $x = 4$	**B1**	
	Equation of normal is $y - 3 = -2(x - 4)$	**M1**	Through (4, 3) with gradient $-\frac{1}{their\ m}$. (Dependent on differentiation used).
	$y = -2x + 11$	**A1**	Only acceptable answer.
		3	
9(b)	$\frac{dy}{dt} = their\ \frac{1}{2} \times 3$	**M1**	Correct use of the chain rule with a numerical gradient.
	$\frac{3}{2}$	**A1**	
		2	
9(c)	Required gradient $\left[= \frac{dy}{dx} \right] = -2$	**B1FT**	SOI. FT from *their* part **(a)** if a normal gradient has been found from $m_1 m_2 = -1$ and differentiation.
	$\frac{dx}{dt} = \frac{1}{their\ normal\ gradient} \times -5$	**M1**	Correct use of the chain rule . Allow method mark also for +5, must be numerical. *Their* normal gradient must come from $m_1 m_2 = -1$ and differentiation in part(a) unless 'restarted' here.
	$\frac{5}{2}$	**A1**	
		3	

Question	Answer	Marks	Guidance
10(a)	Angle $ACO = 0.7$	**B1**	Don't allow AWRT 0.7 .
		1	
10(b)	$[R =]\ 1.53\,r$	**B1**	Allow AWRT 1.53r.
		1	
10(c)	Sector $OAB = \frac{1}{2}r^2 \times 2.8 \quad \left[=1.4r^2\right]$	**B1**	
	Sector $CAB = \frac{1}{2}(\textit{their } R)^2 \times 2 \times \textit{their}\, 0.7$	***M1**	
	$1.638\,r^2$	**A1**	Allow AWRT $1.64\,r^2$.
	$[2]\times\frac{1}{2}r^2\sin(\pi-1.4)$ OR $[2]\times\frac{1}{2}r\times \textit{their}R\sin 0.7$	***M1**	
	$2\times0.4927r^2$	**A1**	Allow AWRT $0.98\,r^2$ to $0.99\,r^2$.
	$1.4r^2-\left(\textit{their}\,1.638r^2-\textit{their}\,0.985r^2\right)$	**DM1**	
	$0.747r^2$ to $0.748r^2$	**A1**	
		7	

Question	Answer	Marks	Guidance
10(c)	**General guidance for alternative methods**		
	Finding any useful sector area of the circle radius, r	**B1**	May be 'nested' in a segment.
	Finding the area of sector CAB	***M1A1**	May be 'nested' in a segment.
	Finding the area of one useful triangle	***M1**	May be 'nested' in a segment.
	Finding the total area of useful triangles	**A1**	May be 'nested' in a segment.
	A correct plan for the shaded area	**DM1**	
	$0.747r^2$ to $0.748r^2$	**A1**	
		7	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx}=\left\{-2\times2\times(2x-1)^{-3}\times2\right\}+\{1\}$	**B1B1**	Expect $\frac{-8}{(2x-1)^3}+1$.
	Substitute $x=\frac{3}{2}$ leading to $\frac{dy}{dx}=\frac{-8}{8}+1=0$. Hence x-coordinate of R is $\frac{3}{2}$	**DB1**	AG. Or correct solution of $\frac{dy}{dx}=0$.
	When $x=\frac{3}{2}, y=\frac{2}{4}+\frac{3}{2}=2$	**B1**	Answer only is acceptable.
		4	

Question	Answer	Marks	Guidance
11(b)	y-coordinate of P = 3, y-coordinate of $Q = \frac{20}{9}$	**B1**	Both required.
	$\left\{\frac{2(2x-1)^{-1}}{-1\times 2}\right\}+\left\{\frac{1}{2}x^2\right\}$	**B1 B1**	Area below curve.
	$\left[\left(-\frac{1}{3}+2\right)-\left(-1+\frac{1}{2}\right)\right]=\frac{5}{3}-\left(-\frac{1}{2}\right)$	**M1**	Apply limits 1→2 to an integral. Expect $\frac{13}{6}$.
	$\frac{1}{2}\left(3+\frac{20}{9}\right)=\frac{47}{18}$	**M1**	Area of trapezium, only allow errors in y-coordinate of Q.
	$\frac{47}{18}-\frac{13}{6}=\frac{4}{9}$	**A1**	Shaded region.
		6	
	Alternative method 1: Changes the award of the first M1		
	Their equation of line PQ:[$y=\frac{-7}{9}x+\frac{34}{9}$]. Integrating between 1 and 2.	**M1**	Must be some evidence of use of limits.
	Alternative method 2: Changes the award of the first M1, a B1 and the second M1		
	Combining line and curve: $\int\left(\frac{-16}{9}x+\frac{34}{9}-\frac{2}{(2x-1)^2}\right)dx$	**M1**	For area under the line if *their* $\frac{34}{9}$ is seen integrated correctly and limits used. Correct first and 3rd terms.
	$=\frac{-8}{9}x^2+\frac{34}{9}x+\frac{1}{(2x-1)}$	**B1 B1**	
	Use of limits on the whole integral	**M1**	

Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER					CANDIDATE NUMBER	

* 9 6 9 9 6 7 5 1 4 5 *

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

1 It is given that θ is an acute angle in degrees such that $\sin\theta = \frac{2}{3}$.

Find the exact value of $\sin(\theta + 60^\circ)$. [3]

2 A curve has equation $y = 3\tan\frac{1}{2}x\cos 2x$.

Find the gradient of the curve at the point for which $x = \frac{1}{3}\pi$. [5]

3 **(a)** Find $\int_4^{10} \frac{4}{2x-5}\,dx$, giving your answer in the form $\ln a$, where a is an integer. [4]

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(b) Find the exact value of $\int_4^{10} e^{2x-5}\,dx$. [2]

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4 **(a)** Sketch, on the same diagram, the graphs of $y = |3x - 5|$ and $y = 2x + 7$. [2]

(b) Solve the equation $|3x - 5| = 2x + 7$. [3]

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(c) Hence solve the equation $\left|3^{y+1} - 5\right| = 2 \times 3^{y} + 7$, giving your answer correct to 3 significant figures. [2]

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5 The polynomial $p(x)$ is defined by

$$p(x) = 6x^3 + ax^2 + bx - 20,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $p(x)$ and that the remainder is -11 when $p(x)$ is divided by $(x + 1)$.

(a) Find the values of a and b. [5]

(b) Hence factorise $p(x)$, and determine the exact roots of the equation $p(3x) = 0$. [4]

6 **(a)** Show that $\operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta) \equiv 4 + 6\cos\theta - 4\cos^2\theta$. [3]

(b) Solve the equation

$$\operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta) + 3 = 0$$

for $-\pi < \theta < 0$. [3]

(c) Find $\int \operatorname{cosec} \theta (3 \sin 2\theta + 4 \sin^3 \theta) \, d\theta$. [3]

7 The curve with equation $e^{2x} - 18x + y^3 + y = 11$ has a stationary point at (p, q).

(a) Find the exact value of p. [4]

(b) Show that $q = \sqrt[3]{2 + 18 \ln 3 - q}$. [2]

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(c) Show by calculation that the value of q lies between 2.5 and 3.0. [2]

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(d) Use an iterative formula, based on the equation in **(b)**, to find the value of q correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS Level

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply that $\cos\theta = \frac{1}{3}\sqrt{5}$	B1	or exact equivalent.
	Substitute appropriate values into $\sin\theta\cos 60° + \cos\theta\sin 60°$	M1	
	Obtain $\frac{1}{3} + \frac{1}{6}\sqrt{15}$	A1	or exact equivalent.
		3	

Question	Answer	Marks	Guidance
2	State or imply derivative of $\tan\frac{1}{2}x$ is $\frac{1}{2}\sec^2\frac{1}{2}x$ or derivative of $\cos 2x$ is $-2\sin 2x$	B1	
	Attempt use of product rule to find first derivative	*M1	
	Obtain correct $\frac{3}{2}\sec^2\frac{1}{2}x\cos 2x - 6\tan\frac{1}{2}x\sin 2x$	A1	or (unsimplified) equivalent.
	Substitute $\frac{1}{3}\pi$ into attempt at first derivative and attempt evaluation to find the gradient	DM1	
	Obtain -4	A1	
		5	

Question	Answer	Marks	Guidance
3(a)	Obtain $2\ln(2x-5)$	**B1**	
	Apply limits correctly	**M1**	For integral of form $k\ln(2x-5)$.
	Use one relevant logarithm property correctly	**M1**	For integral of form $k\ln(2x-5)$.
	Apply second logarithm property correctly and obtain $\ln 25$	**A1**	
		4	
3(b)	Integrate to obtain $\frac{1}{2}e^{2x-5}$	**B1**	
	Obtain final answer $\frac{1}{2}e^{15}-\frac{1}{2}e^{3}$	**B1FT**	or exact equivalent, FT on *their* ke^{2x-5}.
		2	

Question	Answer	Marks	Guidance
4(a)	Draw V-shaped graph with vertex on positive x-axis	**B1**	
	Draw (more or less) correct graph of $y=2x+7$ with smaller gradient	**B1**	And crossing y-axis above y-intercept of modulus graph.
		2	

Question	Answer	Marks	Guidance
4(b)	Solve $3x-5=2x+7$ to obtain $x=12$	**B1**	
	Attempt solution of linear equation where signs of $3x$ and $2x$ are different	**M1**	$3x-5=-2x-7$ OE.
	Obtain $x=-\frac{2}{5}$	**A1**	
	Alternative solution for question 4(b)		
	State or imply non-modulus equation $(3x-5)^2=(2x+7)^2$	**B1**	Must be working with $(3x-5)^2=(2x+7)^2$
	Attempt solution of 3-term quadratic equation	**M1**	
	Obtain $-\frac{2}{5}$ and 12	**A1**	
		3	
4(c)	Apply logarithms and use power law for $3^y=k$ where $k>0$ or correct equivalent	**M1**	Using *their* positive answer from part **(b)** or greater accuracy; and no other values.
	Obtain 2.26	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	Substitute $x=-2$ and equate to zero	*M1	
	Substitute $x=-1$ and equate to -11	*M1	
	Obtain $4a-2b-68=0$ and $a-b-26=-11$ or equivalents	A1	
	Solve a pair of relevant simultaneous linear equations to find a or b	DM1	Dependent at least one M1 mark.
	Obtain $a=19$ and $b=4$	A1	
		5	
5(b)	Divide by $x+2$ at least as far as the x term	M1	or equivalent (inspection, …).
	Obtain $(x+2)^2(6x-5)$	A1	OE
	Replace (or imply replacement of) x by $3x$ in factorised form	M1	
	Obtain $-\frac{2}{3}$ and $\frac{5}{18}$	A1	and no others.
		4	

Question	Answer	Marks	Guidance
6(a)	Use $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	**B1**	
	Express in terms of $\sin\theta$ and $\cos\theta$ only	**M1**	Dependent on B1.
	Obtain given result $4 + 6\cos\theta - 4\cos^2\theta$ with sufficient detail	**A1**	AG
		3	
6(b)	Attempt use of formula to solve 3-term quadratic equation as far as $\cos\theta = k_1$	**M1**	where $-1 < k_1 < 1$.
	Solve $4\cos^2\theta - 6\cos\theta - 7 = 0$ to obtain at least $\cos\theta = -0.770...$	**A1**	or exact equivalent $\cos\theta = \dfrac{6-\sqrt{148}}{8}$.
	Obtain -2.45	**A1**	or greater accuracy; and no others between $-\pi$ and 0.
		3	
6(c)	Express $\cos^2\theta$ term in the form $k_2 + k_3\cos 2\theta$	**M1**	where $k_2 k_3 \neq 0$.
	Obtain integrand $6\cos\theta + 2 - 2\cos 2\theta$	**A1**	Following the 3-term expression in $\cos\theta$ from part **(a)**.
	Integrate to obtain correct $6\sin\theta + 2\theta - \sin 2\theta$	**A1**	Condone absence of $+c$, but all in terms of θ.
		3	

Question	Answer	Marks	Guidance
7(a)	Differentiate y^3 to obtain $3y^2\frac{dy}{dx}$	B1	
	Differentiate complete equation to produce at least one term involving $\frac{dy}{dx}$ using implicit differentiation.	M1	
	Obtain $2e^{2x}-18+3y^2\frac{dy}{dx}+\frac{dy}{dx}=0$	A1	
	Substitute $\frac{dy}{dx}=0$ to obtain either $p=\frac{1}{2}\ln 9$ or $p=\ln 3$	A1	
		4	
7(b)	Substitute value of p in original equation and rearrange as far as $y^3=...$ or $q^3=\ldots$	M1	Allow in terms of $\ln 9$.
	Obtain given result $q=\sqrt[3]{2+18\ln 3-q}$ **or** $y=\sqrt[3]{2+18\ln 3-y}$ with sufficient detail	A1	AG
		2	
7(c)	Consider sign of $q-\sqrt[3]{2+18\ln 3-q}$ or equivalent for 2.5 and 3.0	M1	
	Obtain $-0.18...$ and $0.34...$ with sufficient detail and justify conclusion	A1	OE
		2	

Question	Answer	Marks	Guidance
7(d)	Use iteration process correctly at least once	M1	
	Obtain final answer $q = 2.673$	A1	Answer required to exactly 4 s.f.
	Show sufficient iterations to 6 sf to justify answer or show sign change in the interval $[2.6725,\ 2.6735]$	A1	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5120588436

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

JC23 11_9709_22/2R

1 When the polynomial

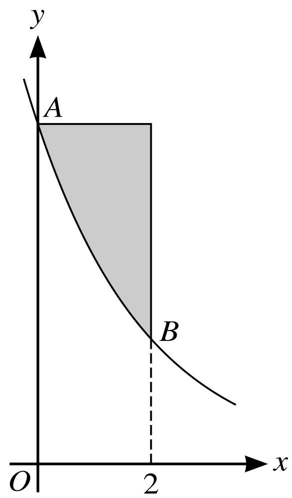
$$ax^3 + 4ax^2 - 7x - 5$$

is divided by $(x + 2)$, the remainder is 33.

Find the value of the constant a. [2]

2 Solve the equation $\sec\theta\cos(\theta - 60°) = 4$ for $-180° < \theta < 180°$. [5]

3



The diagram shows the curve with equation $y = 6e^{-\frac{1}{2}x}$. The points on the curve with x-coordinates 0 and 2 are denoted by A and B respectively. The shaded region is enclosed by the curve, the line through A parallel to the x-axis and the line through B parallel to the y-axis.

(a) Find the exact gradient of the curve at B. [2]

(b) Find the exact area of the shaded region. [3]

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4 **(a)** Sketch, on the same diagram, the graphs of $y = |3 - x|$ and $y = 9 - 2x$. [2]

(b) Solve the inequality $|3 - x| > 9 - 2x$. [3]

...

(c) Use logarithms to solve the inequality $2^{3x-10} < 500$. Give your answer in the form $x < a$, where the value of a is given correct to 3 significant figures. [3]

...

(d) List the integers that satisfy both of the inequalities $|3 - x| > 9 - 2x$ and $2^{3x-10} < 500$. [1]

...

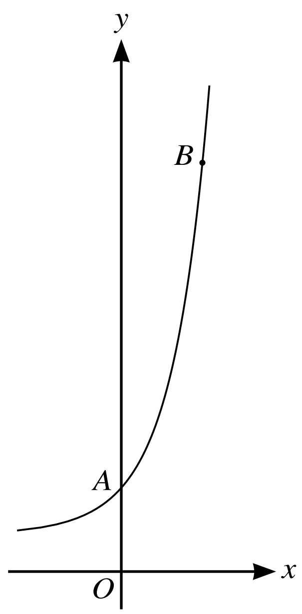
5 **(a)** Find the quotient when $6x^3 - 5x^2 - 24x - 4$ is divided by $(2x + 1)$, and show that the remainder is 6. [3]

(b) Hence find

$$\int_2^7 \frac{6x^3 - 5x^2 - 24x - 4}{2x + 1}\,dx,$$

giving your answer in the form $a + \ln b$, where a and b are integers. [5]

6



The diagram shows the curve with parametric equations

$$x = 3\ln(2t - 3), \qquad y = 4t\ln t.$$

The curve crosses the y-axis at the point A. At the point B, the gradient of the curve is 12.

(a) Find the exact gradient of the curve at A. [5]

(b) Show that the value of the parameter t at B satisfies the equation

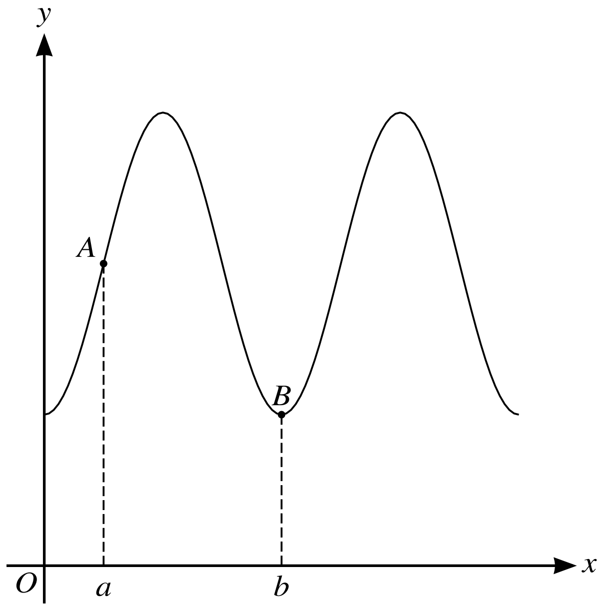
$$t = \frac{9}{1 + \ln t} + \frac{3}{2}. \qquad [2]$$

(c) Use an iterative formula, based on the equation in **(b)**, to find the value of t at B, giving your answer correct to 3 significant figures. Use an initial value of 5 and give the result of each iteration to 5 significant figures. [3]

7 **(a)** Prove that $\sin 2x(\cot x + 3\tan x) \equiv 4 - 2\cos 2x$. [4]

(b) Hence find the exact value of $\cot \frac{1}{12}\pi + 3\tan \frac{1}{12}\pi$. [2]

(c)



The diagram shows the curve with equation $y = 4 - 2\cos 2x$, for $0 < x < 2\pi$. At the point A, the gradient of the curve is 4. The point B is a minimum point. The x-coordinates of A and B are a and b respectively.

Show that $\int_a^b (4 - 2\cos 2x)\,dx = 3\pi + 1$. [5]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS Level

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
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GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Substitute $x = -2$ and equate to 33	**M1**	OE (long or synthetic division). Note: Long division and synthetic division give a remainder of $8a+14-5$. Allow one sign error for M1.
	Obtain $-8a+16a+14-5=33$ and hence $a=3$	**A1**	
		2	

Question	Answer	Marks	Guidance
2	Attempt to express equation in terms of $\cos\theta$ and $\sin\theta$ only, using a correct identity and $\sec\theta = \dfrac{1}{\cos\theta}$	**M1**	
	Obtain $\cos\theta\cos 60 + \sin\theta\sin 60 = 4\cos\theta$	**A1**	OE
	Either divide by $\cos\theta$ to find value of $\tan\theta$ or use another correct method to find $\sin\theta$ or $\cos\theta$	**M1**	May still involve $\cos 60$ and $\sin 60$.
	Obtain $\tan\theta = \frac{7}{\sqrt{3}}$ or $\tan\theta = 4.04...$ or $\sin\theta = \dfrac{7}{\sqrt{52}}$ or $\sin\theta = 0.970...$ or $\cos\theta = \dfrac{\sqrt{3}}{\sqrt{52}}$ or $\cos\theta = 0.240...$	**A1**	
	Obtain -103.9 and 76.1	**A1**	Or greater accuracy; and no others between –180 and 180.
		5	

Question	Answer	Marks	Guidance
3(a)	Differentiate to obtain form $ke^{-\frac{1}{2}x}$	M1	For any non-zero k except 6.
	Substitute $x=2$ to obtain $-3e^{-1}$ or $-\frac{3}{e}$	A1	
		2	
3(b)	Integrate to obtain $-12e^{-\frac{1}{2}x}$	B1	OE
	Use limits 0 and 2 correctly to an integral of the form $ke^{-\frac{x}{2}}$, retaining exactness	M1	For any non-zero k except 6 or equivalent perhaps involving integration of $6-6e^{-\frac{1}{2}x}$.
	Subtract from 12 to obtain final answer $12e^{-1}$ or $\frac{12}{e}$	A1	
		3	

Question	Answer	Marks	Guidance
4(a)	Draw V-shaped graph with vertex on positive x-axis	B1	
	Draw (more or less) correct graph of $y=9-2x$ with steeper negative gradient	B1	Dependent on first B mark, appropriately positioned with respect to first graph.
		2	

Question	Answer	Marks	Guidance
4(b)	Solve linear equation or inequality with signs of x and $2x$ different	**M1**	
	Obtain critical value 4	**A1**	
	Conclude $x > 4$ only	**A1**	6 must be discounted.
	Alternative Method for Question 4(b)		
	State or imply non-modulus equation (or inequality) $(3-x)^2 = (9-2x)^2$	**B1**	
	Attempt solution of three-term quadratic equation (or inequality)	**M1**	Dependent on previous B1.
	Conclude $x > 4$ only	**A1**	6 must be discounted.
		3	
4(c)	State or imply $(3x-10)\ln 2 < \ln 500$	**B1**	Or equivalent perhaps involving different logarithm base.
	Obtain critical value 6.32	**B1**	
	Obtain $x < 6.32$	**B1**	Or greater accuracy.
		3	
4(d)	State 5 and 6 only	**B1**	
		1	

Question	Answer	Marks	Guidance
5(a)	Carry out division at least as far as $3x^2 + k_1x$	**M1**	OE (e.g. by inspection).
	Obtain quotient $3x^2 - 4x - 10$	**A1**	
	Confirm given result of remainder is 6 with sufficient detail	**A1**	AG **SC** If remainder = 6 shown using remainder theorem allow **B1**.
		3	
5(b)	Integrate to obtain at least x^3 and term of form $k_2 \ln(2x+1)$	***M1**	ln term must be added.
	Obtain $x^3 - 2x^2 - 10x + 3\ln(2x+1)$	**A1**	
	Apply limits correctly to expression with four terms	**DM1**	
	Apply appropriate logarithm properties correctly to obtain the form $k_3 \ln a$	**DM1**	
	Obtain $195 + \ln 27$	**A1**	
		5	

Question	Answer	Marks	Guidance
6(a)	Obtain $\dfrac{dx}{dt}=\dfrac{6}{2t-3}$	**B1**	
	Use product rule to find $\dfrac{dy}{dt}$	**M1**	Allow unsimplified.
	Obtain $\dfrac{dy}{dx}=\dfrac{(4\ln t+4)(2t-3)}{6}$	**A1**	OE
	Attempt to find t corresponding to point A using a complete and correct method	**M1**	
	Obtain $t=2$ and hence gradient is $\frac{2}{3}\ln 2+\frac{2}{3}$	**A1**	Or exact equivalent.
		5	
6(b)	Equate $\dfrac{dy}{dx}$ to 12 and attempt rearrangement to $2t-3=\dfrac{k}{4\ln t+4}$	**M1**	
	Confirm given result $t=\dfrac{9}{1+\ln t}+\dfrac{3}{2}$ with sufficient detail	**A1**	AG
		2	
6(c)	Use iteration process correctly at least once	**M1**	Need to see 4.9626 .
	Obtain final answer 4.96	**A1**	Answer required to exactly 3 s.f.
	Show sufficient iterations to 5 sf to justify answer or show sign change in interval [4.955, 4.965]	**A1**	
		3	

Question	Answer	Marks	Guidance
7(a)	Attempt to express LHS in terms of $\sin x$ and $\cos x$ only	**M1***	
	Obtain $2\cos^2 x + 6\sin^2 x$	**A1**	OE
	Use relevant identities to express in terms of $\cos 2x$ only	**DM1**	
	Confirm given result $4 - 2\cos 2x$ with sufficient detail	**A1**	AG
		4	
7(b)	Substitute $x = \frac{1}{12}\pi$ and attempt exact value of $\dfrac{4 - 2\cos 2\left(\frac{\pi}{12}\right)}{\sin 2\left(\frac{\pi}{12}\right)}$	**M1**	
	Obtain $2(4 - 2\cos\frac{1}{6}\pi)$ or equivalent and hence $8 - 2\sqrt{3}$	**A1**	
		2	
7(c)	Differentiate to obtain $4\sin 2x$	**B1**	B2 if both limits obtained by symmetry or observation.
	Equate to 4 and obtain limits $\frac{1}{4}\pi$ and π	**B1**	
	Integrate to obtain $4x - \sin 2x$	**B1**	
	Use limits their *a* and *b* to find exact value for integral of form $k_1 x + k_2 \sin 2x$	**M1**	Angles must be in radians in terms of π .
	Obtain $4\pi - \sin 2\pi - (\pi - \sin\frac{1}{2}\pi)$ and confirm given result $3\pi + 1$ with sufficient detail	**A1**	AG
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

5167909186

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

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1 It is given that θ is an acute angle in degrees such that $\sin\theta = \frac{2}{3}$.

Find the exact value of $\sin(\theta + 60°)$. [3]

2 A curve has equation $y = 3\tan\frac{1}{2}x\cos 2x$.

Find the gradient of the curve at the point for which $x = \frac{1}{3}\pi$. [5]

3 **(a)** Find $\int_4^{10} \frac{4}{2x-5}\,dx$, giving your answer in the form $\ln a$, where a is an integer. [4]

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(b) Find the exact value of $\int_4^{10} e^{2x-5}\,dx$. [2]

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4 **(a)** Sketch, on the same diagram, the graphs of $y = |3x - 5|$ and $y = 2x + 7$. [2]

(b) Solve the equation $|3x - 5| = 2x + 7$. [3]

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(c) Hence solve the equation $\left|3^{y+1} - 5\right| = 2 \times 3^{y} + 7$, giving your answer correct to 3 significant figures. [2]

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5 The polynomial $p(x)$ is defined by

$$p(x) = 6x^3 + ax^2 + bx - 20,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $p(x)$ and that the remainder is -11 when $p(x)$ is divided by $(x + 1)$.

(a) Find the values of a and b. [5]

(b) Hence factorise $p(x)$, and determine the exact roots of the equation $p(3x) = 0$. [4]

6 **(a)** Show that $\operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta) \equiv 4 + 6\cos\theta - 4\cos^2\theta$. [3]

(b) Solve the equation

$$\operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta) + 3 = 0$$

for $-\pi < \theta < 0$. [3]

(c) Find $\int \operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta)\,d\theta$. [3]

7 The curve with equation $e^{2x} - 18x + y^3 + y = 11$ has a stationary point at (p, q).

(a) Find the exact value of p. [4]

(b) Show that $q = \sqrt[3]{2 + 18 \ln 3 - q}$. [2]

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(c) Show by calculation that the value of q lies between 2.5 and 3.0. [2]

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(d) Use an iterative formula, based on the equation in **(b)**, to find the value of q correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS Level

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

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Mathematics Specific Marking Principles

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply that $\cos\theta = \frac{1}{3}\sqrt{5}$	**B1**	or exact equivalent.
	Substitute appropriate values into $\sin\theta\cos 60° + \cos\theta\sin 60°$	**M1**	
	Obtain $\frac{1}{3} + \frac{1}{6}\sqrt{15}$	**A1**	or exact equivalent.
		3	

Question	Answer	Marks	Guidance
2	State or imply derivative of $\tan\frac{1}{2}x$ is $\frac{1}{2}\sec^2\frac{1}{2}x$ or derivative of $\cos 2x$ is $-2\sin 2x$	**B1**	
	Attempt use of product rule to find first derivative	***M1**	
	Obtain correct $\frac{3}{2}\sec^2\frac{1}{2}x\cos 2x - 6\tan\frac{1}{2}x\sin 2x$	**A1**	or (unsimplified) equivalent.
	Substitute $\frac{1}{3}\pi$ into attempt at first derivative and attempt evaluation to find the gradient	**DM1**	
	Obtain -4	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	Obtain $2\ln(2x-5)$	**B1**	
	Apply limits correctly	**M1**	For integral of form $k\ln(2x-5)$.
	Use one relevant logarithm property correctly	**M1**	For integral of form $k\ln(2x-5)$.
	Apply second logarithm property correctly and obtain $\ln 25$	**A1**	
		4	
3(b)	Integrate to obtain $\frac{1}{2}e^{2x-5}$	**B1**	
	Obtain final answer $\frac{1}{2}e^{15}-\frac{1}{2}e^{3}$	**B1FT**	or exact equivalent, FT on *their* ke^{2x-5}.
		2	

Question	Answer	Marks	Guidance
4(a)	Draw V-shaped graph with vertex on positive x-axis	**B1**	
	Draw (more or less) correct graph of $y=2x+7$ with smaller gradient	**B1**	And crossing y-axis above y-intercept of modulus graph.
		2	

Question	Answer	Marks	Guidance
4(b)	Solve $3x-5=2x+7$ to obtain $x=12$	**B1**	
	Attempt solution of linear equation where signs of $3x$ and $2x$ are different	**M1**	$3x-5=-2x-7$ OE.
	Obtain $x=-\frac{2}{5}$	**A1**	
	Alternative solution for question 4(b)		
	State or imply non-modulus equation $(3x-5)^2=(2x+7)^2$	**B1**	Must be working with $(3x-5)^2=(2x+7)^2$.
	Attempt solution of 3-term quadratic equation	**M1**	
	Obtain $-\frac{2}{5}$ and 12	**A1**	
		3	
4(c)	Apply logarithms and use power law for $3^y=k$ where $k>0$ or correct equivalent	**M1**	Using *their* positive answer from part **(b)** or greater accuracy; and no other values.
	Obtain 2.26	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	Substitute $x=-2$ and equate to zero	***M1**	
	Substitute $x=-1$ and equate to -11	***M1**	
	Obtain $4a-2b-68=0$ and $a-b-26=-11$ or equivalents	**A1**	
	Solve a pair of relevant simultaneous linear equations to find a or b	**DM1**	Dependent at least one M1 mark.
	Obtain $a=19$ and $b=4$	**A1**	
		5	
5(b)	Divide by $x+2$ at least as far as the x term	**M1**	or equivalent (inspection, …).
	Obtain $(x+2)^2(6x-5)$	**A1**	OE
	Replace (or imply replacement of) x by $3x$ in factorised form	**M1**	
	Obtain $-\frac{2}{3}$ and $\frac{5}{18}$	**A1**	and no others.
		4	

Question	Answer	Marks	Guidance
6(a)	Use $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	**B1**	
	Express in terms of $\sin\theta$ and $\cos\theta$ only	**M1**	Dependent on B1.
	Obtain given result $4+6\cos\theta-4\cos^2\theta$ with sufficient detail	**A1**	AG
		3	
6(b)	Attempt use of formula to solve 3-term quadratic equation as far as $\cos\theta = k_1$	**M1**	where $-1 < k_1 < 1$.
	Solve $4\cos^2\theta - 6\cos\theta - 7 = 0$ to obtain at least $\cos\theta = -0.770...$	**A1**	or exact equivalent $\cos\theta = \dfrac{6-\sqrt{148}}{8}$.
	Obtain –2.45	**A1**	or greater accuracy; and no others between $-\pi$ and 0.
		3	
6(c)	Express $\cos^2\theta$ term in the form $k_2 + k_3\cos 2\theta$	**M1**	where $k_2 k_3 \neq 0$.
	Obtain integrand $6\cos\theta + 2 - 2\cos 2\theta$	**A1**	Following the 3-term expression in $\cos\theta$ from part **(a)**.
	Integrate to obtain correct $6\sin\theta + 2\theta - \sin 2\theta$	**A1**	Condone absence of $+c$, but all in terms of θ.
		3	

Question	Answer	Marks	Guidance
7(a)	Differentiate y^3 to obtain $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	**B1**	
	Differentiate complete equation to produce at least one term involving $\frac{\mathrm{d}y}{\mathrm{d}x}$ using implicit differentiation.	**M1**	
	Obtain $2e^{2x}-18+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{\mathrm{d}y}{\mathrm{d}x}=0$	**A1**	
	Substitute $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ to obtain either $p=\frac{1}{2}\ln 9$ or $p=\ln 3$	**A1**	
		4	
7(b)	Substitute value of p in original equation and rearrange as far as $y^3=...$ or $q^3=\ldots$	**M1**	Allow in terms of $\ln 9$.
	Obtain given result $q=\sqrt[3]{2+18\ln 3-q}$ **or** $y=\sqrt[3]{2+18\ln 3-y}$ with sufficient detail	**A1**	AG
		2	
7(c)	Consider sign of $q-\sqrt[3]{2+18\ln 3-q}$ or equivalent for 2.5 and 3.0	**M1**	
	Obtain $-0.18...$ and $0.34...$ with sufficient detail and justify conclusion	**A1**	OE
		2	

Question	Answer	Marks	Guidance
7(d)	Use iteration process correctly at least once	**M1**	
	Obtain final answer $q = 2.673$	**A1**	Answer required to exactly 4 s.f.
	Show sufficient iterations to 6 sf to justify answer or show sign change in the interval $[2.6725,\ 2.6735]$	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5051231203

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

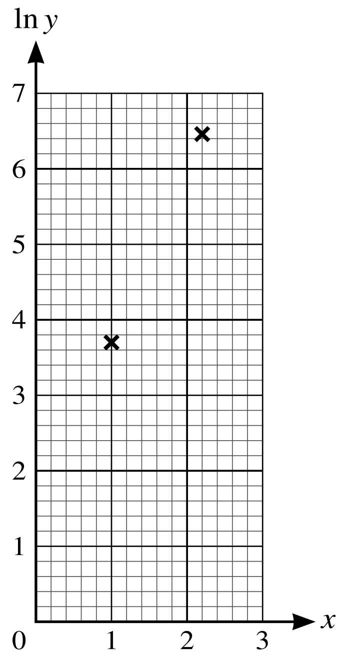
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 Find the exact coordinates of the points on the curve $y = \dfrac{x^2}{1 - 3x}$ at which the gradient of the tangent is equal to 8. [5]

2 On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2\mathrm{i}| \leqslant |z + 2 - \mathrm{i}|$ and $0 \leqslant \arg(z + 1) \leqslant \frac{1}{4}\pi$. [4]

3



The variables x and y are related by the equation $y = ab^x$, where a and b are constants. The diagram shows the result of plotting $\ln y$ against x for two pairs of values of x and y. The coordinates of these points are (1, 3.7) and (2.2, 6.46).

Use this information to find the values of a and b. [4]

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4 The complex number u is defined by $u = \dfrac{3 + 2i}{a - 5i}$, where a is real.

(a) Express u in the Cartesian form $x + iy$, where x and y are in terms of a. [3]

(b) Given that $\arg u = \frac{1}{4}\pi$, find the value of a. [2]

5 **(a)** Given that

$$\sin\left(x + \tfrac{1}{6}\pi\right) - \sin\left(x - \tfrac{1}{6}\pi\right) = \cos\left(x + \tfrac{1}{3}\pi\right) - \cos\left(x - \tfrac{1}{3}\pi\right),$$

find the exact value of $\tan x$. [4]

(b) Hence find the exact roots of the equation

$$\sin\left(x + \tfrac{1}{6}\pi\right) - \sin\left(x - \tfrac{1}{6}\pi\right) = \cos\left(x + \tfrac{1}{3}\pi\right) - \cos\left(x - \tfrac{1}{3}\pi\right)$$

for $0 \leqslant x \leqslant 2\pi$. [2]

6 The parametric equations of a curve are

$$x = \sqrt{t} + 3, \qquad y = \ln t,$$

for $t > 0$.

(a) Obtain a simplified expression for $\dfrac{dy}{dx}$ in terms of t. [3]

(b) Hence find the exact coordinates of the point on the curve at which the gradient of the normal is -2. [3]

7 The variables x and θ satisfy the differential equation

$$\frac{x}{\tan\theta}\frac{dx}{d\theta} = x^2 + 3.$$

It is given that $x = 1$ when $\theta = 0$.

Solve the differential equation, obtaining an expression for x^2 in terms of θ. [7]

8 **(a)** By sketching a suitable pair of graphs, show that the equation

$$\sqrt{x} = e^x - 3$$

has only one root. [2]

(b) Show by calculation that this root lies between 1 and 2. [2]

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(c) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \ln\left(3 + \sqrt{x_n}\right)$$

converges, then it converges to the root of the equation in **(a)**. [1]

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(d) Use the iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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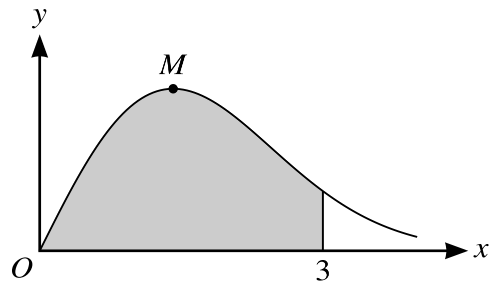


The diagram shows the curve $y = xe^{-\frac{1}{4}x^2}$, for $x \geqslant 0$, and its maximum point M.

(a) Find the exact coordinates of M. [4]

(b) Using the substitution $x = \sqrt{u}$, or otherwise, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 3$. [5]

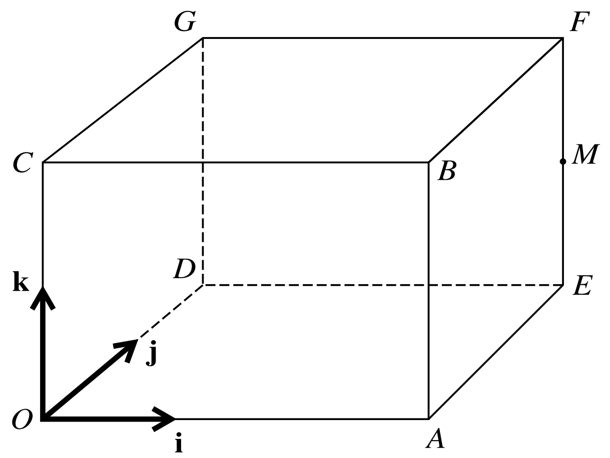
10 Let $f(x) = \dfrac{24x + 13}{(1 - 2x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

(c) State the set of values of x for which the expansion in **(b)** is valid. [1]

11



In the diagram, $OABCDEFG$ is a cuboid in which $OA = 3$ units, $OC = 2$ units and $OD = 2$ units. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OD and OC respectively. M is the midpoint of EF.

(a) Find the position vector of M. [1]

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The position vector of P is $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(b) Calculate angle PAM. [4]

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(c) Find the exact length of the perpendicular from P to the line passing through O and M. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

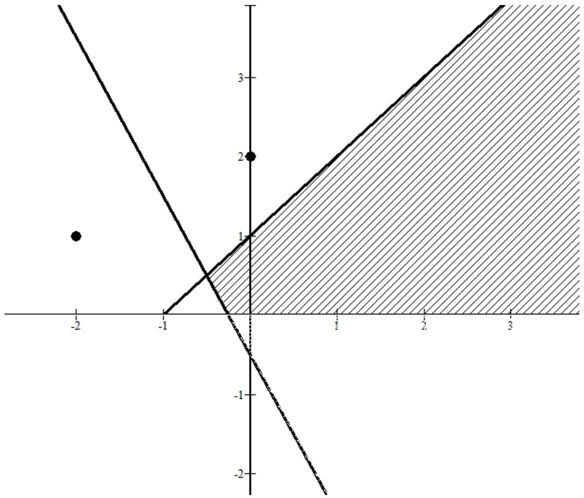
FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use correct quotient or product rule	*M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{(1-3x)2x-x^2(-3)}{(1-3x)^2}\left(=\frac{2x-3x^2}{(1-3x)^2}\right)$ or $3x^2(1-3x)^{-2}+2x(1-3x)^{-1}$.
	Equate derivative to 8 and solve for x	DM1	$75x^2-50x+8=(15x-4)(5x-2)$.
	Obtain answers $x=\frac{2}{5}$ and $\frac{4}{15}$	A1	Exact values required.
	Obtain answers $y=-\frac{4}{5}$ and $\frac{16}{45}$	A1	Allow A1 for one correct point.
		5	

Question	Answer	Marks	Guidance
2	Show points representing 2i and –2 + i	B1	Can be implied if the correct perpendicular is drawn.
	Show perpendicular bisector of *their* (2i and –2 + i)	B1FT	
	Show correct half–line of gradient 1 from point (–1, 0)	B1	Should pass through (0, 1).
	Correct loci and shade correct region	B1	
		4	

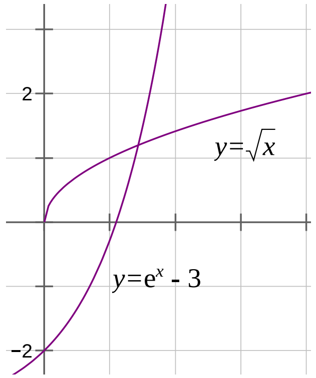
Question	Answer	Marks	Guidance
3	State or imply that $\ln y = \ln a + x \ln b$	**B1**	
	Carry out a completely correct method for finding $\ln a$ or $\ln b$	**M1**	$3.7 = \ln a + \ln b$ and $6.46 = \ln a + 2.2 \ln b$ leading to $\ln a = 1.4$, $\ln b = 2.3$.
	Obtain value $a = 4.06$	**A1**	
	Obtain value $b = 9.97$	**A1**	SC B1 for $a = e^{1.4}$ and $b = e^{2.3}$.
	Alternative Method for Question 3		
	$e^{3.7} = ab^1$ and $e^{6.46} = ab^{2.2}$	**B1**	
	Divide to obtain $e^{2.76} = b^{1.2}$ and state or imply $2.76 = 1.2 \ln b$	**M1**	
	Obtain value $a = 4.06$	**A1**	
	Obtain value $b = 9.97$	**A1**	
		4	

Question	Answer	Marks	Guidance
4(a)	Multiply numerator and denominator by $a + 5\mathrm{i}$	**M1**	OE
	Use $\mathrm{i}^2 = -1$	**M1**	At least once.
	Obtain answer $\dfrac{3a-10}{a^2+25}+\dfrac{2a+15}{a^2+25}\mathrm{i}$	**A1**	
	Alternative Method for Question 4(a)		
	Multiply $x + \mathrm{i}y$ by $a - 5\mathrm{i}$ and use $\mathrm{i}^2 = -1$	**M1**	
	Compare real and imaginary parts	**M1**	$3 = ax + 5y$, $2 = ay - 5x$.
	Obtain answer $\dfrac{3a-10}{a^2+25}+\dfrac{2a+15}{a^2+25}\mathrm{i}$	**A1**	
		3	
4(b)	State or imply Im**(a)** ÷ Re**(a)** = 1	**M1**	Or Im**(a)** = Re**(a)** or equivalent for *their u*.
	Obtain answer $a = 25$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	Use correct trig formulae and obtain an equation in sin x and cos x	*M1	Allow one sign error.
	Obtain a correct equation in any form	A1	e.g $2\cos x \sin\frac{\pi}{6} = -2\sin x \sin\frac{\pi}{3}$.
	Substitute exact trig ratios and obtain an expression for tan x	DM1	Allow one sign error.
	Obtain answer $\tan x = -\dfrac{1}{\sqrt{3}}$	A1	Or exact equivalent.
		4	
5(b)	Obtain answer, e.g. $x = \dfrac{5\pi}{6}$	B1	
	Obtain second answer, e.g. $x = \dfrac{11\pi}{6}$ and no others in the interval	B1FT	FT first answer + π (provided $0 \leqslant$ first answer $\leqslant \pi$). Or FT first answer – π (provided $\pi \leqslant$ first answer $\leqslant 2\pi$). Ignore any answers outside interval.
		2	

Question	Answer	Marks	Guidance
6(a)	State correct derivative of x **or** y with respect to t	**B1**	$\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}, \quad \frac{dy}{dt} = \frac{1}{t}$.
	Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$	**M1**	Use correct chain rule.
	Obtain answer $\frac{dy}{dx} = \frac{2}{\sqrt{t}}$	**A1**	Or simplified equivalent e.g $2t^{\frac{-1}{2}}$ or $\frac{2\sqrt{t}}{t}$.
		3	
6(b)	State or imply *their* $\frac{dy}{dx} = \frac{1}{2}$	**M1**	
	Obtain $\sqrt{t} = 4$	**A1**	Or equivalent.
	Obtain answer (7, ln 16)	**A1**	Or exact equivalent. Can state the two components separately.
		3	

Question	Answer	Marks	Guidance
7	Separate variables correctly	**B1**	$\int \tan\theta \, d\theta = \int \frac{x}{x^2+3} \, dx$. Condone missing integral signs **or** missing dx, dθ. Can be implied by later work.
	Obtain term –ln(cos θ)	**B1**	Or equivalent e.g. ln(sec θ).
	Obtain term of the form $a\ln\left(x^2+3\right)$	**M1**	
	Obtain term $\frac{1}{2}\ln\left(x^2+3\right)$	**A1**	
	Use x = 1, θ = 0 to evaluate a constant or as limits in a solution containing terms of the form $a\ln\left(x^2+3\right)$ and $b\ln(\cos\theta)$	**M1**	If they have rearranged then the constant must be of the correct form.
	Obtain correct answer in any form	**A1**	$\frac{1}{2}\ln\left(x^2+3\right) = -\ln\cos\theta + \ln 2$.
	Obtain final answer $x^2 = \frac{4}{\cos^2\theta} - 3$	**A1**	Or equivalent e.g. $x^2 = 4\sec^2 x - 3$. lns removed.
		7	

Question	Answer	Marks	Guidance
8(a)	Sketch a relevant graph, e.g. $y = e^x - 3$ $y = e^x - 3$ Should cut vertical axis at (0, –2) and have increasing gradient.	**B1**	
	Sketch a second relevant graph, e.g. $y = \sqrt{x}$ and justify the given statement $y = \sqrt{x}$ should start at (0, 0) and have reducing grading	**B1**	Ignore anything outside 1st and 4th quadrants. For second B1 need to mark intersection with a dot, a cross, or say root at point of intersection, or equivalent.
		2	
8(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 2$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	e.g. $1 > -0.28..,\ 1.41 < 4.39..$ $1.28 > 0,\ -2.98 < 0$.
		2	
8(c)	State $x = \ln\left(3 + \sqrt{x}\right)$ and rearrange to the given equation $\sqrt{x} = e^x - 3$	**B1**	Or rearrange $\sqrt{x} = e^x - 3$ to $x = \ln(3 + \sqrt{x})$ and state iterative formula of $x_{n+1} = \ln(3 + \sqrt{x_n})$. AG
		1	

Question	Answer	Marks	Guidance
8(d)	Use the iterative process correctly at least once	**M1**	
	Obtain final answer 1.43	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 1.43 to 2 d.p. or show there is a sign change in the interval (1.425, 1.435) Condone recovery and small differences in the final figure in the iteration	**A1**	e.g. 1, 1.3864, 1.4297, 1.4341, … 1.5, 1.4210, 1.4332, 14.344, 1.4345, … 2, 1.4848, 1.4395, 1.4350, 1.4346, 1.4345, …
		3	

Question	Answer	Marks	Guidance
9(a)	Use the correct product rule	***M1**	Condone error in chain rule.
	Obtain correct derivative in any form	**A1**	e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x^2}{2}\mathrm{e}^{-\frac{x^2}{4}} + \mathrm{e}^{-\frac{x^2}{4}}$.
	Equate derivative to zero and solve for x	**DM1**	
	Obtain answer $\left(\sqrt{2}, \sqrt{2}\mathrm{e}^{-\frac{1}{2}}\right)$	**A1**	Or exact equivalent. Can state the components separately.
		4	

Question	Answer	Marks	Guidance
9(b)	State or imply $dx = \frac{1}{2}u^{-\frac{1}{2}}du$	**B1**	Or equivalent e.g. $du = 2xdx$. Alternative substitution: $u = -\frac{1}{4}x^2$.
	Substitute for x and dx	**M1**	
	Obtain correct integral $\frac{1}{2}\int e^{-\frac{1}{4}u}\,du$	**A1**	OE
	Use correct limits in an integral of the form $ae^{-\frac{1}{4}u}$ or $ae^{-\frac{1}{4}x^2}$	**M1**	$u = 9$ and $u = 0$ or $x = 3$ and $x = 0$.
	Obtain answer $2 - 2e^{-\frac{9}{4}}$	**A1**	Or exact equivalent.
	Alternative Method for Question 9(b)		
	$\int xe^{-\frac{1}{4}x^2}dx = ae^{-\frac{1}{4}x^2}$	**M1**	Recognition used.
	a negative	**A1**	
	$a = -2$	**A1**	
	Use correct limits in an integral of the form $ae^{-\frac{1}{4}x^2}$	**M1**	$x = 3$ and $x = 0$.
	Obtain answer $2 - 2e^{-\frac{9}{4}}$	**A1**	Or exact equivalent.
		5	

Question	Answer	Marks	Guidance
10(a)	State or imply the form $\dfrac{A}{1-2x}+\dfrac{B}{2+x}+\dfrac{C}{(2+x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	$A(2 + x)^2 + B(1 - 2x)(2 + x) + C(1 - 2x)$ $= 24x + 13$.
	Obtain one of $A = 4$, $B = 2$ and $C = -7$	**A1**	If errors in equating still allow A marks for A and C.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Mark the form $\dfrac{A}{1-2x}$ + $\dfrac{Dx+E}{(2+x)^2}$, where $A = 4$, $D = 2$ and $E = -3$, B1 M1 A1 A1 A1 as above. If there are extra term in partial fractions, that is 4 unknowns A, B, D and E then B0 unless recover at end, e.g. by setting $B = 0$. If B set to any value other than 0 and all coefficients correctly found to their new values then allow all A marks, but still B0 for partial fraction expression. Hence A1 for each coefficient, but nothing for coefficient set to specific value. Another case of extra term in partial fraction expression, namely + F, mark as above but need $F = 0$ to recover B1.
		5	

Question	Answer	Marks	Guidance
10(b)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2+x)^{-1}$, $(2+x)^{-2}$, $\left(1+\frac{x}{2}\right)^{-1}$ or $\left(1+\frac{x}{2}\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1.
	Obtain correct un-simplified expansions up to the term in x^2 of each partial fraction	**A1 FT**	$A\left(1+(-1)(-2x)+\frac{(-1)(-2)}{2}(-2x)^2+..\right)$ A = 4.
		A1 FT	$\frac{B}{2}\left(1+(-1)\left(\frac{x}{2}\right)+\frac{(-1)(-2)}{2}\left(\frac{x}{2}\right)^2+...\right)$ B = 2.
		A1 FT	$\frac{C}{4}\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2}\left(\frac{x}{2}\right)^2+...\right)$ C = –7 = 4(1 + 2x + 4x^2) + 2/2(1 – x/2 + x^2/4) – 7/4(1 – x + 3x^2/4) = (4 + 1 – 7/4) + (8 – 1/2 + 7/4) x + (16 + 1/4 – 21/16) x^2 The FT is on A, B, C.

Question	Answer	Marks	Guidance
	Obtain final answer $\frac{13}{4}+\frac{37}{4}x+\frac{239}{16}x^2$	**A1**	OE $(Dx + E)/4\ [1 + (-2)(x/2) + (-2)(-3)(x/2)^2/2\ \dots]$ $D = 2 \quad E = -3$ The FT is on A, D, E. Maclaurin's Series f(0) = 13/4 B1 f'(0) = 37/4 B1 f''(0) = 239/8 B1. $\frac{13}{4}+\frac{37}{4}x+\frac{239}{8}x^2/2$ or equivalent M1 A1. If $1+\frac{37}{4}x+\frac{239}{8}x^2/2$ then M0 A0 unless *their* f(0) actually is 1. For the A, D, E form of fractions, give M1 A1FT A1FT for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer. If final answer has been multiplied throughout (e.g. by 16) then A0 at the end
		5	
10(c)	$\lvert x\rvert < \frac{1}{2}$	**B1**	OE
		1	

Question	Answer	Marks	Guidance
11(a)	Obtain 3**i** + 2**j** + **k**	**B1**	Accept coordinates in place of position vector.
		1	

Question	Answer	Marks	Guidance
11(b)	$\overrightarrow{AM}$ or $\overrightarrow{AP}$ correct soi	**B1**	$\overrightarrow{AM} = 2\mathbf{j} + \mathbf{k}$, or $\overrightarrow{AP} = -2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
	Carry out correct process for evaluating the scalar product of $\overrightarrow{AM}$ and $\overrightarrow{AP}$	**M1**	or $\overrightarrow{MA}$ and $\overrightarrow{PA}$: $0+2+2$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain the inverse cosine of the result	**M1**	For their vectors. $\theta = \cos^{-1}\left(\frac{4}{3\sqrt{5}}\right)$.
	Obtain answer 53.4° or 0.932^c	**A1**	
		4	
11(c)	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on the line passing through O and M,	**B1 FT**	e.g. ***PQ*** = - (**i** + **j** + 2**k**) + μ(3**i** + 2**j** + **k**). Follow *their M*.
	Calculate the scalar product of $\overrightarrow{PQ}$ and a direction vector for the line passing through O and M and equate to zero	***M1**	
	Solve and obtain correct solution e.g. $\mu = -\frac{1}{2}$	**A1**	
	Carry out method to calculate PQ	**DM1**	$\sqrt{.5^2+0+1.5^2}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	**A1**	Or exact equivalent.
	Alternative Method 1 for Question 11(c)		
	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on the line passing through O and M,	**B1 FT**	e.g. ***PQ*** = - (**i** + **j** + 2**k**) + μ(3**i** + 2**j** + **k**). Follow *their M*.
	Use a correct method to express PQ^2 (or PQ) in terms of μ	***M1**	
	Obtain a correct equation in any form	**A1**	e.g. $PQ^2 = (1 + 3\mu)^2 + (1 + 2\mu)^2 + (2 + \mu)^2$

Question	Answer	Marks	Guidance
11(c)	Carry out a complete method for finding its minimum	**DM1**	e.g. $6(1+3\mu)+4(1+2\mu)+2(2+\mu)=0,\ \mu=-\frac{1}{2}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	**A1**	Or exact equivalent.
	Alternative Method 2 for Question 11(c)		
	Calling (0, 0, 0) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i}+\mathbf{j}+2\mathbf{k}$	**B1**	
	Use a scalar product to find the projection of $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) on the line passing through O and M	**M1**	
	Obtain correct answer $\frac{7}{\sqrt{14}}$	**A1**	OE
	Use Pythagoras to find the perpendicular	**M1**	$d=\sqrt{AP^2-AQ^2}=\sqrt{1+1+2^2-\left(\frac{7}{\sqrt{14}}\right)^2}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	**A1**	Or exact equivalent.
	Alternative Method 3 for Question 11(c)		
	Calling (0, 0, 0) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i}+\mathbf{j}+2\mathbf{k}$	**B1**	
	Calculate the vector product of $\overrightarrow{PA}$ and a direction vector for the line passing through O and M	**M1**	
	Obtain correct answer, e.g. $3\mathbf{i}-5\mathbf{j}+\mathbf{k}$	**A1**	

Question	Answer	Marks	Guidance
11(c)	Divide modulus of the product by that of the direction vector	**M1**	e.g. $\frac{\sqrt{3^2+5^2+1^2}}{\sqrt{3^2+2^2+1^2}}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	**A1**	Or exact equivalent.
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 9 1 4 6 9 4 9 6 4 0 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Sketch the graph of $y = |4x - 2|$. [1]

(b) Solve the inequality $1 + 3x < |4x - 2|$. [4]

2 The parametric equations of a curve are

$$x = (\ln t)^2, \qquad y = e^{2-t^2},$$

for $t > 0$.

Find the gradient of the curve at the point where $t = e$, simplifying your answer. [4]

3 The polynomial $2x^3 + ax^2 - 11x + b$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 12.

Find the values of a and b. [5]

4 **(a)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 4 - 3\mathrm{i}| \leqslant 2$ and $\operatorname{Re} z \leqslant 3$. [4]

(b) Find the greatest value of $\arg z$ for points in this region. [2]

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5 Find the exact value of $\displaystyle\int_0^6 \frac{x(x+1)}{x^2+4}\,dx$. [6]

6 **(a)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 2 - \cos x$$

has one root in the interval $0 < x \leqslant \frac{1}{2}\pi$. [2]

(b) Show by calculation that this root lies between 0.6 and 0.8. [2]

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(c) Use the iterative formula $x_{n+1} = \tan^{-1}\left(\dfrac{1}{2 - \cos x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **(a)** By expressing 3θ as $2\theta + \theta$, prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [3]

(b) Hence solve the equation

$$\cos 3\theta + \cos\theta\cos 2\theta = \cos^2\theta$$

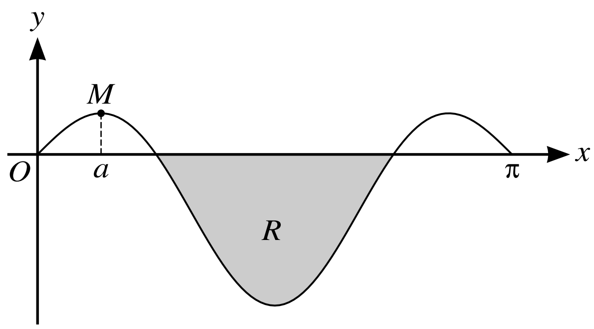
for $0° \leqslant \theta \leqslant 180°$. [5]

8 It is given that $\dfrac{2 + 3a\text{i}}{a + 2\text{i}} = \lambda(2 - \text{i})$, where a and λ are real constants.

(a) Show that $3a^2 + 4a - 4 = 0$. [4]

(b) Hence find the possible values of a and the corresponding values of λ. [3]

9



The diagram shows the curve $y = \sin x \cos 2x$, for $0 \leqslant x \leqslant \pi$, and a maximum point M, where $x = a$. The shaded region between the curve and the x-axis is denoted by R.

(a) Find the value of a correct to 2 decimal places. [5]

(b) Find the exact area of the region R, giving your answer in simplified form. [4]

10 The equations of the lines l and m are given by

$$l: \ \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \qquad \text{and} \qquad m: \ \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 4 \\ c \end{pmatrix},$$

where c is a positive constant. It is given that the angle between l and m is 60°.

(a) Find the value of c. [4]

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(b) Show that the length of the perpendicular from $(6, -3, 6)$ to l is $\sqrt{11}$. [5]

11 The variables x and y satisfy the differential equation

$$x^2\frac{dy}{dx} + y^2 + y = 0.$$

It is given that $x = 1$ when $y = 1$.

(a) Solve the differential equation to obtain an expression for y in terms of x. [8]

(b) State what happens to the value of y when x tends to infinity. Give your answer in an exact form. [1]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2. Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5. Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

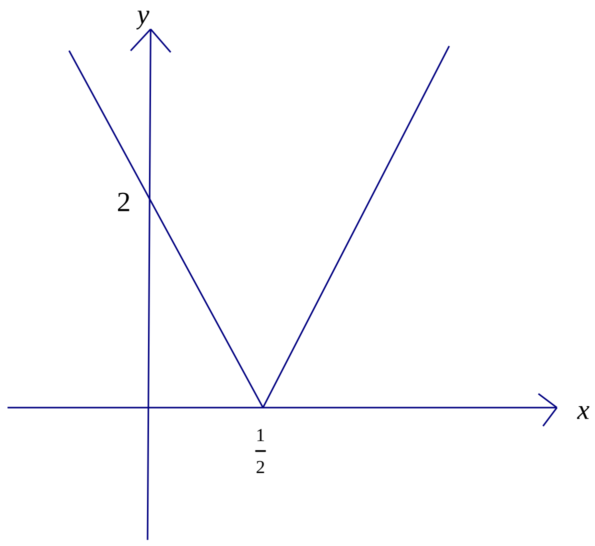
DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

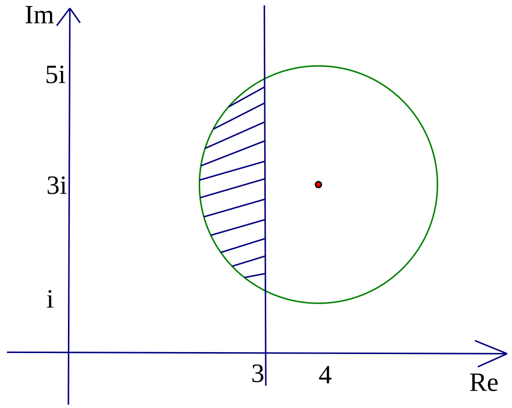
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

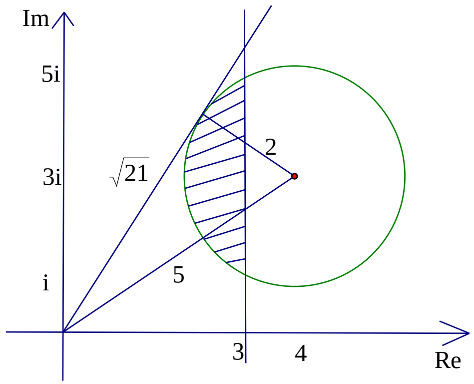
Question	Answer	Marks	Guidance
1(a)		**B1**	Show a recognizable sketch graph of $y = \lvert 4x - 2 \rvert$. Roughly symmetrical. Should extend into the second quadrant. Ignore $y = 4x - 2$ below the axis if intention is clear e.g. dashed or the required lines are clearly bolder. Some indication of scale on **both** axes – accept dashes. Must go beyond (0, 2) and (1, 2). Ignore any attempt to sketch $y = 1 + 3x$.
		1	

Question	Answer	Marks	Guidance
1(b)	Obtain critical value $x = 3$	**B1**	Allow incorrect inequality. Allow if later rejected. Allow $\frac{21}{7}$.
	Solve the linear equation $1 + 3x = 2 - 4x$	**M1**	Or corresponding linear inequality.
	Obtain critical value $\frac{1}{7}$	**A1**	Allow 0.143 or better. Allow incorrect inequality. Allow if later rejected.
	Obtain final answer $x < \frac{1}{7}$ [or] $x > 3$	**A1**	Or equivalent. Allow with a comma, or nothing between. Strict inequalities only. Exact values. A0 for $\frac{1}{7} > x > 3$ A0 for $x < \frac{1}{7}$ **and** $x > 3$.
	Alternative method for question 1(b)		
	Solve the quadratic inequality $(4x-2)^2 > (1+3x)^2$, or corresponding quadratic equation	**M1**	e.g. $7x^2 - 22x + 3 = 0$. Available if they start with the correct equation / inequality, have a correct method for squaring (i.e. not $(a+b)^2 = a^2 + b^2$) and a correct method for solving. Need to obtain at least one critical value.
	Obtain critical value $x = 3$	**A1**	Allow incorrect inequality. Allow if later rejected. Allow $\frac{21}{7}$.
	Obtain critical value $\frac{1}{7}$	**A1**	Allow 0.143 or better. Allow incorrect inequality. Allow if later rejected.
	Obtain final answer $x < \frac{1}{7}$ [or] $x > 3$	**A1**	Or equivalent. Strict inequalities only. Allow with a comma, or nothing between. Exact values. A0 for $\frac{1}{7} > x > 3$ A0 for $x < \frac{1}{7}$ and $x > 3$.
		4	

Question	Answer	Marks	Guidance
2	Obtain $\dfrac{dx}{dt} = \dfrac{2}{t}\ln t$	**B1**	Any equivalent form.
	Obtain $\dfrac{dy}{dt} = -2te^{2-t^2}$	**B1**	Any equivalent form.
	$\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$ and substitute $t = e$	**M1**	Correct use of chain rule for $\dfrac{dy}{dx}\left(\dfrac{-2e^2e^{2-e^2}}{2\ln e}\right)$. Condone an error between correct combination of the derivatives and attempt to substitute e.
	Obtain $-e^{4-e^2}$	**A1**	ISW Accept $-0.0337(405..)$. Accept $-e^4e^{-e^2}$, $\dfrac{-e^4}{e^{e^2}}$ and $-e^2e^{2-e^2}$. Allow M1A1 for a correct decimal answer following B1B1 seen.
		4	

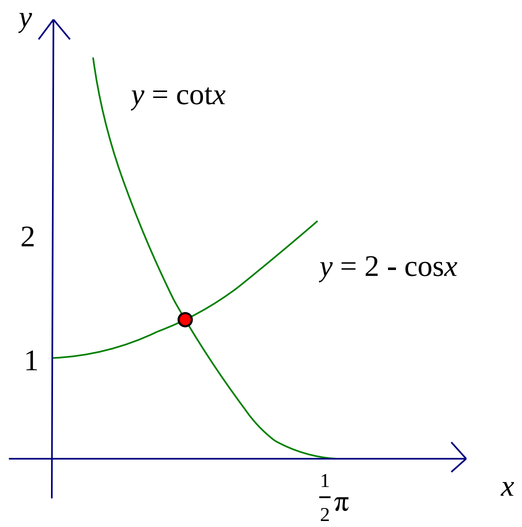
Question	Answer	Marks	Guidance
3	Substitute $x = \frac{1}{2}$ and equate the result to zero	**M1**	Or divide as far as $(2x-1)(x^2+px+q)$ and equate constant remainder to zero.
	Obtain a correct equation with powers evaluated	**A1**	e.g. $\frac{1}{4}+\frac{a}{4}-\frac{11}{2}+b=0$ or $a+4b=21$.
	Substitute $x=-1$ and equate result to 12	**M1**	Or divide as far as $(x+1)(2x^2+rx+s)$ and equate constant remainder to 12.
	Obtain a correct equation with powers evaluated	**A1**	e.g. $-2+a+11+b=12$ or $a+b=3$.
	Obtain $a=-3, \quad b=6$	**A1**	
		5	
	$$\begin{array}{r\|llll} & & x^2 & \left(\frac{a+1}{2}\right)x & \frac{1}{2}\left(-\frac{21}{2}+\frac{a}{2}\right) \\ \hline 2x-1 & 2x^3 & ax^2 & -11x & b \\ & 2x^3 & -x^2 & & \\ \hline & & (a+1)x^2 & -11x & \\ & & (a+1)x^2 & -\left(\frac{a+1}{2}\right)x & \\ \hline & & & \left(-\frac{21}{2}+\frac{a}{2}\right)x & b \\ & & & \left(-\frac{21}{2}+\frac{a}{2}\right)x & -\frac{1}{2}\left(-\frac{21}{2}+\frac{a}{2}\right) \\ \hline & & & & b-\frac{21}{4}+\frac{a}{4} \\ \hline \end{array}$$		$$\begin{array}{r\|llll} & & 2x^2 & (a-2)x & (-9-a) \\ \hline x+1 & 2x^3 & ax^2 & -11x & b \\ & 2x^3 & 2x^2 & & \\ \hline & & (a-2)x^2 & -11x & \\ & & (a-2)x^2 & (a-2)x & \\ \hline & & & (-9-a)x & b \\ & & & (-9-a)x & (-9-a) \\ \hline & & & & b+9+a \\ \hline \end{array}$$

Question	Answer	Marks	Guidance
4(a)	Show a circle with centre $4+3\mathrm{i}$. Accept a curved shape with correct point roughly in the middle.	**B1**	
	Show a circle with radius 2 and centre not at the origin. The shape should be consistent with their scales	**B1**	
	Show correct vertical line. Enough to meet correct circle twice or complete line for any other circle.	**B1**	
	Shade the correct region on a correct diagram Any other shading must be accompanied by words to explain which region is required	**B1**	Need some indication of scale e.g. label the centre, mark key points on the axes or dashes on the axes. Condone dotted lines in place of solid lines Condone correct shaded shape but not an entire circle
		4	

Question	Answer	Marks	Guidance
4(b)	Carry out a complete method for finding the greatest value of $\arg(z)$ e.g $\tan^{-1}\frac{3}{4}+\sin^{-1}\frac{2}{5}$ $(0.6435+0.4115)$	**M1**	
	Obtain answer 1.06 (accept 1.055 or 1.056) radians or $60.45°$ (accept $60.4°$ or $60.5°$)	**A1**	
	Alternative method for question 4(b)		
	Tangent to circle passing through origin has equation $y=mx$. The equation $(x-4)^2+(y-3)^2=4$ will have one root. Hence $(1+m^2)x^2-(8+6m)x+21=0$, discriminant = 0 = $48m^2-96m+20$ and $m=\dfrac{6\pm\sqrt{21}}{6}$ with the larger value needed to give greatest arg(z). Required angle is $\tan^{-1}m$.	**M1**	Complete method for finding the greatest value of arg(z).
	Obtain answer 1.06 radians or $60.45°$	**A1**	Accept 1.055 or 1.056 radians. Accept $60.4°$ or $60.5°$.
		2	

Question	Answer	Marks	Guidance
5	Split fraction to obtain $1+\dfrac{x-4}{x^2+4}$	**B1**	
	Attempt integration and obtain $p\ln\left(x^2+4\right)$ or $q\tan^{-1}\left(\dfrac{x}{2}\right)$ from correct working	**M1**	Allow for $p\ln\left(x^2+4\right)$ from $\int\dfrac{x}{x^2+4}\mathrm{d}x$ but only if a correct method for splitting has been used.
	Obtain $\frac{1}{2}\ln\left(x^2+4\right)$	**A1 FT**	Follow through is on their coefficients in the partial fraction. Allow from $\dfrac{x^2}{x^2+4}+\dfrac{x}{x^2+4}$ even if the split of the fraction is not complete. If $1-\dfrac{4}{x^2+4}+\dfrac{x}{x^2+4}$ later seen or implied, award the B1. Only available from a correct split, not from an approach using parts that is incomplete.
	Obtain $-2\tan^{-1}\left(\dfrac{x}{2}\right)$	**A1 FT**	Only available from a correct split, not from an approach using parts that is incomplete.
	Correct use of correct limits 0 and 6 in an expression involving $p\ln\left(x^2+4\right)$, $q\tan^{-1}\left(\dfrac{x}{2}\right)$ and no incorrect terms.	**M1**	p and q should be constants. The x term is not required at this stage.
	Obtain $6+\frac{1}{2}\ln 10-2\tan^{-1}3$	**A1**	ISW Or three term equivalent. (Must combine the ln terms.) Accept with $\frac{1}{2}\ln\lvert 10\rvert$.

Question	Answer	Marks	Guidance
5	**Alternative method for question 5**		
	Use the substitution $x = 2\tan\theta$ to obtain $\int 2\tan^2\theta + \tan\theta \, d\theta$	**B1**	
	Attempt integration and obtain $p\tan\theta$ or $r\ln(\sec\theta)$ from correct working	**M1**	
	Obtain $2\tan\theta(-2\theta)$ and	**A1 FT**	Follow through on *their* coefficients after the substitution.
	Obtain $\ln\sec\theta$	**A1 FT**	Follow through on *their* coefficients after the substitution.
	Use correct limits 0 and $\tan^{-1} 3$in an expression involving $u \tan\theta$, $v \ln\sec\theta$ and no incorrect terms	**M1**	u and v should be constants. The θ term is not required at this stage.
	Obtain $6 + \ln\left\|\sec\left(\tan^{-1} 3\right)\right\| - 2\tan^{-1} 3$	**A1**	ISW Or three term equivalent. Not required to simplify $\ln\left\|\sec\left(\tan^{-1} 3\right)\right\|$.
		6	

Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph. e.g. $y = \cot x$: x intercept should be correct. Not touching the y-axis. No incorrect curvature. Ignore anything outside $0 < x \leqslant \frac{1}{2}\pi$.	**B1**	
	Sketch a second relevant graph **and justify the given statement** e.g. $y = 2 - \cos x$: Condone if looks almost straight, but not if drawn with a ruler and not incorrect curvature. Correct y intercept. Needs to be drawn for $0 < x \leqslant \frac{1}{2}\pi$. Ignore outside this.	**B1**	
		2	2nd B1 requires a mark at the point of intersection or a suitable comment for the justification.
6(b)	Calculate the value of a relevant expression or values of a pair of expressions at $x = 0.6$ and $x = 0.8$ Must be working in radians. Values correct to at least 2 significant figures. Need all relevant values but only one (pair) needs to be correct to award M1. Complete set of values for their expression. If not comparing with 0 or 1 then the pairing must be clear, not just embedded values.	**M1**	e.g. $1.17 < 1.46$, $1.30 > 0.971$, $-0.29 < 0$, $0.33 > 0$. $-0.20 < 0$, $0.342 > 0$ from $\tan x(2 - \cos x) - 1 = 0$. $0.80 < 1$, $1.34 > 1$ from $\tan x(2 - \cos x) = 1$. $0.146 > 0, -0.105 < 0$ from $x - \tan^{-1}\left(\frac{1}{2-\cos x}\right)$.
	Complete the argument correctly with correct calculated values (awrt 2 s.f.). Clear comparison for their expression. Allow work on a smaller interval.	**A1**	Accept truncated values. If comparing with 0 can either indicate different signs or a negative product.
		2	

Question	Answer	Marks	Guidance
6(c)	Use the iterative process correctly at least once. Must be working in radians	**M1**	
	Obtain final answer 0.68	**A1**	Must be a clear conclusion.
	Show sufficient iterations to at least 4 d.p. to justify 0.68 to 2 d.p. or show there is a sign change in the interval (0.675, 0.685). Allow recovery. Allow truncation. Allow small differences in the 4th s.f.	**A1**	e.g. 0.7, 0.6806, 0.6855, 0.6843, 0.6846 0.6, 0.7053, 0.6792, 0.6858, 0.6842, 0.6846 0.8, 0.6545, 0.6920, 0.6826, 0.6850, 0.6844, 0.6845 .
		3	

Question	Answer	Marks	Guidance
7(a)	Use correct expansion for $\cos(2\theta+\theta)$	***M1**	
	Use correct double angle formulae to express $\cos 3\theta$ in terms of $\cos\theta$ and $\sin\theta$	**DM1**	
	Show sufficient working to confirm $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$	**A1**	AG
		3	

Question	Answer	Marks	Guidance
7(b)	Use the identity and correct double angle formula to obtain an equation in $\cos\theta$ only. Must come from using all three terms in the given equation.	***M1**	e.g. $4\cos^3\theta - 3\cos\theta + \cos\theta(2\cos^2\theta - 1) = \cos^2\theta$ $6\cos^3\theta - \cos^2\theta - 4\cos\theta = 0$ or $6\cos^2\theta - \cos\theta - 4 = 0$.
	Obtain $\theta = 90°$	**B1**	Allow if $\cos\theta$ obtained correctly as a factor of *their* expression (even if there is an error in the quadratic factor). Can follow M0.
	Solve a 3-term quadratic in $\cos\theta$ to obtain a value of θ	**DM1**	
	Obtain one value e.g. $25.3°$	**A1**	Accept awrt $25.3°$.
	Obtain a second value e.g. $137.5°$ and no extras in range	**A1**	Accept awrt $137.5°$. Ignore values outside the range. Mark solutions in radians as a misread $(0.442, 1.57, 2.40)$.
		5	

Question	Answer	Marks	Guidance
8(a)	Multiply both sides by $a+2\text{i}$ and attempt expansion of right-hand side	*M1	
	Use of $\text{i}^2=-1$ seen at least once (or implied)	DM1	e.g. $2+3a\text{i}=\lambda(2a+2)+\lambda\text{i}(-a+4)$
	Compare real and imaginary parts to obtain an equation in *a* only $[2=\lambda(2a+2),\quad 3a=\lambda(-a+4)]$	M1	e.g. $\dfrac{3a}{2}=\dfrac{-a+4}{2a+2}$. Any equivalent form.
	Obtain $3a^2+4a-4=0$ from correct working	A1	AG
	Alternative method for question 8(a)		
	Multiply top and bottom of the left-hand side by $a-2\text{i}$ and attempt both expansions	*M1	Do not need the right-hand side at this stage.
	Use of $\text{i}^2=-1$ seen at least once or implied	DM1	e.g. $[\lambda(2-\text{i})=]\dfrac{8a+\text{i}(3a^2-4)}{a^2+4}$.
	Compare real and imaginary parts to obtain an equation in *a* only	M1	e.g. $8a=-2(3a^2-4)$. Any equivalent form.
	Obtain $3a^2+4a-4=0$ from correct working	A1	AG
		4	
8(b)	Solve given quadratic to obtain a value of *a* and use this to form an equation in λ only (based on an equation seen in *their* working in **(a)** or **(b)**)	M1	Can be implied by relevant working seen or a correct value for λ seen.
	Obtain $a=-2,\ \lambda=-1$ or $a=\frac{2}{3},\ \lambda=\frac{3}{5}$	A1	Allow $\frac{6}{10}$ and 0.6.
	Obtain second correct pair of values	A1	
		3	

Question	Answer	Marks	Guidance
9(a)	Use correct product rule	*M1	As far as $p\cos x\cos 2x + q\sin x\sin 2x$ or full working (u, v, du/dx, dv/dx) shown.
	Obtain $\frac{dy}{dx} = \cos x\cos 2x - 2\sin x\sin 2x$	A1	OE
	Equate derivative to zero and use correct double angle formulae	DM1	Allow if only have one double angle in their derivative.
	Obtain $\cos x\left(1-6\sin^2 x\right)=0$ or equivalent	A1	e.g. $\cos x\left(6\cos^2 x-5\right)=0$, $5\tan^2 x=1$. Simplified but not necessarily factorised - like terms must be collected.
	Obtain $a = 0.42$	A1	Only. Accept $x = 0.42$.
	Alternative method for question 9(a)		
	Use correct double angle formula	*M1	
	Obtain $\sin x - 2\sin^3 x$ or equivalent	A1	
	Use correct chain rule or product rule to differentiate and equate the derivative to zero	DM1	
	Obtain $\cos x\left(1-6\sin^2 x\right)=0$	A1	OE
	Obtain $a = 0.42$	A1	Only. Accept $x = 0.42$.
		5	

Question	Answer	Marks	Guidance
9(b)	Use double angle formula and obtain $p\cos^3 x + q\cos x$ correctly	*M1	e.g. from $\int 2\cos^2 x\sin x - \sin x \, dx$.
	Obtain $\pm\left(-\frac{2}{3}\cos^3 x + \cos x\right)$	A1	Correct for *their* integral.
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	DM1	OE $\pm\left(-\frac{2}{3}\left[\left(\frac{-1}{\sqrt{2}}\right)^3 - \left(\frac{1}{\sqrt{2}}\right)^3\right] - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$.
	Obtain $\frac{2\sqrt{2}}{3}$	A1	Or simplified exact equivalent. Final answer must be positive.
	Alternative method 1 for question 9(b)		
	Use integration by parts **twice** and obtain $r\cos x\cos 2x + s\sin x\sin 2x$	*M1	Seen, not just implied.
	Obtain $\frac{1}{3}\cos x\cos 2x + \frac{2}{3}\sin x\sin 2x$	A1	Accept $\pm$ (correct for *their* integral).
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	DM1	OE $\pm\frac{1}{3}\left(0 + 2\times\frac{1}{\sqrt{2}}\times -1 - 0 - 2\times\frac{1}{\sqrt{2}}\times 1\right)$.
	Obtain $\frac{2\sqrt{2}}{3}$	A1	Or simplified exact equivalent. Final answer must be positive.

Question	Answer	Marks	Guidance
	Alternative method 2 for question 9(b)		
	Use factor formula and integrate to obtain $g\cos 3x + h\cos x$	***M1**	$\int \frac{1}{2}(\sin 3x - \sin x)\,dx$.
	Obtain $\pm\left(-\frac{1}{6}\cos 3x + \frac{1}{2}\cos x\right)$	**A1**	Correct for *their* integral.
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	**DM1**	OE $\mp\frac{1}{\sqrt{2}}\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{6}+\frac{1}{2}\right)$.
	Obtain $\dfrac{2\sqrt{2}}{3}$	**A1**	Or exact equivalent. Final answer must be positive.
		4	

Question	Answer	Marks	Guidance
10(a)	Use the correct process to calculate the scalar product of the direction vectors	**M1**	$(-2+4+2c)$.
	Divide the scalar product by the product of the moduli and equate the result to $\cos 60°$	**M1**	Or equivalent e.g. $2+2c=\sqrt{6}\sqrt{20+c^2}\cos 60°$. Allow for the correct process using $60°$ but the wrong vectors.
	Obtain correct equation in c	**A1**	e.g. $\dfrac{2+2c}{\sqrt{6}\sqrt{20+c^2}}=\dfrac{1}{2}$ or $10c^2+32c-104=0$.
	Obtain $c=2$	**A1**	Only.
		4	

Question	Answer	Marks	Guidance
10(b)	Calling $(6, -3, 6)$ *A*, find $\overrightarrow{AP}$ for a general point *P* on *l*	**B1**	e.g. $\begin{pmatrix} -3+\lambda \\ 1+\lambda \\ -5+2\lambda \end{pmatrix}$.
	Equate the scalar product of *their* $\overrightarrow{AP}$ and a direction vector for *l* to zero and obtain an equation in λ	***M1**	e.g. $(-3+\lambda)+(1+\lambda)+(-10+4\lambda)=0$.
	Solve and obtain $\lambda = 2$	**A1**	
	Carry out a method to calculate $\lvert\overrightarrow{AP}\rvert$	**DM1**	e.g. $(-1)^2+3^2+(-1)^2$ or $1^2+3^2+1^2$.
	Obtain $\sqrt{11}$ from correct working	**A1**	AG
	Alternative method 1 for question 10(b)		
	Calling $(6, -3, 6)$ *A*, find $\overrightarrow{AP}$ for a general point *P* on *l*	**B1**	e.g. $\begin{pmatrix} -3+\lambda \\ 1+\lambda \\ -5+2\lambda \end{pmatrix}$
	Differentiate the modulus of $\overrightarrow{AP}$ or the square of the modulus and equate the derivative to zero	***M1**	e.g. $2(-3+\lambda)+2(1+\lambda)+4(-5+2\lambda)=0$
	Solve and obtain $\lambda = 2$	**A1**	
	Carry out a method to calculate $\lvert\overrightarrow{AP}\rvert$	**DM1**	e.g. $(-1)^2+3^2+(-1)^2$ or $1^2+3^2+1^2$
	Obtain $\sqrt{11}$ from correct working	**A1**	AG

Question	Answer	Marks	Guidance
	Alternative method 2 for question 10(b)		
	Vector from $(6,-3,6)$ to $(3,-2,1)$ is $-3\mathbf{i}+\mathbf{j}-5\mathbf{k}$	**B1**	The method works for vector from $(6,-3,6)$ to any point on *l*.
	Use scalar product to find the angle between *their* vector and the direction of *l*	**M1**	
	Obtain $\cos\theta=\dfrac{3-1+10}{\sqrt{35}\sqrt{6}}\left(=\sqrt{\dfrac{24}{35}}\right)$ or $\sin\theta=\sqrt{\dfrac{11}{35}}$	**A1**	
	Correct use of trig to find the projection of their vector on the normal to *l*	**M1**	$\sqrt{35}\sin\theta=\sqrt{35}\times\sqrt{\dfrac{11}{35}}$.
	Obtain $\sqrt{11}$ from correct working	**A1**	AG
	Alternative method 3 for question 10(c)		
	Vector from $(6,-3,6)$ to $(3,-2,1)$ is $-3\mathbf{i}+\mathbf{j}-5\mathbf{k}$	**B1**	
	Find the vector product of *their* vector and the direction of *l*	**M1**	
	Obtain $\mathbf{i}(2+5)-\mathbf{j}(-6+5)+\mathbf{k}(-3-1)\,(=7\mathbf{i}+\mathbf{j}-4\mathbf{k})$	**A1**	
	Correct use of trig to find the perpendicular distance	**M1**	$\dfrac{\lvert \textit{their}\text{ vector product}\rvert}{\lvert\text{direction vector}\rvert}$.
	Distance $=\dfrac{\sqrt{66}}{\sqrt{6}}=\sqrt{11}$	**A1**	
		5	

Question	Answer	Marks	Guidance
11(a)	Correct separation of variables.	**B1**	$\int \frac{1}{y^2+y}\,dy = \int -\frac{1}{x^2}\,dx$. Condone missing integral signs or missing dx, dy, but not both.
	Obtain $\frac{1}{x}$	**B1**	
	Express $\frac{1}{y^2+y}$ in partial fractions or express the denominator of the fraction as a difference of two squares	***M1**	Allow for the correct split of $\frac{\pm 1}{(y^2 \pm y)}$.
	Obtain $\frac{1}{y} - \frac{1}{y+1}$ or $\frac{1}{\left(y+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$	**A1**	Allow if coefficients for the partial fractions are correct but followed by an error.
	Obtain $\ln y - \ln(y+1)$	**A1**	Or equivalent, dependent on where they left the minus sign.
	Use $x=1, y=1$ to find constant of integration or as limits in a definite integral in an expression containing terms of the form $\frac{p}{x}, q\ln y$ and $r\ln(1+y)$	**DM1**	$\ln\frac{1}{2} = 1 + C$. If they rearrange the equation before finding the constant of integration then the constant must be of the correct form.
	Correct equation in x and y	**A1**	$\ln\frac{y}{1+y} = \frac{1}{x} - 1 + \ln\frac{1}{2}$.
	Obtain $y = \frac{e^{\frac{1}{x}-1}}{2 - e^{\frac{1}{x}-1}}$	**A1**	Or equivalent e.g. $y = \frac{1}{2e^{1-\frac{1}{x}} - 1}$, $y = \frac{1}{e^{1-\frac{1}{x}+\ln 2} - 1}$. Accept with decimal value for e^{-1}.
		8	

Question	Answer	Marks	Guidance
11(b)	State that y approaches $\dfrac{1}{2\mathrm{e}-1}$	**B1 FT**	Or exact equivalent. Condone $y=\dfrac{1}{2\mathrm{e}-1}$. FT on an expression in $\mathrm{e}^{\frac{1}{x}}$.
		1	

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* 7 3 5 4 6 4 7 7 5 5 *

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find the set of values of x satisfying the inequality $\left|2^{x+1} - 2\right| < 0.5$, giving your answer to 3 significant figures. [4]

2 On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 1 + 2\mathrm{i}| \leqslant |z|$ and $|z - 2| \leqslant 1$. [5]

3 The polynomial $2x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $(x + 2)$ the remainder is -38 and when $p(x)$ is divided by $(2x - 1)$ the remainder is $\frac{19}{2}$.

Find the values of a and b. [5]

4 Solve the quadratic equation $(3 + \mathrm{i})w^2 - 2w + 3 - \mathrm{i} = 0$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

5 Find the exact coordinates of the stationary points of the curve $y = \dfrac{e^{3x^2-1}}{1-x^2}$. [6]

6 **(a)** Show that the equation $\cot^2\theta + 2\cos 2\theta = 4$ can be written in the form

$$4\sin^4\theta + 3\sin^2\theta - 1 = 0.$$ [3]

(b) Hence solve the equation $\cot^2\theta + 2\cos 2\theta = 4$, for $0° < \theta < 360°$. [3]

7 The equation of a curve is $x^3 + y^2 + 3x^2 + 3y = 4$.

(a) Show that $\dfrac{dy}{dx} = -\dfrac{3x^2 + 6x}{2y + 3}$. [3]

(b) Hence find the coordinates of the points on the curve at which the tangent is parallel to the x-axis. [5]

8 The variables x and y satisfy the differential equation

$$e^{4x}\frac{dy}{dx} = \cos^2 3y.$$

It is given that $y = 0$ when $x = 2$.

Solve the differential equation, obtaining an expression for y in terms of x. [7]

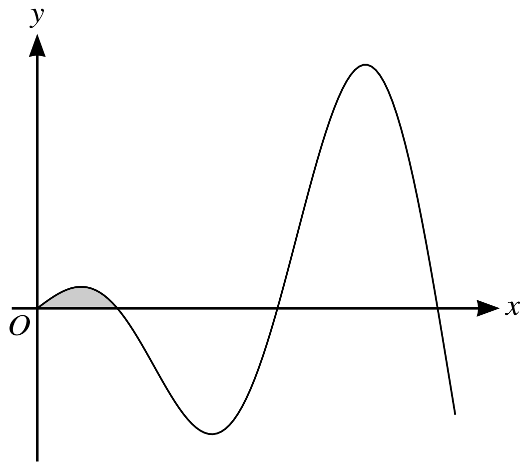
9 Let $f(x) = \dfrac{17x^2 - 7x + 16}{(2 + 3x^2)(2 - x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [5]

(c) State the set of values of x for which the expansion in **(b)** is valid. Give your answer in an exact form. [1]

10



The diagram shows the curve $y = x\cos 2x$, for $x \geqslant 0$.

(a) Find the equation of the tangent to the curve at the point where $x = \frac{1}{2}\pi$. [4]

(b) Find the exact area of the shaded region shown in the diagram, bounded by the curve and the x-axis. [5]

11 The line l has equation $\mathbf{r} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$. The points A and B have position vectors $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively.

(a) Find a unit vector in the direction of l. [2]

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The line m passes through the points A and B.

(b) Find a vector equation for m. [2]

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(c) Determine whether lines l and m are parallel, intersect or are skew. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

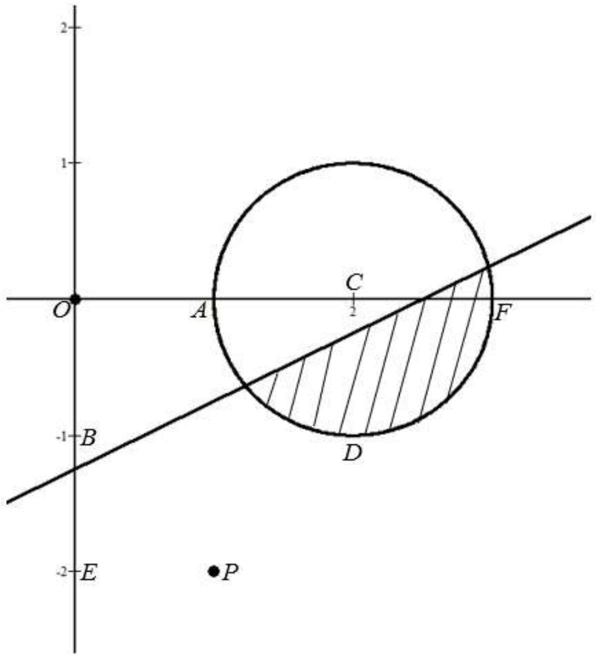
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $-0.5 < 2^{x+1} - 2 < 0.5$, can be in two separate statements, or $\left(2^{x+1} - 2\right)^2 < 0.5^2$ or corresponding pair of linear equations $0.5 = 2^{x+1} - 2$ and $-0.5 = 2^{x+1} - 2$ or quadratic equation $\left(2^{x+1} - 2\right)^2 = 0.5^2$	**B1**	$-0.25 < 2^x - 1 < 0.25$, can be in two separate statements, or $\left(2^x - 1\right)^2 < 0.25^2$ or corresponding pair of linear equations $0.25 = 2^x - 1$ and $-0.25 = 2^x - 1$ or quadratic equation $\left(2^x - 1\right)^2 = 0.25^2$. Incorrect inequality mark recoverable by correct final answer or $x < 0.32$ and $x > -0.42$.
	Use correct method for solving an equation or inequality of the form $2^{x+1} = a$ or $2^x = b$ where $a, b > 0$	**M1**	Reach $(x + 1)\ln 2 = \ln a$ or equivalent, do not need to reach $x = \ldots$
	Obtain critical values $x = 0.322$ and -0.415 or awrt $x = 0.32$ and -0.42 or exact equivalents	**A1**	e.g. $\frac{\ln 2.5}{\ln 2} - 1$ and $\frac{\ln 1.5}{\ln 2} - 1$.
	State final answer $-0.415 < x < 0.322$ or $(-0.415, 0.322)$	**A1**	Need 3 significant figures. Need combined result, not $x < 0.32$ and $x > -0.42$. Must be strict inequalities. No working, 0/4.
	Alternative method for Question 1		
	Use correct method for solving an equation or inequality of the form $2^{x+1} = a$ or $2^x = b$ where $a, b > 0$	**M1**	May see $2^{x+1} = 1.5$ and $2^{x+1} = 2.5$. Reach $(x + 1)\ln 2 = \ln a$ or equivalent, don't need to reach $x = \ldots$
	Obtain one critical value, e.g. 0.322 or awrt $x = 0.32$ or exact equivalent	**A1**	e.g. $\frac{\ln 2.5}{\ln 2} - 1$.
	Obtain the other critical value e.g. -0.415 or awrt $x = -0.42$ or exact equivalent	**A1**	e.g. $\frac{\ln 1.5}{\ln 2} - 1$.

Question	Answer	Marks	Guidance
1	State final answer $-0.415 < x < 0.322$ or $(-0.415, 0.322)$	**A1**	Need 3 significant figures. Need combined result, not $x < 0.32$ and $x > -0.42$. Must be strict inequalities. No working, 0/4.
		4	

Question	Answer	Marks	Guidance
2	Show a circle centre (2, 0)	**B1**	
	Show the relevant part of a circle with radius 1	**B1 FT**	FT centre not at the origin even if centre at 1 – 2i. Must clearly go through (1, 0) or (3, 0) (oe for FT mark).
	Show the point representing 1 – 2i	**B1**	Can be implied by correct perpendicular bisector
	Show the perpendicular bisector of the line joining 1 – 2i and the origin. Perpendicular to *OP* by eye and at midpoint of *OP* by eye sufficient. Must reach midpoint of *OP* and if extended will cut *BE.*	**B1 FT**	FT on the position of 1 – 2i.

Question	Answer	Marks	Guidance
2	Shade the correct region. Dependent on all previous marks, except in case 3 below, and the perpendicular must cut axes between *CF* and *BE*, but not actually through *C* or *F* and not through *B* or *E* Scale can be implied by dashes 1 Scale only on y-axis and $2OA = OC$ B1, B1FT, B1, B1FT, B1 2 Scale only on x-axis and $2OB = OE$ B1, B1FT, B1, B1FT, B1 3 No scale on either axis, but $2OA = OC$ B0, B1FT, B0, B1FT, B1 then $2OB = OE$	**B1**	
		5	

Question	Answer	Marks	Guidance
3	$2(-2)^3 + a(-2)^2 + b(-2) + 6 = -38$ Allow errors $x+2\,\overline{)\,2x^3 + ax^2 \quad + bx \quad + 6}$ quotient $2x^2 + (a-4)x + b - 2a + 8$ $\underline{2x^3 + 4x^2}$ $(a-4)x^2 + bx$ $\underline{(a-4)x^2 + (2a-8)x}$ $(b-2a+8)x + 6$ $\underline{(b-2a+8)x + 2b - 4a + 16}$ $4a - 2b - 10$	**M1**	Substitute $x = -2$ and equate the result to -38 or divide by $x + 2$ to obtain quadratic quotient, and equate constant remainder to -38.

Question	Answer	Marks	Guidance
3	Obtain a correct evaluated equation, e.g. $-16 + 4a - 2b + 6 = -38$ or $4a - 2b = -28$	**A1**	
	$2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 6 = \frac{19}{2}$ Allow errors $\begin{array}{rl} & x^2+\frac{a+1}{2}x+\frac{b}{2}+\frac{a}{4}+\frac{1}{4} \\ 2x-1 & \overline{)\,2x^3+ax^2 \quad +bx \quad +6} \\ & \underline{2x^3-x^2} \\ & (a+1)x^2+bx \\ & \underline{(a+1)x2-\left(\frac{a}{2}+\frac{1}{2}\right)x} \\ & \left(b+\frac{a}{2}+\frac{1}{2}\right)x+6 \\ & \underline{\left(b+\frac{a}{2}+\frac{1}{2}\right)x-\left(\frac{b}{2}+\frac{a}{4}+\frac{1}{4}\right)} \\ & 6+\frac{b}{2}+\frac{a}{4}+\frac{1}{4} \end{array}$	**M1**	Substitute $x = \frac{1}{2}$ and equate the result to $\frac{19}{2}$ or divide by $2x - 1$ to obtain quadratic quotient, and equate constant remainder to $\frac{19}{2}$.
	Obtain a correct evaluated equation, e.g. $\frac{1}{4}+\frac{a}{4}+\frac{b}{2}+6=\frac{19}{2}$ or $\frac{a}{4}+\frac{b}{2}=\frac{13}{4}$	**A1**	
	Obtain $a = -3$ and $b = 8$	**A1**	ISW
		5	

Question	Answer	Marks	Guidance
4	$\dfrac{2 \pm \sqrt{(-2)^2 - 4(3+i)(3-i)}}{2(3+i)}$	M1	Use quadratic formula to solve for w
	Use $i^2 = -1$ in $(3+i)(3-i)$	M1	
	Obtain one of the answers $w = \dfrac{2+6i}{6+2i}$ or $w = \dfrac{2-6i}{6+2i}$	A1	Must be simplified to this form.
	Show **intention** to multiply numerator and denominator by conjugate of their denominator.	M1	Independent of previous M marks but must be of the same form, e.g. $\dfrac{a}{b+ci}$.
	Obtain final answers $\dfrac{3}{5}+\dfrac{4}{5}i$ and $-i$ Accept $0.6 + 0.8i$ and $0 - i$	A1	**SC** Both correct final answers from $w = \dfrac{2+6i}{6+2i}$ and $w = \dfrac{2-6i}{6+2i}$ seen, no evidence of conjugate, then **SC B1** for both. Allow $x = \dfrac{3}{5}$, $y = \dfrac{4}{5}$ or $x = 0$, $y = -1$. A0 for $\dfrac{3+4i}{5}$.
	Alternative method for Question 4		
	Multiply the equation by $3 - i$	M1	
	Use $i^2 = -1$ in $(3+i)(3-i)$	M1	
	Obtain $10w^2 - 2(3-i)w + (3-i)^2 = 0$ or equivalent	A1	
	Use quadratic formula or factorise to solve for w	M1	

Question	Answer	Marks	Guidance
4	Obtain final answers $\frac{3}{5}+\frac{4}{5}\mathrm{i}$ and $-\mathrm{i}$ Accept $0.6 + 0.8\mathrm{i}$ and $0 - \mathrm{i}$	A1	**SC** Both correct final answers from $10w^2 - 2(3 - \mathrm{i})w + (3 - \mathrm{i})^2 = 0$ with no working then **SC B1** for both. Allow $x = \frac{3}{5}$, $y = \frac{4}{5}$ or $x = 0$, $y = -1$. A0 for $\frac{3+4\mathrm{i}}{5}$.
	Alternative method for Question 4		
	Substitute $w = x + \mathrm{i}y$ and form equations for real and imaginary parts	M1	
	Use $\mathrm{i}^2 = -1$ in $(x + \mathrm{i}y)^2$	M1	
	Obtain $3(x^2 - y^2) - 2xy - 2x + 3 = 0$ and $x^2 - y^2 + 6xy - 2y - 1 = 0$	A1	OE
	Form quartic equation in x only or y only using the correct substitution and solve for x or y	M1	Use correct $y = \frac{(3-x)}{10x-3}$ to attempt to form and solve $50x^4 - 60x^3 + 63x^2 - 27x = 0$ $x(5x - 3)(10x^2 - 6x + 9) = 0$. Use correct $x = \frac{3(1+y)}{10y+1}$ to attempt to form and solve $100y^4 + 40y^3 - 66y^2 - 14y - 8 = 0$ $(y + 1)(5y - 4)(20y^2 + 4y + 2) = 0$.
	Obtain final answers $\frac{3}{5}+\frac{4}{5}\mathrm{i}$ and $-\mathrm{i}$ Accept $0.6 + 0.8\mathrm{i}$ and $0 - \mathrm{i}$	A1	**SC** Both correct final answers from $3(x^2 - y^2) - 2xy - 2x + 3 = 0$ and $x^2 - y^2 + 6xy - 2y - 1 = 0$ with no working then **SC B1** for both. Allow $x = \frac{3}{5}$, $y = \frac{4}{5}$ or $x = 0$, $y = -1$. A0 for $\frac{3+4\mathrm{i}}{5}$.
		5	

Question	Answer	Marks	Guidance
5	Use correct product or quotient rule	**M1**	Need attempt at both derivatives condone errors in chain rule. In quotient rule allow BOD in formula if $\pm 2x$ seen unless clear that incorrect formula has been used. If omit denominator or forget to square or complete reversal of signs then M0 A0 M1 A1 A1 A1.
	Obtain correct derivative in any form, e.g. $\dfrac{6x(1-x^2)e^{3x^2-1}+2xe^{3x^2-1}}{\left(1-x^2\right)^2}$	**A1**	If $6x\left(1-x^2\right)e^{3x^2-1}+2xe^{3x^2-1}=0$ from the start, with no wrong formula seen, award M1A1.
	Equate derivative (or its numerator) to zero and solve for x	**M1**	$6x - 6x^3 + 2x = 0$ and solve. Allow for just one x value. Allow if from solution of 3 term quadratic equation, but if they get $x = 0$ the x must factorise out
	Obtain the point $(0, e^{-1})$ or exact equivalent	**A1**	Or for all three x coordinates found 0, $\pm\dfrac{2\sqrt{3}}{3}$ oe and no extras but if this is the case then one pair of correct coordinates A1 and both other pairs of correct coordinates A1. Accept, e.g. $x = 0$, $y = e^{-1}$ ISW for last 3 marks.
	Obtain the point $\left(\dfrac{2\sqrt{3}}{3}, -3e^3\right)$ or exact equivalent	**A1**	Allow $\sqrt{(4/3)}$.
	Obtain the point $\left(-\dfrac{2\sqrt{3}}{3}, -3e^3\right)$ or exact equivalent	**A1**	
		6	

Question	Answer	Marks	Guidance
6(a)	Use correct Pythagoras $\cot^2\theta = \operatorname{cosec}^2\theta - 1$ or $\cot^2\theta = 1/\sin^2\theta - 1$ or $\cot^2\theta = \cos^2\theta/\sin^2\theta$ and then $\cos^2\theta = 1 - \sin^2\theta$, together with double angle formula $\cos 2\theta = 1 - 2\sin^2\theta$, to obtain an equation in $\sin\theta$ or $\sin\theta$ and $\operatorname{cosec}^2\theta$	**M1**	If consistent omission of brackets, e.g. $(\sin\theta)^2$ written as $\sin\theta^2$ then **SC B1** in place of M1A1.
	Obtain a correct equation in $\sin\theta$ in any form	**A1**	e.g. $1/\sin^2\theta - 1 + 2(1 - 2\sin^2\theta) = 4$ or $\dfrac{1-\sin^2}{\sin^2} + 2\left(1 - 2\sin^2\right) = 4$. If $\dfrac{\cos^2}{\sin^2} + 2\left(1-2\sin^2\right) = 4$ then e.g. $1 - \sin^2 + 2\left(1 - 2\sin^2\right)\sin^2 = 4$. (missing $\sin^2$ on right) allow M1A1A0.
	Reduce to the given answer of $4\sin^4\theta + 3\sin^2\theta - 1 = 0$ correctly	**A1**	AG Must follow from a horizontal equation (no denominators). If $s = \sin\theta$ used and defined, allow all marks. If not defined, award M1A1A0.
		3	

Question	Answer	Marks	Guidance
6(b)	Solve the given quadratic to obtain a value for θ	**M1**	$(4\sin^2\theta - 1)(\sin^2\theta + 1) = 0$ and solve for θ. Incorrect sign in solution of quadratic seen, e.g. $(4\sin^2\theta - 1)(\sin^2\theta - 1) = 0$ then M0 A0 A0 but if only see $(4\sin^2\theta - 1) = 0$ and nothing incorrect seen allow 3/3.
	Obtain answer, e.g. $\theta = 30°$	**A1**	$\pi/6$ award A0
	Obtain three further answers, e.g. $\theta = 150°$, $210°$ and $330°$ and no others in the interval	**A1**	Ignore any answers outside interval. $5\pi/6$ $7\pi/6$ $11\pi/6$ award A1.
		3	

Question	Answer	Marks	Guidance
7(a)	State or imply $2y\dfrac{dy}{dx}$ as the derivative of y^2	**B1**	Allow for $3x^2dx + 2ydy$ or $F_x = 3x^2 + 6x$ and $F_y = 2y + 3$.
	Equate derivative of LHS to zero and solve for $\dfrac{dy}{dx}$	**M1**	$3x^2 + 2y\dfrac{dy}{dx} + 6x + 3\dfrac{dy}{dx} = 0$ or $3x^2dx + 2ydy + 6xdx + 3dy = 0$ or $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y}$ need evidence from B1 mark or formula must be seen. Allow errors.
	Obtain the given answer	**A1**	AG $\dfrac{dy}{dx} = -\dfrac{3x^2 + 6x}{2y+3}$ not $\dfrac{-3x^2 - 6x}{2y+3}$. Must factorise with $\dfrac{dy}{dx}$ e.g. $3x^2 + 6x + \dfrac{dy}{dx}(2y+3) = 0$ or $3x^2dx + 6xdx + dy(2y+3) = 0$.
		3	

Question	Answer	Marks	Guidance
7(b)	Equate numerator to zero and solve for x	***M1**	Allow for just one x value.
	Obtain $x = 0$ and $x = –2$ only	**A1**	
	Substitute their x, [$x = 0$ or $x = –2$] in curve equation to obtain quadratic equation in y equal to 0	**DM1**	$y^2 + 3y – 4 = 0$ or $y^2 + 3y = 0$.
	Obtain $y = 1$ and $y = –4$ [when $x = 0$]	**A1**	
	Obtain $y = 0$ and $y = –3$ [when $x = –2$]	**A1**	ISW If forget $x = 0$ then max 3/5.
		5	

Question	Answer	Marks	Guidance
8	Separate variables correctly and reach $a\sec^2 3y$ or $b\mathrm{e}^{-4x}$	**B1**	Condone missing integral signs or dy and dx, but allow if recognisable integrals follow. Not for $1/\cos^2 3y$ and $1/\mathrm{e}^{4x}$.
	Obtain term $-\frac{1}{4}\mathrm{e}^{-4x}$	**B1**	Can recover the previous B1 if $d\mathrm{e}^{-4x}$ seen here.
	Obtain only a term of the form $a\tan 3y$	**M1**	Can recover the first B1 if $a\tan 3y$ seen here.
	Obtain term $\frac{1}{3}\tan 3y$	**A1**	
	Use $x = 2$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\tan by$ and $c\mathrm{e}^{\pm 4x}$	**M1**	May see $\tan by$ and $\mathrm{e}^{\pm 4x}$ here.
	Obtain correct answer in any form	**A1**	e.g. $\frac{1}{3}\tan 3y = -\frac{1}{4}\mathrm{e}^{-4x} + \frac{1}{4}\mathrm{e}^{-8}$ or $\frac{1}{3}\tan 3y = -\frac{1}{4}\mathrm{e}^{-4x} + 8.39 \times 10^{-5}$
	Obtain final answer $y = \frac{1}{3}\tan^{-1}\left(\frac{3}{4}\mathrm{e}^{-8} - \frac{3}{4}\mathrm{e}^{-4x}\right)$	**A1**	ISW OE e.g. $y = \frac{1}{3}\tan^{-1}\left(2.52\times10^{-4} - \frac{3}{4}\mathrm{e}^{-4x}\right)$
		7	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{Ax+B}{2+3x^2}+\dfrac{C}{2-x}$	**B1**	If incorrect partial fractions e.g. $A = 0$ or $Ax^2 + B$ then M1, A1 A0 for correct C. Only allow single A1 even if other coefficients correct. B1 recoverable by a correct form end statement.
	Use a correct method for finding a coefficient	**M1**	e.g. $(Ax + B)(2 - x) + C(2 + 3x^2)$ $= (3C - A)x^2 + (2A - B)x + (2B + 2C)$ $= 17x^2 - 7x + 16$.
	Obtain one of $A = -2$, $B = 3$ and $C = 5$	**A1**	If error present in above still allow A1 for C.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Extra term in partial fractions, $D/(2 + 3x^2)$, that is 4 unknowns A, B, C and D then B0 unless recover at end, e.g. by setting B or $D = 0$. If B or D set to any value other than 0 and all coefficients correctly found to their new values then allow all A marks, but still B0 for partial fraction expression unless $B + D$ combined. Hence A1 for each coefficient, but nothing for coefficient set to specific value. Another case of extra term in partial fraction expression, namely $+ F$, mark as above but need $F = 0$ to recover B1.
		5	

Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion $(2-x)^{-1} = 2^{-1} + (-1)\,2^{-2}(-x) + [(-1)(-2)2^{-3}(-x)^2/2!]$, $\left(1+\frac{3x^2}{2}\right)^{-1} = 1 - \frac{3x^2}{2}$ or $\left(1-\frac{x}{2}\right)^{-1} = 1 - \left(\frac{-x}{2}\right)$	**M1**	Symbolic coefficients are not sufficient for the M1.
	$\frac{Ax+B}{2}\left[1+(-1)\frac{3x^2}{2}\dots\right]$ $A = -2$ $B = 3$ $\frac{C}{2}\left[1+(-1)\left(\frac{-x}{2}\right)+\frac{(-1)(-2)}{2}\left(\frac{-x}{2}\right)^2 + \frac{(-1)(-2)(-3)}{6}\left(\frac{-x}{2}\right)^3\dots\right]$ $C = 5$	**A1 FT**	Obtain correct un-simplified expansions up to the term in x^3 of each partial fraction.
	$= \frac{3-2x}{2}\left(1-\frac{3x^2}{2}\right) + \frac{5}{2}\left(1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}\right)$ $=\left(\frac{3}{2}+\frac{5}{2}\right)+\left(-1+\frac{5}{4}\right)x+\left(-\frac{9}{4}+\frac{5}{8}\right)x^2+\left(\frac{3}{2}+\frac{5}{16}\right)x^3$	**A1 FT**	Unsimplified $(2-x)^{-1}$ expanded correctly, error in simplifying before their C is involved in the expression, allow A1FT when their C is introduced. The FT is on A, B, C.
	Multiply expansion of $\left(1+\frac{3x^2}{2}\right)^{-1}$ $\left(\text{must reach } 1\pm\frac{3x^2}{2}\right)$ by $Ax + B$, where $AB \neq 0$, up to the term in x^3. Allow if used $Cx + D$ ($Ax + B$ miscopied).	**M1**	Allow either ± 2 or $\pm 2^{-1}$ outside bracket or missing. Allow one error in actual multiplication to acquire the 4 terms [all terms needed]. Ignore errors in higher powers.

Question	Answer	Marks	Guidance
9(b)	Obtain final answer $4+\frac{1}{4}x-\frac{13}{8}x^2+\frac{29}{16}x^3$, or equivalent [If final answer has been multiplied throughout, e.g. by 16 then A0 at the end]	**A1**	Maclaurin's Series: f(0) = 4 B1 f'(0) = 1/4 B1. f''(0) = –13/4 and f'''(0) = 87/8 B1. $4+\frac{1}{4}x-\frac{\frac{13}{4}x^2}{2}+\frac{\frac{87}{8}x^3}{6}$ or equivalent M1 A1. If $1+\frac{1}{4}x-\frac{\frac{13}{4}x^2}{2}+\frac{\frac{87}{8}x^3}{6}$ then M0 A0 unless their f(0) actually is 1.
		5	
9(c)	State answer $\lvert x\rvert<\sqrt{\frac{2}{3}}$ or $-\sqrt{\frac{2}{3}}<x<\sqrt{\frac{2}{3}}$ clear conclusion required	**B1**	Or exact equivalent. Strict inequality.
		1	

Question	Answer	Marks	Guidance
10(a)	Use the product rule correctly on $y = x\cos 2x$	**M1**	dx/dx cos 2x + x d/dx(cos 2x) attempted.
	Obtain the correct derivative in any form	**A1**	e.g. cos 2x – 2x sin 2x. If cos 2x + x–2sin 2x, not recovered, max M1A0A1FTA0 but can recover for full marks by seeing correct substitution.
	Obtain $y=-\frac{\pi}{2}$ and $\frac{dy}{dx}=-1$ when $x=\frac{\pi}{2}$	**A1FT**	FT *their* $\frac{dy}{dx}$ with $x=\frac{\pi}{2}$ substituted.
	Obtain answer $x + y = 0$	**A1**	OE CWO Need to see y and dy/dx at $x=\frac{\pi}{2}$.
		4	

Question	Answer	Marks	Guidance
10(b)	Integrate by parts and reach $ax\sin 2x + b\int \sin 2x\,\mathrm{d}x$	*M1	
	Obtain $\frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x\,\mathrm{d}x$	A1	OE
	Complete integration and obtain $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$	A1	OE
	Use limits of $x = 0$ and $x = \frac{\pi}{4}$ in the correct order, having integrated twice to obtain ax sin $2x$ + $c$cos $2x$	DM1	If correct, $\frac{1}{2}\left(\frac{\pi}{4}\right)\sin\frac{2\pi}{4} + \frac{1}{4}\cos\frac{2\pi}{4} - \frac{1}{4}\cos 0$ or $\frac{1}{2}\left(\frac{\pi}{4}\right)\sin\frac{2\pi}{4} - \frac{1}{4}\cos 0$. Max one substitution error.
	Obtain answer $\frac{\pi}{8} - \frac{1}{4}$ or exact simplified two term equivalent	A1	ISW Accept $\frac{\pi - 2}{8}$. Accept $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$ then final answer.
		5	

Question	Answer	Marks	Guidance
11(a)	Use correct process for modulus on direction vector of l, e.g. $\sqrt{(-1)^2 + 1^2 + 2^2}$	M1	SOI Allow -1^2. Allow $\sqrt{(-\lambda)^2 + \lambda^2 + (2\lambda)^2}$.
	$[\pm]\frac{1}{\sqrt{6}}(-\mathrm{i} + \mathrm{j} + 2\mathrm{k})$	A1	OE Allow coordinates as row or column, but not row or column with **i**, **j** and **k** included.
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>11(b)</td><td>Use a correct method to form an equation for line m</td><td>M1</td><td>Allow even if all signs of point incorrect, namely use $+2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ or $-3\mathbf{i} + \mathbf{j} - \mathbf{k}$.</td></tr>
<tr><td></td><td>Obtain $\mathbf{r} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \mu_1(-5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$</td><td>A1</td><td>OE, e.g. $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} + \mu_2(-5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$
Must have $\mathbf{r} = \ldots$</td></tr>
<tr><td></td><td></td><td>2</td><td></td></tr>
<tr><td>11(c)</td><td>Justify lines are not parallel</td><td>B1</td><td>$(-5, 3, -2) \neq d\,(-1, 1, 2)$ or $(-5, 3, -2)\mathbf{x}(-1, 1, 2) \neq 0$.
Can find angle (105°, 74.6°, 1.84^c or $1.3(0)^c$) instead but if incorrect B0 and A0 at end.
Accept direction vectors don’t have common factor
but not direction vectors are not equal or direction vectors are different or $\mu \neq \lambda$ or scalar product $\neq 0$.
Not the line equations are not multiples of each other.</td></tr>
<tr><td></td><td>Express l or m in component form
e.g. $(-2 - 5\mu_1, 2 + 3\mu_1, -1 - 2\mu_1)$ or $(3 - 5\mu_2, -1 + 3\mu_2, 1 - 2\mu_2)$ or
$(1 - \lambda, -2 + \lambda, -3 + 2\lambda)$</td><td>B1</td><td></td></tr>
<tr><td></td><td>Equate two pairs of components of general points on l and their m and solve simultaneously for λ or for μ</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain correct answer for λ or μ, e.g. $\lambda = \frac{11}{2}$, $\mu_1 = \frac{1}{2}$</td><td>A1</td><td></td></tr>
<tr><td></td><td>Determine that all three equations are not satisfied and the lines fail to intersect and conclude the lines are skew.
Conclusion needs to follow correct working</td><td>A1</td><td><table>
<tr><td>1</td><td>λ</td><td>μ_1</td><td></td><td>2</td><td>λ</td><td>μ_2</td><td></td></tr>
<tr><td>ij</td><td>11/2</td><td>1/2</td><td>$8 \neq -2$</td><td>ij</td><td>11/2</td><td>3/2</td><td>$8 \neq -2$</td></tr>
<tr><td>ik</td><td>4/3</td><td>–1/3</td><td>$-2/3 \neq 1$</td><td>ik</td><td>4/3</td><td>2/3</td><td>$-2/3 \neq 1$</td></tr>
<tr><td>jk</td><td>7/4</td><td>–3/4</td><td>$-3/4 \neq 7/4$</td><td>jk</td><td>7/4</td><td>1/4</td><td>$-3/4 \neq 7/4$</td></tr>
</table>Dependent on 4 previous marks gained.</td></tr>
<tr><td></td><td></td><td>5</td><td></td></tr>
</table>

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

6303252032

MATHEMATICS **9709/41**

Paper 4 Mechanics **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

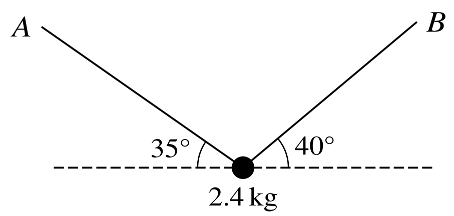
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 A particle of mass 1.6 kg is projected with a speed of $20\,\mathrm{m\,s^{-1}}$ up a line of greatest slope of a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

Use an energy method to find the distance the particle moves up the plane before coming to instantaneous rest. [3]

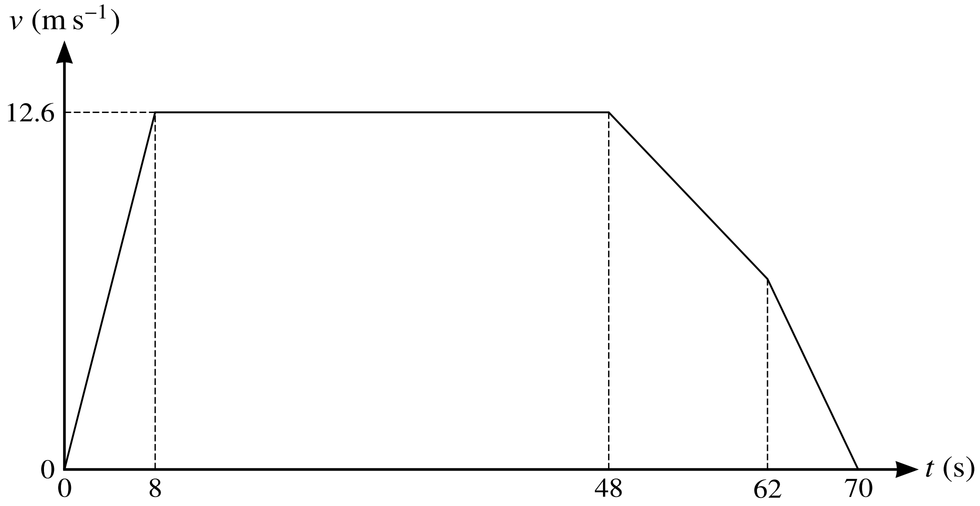
2



A particle of mass 2.4 kg is held in equilibrium by two light inextensible strings, one of which is attached to point A and the other attached to point B. The strings make angles of 35° and 40° with the horizontal (see diagram).

Find the tension in each of the two strings. [5]

3



The diagram shows the velocity-time graph for the motion of a bus. The bus starts from rest and accelerates uniformly for 8 seconds until it reaches a speed of $12.6\,\text{m s}^{-1}$. The bus maintains this speed for 40 seconds. It then decelerates uniformly in two stages. Between 48 and 62 seconds the bus decelerates at $a\,\text{m s}^{-2}$ and between 62 and 70 seconds it decelerates at $2a\,\text{m s}^{-2}$ until coming to rest.

(a) Find the distance covered by the bus in the first 8 seconds. [1]

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(b) Find the value of a. [3]

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(c) Find the average speed of the bus for the whole journey. [4]

4 Two particles P and Q, of masses 6 kg and 2 kg respectively, lie at rest 12.5 m apart on a rough horizontal plane. The coefficient of friction between each particle and the plane is 0.4. Particle P is projected towards Q with speed $20\,\mathrm{m\,s^{-1}}$.

(a) Show that the speed of P immediately before the collision with Q is $10\sqrt{3}\,\mathrm{m\,s^{-1}}$. [3]

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In the collision P and Q coalesce to form particle R.

(b) Find the loss of kinetic energy due to the collision. [4]

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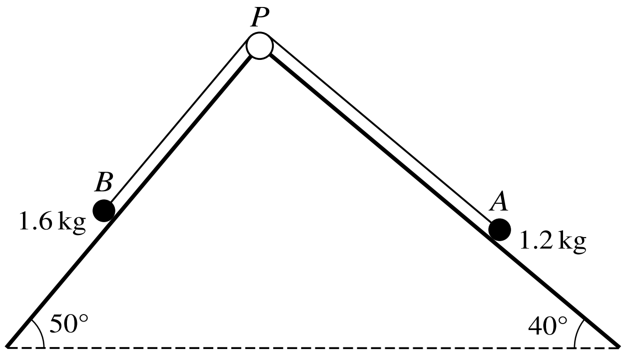
The coefficient of friction between R and the plane is 0.4.

(c) Find the distance travelled by particle R before coming to rest. [2]

5



The diagram shows a particle A, of mass 1.2 kg, which lies on a plane inclined at an angle of 40° to the horizontal and a particle B, of mass 1.6 kg, which lies on a plane inclined at an angle of 50° to the horizontal. The particles are connected by a light inextensible string which passes over a small smooth pulley P fixed at the top of the planes. The parts AP and BP of the string are taut and parallel to lines of greatest slope of the respective planes. The two planes are rough, with the same coefficient of friction, μ, between the particles and the planes.

Find the value of μ for which the system is in limiting equilibrium. [7]

6 A car of mass 1300 kg is moving on a straight road.

(a) On a horizontal section of the road, the car has a constant speed of $30\,\mathrm{m\,s^{-1}}$ and there is a constant force of 650 N resisting the motion.

(i) Calculate, in kW, the power developed by the engine of the car. [2]

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(ii) Given that this power is suddenly increased **by** 9 kW, find the instantaneous acceleration of the car. [3]

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(b) On a section of the road inclined at $\sin^{-1} 0.08$ to the horizontal, the resistance to the motion of the car is $(1000 + 20v)$ N when the speed of the car is $v\,\mathrm{m\,s^{-1}}$. The car travels downwards along this section of the road at constant speed with the engine working at 11.5 kW.

Find this constant speed. [4]

7 A particle moves in a straight line starting from a point O before coming to instantaneous rest at a point X. At time t s after leaving O, the velocity $v\,\mathrm{m\,s^{-1}}$ of the particle is given by

$$\begin{aligned} v &= 7.2t^2 && 0 \leqslant t \leqslant 2, \\ v &= 30.6 - 0.9t && 2 \leqslant t \leqslant 8, \\ v &= \frac{1600}{t^2} + kt && 8 \leqslant t, \end{aligned}$$

where k is a constant. It is given that there is no instantaneous change in velocity at $t = 8$.

Find the distance OX. [9]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/41**

Paper 4 Mechanics **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only

ISW Ignore Subsequent Working

SOI Seen Or Implied

SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

WWW Without Wrong Working

AWRT Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\pm 1.6g \times x \times \frac{3}{5}$ [=9.6x] or $\pm \frac{1}{2} \times 1.6 \times 20^2$ [= 320]	**B1**	For either the correct potential energy or kinetic energy term. Need not be evaluated.
	$\frac{1}{2} \times 1.6 \times 20^2 = 1.6g \times x \sin\alpha$ where $\sin\alpha = \frac{3}{5}$	**M1**	Attempt at energy equation; 2 relevant terms. Dimensionally correct but allow sign errors. Allow sin/cos mix and sin(36.869…) but $\sin\alpha$ (oe) must have been substituted. M0 for $1.6g \times x \times \frac{3}{4}$.
	$x = \frac{100}{3}$	**A1**	Allow 33.3.
		3	

Question	Answer	Marks	Guidance
2	Attempt to resolve horizontally or vertically to form an equation.	***M1**	Correct number of terms; allow sin/cos mix; allow sign errors – do not award this mark if using T for both (see **SC** later).
	$T_1 \cos 35 = T_2 \cos 40$	**A1**	Must be different Ts.
	$T_1 \sin 35 + T_2 \sin 40 = 2.4g$	**A1**	If same Ts, then **SC B2** only for this equation.
	Attempt to solve for either tension.	**DM1**	From equations with correct number of relevant terms. Must get a value for at least one tension. E.g. $T_2\left(\frac{\cos 40}{\cos 35} \times \sin 35 + \sin 40\right) = 24$
	$T_1 = 20.4$ N and $T_2 = 19.0$ N	**A1**	$T_1 = 19.033621\ldots$ $T_2 = 20.353166\ldots$ awrt 20.4 for T_1 www and 19(.0) for T_2.
		5	

Question	Answer	Marks	Guidance
3(a)	Distance $= 50.4\,\text{m}$	**B1**	Allow $\frac{252}{5}$.
		1	
3(b)	$v_1 = 12.6 - (62-48)a$	**M1**	Use of suvat for first section of deceleration. $12.6 \pm (62-48)a$ only.
	$0 = v_1 - 2a \times (70-62)$	**M1**	Use of suvat for second section of deceleration. An expression for the velocity at 62 seconds must be $\pm 2a \times (70-62)$.
	$a = 0.42$	**A1**	–0.42 scores A0.
		3	
3(c)	Speed at time $t = 62$ is $6.72\,\text{m}\,\text{s}^{-1}$	**B1**	This may be seen in part **(b)** but must be used in part **(c)** to get this mark.
	$s_2 = (48-8)\times 12.6\ [=504]$ $s_3 = 0.5\times(12.6 + \textit{their}\ 6.72)\times(62-48)\ \left[=135.24 \text{ or } \frac{3381}{25} \text{ oe}\right]$ or $\textit{their}\ 6.72\times(62-48) + 0.5\times(62-48)\times(12.6 - \textit{their}\ 6.72)$ $s_4 = 0.5\times \textit{their}\ 6.72\times(70-62)\left[=26.88 \text{ or } \frac{672}{25} \text{ oe}\right]$	**B2FT**	B2 FT for any 2 correct, B1 FT for any 1 correct – follow through *their* value of v_1 where $0 < v_1 < 12.6$ but must have come from the correct equations seen in part **(b)**. Allow correct value of v_1 from $a = -0.42$ where $v_1 = 12.6 + (62-48)a$ and $v_1 = -2a\times(70-62)$.
	Average speed $= 10.236\,\text{m}\,\text{s}^{-1}$	**B1**	Allow 10.2 or better oe e.g. $\frac{2559}{250}$, $10\frac{59}{250}$.
		4	

Question	Answer	Marks	Guidance
4(a)	$-0.4\times 6g = 6a$	*B1	Resolve horizontally using Newton's second law; 2 relevant terms; must be either $-0.4\times 6g = 6a$ or $0.4\times 6g = 6a$.
	$v^2 = 20^2 + 2\times(-4)\times 12.5$	DM1	Use complete suvat method to get an equation in v or v^2 – must be using u = 20, s = 12.5 and *their* a.
	$v^2 = 300 \Rightarrow v = 10\sqrt{3}$	A1	AG. Condone correct expression for v or v^2 followed by correct answer.
	Alternative method for Question 4(a)		
	$RF = 0.4\times 6g$	*B1	Correct application of $F = \mu R$ for P.
	$0.5\times 6\times 20^2 - 0.5\times 6\times v^2 = 12.5\times(0.4\times 6g)$	DM1	3 relevant terms; dimensionally correct; allow sign errors only.
	$v^2 = 300 \Rightarrow v = 10\sqrt{3}$	A1	AG. Condone correct expression for v or v^2 followed by correct answer.
		3	
4(b)	$6\times 10\sqrt{3} = (6+2)v'$	M1	For use of conservation of momentum, 3 non-zero terms, allow sign errors. Use of 20 is M0.
	$v' = 7.5\sqrt{3}$	A1	12.99038…
	Initial KE $= \frac{1}{2}\times 6\times(10\sqrt{3})^2\ [=900]$ Final KE $= \frac{1}{2}\times 8\times(7.5\sqrt{3})^2\ [=675]$	B1	Either initial kinetic energy or final kinetic energy correct. Allow unsimplified.
	Loss of KE = 225 J	A1	
		4	

Question	Answer	Marks	Guidance
4(c)	$0 = \left(\textit{their}\; 7.5\sqrt{3}\right)^2 + 2\times\left(\textit{their}\; -4\right)\times s$	**M1**	Use complete suvat method to find distance. This must be using *their* v' from part **(b)**, so it is dependent on scoring the first M mark in part **(b)** and either *their a* from part **(a)**, or from $\pm 0.4\times 8g = 8a$.
	[Distance =] 21.1 m	**A1**	21.1 or better (21.09375).
		2	

Question	Answer	Marks	Guidance
5	Resolving parallel to the slope at *A* or *B* to form an equation.	***M1**	Correct number of terms; allow sign errors; allow sin/cos mix.
	$1.6g\sin 50 - T - F_B = 0$	**A1**	If using the same *F*s, then M1A1A0B1 max.
	$T - F_A - 1.2g\sin 40 = 0$	**A1**	System equation (must be four different terms): $1.6g\sin 50 - F_B - F_A - 1.2g\sin 40 = 0$ only scores M1A1A1. Any sign errors scores M1 only.
	$R_A = 1.2g\cos 40$ or $R_B = 1.6g\cos 50$	***B1**	Either correct. Must be explicitly linked to the correct contact (so could be seen on a diagram), or as part of a resolving parallel to the slope equation(s) (so must be combined with μ).
	$F_A = 1.2g\mu\cos 40$ or $F_B = 1.6g\mu\cos 50$	***M1**	Use of $F = \mu R$ at either *A* or *B*. Must be explicitly linked to the correct contact (could be seen on a diagram) or as part of a resolving parallel to the slope equation(s). Allow sin/cos mix error only.
	$1.6g\sin 50 - 1.6g\mu\cos 50 = 1.2g\sin 40 + 1.2g\mu\cos 40$	**DM1**	Eliminating *T*, F_A and F_B to form an equation in μ only.
	$\left[\mu = \dfrac{1.6g\sin 50 - 1.2g\sin 40}{1.2g\cos 40 + 1.6g\cos 50} \Rightarrow\right] \mu = 0.233$	**A1**	0.23326119…
		7	

Question	Answer	Marks	Guidance
6(a)(i)	Power $= 19.5$ kW	**B2**	Or B1 for either 650×30 or 19 500.
		2	
6(a)(ii)	$\textit{their}\, 19500 + 9000 = DF \times 30$	**B1FT**	Oe FT *their* 19.5 in watts only.
	$DF - 650 = 1300a$	**M1**	Newton's second law horizontally; 3 relevant terms; dimensionally correct but allow sign errors; allow with *their* driving force or just *DF*.
	$a = \frac{3}{13} = 0.231\,\text{m}\,\text{s}^{-2}$	**A1**	0.23076923…
	Alternative scheme for 6(a)(ii)		
	$9000 = DF \times 30$	***B1**	oe e.g. $DF = \frac{9000}{30}$.
	$DF = 1300a$	**DM1**	Resolving horizontally using Newton's second law; 2 relevant terms; dimensionally correct but allow sign errors.
	$a = \frac{3}{13} = 0.231\,\text{m}\,\text{s}^{-2}$	**A1**	0.23076923…
		3	

Question	Answer	Marks	Guidance
6(b)	$DF = \frac{11500}{v}$	**B1**	oe e.g. $DF \times v = 11500$.
	Attempt at Newton's second law.	**M1**	4 relevant terms, *their DF* or just *DF*; allow sign errors: allow sin/cos mix; allow *g* missing.
	$\frac{11500}{v} + 1300 \times g \times 0.08 - (1000 + 20v) = 0$	**A1**	Correct equation.
	Speed = $25\,\mathrm{m\,s^{-1}}$	**A1**	
		4	

Question	Answer	Marks	Guidance
7	$30.6 - 0.9 \times 8 = \dfrac{1600}{8^2} + 8k$	*M1	Use velocity at $t = 8$ to set up a linear equation in k only. Allow a slip in one value or sign only.
	$k = -0.2$	A1	
	$\dfrac{1600}{t^2} + (\textit{their}\ k) \times t = 0 \Rightarrow t = \ldots$	DM1	Attempt to find the value of t when the particle comes to rest using the correct expression for v, set equal to zero with *their* negative value of k. Must find a positive value for t (for reference, t = 20).
	Attempt to integrate v for one of the 3 intervals	*M1	Increase power by 1 and a change in coefficient in at least one term (which must be the same term); $s = vt$ is M0.
	$s = \dfrac{7.2}{3}t^3 (+c)$	A1	May be unsimplified (for reference, limits are from 0 to 2).
	$s = 30.6t - \dfrac{0.9}{2}t^2 (+c)$	A1	May be unsimplified (for reference, limits are from 2 to 8).
	$s = \dfrac{1600}{-1}t^{-1} + \dfrac{k}{2}t^2 (+c)$	A1FT	May be unsimplified (for reference limits are from 8 to 20). Follow through *their* value of k or just k only.
	Either 19.2 or 156.6 or ± 86.4	B1	One correct distance found. Allow unsimplified e.g. $(216 - 59.4)$ or $\dfrac{1}{2} \times (8-2) \times (28.8 + 23.4)$ etc.
	Distance $= 19.2 + (216 - 59.4) + (-120 - (-206.4)) = 262.2$ m	B1	This mark can be awarded if no integration is shown oe. e.g. $\dfrac{1311}{5}$. Condone 262 www.
		9	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

* 1 0 7 9 2 4 8 2 6 6 *

MATHEMATICS **9709/42**

Paper 4 Mechanics **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

JC23 11_9709_42/2R

1 A block of mass 15 kg slides down a line of greatest slope of an inclined plane. The top of the plane is at a vertical height of 1.6 m above the level of the bottom of the plane. The speed of the block at the top of the plane is $2\,\mathrm{m\,s^{-1}}$ and the speed of the block at the bottom of the plane is $4\,\mathrm{m\,s^{-1}}$.

Find the work done against the resistance to motion of the block. [4]

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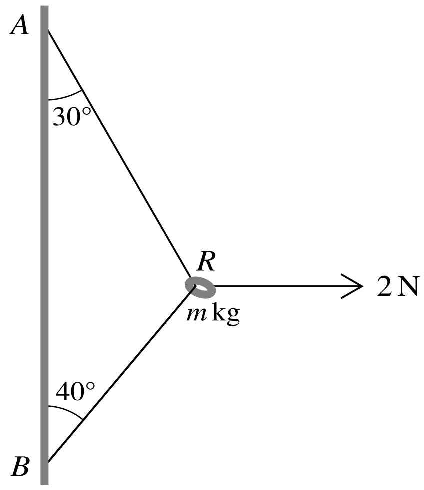

The diagram shows a smooth ring R, of mass m kg, threaded on a light inextensible string. A horizontal force of magnitude 2 N acts on R. The ends of the string are attached to fixed points A and B on a vertical wall. The part AR of the string makes an angle of 30° with the vertical, the part BR makes an angle of 40° with the vertical and the string is taut. The ring is in equilibrium.

Find the tension in the string and find the value of m. [5]

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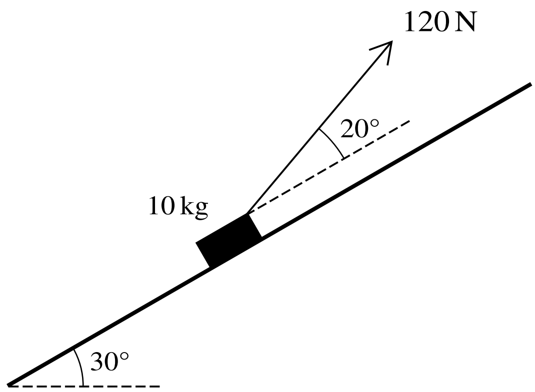

A block of mass 10 kg is at rest on a rough plane inclined at an angle of 30° to the horizontal. A force of 120 N is applied to the block at an angle of 20° above a line of greatest slope (see diagram). There is a force resisting the motion of the block and 200 J of work is done against this force when the block has moved a distance of 5 m up the plane from rest.

Find the speed of the block when it has moved a distance of 5 m up the plane from rest. [5]

4 A particle P of mass 0.2 kg lies at rest on a rough horizontal plane. A horizontal force of 1.2 N is applied to P.

(a) Given that P is in limiting equilibrium, find the coefficient of friction between P and the plane. [3]

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(b) Given instead that the coefficient of friction between P and the plane is 0.3, find the distance travelled by P in the third second of its motion. [4]

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5 A particle A of mass 0.5 kg is projected vertically upwards from horizontal ground with speed $25\,\mathrm{m\,s^{-1}}$.

(a) Find the speed of A when it reaches a height of 20 m above the ground. [2]

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When A reaches a height of 20 m, it collides with a particle B of mass 0.3 kg which is moving downwards in the same vertical line as A with speed $32.5\,\mathrm{m\,s^{-1}}$. In the collision between the two particles, B is brought to instantaneous rest.

(b) Show that the velocity of A immediately after the collision is $4.5\,\mathrm{m\,s^{-1}}$ downwards. [2]

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(c) Find the time interval between A and B reaching the ground. You should assume that A does not bounce when it reaches the ground. [4]

6 A railway engine of mass 120 000 kg is towing a coach of mass 60 000 kg up a straight track inclined at an angle of α to the horizontal where $\sin\alpha = 0.02$. There is a light rigid coupling, parallel to the track, connecting the engine and coach. The driving force produced by the engine is 125 000 N and there are constant resistances to motion of 22 000 N on the engine and 13 000 N on the coach.

(a) Find the acceleration of the engine and find the tension in the coupling. [5]

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At an instant when the engine is travelling at $30\,\mathrm{m\,s^{-1}}$, it comes to a section of track inclined upwards at an angle β to the horizontal. The power produced by the engine is now 4 500 000 W and, as a result, the engine maintains a constant speed.

(b) Assuming that the resistance forces remain unchanged, find the value of β. [4]

7 A particle X travels in a straight line. The velocity of X at time t s after leaving a fixed point O is denoted by $v\,\mathrm{m\,s^{-1}}$, where

$$v = -0.1t^3 + 1.8t^2 - 6t + 5.6.$$

The acceleration of X is zero at $t = p$ and $t = q$, where $p < q$.

(a) Find the value of p and the value of q. [4]

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It is given that the velocity of X is zero at $t = 14$.

(b) Find the velocities of X at $t = p$ and at $t = q$, and hence sketch the velocity-time graph for the motion of X for $0 \leqslant t \leqslant 15$. [3]

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(c) Find the total distance travelled by X between $t = 0$ and $t = 15$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/42**

Paper 4 Mechanics **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\pm\frac{1}{2}\times 15\times 2^2\left[=\pm 30\right]$ $\pm\frac{1}{2}\times 15\times 4^2\left[=\pm 120\right]$	**B1**	For KE at top or bottom. Need not be evaluated. $\frac{1}{2}\times 15\times\left(4-2\right)^2$ is B0.
	$\pm 15g\times 1.6\left[=\pm 240\right]$	**B1**	For PE change. Need not be evaluated.
	$240+30=120+W$	**M1**	Attempt at work energy equation; 4 relevant terms; dimensionally correct; allow sign errors. $\frac{1}{2}\times 15\times\left(4-2\right)^2$ is M0. If $W = F$ times a numerical distance seen, then M0.
	Work done = 150 J	**A1**	

Question	Answer	Marks	Guidance
1	**Alternative method for Q1**		
	$4^2 = 2^2 + 2a \times \frac{1.6}{\sin\theta}$	***M1**	Attempt to use $v^2 = u^2 + 2as$ with $s = \frac{1.6}{\sin\theta}$ or $\frac{1.6}{\cos\theta}$ but not $s = 1.6\sin\theta$ or $1.6\cos\theta$ or 1.6. If θ is given a value, then M0. Must be using speeds 2 and 4 here.
	$15g\sin\theta - R = 15a$	**DM1**	3 terms; allow sign errors; allow sin/cos mix but weight must be resolved; dimensionally correct.
	$R = 93.75\sin\theta$	**A1**	$R = 93.75\cos\theta$ Must be consistent with their s .
	Work done $\left[= 93.75\sin\theta \times \frac{1.6}{\sin\theta}\right] = 150$ J	**A1**	
		4	

Question	Answer	Marks	Guidance
2	Attempt to resolve in at least one direction to form an equation.	*M1	Correct number of terms; allow sign errors; allow sin/cos mix; allow with different T 's.
	$T\sin 30 + T\sin 40 - 2 = 0$	A1	If different T's then allow M1A1A0 max.
	$T\cos 30 - T\cos 40 - mg = 0$	A1	Allow with their T.
	Attempt to solve for T or m	DM1	From equation(s) with correct number of relevant terms.
	Tension T = 1.75, m = 0.0175	A1	$T = 1.7501\ldots$ $m = 0.017497\ldots$ awrt 1.75 for T www, and awrt 0.0175 for m www.
		5	

Question	Answer	Marks	Guidance
3	Work done by 120 N force = $120 \times 5\cos 20 [= 563.81557....]$	B1	
	$(\text{PE change} =) 10g \times 5\sin 30 [= 250]$	B1	For PE change.
	Attempt at work energy equation	M1	4 relevant terms; dimensionally correct; allow sign errors; allow sin/cos mix in relevant resolved terms.
	$120 \times 5\cos 20 - 10g \times 5\sin 30 - 200 = \frac{1}{2} \times 10 \times v^2$ $[563.815... - 250 - 200 = 5v^2]$	A1	
	Speed = 4.77 m s^{-1}	A1	awrt 4.77.

Question	Answer	Marks	Guidance
3	**Alternative method for Question 3**		
	Resistive force = $\frac{200}{5}[=40]$	***B1**	oe e.g. $5 \times RF = 200$.
	$120\cos 20 - RF - 10g\sin 30 = 10a$	***M1**	4 relevant terms; dimensionally correct; allow sign errors; allow sin/cos mix; allow with their resistive force or just *RF*.
	$a = 2.276\ldots$	**A1**	Allow arwt 2.3 to 2sf from correct work.
	$v^2 = 0 + 2 \times (2.276\ldots) \times 5$	**DM1**	Use of $v^2 = u^2 + 2as$ using $u = 0$, $s = 5$ and their positive a which has come from a resistive force using work done.
	Speed = $4.77\,\text{m s}^{-1}$	**A1**	awrt 4.77.
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Question	Answer	Marks	Guidance
4(a)	$R = 0.2g$	**B1**	
	$1.2 = \mu \times 0.2g$	**M1**	Resolve horizontally and using $F = \mu R$ to get an equation in μ; 2 relevant terms.
	$\mu = 0.6$	**A1**	oe
		3	
4(b)	$1.2 - 0.3 \times 0.2g = 0.2a$	***M1**	Resolve horizontally using Newton's Second Law; 3 relevant terms; allow sign errors; $R = 0.2g$ only.
	$a = 3$	**A1**	$0.6 = 0.2a$ only seen, allow with BOD, but if 0.6 as friction being used as resultant force, this is M0A0.
	$s_3 = 0 + \frac{1}{2} \times 3 \times 3^2 \; [=13.5] \; s_2 = 0 + \frac{1}{2} \times 3 \times 2^2 \; [=6]$	**DM1**	For use of $s = ut + \frac{1}{2}at^2$ (or a complete method) to find a distance at least once with $u = 0$ and their positive a and $t = 2$ or $t = 3$.
	Distance = $13.5 - 6 = 7.5$ m	**A1**	www
		4	

Question	Answer	Marks	Guidance
5(a)	$v^2 = 25^2 + 2(-g)\times 20$ OR $\frac{1}{2}\times 0.5\times v^2 = \frac{1}{2}\times 0.5\times 25^2 - 0.5\times g\times 20$	**M1**	Use of $v^2 = u^2 + 2as$ with $u = 25$, $s = 20$ and $a = \pm g$. OR using change in KE = ± change in PE.
	$\text{Speed} = 15\text{ m s}^{-1}$	**A1**	
		2	
5(b)	Taking up as positive direction: $0.5\times 15 + 0.3\times(-32.5) = 0.5v + 0$ or Taking down as positive direction: $0.5\times(-15) + 0.3\times 32.5 = 0.5v + 0$	**M1**	For use of conservation of momentum, 3 non-zero terms, allow sign errors, using their speed 15 m s^{-1}. Must show how ± 2.25 is obtained.
	[Taking up as positive direction: velocity of A = -4.5 m s^{-1}] [Taking down as positive direction: velocity of A = 4.5 m s^{-1}] Speed = 4.5 m s^{-1} direction downwards	**A1**	Any error seen in calculating v is A0. Must explicitly say 4.5 m s^{-1} and downwards.
		2	

Question	Answer	Marks	Guidance
5(c)	Downwards to be positive, for A $20 = 4.5t_A + \frac{1}{2}gt_A^{\ 2}$ and solve for t_A Upwards to be positive, for A $-20 = -4.5t_A - \frac{1}{2}gt_A^{\ 2}$ and solve for t_A	**M1**	Using constant acceleration formula(e) to get a correct equation in t_A and solve for t_A. If using quadratic formula, must be the correct formula. If factorising, when brackets expanded, 2 terms correct.
	For B $20 = 0 + \frac{1}{2}gt_B^{\ 2}$ $[t = 2]$ and solve for t_B	**M1**	Using constant acceleration formula(e) to get a correct equation in t_B and solve for t_B.
	$t_A = 1.6$ or $t_B = 2$	**A1**	
	Difference = 0.4 s only	**A1**	
		4	

Question	Answer	Marks	Guidance
6(a)	Engine: $125000-120000g\times 0.02-22000-T=120000a$ $125000-120000g\sin(1.145\ldots..)-22000-T=120000a$ $[125000-24000-22000-T=120000a \Rightarrow 79000-T=120000a]$ Coach: $T-60000g\times 0.02-13000=60000a$ $T-60000g\sin(1.145\ldots..)-13000=60000a$ $[T-12000-13000=60000a \Rightarrow T-25000=60000a]$	***M1**	Attempt at Newton's second law at least once; correct number of relevant terms; allow sign errors; allow sin/cos mix; allow g missing; a value for α or $\sin\alpha$ must be substituted. Allow with $\alpha=1.1$ or better $[\alpha=1.145991998]$.
		A1	Any equations correct.
	System: $125000-120000g\times 0.02-60000g\times 0.02-22000-13000=(120000+60000)a$ $125000-120000g\sin(1.145\ldots..)-60000g\sin(1.145\ldots..)-22000-13000=(120000+60000)a$ $[125000-24000-12000-22000-13000=(120000+60000)a \quad \Rightarrow 54000=180000a]$	**A1**	Two equations correct. If using separate equations for engine and coach and different T's, then allow M1A1A0 max.
	Solve for T or a	**DM1**	Using equations with the correct number of relevant terms. If no working seen, must be solutions to their equation(s) to be awarded M1.
	Acceleration = $0.3\,\text{m}\,\text{s}^{-2}$ and Tension = 43 000 N	**A1**	Allow 0.299 from use of $\alpha=1.15$. Awrt 43000 to 3sf from correct work.
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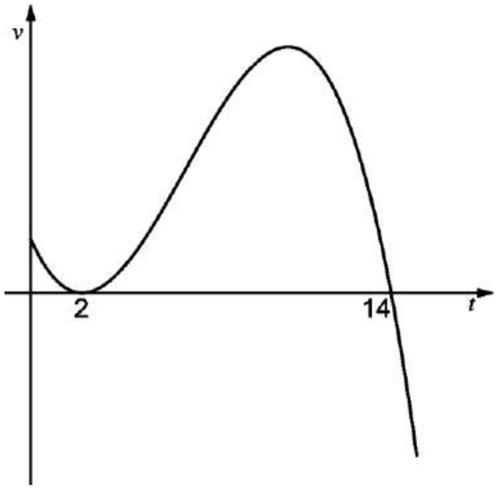
Question	Answer	Marks	Guidance
6(b)	Driving force, $DF = \frac{4500000}{30}\left[=150000\right]$	**B1**	Use of $F = \frac{P}{v}$, oe e.g. $DF \times 30 = 4500000$.
	Attempt to resolve parallel to the track once if using system equation, twice if using equations for engine and coach separately	**M1**	Correct number of relevant terms; allow sign errors; allow sin/cos mix; allow g missing. Must be correct number of equations depending on method.
	System: $150000 - 120000g\sin\beta - 60000g\sin\beta - 22000 - 13000 = 0$ or for Engine: $150000 - 120000g\sin\beta - 22000 - T' = 0$ and Coach: $T' - 60000g\sin\beta - 13000 = 0$	**A1**	Allow *DF* or their *DF*. Must be using same T'.
	Solve to get $\beta = 3.7°$	**A1**	3.663058552 awrt 3.7° www.
		4	

Question	Answer	Marks	Guidance
7(a)	Attempt to differentiate v	*M1	Decrease power by 1 and a change in coefficient in at least one term (which must be the same term); allow unsimplified; allow p or q for t. $a = \frac{v}{t}$ is M0.
	$\left(a = \frac{dv}{dt} =\right) 3 \times -0.1t^{3-1} + 2 \times 1.8t^{2-1} - 6t^{1-1} = -0.3t^2 + 3.6t - 6$	A1	May be unsimplified. Allow p or q for t.
	Setting $a = \frac{dv}{dt} = 0$ and attempt to solve a 3 term quadratic for t. $\left[a = \frac{dv}{dt} = 0 \Rightarrow 3t^2 - 36t + 60 = 0 \Rightarrow t^2 - 12t + 20 = 0\right]$	DM1	Allow p or q for t. Must get 2 values or numerical expressions for t from their three term quadratic. If using quadratic formula, must be the correct formula. If factorising, when brackets expanded, 2 terms correct.
	$p = 2,\ q = 10$	A1	
		4	

Question	Answer	Marks	Guidance
7(b)	Velocities are $0\,\mathrm{m\,s^{-1}}$ and $25.6\,\mathrm{m\,s^{-1}}$	**B1**	SOI
	Curve with single minimum turning point followed by single maximum turning point	***B1**	Ignore placement of graph on axes Not a cusp for the minimum point or maximum point.
	 	**DB1**	All correct in 1st and 4th quadrant. Must go from convex to concave. Need to label 2 and 14 on the t-axis where the curve meets the t-axis. Do not need to show exact velocities at t = 0 or t = 10 or t = 15. Ignore graph outside $0 \leqslant t \leqslant 15$.
		3	

Question	Answer	Marks	Guidance
7(c)	Attempt to integrate v	*M1	Increase power by 1 and a change in coefficient in at least one term (which must be the same term); $s = vt$ is M0.
	$(s=) -\frac{0.1}{4}t^{3+1} + \frac{1.8}{3}t^{2+1} - \frac{6}{2}t^{1+1} + 5.6t(+c)$ $= -0.025t^4 + 0.6t^3 - 3t^2 + 5.6t\ [+c]$	A1	May be unsimplified.
	Attempt distance from $t=0$ to $t=14$ [= 176.4]	DM1	Correct use of limits 0 and 14 for their s, i.e. $F(14) - F(0)$ May see limits 0 to 2 and 2 to 14 used but must be $\left(F(14) - F(2)\right) + \left(F(2) - F(0)\right)$.
	Attempt distance from $t=14$ to $t=15$ [= (-) 8.025]	DM1	Correct use of limits 14 and 15 for their s, i.e. $\pm\left(F(15) - F(14)\right)$. For reference $F(2) = \frac{18}{5} = 3.6$, $F(14) = \frac{882}{5} = 176.4$ and $F(15) = \frac{1347}{8} = 168.375$.
	Total distance $= 176.4 + 8.025 = 184.425$ m $= \frac{7377}{40}$	A1	www Condone 184 or better.

Question	Answer	Marks	Guidance
7(c)	**SC for those who show no integration. Max 3 marks.**		
	$\int_0^{14}\left(-0.1t^3+1.8t^2-6t+5.6\right)dt=176.4$	**B1**	
	$\int_{14}^{15}\left(-0.1t^3+1.8t^2-6t+5.6\right)dt=-8.025$ OR $\left\|\int_{14}^{15}\left(-0.1t^3+1.8t^2-6t+5.6\right)dt\right\|=8.025$	**B1**	
	Total distance $=176.4+8.025=184.425$ m $=\frac{7377}{40}$	**B1**	Condone 184 or better.
	SC for those who show no integration and don't consider the 2 areas. Max 1 mark.		
	$\int_0^{15}\left\|-0.1t^3+1.8t^2-6t+5.6\right\|dt=184.425$ m $=\frac{7377}{40}$	**B1**	Condone 184 or better.
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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 9 3 0 2 2 9 3 0 8 4 *

MATHEMATICS **9709/43**

Paper 4 Mechanics **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

1 A particle is projected vertically upwards from horizontal ground with a speed of $u\,\text{m}\,\text{s}^{-1}$. The particle has height s m above the ground at times 3 seconds and 4 seconds after projection.

Find the value of u and the value of s. [3]

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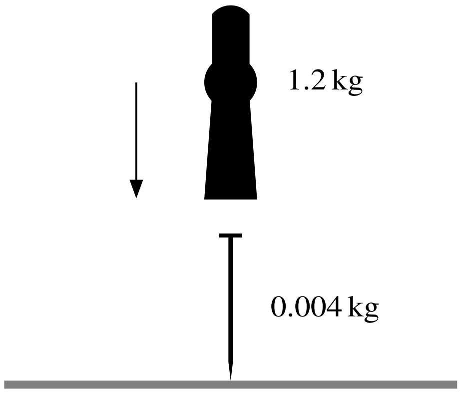

A machine for driving a nail into a block of wood causes a hammerhead to drop vertically onto the top of a nail. The mass of the hammerhead is 1.2 kg and the mass of the nail is 0.004 kg (see diagram). The hammerhead hits the nail with speed $v\,\mathrm{m\,s^{-1}}$ and remains in contact with the nail after the impact. The combined hammerhead and nail move immediately after the impact with speed $40\,\mathrm{m\,s^{-1}}$.

(a) Calculate v, giving your answer as an exact fraction. [2]

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(b) The nail is driven 4 cm into the wood.

Find the constant force resisting the motion. [3]

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3 A block of mass 8 kg slides down a rough plane inclined at 30° to the horizontal, starting from rest. The coefficient of friction between the block and the plane is μ. The block accelerates uniformly down the plane at $2.4\,\mathrm{m\,s^{-2}}$.

(a) Draw a diagram showing the forces acting on the block. [1]

(b) Find the value of μ. [4]

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(c) Find the speed of the block after it has moved 3 m down the plane. [1]

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4 A car has mass 1600 kg.

(a) The car is moving along a straight horizontal road at a constant speed of $24\,\mathrm{m\,s^{-1}}$ and is subject to a constant resistance of magnitude 480 N.

Find, in kW, the rate at which the engine of the car is working. [2]

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The car now moves down a hill inclined at an angle of θ to the horizontal, where $\sin\theta = 0.09$. The engine of the car is working at a constant rate of 12 kW. The speed of the car is $24\,\mathrm{m\,s^{-1}}$ at the top of the hill. Ten seconds later the car has travelled 280 m down the hill and has speed $32\,\mathrm{m\,s^{-1}}$.

(b) Given that the resistance is not constant, use an energy method to find the total work done against the resistance during the ten seconds. [5]

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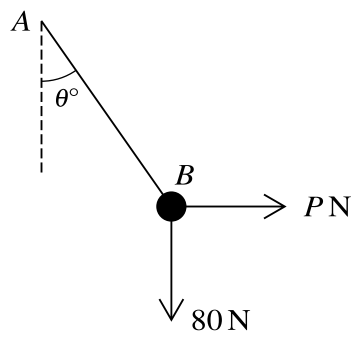

A light string AB is fixed at A and has a particle of weight 80 N attached at B. A horizontal force of magnitude P N is applied at B such that the string makes an angle $\theta°$ to the vertical (see diagram).

(a) It is given that $P = 32$ and the system is in equilibrium.

Find the tension in the string and the value of θ. [4]

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(b) It is given instead that the tension in the string is 120 N and that the particle attached at B still has weight 80 N.

Find the value of P and the value of θ. [4]

6 A particle moves in a straight line. At time t s, the acceleration, $a\,\mathrm{m\,s^{-2}}$, of the particle is given by $a = 36 - 6t$. The velocity of the particle is $27\,\mathrm{m\,s^{-1}}$ when $t = 2$.

(a) Find the values of t when the particle is at instantaneous rest. [4]

(b) Find the total distance the particle travels during the first 12 seconds. [4]

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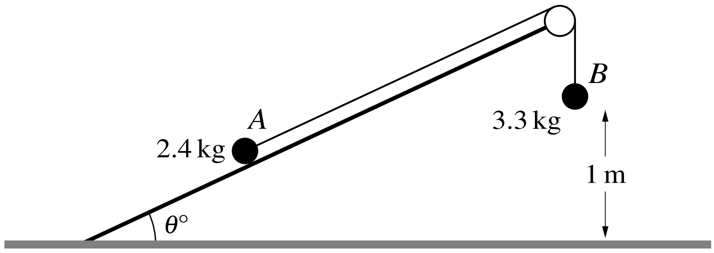

Particles A and B, of masses 2.4 kg and 3.3 kg respectively, are connected by a light inextensible string that passes over a smooth pulley which is fixed to the top of a rough plane. The plane makes an angle of $\theta°$ with horizontal ground. Particle A is on the plane and the section of the string between A and the pulley is parallel to a line of greatest slope of the plane. Particle B hangs vertically below the pulley and is 1 m above the ground (see diagram). The coefficient of friction between the plane and A is μ.

(a) It is given that $\theta = 30$ and the system is in equilibrium with A on the point of moving directly up the plane.

Show that $\mu = 1.01$ correct to 3 significant figures. [5]

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(b) It is given instead that $\theta = 20$ and $\mu = 1.01$. The system is released from rest with the string taut.

Find the total distance travelled by A before coming to instantaneous rest. You may assume that A does not reach the pulley and that B remains at rest after it hits the ground. [8]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/43**

Paper 4 Mechanics **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Due to a series-specific issue during the live exam series, all candidates were awarded full marks for questions 1 and 2a. This published mark scheme for these questions was created alongside the question paper, but has not been used by examiners.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

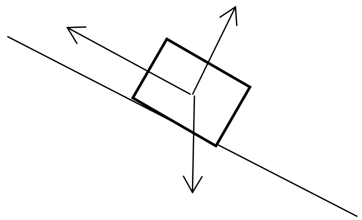
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use of suvat to find expressions for s	**M1**	
	$s = 3u - 5 \times 9$ $s = 4u - 5 \times 16$	**A1**	
	$u = 35, s = 60$	**A1**	
	Alternative method for Question 1:		
	Use of suvat to find expressions for u at max height	**M1**	
	$0 = u - 10 \times 3.5$	**A1**	
	$u = 35, s = 60$	**A1**	
		3	

Question	Answer	Marks	Guidance
2(a)	Attempt at conservation of momentum $[1.2v = (1.2 + 0.004) \times 40]$	**M1**	
	$v = \frac{602}{15}$	**A1**	oe
		2	
2(b)	$0^2 = (40)^2 + 2 \times 0.04 \times a \quad [a = -20000]$ or $0.04 = \frac{0+40}{2}t$ gets $t = 0.002$, so $0 = 40 + 0.002a\,[a = -20000]$	**M1**	Use of a 'suvat' method to get an equation in a. Allow sign errors. Allow ± 20000. Do not allow 4 in place of 0.04. Allow use of 40.1 or $\frac{602}{15}$ for velocity in place of 40.
	Attempt to use Newton's Second Law vertically. $[-R + (1.2 + 0.004)g = (1.2 + 0.004) \times a]$ $[-R + 12.04 = 1.204a]$	**M1**	Must have the correct number of relevant terms. Allow sign errors, but terms including masses must be effectively added. Do not allow any mass other than (1.2 + 0.004).
	R = 24 100 N [24 092.04 = $\frac{602301}{25}$]	**A1**	WWW. Note: use of wrong sign for g leads to answers 24 067.96 which gets max M1M1A0. Note: Missing weight term gets 24 080 which gets Max M1M0A0.
		3	

Question	Answer	Marks	Guidance
2(b)	**Alternative method for Question 2(b) using energy**		
	[Change in PE =] $1.204g \times 0.04[=0.4816]$ or [change in KE =] $\frac{1}{2}\times 1.204 \times (40)^2 [=963.2]$	**B1**	Allow use of 40.1 or $\frac{602}{15}$ for velocity in place of 40. B0 for kinetic energy, if extra kinetic energy terms present.
	$1.204g \times 0.04 + \frac{1}{2}\times 1.204 \times (40)^2 = 0.04R$	**M1**	Attempt at work energy equation. Must have correct number of relevant terms. dimensionally correct; allow sign errors. Do not allow 4 in place of 0.04. Allow use of 40.1 or $\frac{602}{15}$ for velocity in place of 40.
	R = 24 100 N [24 092.04 = $\frac{602301}{25}$]	**A1**	WWW Note: use of wrong sign for g leads to answers 24 067.96 which gets max B1M1A0. Note: Missing potential energy term gets 24 080, which gets maximum of B1M0A0.

Question	Answer	Marks	Guidance
3(a)	Correct force diagram with 3 forces in the correct directions. 	**B1**	No labels required on the 3 forces and ignore wrong labels. Arrows needed. Allow either or both components of weight if fully labelled. Allow sin/cos mix. If forces are not connected to the block, then the line of action of each force must go through the block.
		1	
3(b)	$R = 8g\cos 30 \ \left[= 40\sqrt{3} = 69.282\ldots\right]$	**B1**	Resolving perpendicular to the plane.
	Resolving parallel to the plane and attempt to apply Newton's second law. $\left[8g\sin 30 - F = 8\times 2.4 \quad \Rightarrow F = 20.8\right]$	**M1***	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
	Use of $F = \mu R$ to get an equation in μ only. $\left[8g\sin 30 - 8g\mu\cos 30 = 8\times 2.4 \qquad 40 - 40\sqrt{3}\mu = 19.2\right]$	**DM1**	Allow g missing in either or both of F and R. Allow sign errors, consistent sin/cos mix. R must be a single component of a force. Allow the 3 masses to be cancelled.
	$\mu = 0.3[0\ldots]$ $\quad \left[\text{May first see } \frac{20.8}{40\sqrt{3}} \text{ or } \frac{20.8}{69.282\ldots}\right]$	**A1**	Allow exact value $\frac{13\sqrt{3}}{75}$ or $\frac{104\sqrt{3}}{600}$ oe.
		4	

Question	Answer	Marks	Guidance
3(c)	[$v^2 = 2\times 2.4\times 3 \;\Rightarrow$ greatest speed =] 3.79 $\text{ms}^{-1} = \dfrac{6\sqrt{10}}{5}$	**B1**	3.79473… (3.8 without a more accurate value seen gets B0 and should be annotated SF).
		1	

Question	Answer	Marks	Guidance
4(a)	$P = 480\times 24$ or, e.g. $\dfrac{P}{24} - 480 = 0$	**M1**	For $\dfrac{P}{v} - F = 0$ or $P = Fv$ oe.
	$P = 11.52$ [kW]	**A1**	Allow 11.5 M1A0 for 11 520 or 11 500.
		2	

Question	Answer	Marks	Guidance
4(b)	$\left[\text{KE}_{before}\right]=\frac{1}{2}\times1600\times24^2\,[=460800]$ $\left[\text{KE}_{after}\right]=\frac{1}{2}\times1600\times32^2\,[=819200]$	**B1**	For either correct. Do not allow $\frac{1}{2}\times1600\times(32-24)^2$.
	$\left[\text{PE}_{loss}\right]=1600g\times280\times0.09[=1600g\times25.2=403200]$	**B1**	Allow $1600g\times280\times\sin5.16°$ or $1600g\times280\times\sin5.2°$ but not simply $1600g\times280\times\sin\theta$ (unless implied by correct final answer).
	Total WD $=12000\times10\;[=120000]$	**B1**	oe, e.g. $12\,000=\frac{\text{WD}}{10}$.
4(b)	Work done against resistance = or $280F=$ or WD = or W = oe $12000\times10+1600g\times280\times0.09-\frac{1}{2}\times1600\times32^2+\frac{1}{2}\times1200\times24^2$ $[=120000+403200-819200+460800]$	**M1**	Attempt at work energy equation with 5 relevant terms (4 relevant terms plus work done against resistance); dimensionally correct. Allow sign errors. M0 for use of constant acceleration. Do not allow $\frac{1}{2}\times1600\times(32-24)^2$.
	WD = 164 800 [J]	**A1**	Or 164.8 kJ CAO but condone 165 kJ or 165 000 [J] Not from use of constant acceleration or Newton's second law. ISW attempt to find force after correct WD found.
		5	

Question	Answer	Marks	Guidance
5(a)	Attempt to resolve in one direction and form equation.	**M1**	Must have correct number of relevant terms (forces must have components as required). Allow sin/cos mix. Allow sign errors. If only one equation shown and it involves 32, it must be 32, not *P*.
	$T\sin\theta = 32$ and $T\cos\theta = 80$ or $0 = 80\sin\theta - 32\cos\theta$ and $T = 80\cos\theta + 32\sin\theta$	**A1**	For both horizontal and vertical, or both parallel and perpendicular.
	Attempt to solve for T or θ	**M1**	Must get to T or θ; e.g. $T = \sqrt{32^2 + 80^2}$ or $\theta = \tan^{-1}\left(\frac{32}{80}\right)$. Condone, e.g. $\theta = \tan^{-1}\left(\frac{80}{32}\right)$. Must come from equations with correct number of relevant terms.
	$T = 86.2$ [N 86.1626….] or $16\sqrt{29}$ or $\sqrt{7424}$ and $\theta = 21.8$ [21.801…]	**A1**	For both.
		4	

Question	Answer	Marks	Guidance
5(a)	**Alternative method using triangle of forces**		
	$T^2 = 80^2 + 32^2\left[-2\times 80\times 32\cos 90\right]$ or $T\sin\theta = 32$ or $T\cos\theta = 80$	**M1**	For any of the five; allow sign errors.
	$80\tan\theta = 32$ or $T = 80\cos\theta + 32\sin\theta$ oe	**A1**	For any two equations.
	Attempt to solve for T or θ	**M1**	Must get to T or θ; e.g. $T = \sqrt{32^2 + 80^2}$ or $\theta = \tan^{-1}\left(\frac{32}{80}\right)$.
	$T = 86.2$ [N 86.1626….] or $16\sqrt{29}$ or $\sqrt{7424}$ and $\theta = 21.8$ [21.801…]	**A1**	
	Alternative Triangle of forces method using sine rule		
	$\frac{T}{\sin 90} = \frac{32}{\sin(\theta)} = \frac{80}{\sin(90-\theta)}$	**M1**	For any two.
		A1	For all three.
	Attempt to solve for T or θ	**M1**	e.g. $\theta = \tan^{-1}\left(\frac{32}{80}\right)$.
	$T = 86.2$ [N 86.1626….] or $16\sqrt{29}$ or $\sqrt{7424}$ and $\theta = 21.8$ [21.801…]	**A1**	For both.

Question	Answer	Marks	Guidance
5(b)	Attempt to resolve in one direction and form equation	**M1**	Must have correct number of relevant terms (forces must have components as required). Allow sin/cos mix. Allow sign errors. Must use 120, not *T*.
	$120\sin\theta = P$ and $120\cos\theta = 80$ or $0 = 80\sin\theta - P\cos\theta$ and $120 = 80\cos\theta + P\sin\theta$	**A1**	For both horizontal and vertical, or both parallel and perpendicular.
	Attempt to solve for P or θ	**M1**	Must get to P or θ; e.g. $P = \sqrt{120^2 - 80^2}$ or $\theta = \cos^{-1}\left(\frac{80}{120}\right)$. Must come from equations with correct number of relevant terms.
	$P = 89.4$ $[89.4427]$ or $40\sqrt{5}$ or $\sqrt{8000}$ $\theta = 48.2$ $[48.1896\ldots]$	**A1**	For both; allow $P = 89.5$ (from $120\sin 48.2$).
		4	
	Alternative method using triangle of forces		
	$120^2 = P^2 + 80^2 [-2\times 80\times P\cos 90]$ or $120\sin\theta = P$ or $120\cos\theta = 80$ or $80\tan\theta = P$ or $120 = 80\cos\theta + P\sin\theta$ oe	**M1**	For any of the five; allow sign errors.
		A1	For any two equations.
	Attempt to solve for P or θ	**M1**	Must get to P or θ; e.g. $P = \sqrt{120^2 - 80^2}$ or $\theta = \cos^{-1}\left(\frac{80}{120}\right)$, oe.
	$P = 89.4$ $[89.4427]$ or $40\sqrt{5}$ or $\sqrt{8000}$ $\theta = 48.2$ $[48.1896\ldots]$	**A1**	For both; allow $P = 89.5$ (from $120\sin 48.2$) .

Question	Answer	Marks	Guidance
5(b)	**Alternative Triangle of forces method using sine rule**		
	$\frac{120}{\sin 90}=\frac{P}{\sin(\theta)}=\frac{80}{\sin(90-\theta)}$	**M1**	For any two.
		A1	For all three.
	Attempt to solve for P or θ	**M1**	Must get to P or θ; e.g. $\theta=90-\sin^{-1}\left(\frac{80}{120}\right)$ or $\theta=\cos^{-1}\left(\frac{80}{120}\right)$.
	$P=89.4\ [89.4427]$ or $40\sqrt{5}$ or $\sqrt{8000}$ $\theta=48.2\ [48.1896\ldots]$	**A1**	For both; allow $P=89.5$ (from $120\sin 48.2$).
6(a)	Attempt to integrate a	**M1***	The power of t must increase by 1 with a change of coefficient in the t^2 term. Do not penalise missing c. Use of $v=at$ scores M0.
	$[v=]36t-3t^2[+c]$ or $[v=]36t-\frac{6t^2}{2}[+c]$	**A1**	Condone an integral sign in front of correct answer.
	$0=36t-3t^2-33$ $\left[27=36\times 2-3\times 2^2+c \quad \Rightarrow c=-33\right]$	**DM1**	Use $t=2$ and $v=27$ to find c. Must get to c = and set 3 term quadratic equal to zero.
	Solve $0=36t-3t^2-33$ to get $t=1$ and $t=11$	**A1**	Allow $t=1$ or $t=11$; $t=1,\ t=11$ oe.
		4	

Question	Answer	Marks	Guidance
6(b)	Attempt to integrate an expression of the form $at + bt^2 [+c]$ with non-zero *a* and *b*. If correct $[s =]\frac{36t^2}{2} - \frac{3t^3}{3} - 33t[+c']$ or $[s =]18t^2 - t^3 - 33t\ [+c']$	M1*	The power of *t* must increase by 1 with a change of coefficient in the same term. Use of $s = vt$ scores M0.
	Attempt to evaluate *their* $\left[18t^2 - t^3 - 33t\right]$ for $t = 0$ to $t = 1$ or $t = 1$ to $t = 11$ or $t = 11$ to $t = 12$ 0 to 1: –16 – 0 = –16 1 to 11: 484 – (–16) = 500 11 to 12: 468 – 484 = –16	DM1	Attempt using their limits (at least one strictly between 0 and 12) correctly.
	For all three	DM1	Allow 11 to 12 implied by symmetry instead of found separately.
	Distance $= [16 + 500 + 16] = 532\,\text{m}$	A1	
		4	

Question	Answer	Marks	Guidance
7(a)	Resolving for both particles or for the system to form equation(s)	**M1***	Must have correct number of terms. Allow sign errors. Allow sin/cos mix. Allow g missing. M0 if acceleration included unless subsequently equated to zero. Masses must be appropriate for their equation(s). Forces must have components (or not) as required.
	Either $T - F - 2.4g\sin 30 = 0$ AND $3.3g - T = 0$ Or $3.3g - F - 2.4g\sin 30 = 0$	**A1**	Both correct or system correct. May get $F = 21$. Can be with a wrong non-zero F.
	$R = 2.4g\cos 30 \ \left[= 12\sqrt{3} = 20.7846\ldots\right]$	**B1**	
	Use of $F = \mu R$ to get an equation in μ only $[3.3g - 2.4g\mu\cos 30 - 2.4g\sin 30 = 0]$	**DM1**	Must be from F dimensionally correct and single term R which is equal to a component the 2.4 kg weight. Allow consistent sin/cos mix but must be different components of weight. F and R must be numerical expressions.
	$\mu = 1.01$ [sight of 1.01036… or 1.0104]	**A1**	AG perhaps from one of $\mu = \dfrac{3.3g - 24\sin 30}{2.4g\cos 30} = \dfrac{33-12}{12\sqrt{3}} = \dfrac{21}{12\sqrt{3}} = \dfrac{7\sqrt{3}}{12} = \dfrac{21}{20.7846\ldots}$ $= \dfrac{21}{20.8}$ Do not allow unless evidence of 30 substituted for θ. E.g.: sight of 1.01036… or 1.0104.
		5	

Question	Answer	Marks	Guidance
7(b)	Using Newton's second law for both particles or the system	**M1***	Must have correct number of terms. Allow sign errors. Allow sin/cos mix. Allow g missing. Masses must be appropriate for their equation(s). Forces must have components (or not) as required.
	Either $3.3g - T = 3.3a$ and $T - F - 2.4g\sin 20 = 2.4a$ $[T - 22.778 - 8.208 = 2.4a]$ or $[T - 30.986 = 2.4a]$ or $3.3g - F - 2.4g\sin 20 = (2.4 + 3.3)a$ $[2.013367\ldots = 5.7a]$	**A1**	Both correct or system equation correct. Can be with a wrong non-zero F.
	$F = 1.01 \times 2.4g\cos 20$ $[= 22.778]$	**B1**	For correct expression for F.
	Attempt to solve for a $a = 0.353$ $[0.353222\ldots]$	**DM1**	Using their F Must get to 'a ='. If sin/cos mix must be consistent.
	$v^2 = 2 \times 0.353 \times 1$ $[= 0.706444\ldots]$ or $[v = 0.841]$ Or $1 = 0 + \frac{1}{2} \times 0.353t^2 \Rightarrow t = 2.3795 \Rightarrow v = 0.353 \times 2.38$	**A1FT**	FT their value of $a \neq \pm g$ to get an expression for v^2 or v. Can be implied by awrt 0.84 for v or awrt 0.71 for v^2. This mark does not depend on previous A or B mark, but both Ms must have been awarded.
	Using Newton's second law on A after B reaches the ground $-F - 2.4g\sin 20 = 2.4a$ $[-1.01 \times 2.4g\cos 20 - 2.4g\sin 20 = 2.4a]$ $[-22.78814\ldots - 8.20848\ldots = 2.4a]$	**M1***	Must have correct number of terms. Allow sign errors. Allow sin/cos mix. Allow g missing. $[\Rightarrow a = -12.911\ldots]$
	Use of suvat to find s $\left[0 = \mathit{their}\,0.841^2 + 2 \times \mathit{their} -12.911\ldots \times s \Rightarrow s = 0.027358\ldots.\right]$	**DM1**	Using their $a \neq \pm g$. Must get to 's ='. May find and use t = 0.0651.

Question	Answer	Marks	Guidance
7(b)	Total distance = 1.03 m	**A1**	
		8	
	Alternative method using energy for first 5 marks		
	$\left[\text{KE gained} =\right]\frac{1}{2}\times(2.4+3.3)v^2\left[=2.85v^2\right]$	**B1**	
	$\left[\text{PE lost} =\right]3.3g\times1-2.4g\times1\sin 20\left[=24.791\ldots\right] =$	**B1**	Allow omission of 1 in either or both terms.
	[Friction =] $1.01\times2.4g\cos 20\left[=22.778\ldots\right]$	**B1**	For correct expression for *F*.
	$\frac{1}{2}\times(2.4+3.3)v^2=3.3g\times1-2.4g\times1\sin 20-1.01\times2.4g\cos 20\times1$ Or $2.85v^2=24.791\ldots-22.778\ldots$	**M1**	For attempt at energy equation. Allow sign errors, allow sin/cos mix but must have sin/cos where needed. Correct number of terms, dimensionally correct. Allow omission of 1 in any or all the three relevant terms. Must have $\cos 20$ and $\sin 20$.
	To get a correct expression for v^2 $\left[v^2=0.706444\ldots \text{ or } v=0.841\ldots\right]$	**A1**	Can be implied by awrt 0.84 for v or awrt 0.71 for v^2 if expression not seen.

Question	Answer	Marks	Guidance
7(b)	**Alternative method using energy for final 3 marks**		
	KE = $\frac{1}{2} \times 2.4 \times 0.841^2$	**M1**	Using their v^2.
	$1.01 \times 2.4g \cos 20 \times s + 2.4g \sin 20 \times s = \frac{1}{2} \times 2.4 \times 0.841^2$ $[\Rightarrow s = 0.027358..]$	**M1**	For attempt at 3 term energy equation and solved to get to 's ='. Allow sign errors, allow consistent sin/cos mix but must have sin/cos where needed. Correct number of terms, dimensionally correct.
	Total distance = 1.03 m	**A1**	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0775527237

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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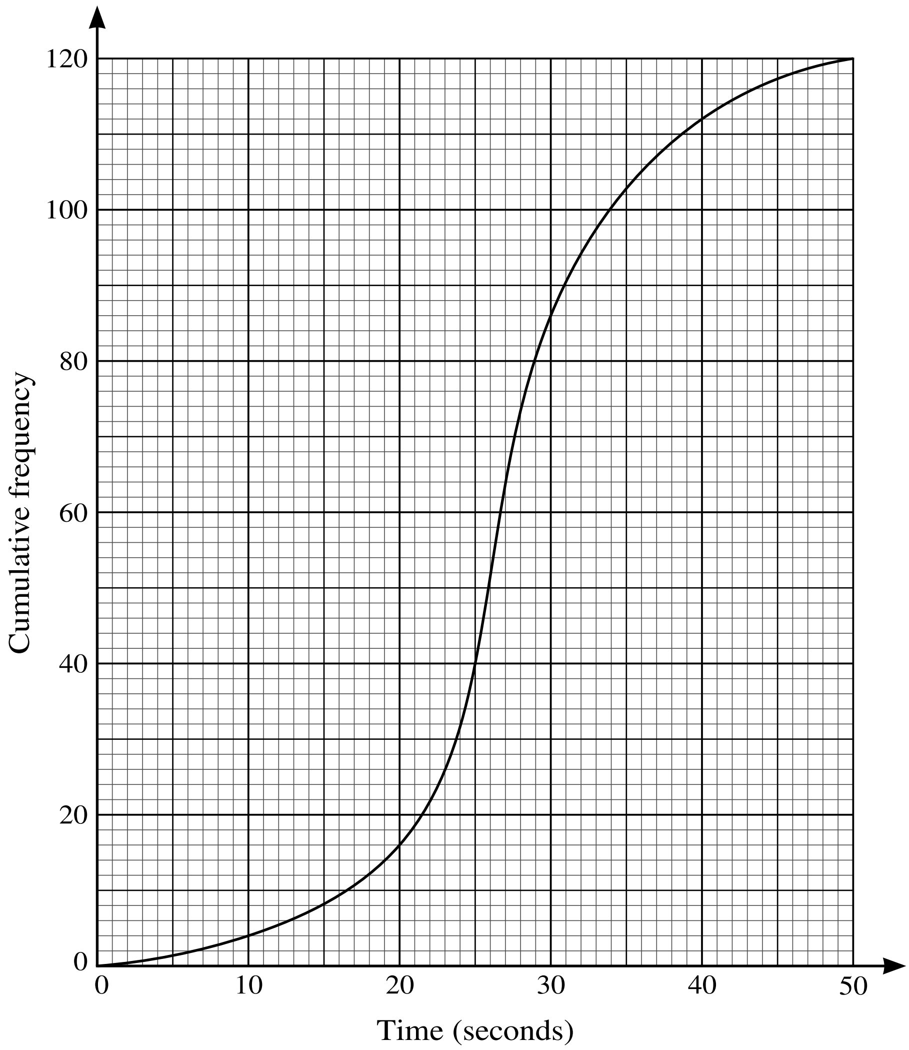
1



The times taken by 120 children to complete a particular puzzle are represented in the cumulative frequency graph.

(a) Use the graph to estimate the interquartile range of the data. [2]

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35% of the children took longer than T seconds to complete the puzzle.

(b) Use the graph to estimate the value of T. [2]

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2 Hazeem repeatedly throws two ordinary fair 6-sided dice at the same time. On each occasion, the score is the sum of the two numbers that she obtains.

(a) Find the probability that it takes exactly 5 throws of the two dice for Hazeem to obtain a score of 8 or more. [2]

(b) Find the probability that it takes no more than 4 throws of the two dice for Hazeem to obtain a score of 8 or more. [2]

(c) For 8 randomly chosen throws of the two dice, find the probability that Hazeem obtains a score of 8 or more on fewer than 3 occasions. [3]

3 A farmer sells eggs. The weights, in grams, of the eggs can be modelled by a normal distribution with mean 80.5 and standard deviation 6.6. Eggs are classified as small, medium or large according to their weight. A small egg weighs less than 76 grams and 40% of the eggs are classified as medium.

(a) Find the percentage of eggs that are classified as small. [3]

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(b) Find the least possible weight of an egg classified as large. [3]

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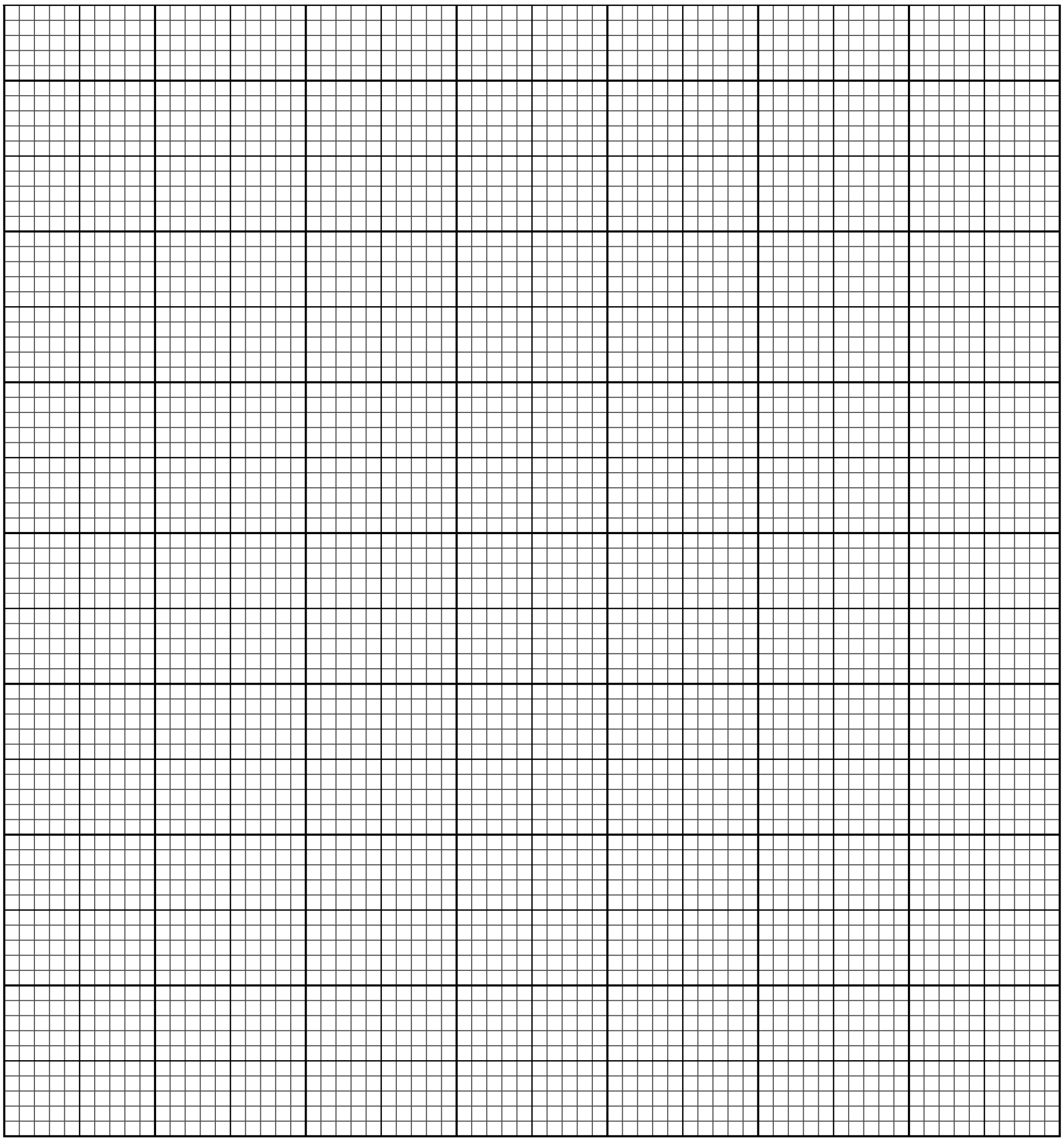
150 of the eggs for sale last week were weighed.

(c) Use an approximation to find the probability that more than 68 of these eggs were classified as medium. [5]

4 The times, to the nearest minute, of 150 athletes taking part in a charity run are recorded. The results are summarised in the table.

Time in minutes	101 – 120	121 – 130	131 – 135	136 – 145	146 – 160
Frequency	18	48	34	32	18

(a) Draw a histogram to represent this information. [4]



(b) Calculate estimates for the mean and standard deviation of the times taken by the athletes. [5]

5 A red spinner has four sides labelled 1, 2, 3, 4. When the spinner is spun, the score is the number on the side on which it lands. The random variable X denotes this score. The probability distribution table for X is given below.

x	1	2	3	4
$P(X = x)$	0.28	p	$2p$	$3p$

(a) Show that $p = 0.12$. [1]

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A fair blue spinner and a fair green spinner each have four sides labelled 1, 2, 3, 4. All three spinners (red, blue and green) are spun at the same time.

(b) Find the probability that the sum of the three scores is 4 or less. [3]

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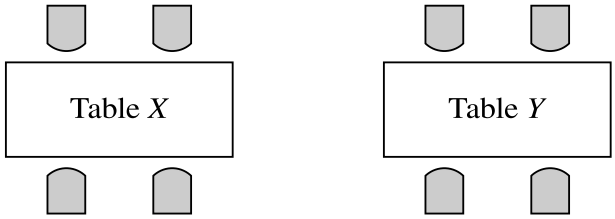
(c) Find the probability that the product of the three scores is 4 or less given that X is odd. [5]

6



In a restaurant, the tables are rectangular. Each table seats four people: two along each of the longer sides of the table (see diagram). Eight friends have booked two tables, X and Y. Rajid, Sue and Tan are three of these friends.

(a) The eight friends will be divided into two groups of 4, one group for table X and one group for table Y.

Find the number of ways in which this can be done if Rajid and Sue must sit at the same table as each other and Tan must sit at the other table. [3]

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When the friends arrive at the restaurant, Rajid and Sue now decide to sit at table X on the same side as each other. Tan decides that he does not mind at which table he sits.

(b) Find the number of different seating arrangements for the 8 friends. [3]

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As they leave the restaurant, the 8 friends stand in a line for a photograph.

(c) Find the number of different arrangements if Rajid and Sue stand next to each other, but neither is at an end of the line. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **18** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	[IQR =] 31 – 23.7	**M1**	30.5 < UQ < 31.25 – 23.25 < LQ $\leqslant$ 24 Evidence of graph use must be seen at least once.
	7.3	**A1**	7.0 $\leqslant$ IQR $\leqslant$ 7.5 If M0 scored, **SC B1** for 7.0 $\leqslant$ IQR $\leqslant$ 7.5 www.
		2	
1(b)	[65% of 120 =]78	**B1**	Seen or implied by use on graph.
	28.5	**B1**	28 < ans < 29
		2	

Question	Answer	Marks	Guidance
2(a)	$\left(\frac{21}{36}\right)^4\left(\frac{15}{36}\right)$	**M1**	$(1-p)^4 \times p$, $0 < p < 1$
	$= \frac{12005}{248832}$, 0.0482	**A1**	0.0482454… to at least 3SF.
		2	

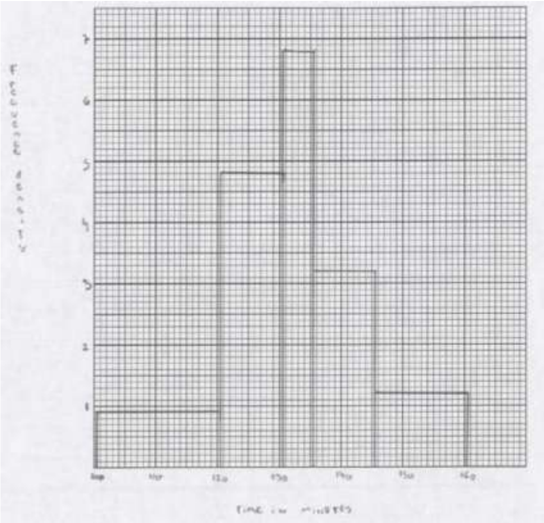
Question	Answer	Marks	Guidance
2(b)	**Method 1**		
	$[\mathrm{P}(X \leqslant 4) =]\ 1-\left(\frac{21}{36}\right)^4$	**M1**	$1-b^r$, b = *their* $(1-p)$ in **2(a)** or correct; r = 4, 5.
	$= \frac{18335}{20736}$, 0.884	**A1**	0.884211… to at least 3SF.
		2	
	Method 2		
	$[\mathrm{P}(X \leqslant 4) =]\ \frac{15}{36}+\frac{15}{36}\times\frac{21}{36}+\frac{15}{36}\times\left(\frac{21}{36}\right)^2+\frac{15}{36}\times\left(\frac{21}{36}\right)^3$	**M1**	$p + p(1-p) + p(1-p)^2 + p(1-p)^3$ $\left[+p\left(1-p\right)^4\right]$ FT from **2(a)** or correct.
	$= \frac{18335}{20736}$, 0.884	**A1**	0.884211… to at least 3SF.
		2	

Question	Answer	Marks	Guidance
2(c)	**Method 1**		
	[P(0,1,2) =] $^8C_0\left(\frac{5}{12}\right)^0\left(\frac{7}{12}\right)^8 + {}^8C_1\left(\frac{5}{12}\right)^1\left(\frac{7}{12}\right)^7 + {}^8C_2\left(\frac{5}{12}\right)^2\left(\frac{7}{12}\right)^6$ 0.01341 + 0.07661 + 0.1915	**M1**	One term $^8C_x(q)^x(1-q)^{8-x}$, $0<q<1,\ x\neq 0,8$.
		A1 FT	Correct expression, accept unsimplified, no terms omitted leading to final answer. FT only with unsimplified expression.
	= 0.282	**B1**	$0.2815 \leqslant q \leqslant 0.282$
	Method 2		
	[1 – P(3,4,5,6,7,8) =] 1 – ($^8C_3\left(\frac{5}{12}\right)^3\left(\frac{7}{12}\right)^5 + {}^8C_4\left(\frac{5}{12}\right)^4\left(\frac{7}{12}\right)^4 + \ldots + {}^8C_7\left(\frac{5}{12}\right)^7\left(\frac{7}{12}\right)^1 + {}^8C_8\left(\frac{5}{12}\right)^8\left(\frac{7}{12}\right)^0$) = 1 – (0.2736 + 0.2443 + … + 0.01017 + 9.084×10^{-4})	**M1**	One term $^8C_x(q)^x(1-q)^{8-x}$, $0<q<1,\ x\neq 0,8$.
		A1 FT	Correct expression, accept unsimplified, no terms omitted leading to final answer. FT only with unsimplified expression.
	= 0.282	**B1**	$0.2815 \leqslant q \leqslant 0.282$
		3	

Question	Answer	Marks	Guidance
3(a)	[P(X < 76) =] P($Z < \frac{76-80.5}{6.6}$)	**M1**	Use of ± standardisation formula with 76, 80.5 and 6.6, condone 6.6^2 or $\sqrt{6.6}$, no continuity correction.
	[= Φ(−0.6818) = 1 −Φ(0.6818) =] 1 – 0.7524 = 0.2476	**M1**	Calculating the appropriate probability area (leading to their final answer).
	24.8%	**A1**	24.75% < ans ⩽ 24.8% (percentage value required). If A0 scored, **SC B1** for 24.75% < ans ⩽ 24.8% www.
		3	
3(b)	[% of large eggs = 100 – 40 – 24.76 = 35.24] [P(Z > $\frac{x-80.5}{6.6}$) $= 0.40 + 0.2476 = 0.6476$] $\frac{x-80.5}{6.6} = 0.378$	**B1**	0.378 ⩽ z < 0.3791 or −0.3791 < z ⩽ −0.378 seen.
		M1	Use of ± standardisation formula with x, 80.5, 6.6 and a z-value (not 0.6476, 0.3524, 0.4, 0.2476) (treat ±0.38 as a z-value), not 6.6^2, not $\sqrt{6.6}$, no continuity correction.
	$x = 83[.0]$	**A1**	awrt 83.0
		3	

Question	Answer	Marks	Guidance
3(c)	Mean $= 150 \times 0.4 = 60$ Var $= 150 \times 0.4 \times 0.6 = 36$	**B1**	60 and 36 seen, allow unsimplified.
	P(X > 68) = $\text{P}\left(Z > \frac{68.5 - 60}{\sqrt{36}}\right)$	**M1**	Substituting *their* 60 and *their* 6 into ±standardisation formula (any number for 68.5), condone *their* σ^2 and *their* $\sqrt{\sigma}$.
		M1	Using continuity correction 67.5 or 68.5 in *their* standardisation formula.
	$\text{P}(Z > 1.417) = 1 - \Phi(1.417)$ [= 1 – 0.9217]	**M1**	Appropriate area Φ, from final process, must be a probability.
	0.0783	**A1**	$0.07825 < p \leqslant 0.0783$ If A0 scored, **SC B1** for $0.07825 < p \leqslant 0.0783$.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">4(a)</td><td>
<table>
<tr><td>Class width</td><td>20</td><td>10</td><td>5</td><td>10</td><td>15</td></tr>
<tr><td>Frequency density</td><td>0.9</td><td>4.8</td><td>6.8</td><td>3.2</td><td>1.2</td></tr>
</table>
</td><td>M1</td><td>At least 4 frequency densities calculated by $\frac{f}{cw}$ e.g. $\frac{18}{20}$ (condone $\frac{f}{cw \pm 0.5}$ if unsimplified).

Accept unsimplified, may be read from graph using their scale, no lower than 1cm = 1 fd.</td></tr>
<tr><td rowspan="3"></td><td>A1</td><td>All bar heights correct on graph (no FT), using their suitable linear scale with at least 3 values indicated, no lower than 1cm = 1 fd.</td></tr>
<tr><td>B1</td><td>Bar ends at 120.5, 130.5, 135.5, 145.5, 160.5. 5 bars drawn with a horizontal linear scale, no lower than 1 cm = 10 min, with at least 3 values indicated. 100 $\leqslant$ horizontal scale $\leqslant$ 160.</td></tr>
<tr><td>B1</td><td>Axes labelled frequency density (fd), time (t) and minutes (min, m) oe, or an appropriate title.
(Axes may be reversed).</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(b)	[Midpoints] 110.5 125.5 133 140.5 153	**B1**	At least 4 correct mid-points seen, may be by data table or used in formula.
	Mean = $\frac{18\times110.5+48\times125.5+34\times133+32\times140.5+18\times153}{150}$ $=\frac{1989+6024+4522+4496+2754}{150}$	**M1**	Correct formula for mean using midpoints ±0.5, condone 1 midpoint error within class.
	= 131.9	**A1**	Accept 132, $131\frac{9}{10}$, or $\frac{1319}{10}$. Must be identified.
	Variance = $\frac{18\times110.5^2+48\times125.5^2+34\times133^2+32\times140.5^2+18\times153^2}{150}-(their\,131.9)^2$	**M1**	Appropriate variance formula with *their* 5 midpoints within class (not upper bound, lower bound, class width, frequency density, frequency or cumulative frequency). Condone 1 error. If correct midpoints seen, accept $\left\{\frac{3200+41400+194400+157300+153600}{150}\ or\ \frac{2630272.5}{150}\right\}$ $-\{131.9^2\ or\ 17397.61\}$.
	[= 137.54] [Standard deviation =] 11.7	**A1**	11.7277448… to at least 3SF. Accept 11.6 ⩽ σ < 11.95 www. If M0 awarded, **SC B1** 11.6 ⩽ σ < 11.95 www.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>5(a)</td><td>$0.28 + 6p = 1, \quad p = 0.12$</td><td>B1</td><td>Using sum of probabilities = 1 to form an equation.
Accept 0.28 + p + 2p + 3p = 1, p = 0.12.
Substitution of 0.12 into the expression scores B0.</td></tr>
<tr><td></td><td></td><td>1</td><td></td></tr>
<tr><td>5(b)</td><td>[For fair spinners (blue and green), probability of any score is 0.25
Scenarios to give total 4 or less:]</td><td>B1</td><td>Correct probability for 1 identified scenario, accept unsimplified, www.</td></tr>
<tr><td></td><td><table><tr><td>R</td><td>B</td><td>G</td><td></td><td></td></tr><tr><td>1</td><td>1</td><td>2</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>2</td><td>1</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>1</td><td>1</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>2</td><td>1</td><td>1</td><td>$0.12 \times (0.25)^2$</td><td>= 0.0075</td></tr></table></td><td>M1</td><td>Add values of 4 correct scenarios, may be implied by correct unsimplified expressions. No incorrect/repeated scenarios.</td></tr>
<tr><td></td><td>0.06</td><td>A1</td><td>If A0 scored, SC B1 for 0.06 www.</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
5(c)	[P(X is odd) = 0.28 + 2×0.12 *or* 0.24]= 0.52[0]	**B1**	Seen alone or as the denominator of a conditional probability fraction. Accept unsimplified.
	(table below)	**M1**	Values of at least 5 identified correct scenarios added, accept unsimplified, condone incorrect scenarios in calculation.
	[P(product of 3 scores $\leqslant$ 4 ∩ X is odd) =] $0.28\times(0.25)^2\times 8+0.24\times(0.25)^2$	**M1**	$0.28\times(0.25)^2\times x+0.24\times(0.25)^2$, or 0.0175 × x + 0.015 where x = 4, 5, 6, 7, or 8. Seen alone or as numerator/denominator of a conditional probability fraction.

R	B	G		
1	1	1	$0.28\times(0.25)^2$	= 0.0175
1	1	2	$0.28\times(0.25)^2$	= 0.0175
1	1	3	$0.28\times(0.25)^2$	= 0.0175
1	1	4	$0.28\times(0.25)^2$	= 0.0175
1	2	1	$0.28\times(0.25)^2$	= 0.0175
1	2	2	$0.28\times(0.25)^2$	= 0.0175
1	3	1	$0.28\times(0.25)^2$	= 0.0175
1	4	1	$0.28\times(0.25)^2$	= 0.0175
3	1	1	$0.24\times(0.25)^2$	= 0.015

Question	Answer	Marks	Guidance
5(c)	$\left[\text{P}\left(\text{product of 3 scores} \leqslant 4 \mid X \text{ is odd}\right) = \dfrac{\text{P}\left(\text{product of 3 scores} \leqslant 4 \cap X \text{ is odd}\right)}{\text{P}\left(X \text{ is odd}\right)} = \right]$ $\dfrac{0.155}{0.52}$	**M1**	$\dfrac{0.28\times(0.25)^2\times x+0.24\times(0.25)^2}{0.28+0.24}$ x = 4, 5, 6, 7, 8 or $\dfrac{\textit{their}\text{ identified P}\left(\text{product of 3 scores is 4 or less and } X \text{ is odd}\right)}{\textit{their}\text{ identified P}\left(\text{odd}\right)}$.
	$= 0.298, \dfrac{155}{520}, \dfrac{31}{104}$	**A1**	0.2980769… to at least 3SF.
		5	

Question	Answer	Marks	Guidance
6(a)	${}^5C_2 \times 2$	**M1**	${}^5C_2 \times r$, r = positive integer, 1 implied, no addition.
		M1	$s \times 2$, $s = {}^5C_2$ or ${}^5P_2$ or if ${}^5C_2$ or ${}^5P_2$ not present, s = a single integer > 1 or $t! \times 2$, $2 \leqslant t \leqslant 8$, no other terms.
	20	**A1**	
		3	

Question	Answer	Marks	Guidance
6(b)	**Method 1**		
	${}^6C_2 \times 2 \times 2 \times 2 \times 4!$	**M1**	${}^6C_2 \times 2 \times 2 \times 2 \times t$, t = positive integer $\geqslant 1$. ${}^6P_2 \times 2 \times 2 \times t$, t = positive integer $\geqslant 1$.
		M1	$u \times 4!$, u = positive integer > 1.
	2880	**A1**	If A0 scored, **SC B1** for 2880 nfww.
	Method 2		
	$6! \times 2 \times 2$	**M1**	$6! \times v$, v = positive integer $\geqslant 1$.
		M1	$w \times 2 \times 2$, w = positive integer > 1. condone $w \times 4$, w = positive integer > 1.
	2880	**A1**	If A0 scored, **SC B1** for 2880 nfww.
		3	

Question	Answer	Marks	Guidance
6(c)	**Method 1:** Number of arrangements with Rajid and Sue together – Number of arrangements with Rajid and Sue together and at end of line		
	$7!\times 2-6!\times 4$	**M1**	$7! \times 2 - a$, a = positive integer > 1.
		M1	$b - 6! \times 4$, b = positive integer > 2880.
		M1	$7! \times c - 6! \times d$, c = 1,2 and d = 1, 4.
	= 7200	**A1**	If A0 scored, **SC B1** for 7200 nfww.
	Method 2: Arrangements of 6 people and then place Rajid and Sue		
	$6!\times 2\times 5$	**M1**	$6! \times e \times f$, e, f = positive integers $\geqslant$ 1.
		M1	$6! \times 2 \times f$, f = positive integer $\geqslant$ 1. If 5! Used, **SC B1** $5! \times 2 \times f$, f = positive integer > 1.
		M1	$6! \times e \times 5$, e = positive integer $\geqslant$ 1.
	7200	**A1**	If A0, scored **SC B1** for 7200 nfww.
	Method 3: Friends at ends picked first F ^ RS ^ ^ ^ F		
	${}^6P_2 \times 5! \times 2$	**M1**	${}^6P_2 \times e \times f$, e, f = positive integers $\geqslant$ 1.
		M1	${}^6P_2 \times 5! \times f$, f = positive integer $\geqslant$ 1. Condone ${}^6C_2 \times 5! \times f$, f = positive integer $\geqslant$ 1.
		M1	${}^6P_2 \times e \times 2$, e = positive integer $\geqslant$ 1. Condone ${}^6C_2 \times e \times 2$, e = positive integer $\geqslant$ 1.
	7200	**A1**	If A0 scored, **SC B1** for 7200 nfww.

Question	Answer	Marks	Guidance
6(c)	**Method 4:** RS placed in different possible positions		
	^ RS ^ ^ ^ ^ ^ $^6P_1 \times 2 \times 5! = 1440$ ^ ^ RS ^ ^ ^ ^ $^6P_2 \times 2 \times 4! = 1440$ ^ ^ ^ RS ^ ^ ^ $^6P_3 \times 2 \times 3! = 1440$ ^ ^ ^ ^ RS ^ ^ $^6P_4 \times 2 \times 2! = 1440$ ^ ^ ^ ^ ^ RS ^ $^6P_5 \times 2 \times 1! = 1440$	**M1**	$^6P_n \times a \times (6-n)!$, a = positive integer, $1 \leqslant n \leqslant 5$ seen once.
		M1	$^6P_n \times 2 \times (6-n)!$, a = positive integer, $1 \leqslant n \leqslant 5$ seen at least 3 times in identified scenarios.
		M1	Add 5 values of appropriate scenarios only. No additional, incorrect or repeated scenarios. Accept unsimplified.
	7200	**A1**	If A0 scored, **SC B1** for 7200 nfww.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8016065171

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

1 A competitor in a throwing event has three attempts to throw a ball as far as possible. The random variable X denotes the number of throws that exceed 30 metres. The probability distribution table for X is shown below.

x	0	1	2	3
$\mathrm{P}(X = x)$	0.4	p	r	0.15

(a) Given that $\mathrm{E}(X) = 1.1$, find the value of p and the value of r. [3]

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(b) Find the numerical value of $\mathrm{Var}(X)$. [2]

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2 George has a fair 5-sided spinner with sides labelled 1, 2, 3, 4, 5. He spins the spinner and notes the number on the side on which the spinner lands.

(a) Find the probability that it takes fewer than 7 spins for George to obtain a 5. [2]

George spins the spinner 10 times.

(b) Find the probability that he obtains a 5 more than 4 times but fewer than 8 times. [3]

3 A factory produces a certain type of electrical component. It is known that 15% of the components produced are faulty. A random sample of 200 components is chosen.

Use an approximation to find the probability that more than 40 of these components are faulty. [5]

4 The heights, in cm, of the 11 players in each of two teams, the Aces and the Jets, are shown in the following table.

Aces	180	174	169	182	181	166	173	182	168	171	164
Jets	175	174	188	168	166	174	181	181	170	188	190

(a) Draw a back-to-back stem-and-leaf diagram to represent this information with the Aces on the left-hand side of the diagram. [4]

(b) Find the median and the interquartile range of the heights of the players in the Aces. [3]

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(c) Give one comment comparing the spread of the heights of the Aces with the spread of the heights of the Jets. [1]

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5 **(a)** The heights of the members of a club are normally distributed with mean 166 cm and standard deviation 10 cm.

(i) Find the probability that a randomly chosen member of the club has height less than 170 cm. [2]

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(ii) Given that 40% of the members have heights greater than h cm, find the value of h correct to 2 decimal places. [3]

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(b) The random variable X is normally distributed with mean μ and standard deviation σ.

Given that $\sigma = \frac{2}{3}\mu$, find the probability that a randomly chosen value of X is positive. [3]

6 Freddie has two bags of marbles.

Bag X contains 7 red marbles and 3 blue marbles.

Bag Y contains 4 red marbles and 1 blue marble.

Freddie chooses one of the bags at random. A marble is removed at random from that bag and not replaced. A new red marble is now added to each bag. A second marble is then removed at random from the same bag that the first marble had been removed from.

(a) Draw a tree diagram to represent this information, showing the probability on each of the branches. [3]

(b) Find the probability that both of the marbles removed from the bag are the same colour. [4]

(c) Find the probability that bag Y is chosen given that the marbles removed are **not** both the same colour. [2]

7 **(a)** Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which no consonant is next to another consonant. (The letters D, M, N and R are consonants and the letters A, E and O are **not** consonants.) [3]

(b) Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which there is an A at each end and the Ds are **not** together. [3]

Four letters are selected at random from the 9 letters in the word ANDROMEDA.

(c) Find the probability that this selection contains at least one D and exactly one A. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2. Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5. Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$p+r+0.55=1$	**M1**	Using sum of probabilities = 1 to form an equation. Accept $p + r = 0.45$ oe.
	$p+2r+0.45=1.1$	**M1**	Use $\mathrm{E}(X) = 1.1$ to form an equation. Accept $p + 2r = 0.65$ oe. NB: These marks can be gained in either order; the second M may have an algebraic substitution.
	$p=0.25,\ r=0.2$	**A1**	If both Ms not awarded, **SC B1** for $p=0.25,\ r=0.2$ stated.
		3	
1(b)	[Var(X) =] $\left[0.4\times0^2+\right] \textit{their}\ 0.25\left[\times1^2\right]+\left(\textit{their}\ 0.2\right)\times2^2+0.15\times3^2-1.1^2$ [=[0+] 0.25 + 0.8 + 1.35 – 1.21]	**M1**	Correct formula for variance method using their probability distribution table, 0 < their P(x) < 1.
	= 1.19, $1\frac{19}{100}$	**A1**	If M0 awarded, **SC B1** for 1.19 www. $\frac{119}{100}$ is A0.
		2	

Question	Answer	Marks	Guidance
2(a)	**Method 1:**		
	[P(5) = 0.2] [P(X < 7) =] $1-0.8^6$	**M1**	$1 - 0.8^n$, n = 6, 7.
	$= 0.738,\ \frac{11529}{15625}$	**A1**	0.737856 to at least 3SF.
	Method 2:		
	[P(X < 7) =] $0.2+0.2\times0.8+0.2\times0.8^2+0.2\times0.8^3+0.2\times0.8^4+0.2\times0.8^5$	**M1**	$0.2+0.2\times0.8+0.2\times0.8^2+0.2\times0.8^3+$ $0.2\times0.8^4+0.2\times0.8^5\left(+0.2\times0.8^6\right)$
	$= 0.738,\ \frac{11529}{15625}$	**A1**	0.737856 to at least 3SF.
		2	

Question	Answer	Marks	Guidance
2(b)	**Method 1:**		
	[P(5, 6, 7) =] ${}^{10}C_5\,(0.2)^5(0.8)^5 + {}^{10}C_6\,(0.2)^6(0.8)^4 + {}^{10}C_7\,(0.2)^7(0.8)^3$	**M1**	One term: ${}^{10}C_x\,(p)^x(1-p)^{10-x},\ 0<p<1,\ x\neq 0,10.$
	$[0.02642 + 5.505\times10^{-3} + 7.864\times10^{-4}]$	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.0327	**B1**	awrt
	Method 2:		
	[P(X < 8) – P($X \leqslant$ 4) = 1 – P($X \geqslant$ 8) – P($X \leqslant$ 4) =] $1-\{{}^{10}C_8(0.2)^8(0.8)^2 + {}^{10}C_9(0.2)^9 0.8 + (0.2)^{10}\}$ $-\{(0.8)^{10} + {}^{10}C_1(0.2)(0.8)^9 + {}^{10}C_2(0.2)^2(0.8)^8 + {}^{10}C_3(0.2)^3(0.8)^7 + {}^{10}C_4(0.2)^4(0.8)^6\}$	**M1**	One term: ${}^{10}C_x\,(p)^x(1-p)^{10-x},\ 0<p<1,\ x\neq 0,10.$
	$[1-\{7.373\times10^{-5}+4.096\times10^{-6}+1.024\times10^{-7}\}-$ $\{0.1074+0.2684+0.3020+0.2013+0.08808\}$	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.0327	**B1**	awrt
		3	

Question	Answer	Marks	Guidance
3	[Mean = $200 \times 0.15 =$] 30 [Var = $200 \times 0.15 \times 0.85 =$] 25.5	**B1**	30 and 25.5, $25\frac{1}{2}, \frac{51}{2}$ seen, allow unsimplified. May be seen in standardisation formula. [σ =] $5.049 \leqslant \sigma \leqslant 5.05[0]$, $\frac{\sqrt{102}}{2}$ implies correct variance. Correct notation is required.
	[P(X > 40) =] P(Z > $\frac{40.5-30}{\sqrt{25.5}}$)	**M1**	Substituting *their* mean and *their* positive 5.04975 into ±standardisation formula (any number for 40.5), not *their* σ^2 or $\sqrt{their\ \sigma}$.
		M1	Using continuity correction 39.5 or 40.5 in *their* standardisation formula.
	[1 – Φ (2.079)] 1 – 0.9812	**M1**	Appropriate area Φ, from final process, must be a probability.
	= 0.0188	**A1**	$0.01875 < p \leqslant 0.0188$
		5	

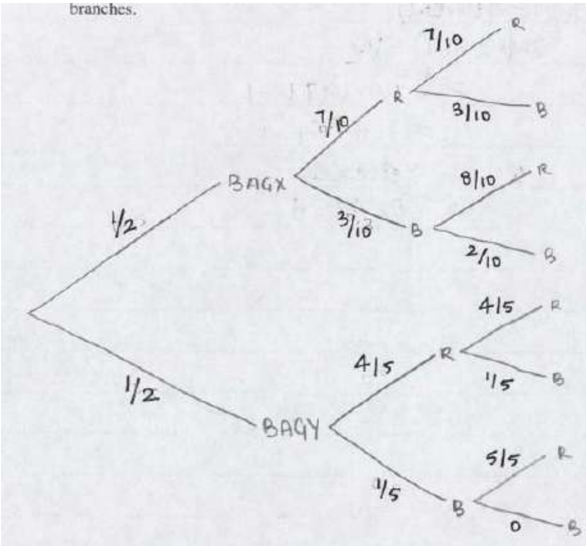
<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">4(a)</td><td rowspan="2"><table><tr><th>Aces</th><th></th><th>Jets</th></tr><tr><td>9 8 6 4</td><td>16</td><td>6 8</td></tr><tr><td>4 3 1</td><td>17</td><td>0 4 4 5</td></tr><tr><td>2 2 1 0</td><td>18</td><td>1 1 8 8</td></tr><tr><td></td><td>19</td><td>0</td></tr></table></td><td>B1</td><td>Correct stem, ignore extra values (not in reverse, not split).

If a split stem-and-leaf plot is used (i.e., stem values are repeated), the remaining B marks are available.</td></tr>
<tr><td>B1</td><td>Correct Aces labelled on left, leaves in order from right to left and lined up vertically, no commas or other punctuation.</td></tr>
<tr><td rowspan="2">Key: 1 | 17 | 0 means 171 cm for the Aces and 170 cm for the Jets</td><td>B1</td><td>Correct Jets labelled on same diagram, leaves in order and lined up vertically, no commas or other punctuation.</td></tr>
<tr><td>B1</td><td>Correct key for their diagram, need both teams labelled and ‘cm’ stated at least once here, or in leaf headings or title.</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(b)	Median = 173 [cm]	**B1**	Accept Q_2; must be identified.
	[IQR =] 181 – 168	**M1**	$180 \leqslant UQ \leqslant 182 - 166 \leqslant LQ \leqslant 169$ Implied if both quartile values are stated and an appropriate IQR calculated accurately.
	13 [cm]	**A1**	www If M0 scored **SC B1** for 13 www.
		3	
4(c)	Jets have a greater variety of heights. Jets have a wider range of height. Jets have a greater/larger/bigger/wider/'more' spread of heights. Aces have a smaller variety of height etc…	**B1**	[Jets IQR = 18 cm, Range = 24 cm Aces IQR = *their* **4(b)**, Range = 18 cm] Comment about spread in context, must include height. Comparison of values does not score until a comment in context is made. If values for range or IQR are stated, they must be correct or FT from **4(b)**. If more than one comment about spread, mark the final comment. Additional comments about central tendency score B0.
		1	

Question	Answer	Marks	Guidance
5(a)(i)	[P(X < 170) =] P(Z < $\frac{170-166}{10}$)	**M1**	Use of ± standardisation formula with 170, 166 and 10 substituted appropriately, condone $10^2, \sqrt{10}$, condone continuity correction ±0.5.
	[= P(Z < 0.4) =] 0.655	**A1**	$0.655 \leqslant p < 0.6555$ If M0 awarded, **SC B1** for correct answer www.
		2	
5(a)(ii)	$\left[P\left(Z > \frac{h-166}{10}\right) = 0.4\right]$	**B1**	$0.253 \leqslant z \leqslant 0.2535$ or $-0.2535 \leqslant z \leqslant -0.253$ seen.
	$\frac{h-166}{10} = 0.253$	**M1**	Use of the ±standardisation formula with *h*, 166, 10 and a *z*-value (not 1 – *z*-value), not $10^2, \sqrt{10}$, no continuity correction.
	$h = 168.53$	**A1**	If M0 scored, **SC B1** for $168.53 \leqslant h \leqslant 168.535$, 168.5. **SC B1** for 168.54 from *z* = 0.254.
		3	

Question	Answer	Marks	Guidance
5(b)	$\left[P(X>0)=P\left(Z>\dfrac{0-\mu}{\sigma}\right)=\right]P\left(Z>\dfrac{[0]-\mu}{\frac{2}{3}\mu}\right)$ Or $P\left(Z>\dfrac{[0]-\frac{3}{2}\sigma}{\sigma}\right)$	**M1**	Use of the ±standardisation formula with 0, μ and $\frac{2}{3}\mu$ substituted for σ. Or use of the ±standardisation formula with 0, σ and $\frac{3}{2}\sigma$ substituted for μ.
	= P($Z>-1.5$)	**A1**	-1.5 seen, no additional terms (e.g. x – 1.5 A0). Condone $Z<1.5$. If M0 scored, **SC B1** $Z>-1.5$ or $Z<1.5$ seen www.
	= 0.933 final answer	**A1**	$0.933 \leqslant p < 0.9333$. If M0 scored, **SC B1** $0.933 \leqslant p < 0.9333$ seen www.
		3	

Question	Answer	Marks	Guidance
6(a)	 .	**B1**	1st column, 2 branches identified *X*, *Y* with probabilities ½, ½ indicated.
		B1	2nd column (1st marble pick) of 4 branches identified *R B R B* (oe) and probabilities $\frac{7}{10}, \frac{3}{10}, \frac{4}{5}, \frac{1}{5}$ indicated appropriately.
		B1	3rd column (2nd marble pick) of 8 branches identified *R B R B R B R* [*B*] (oe) and probabilities $\frac{7}{10}, \frac{3}{10}, \frac{8}{10}, \frac{2}{10}, \frac{4}{5}, \frac{1}{5}, 1, [0]$. Condone omission of *YBB* branch if *YBR* branch is fully correct. Ignore any additional columns of branches. If separate tree diagrams for bags *X* and *Y*, B0B1B1 max if bags clearly identified.
		3	

Question	Answer	Marks	Guidance
6(b)	[P(both same colour) = P(BB) + P(RR) = P(XBB) + P(XRR) + P(YRR) =] $\frac{1}{2}\times\frac{3}{10}\times\frac{2}{10}+\frac{1}{2}\times\frac{7}{10}\times\frac{7}{10}+\frac{1}{2}\times\frac{4}{5}\times\frac{4}{5}$ $\left[=\frac{6}{200}+\frac{49}{200}+\frac{16}{50},\ 0.03+0.245+0.32\right]$	**B1 FT**	$\left[\mathrm{P(BB)}=\right]\left[\frac{1}{2}\times\frac{3}{10}\times\frac{2}{10}\left[+\frac{1}{2}\times\frac{1}{5}\times 0\right]=\right]\frac{6}{200}$ seen. Accept unsimplified. FT from **6(a)** unsimplified only with 3 term probabilities.
		B1 FT	Either [P(XRR) =] $\frac{1}{2}\times\frac{7}{10}\times\frac{7}{10}$ or [P(YRR) =] $\frac{1}{2}\times\frac{4}{5}\times\frac{4}{5}$ seen. FT from **6(a)** unsimplified only with 3 term probabilities.
		M1	[P(BB) + P(XRR) + P(YRR) =] $\textit{their}\ \frac{6}{200}+\textit{their}\ \frac{49}{200}+\textit{their}\ \frac{16}{50}$ Accept unsimplified, consistent with tree diagram if not clearly identified by notation.
	$=\frac{119}{200}$, 0.595	**A1**	
		4	Special case: if ½ omitted consistently in the tree diagram and the calculation (i.e., no probability for picking the bags), no FT. **SC B1** [P(BB) =] $\frac{3}{10}\times\frac{2}{10}\left[+\frac{1}{5}\times 0\right]$ **SC B1** [P(RR) =] $\frac{7}{10}\times\frac{7}{10}+\frac{4}{5}\times\frac{4}{5}$ **SC B1** $\frac{3}{10}\times\frac{2}{10}\left[+\frac{1}{5}\times 0\right]+\frac{7}{10}\times\frac{7}{10}+\frac{4}{5}\times\frac{4}{5}$

Question	Answer	Marks	Guidance
6(c)	$\left[P(\text{ bag } Y \mid \text{different colours}) = \left(\dfrac{P(\text{bag } Y \cap \text{different colours})}{P(\text{different colours})}\right)\right]$ $\dfrac{\frac{1}{2}\times\frac{4}{5}\times\frac{1}{5}+\frac{1}{2}\times\frac{1}{5}[\times 1]}{1-\textit{their}\left(\frac{119}{200}\right)}$ or $\dfrac{\frac{1}{2}\times\frac{4}{5}\times\frac{1}{5}+\frac{1}{2}\times\frac{1}{5}[\times 1]}{\frac{1}{2}\times\frac{7}{10}\times\frac{3}{10}+\frac{1}{2}\times\frac{3}{10}\times\frac{8}{10}+\frac{1}{2}\times\frac{4}{5}\times\frac{1}{5}+\frac{1}{2}\times\frac{1}{5}[\times 1]}$	**M1**	FT from *their* **6(a)** and *their* **6(b)** with 3 term probabilities unsimplified only or correct. Accept $\dfrac{\frac{4}{50}+\frac{1}{10}}{\frac{81}{200}}$, $\dfrac{\frac{2}{25}+\frac{1}{10}}{\frac{81}{200}}$, $\dfrac{0.08+0.1}{0.405}$.
	$=\left[\dfrac{\frac{9}{50}}{\frac{81}{200}}\right] = \dfrac{4}{9}$, 0.444	**A1**	Accept $\dfrac{36}{81}$, $0.\dot{4}$.
		2	Special case: if ½ omitted consistently in the tree diagram and the calculation (ie no probability for picking the bags), no FT. **SC B1** $\dfrac{\frac{4}{5}\times\frac{1}{5}+\frac{1}{5}[\times 1]}{1-\textit{their}\ \mathbf{6(b)}}$, or $\dfrac{\frac{4}{5}\times\frac{1}{5}+\frac{1}{5}[\times 1]}{\frac{7}{10}\times\frac{3}{10}+\frac{3}{10}\times\frac{8}{10}+\frac{4}{5}\times\frac{1}{5}+\frac{1}{5}[\times 1]}$.

Question	Answer	Marks	Guidance
7(a)	$\dfrac{5!\times 4!}{2!\times 2!}$	**M1**	$\dfrac{5!\times 4!}{e}$, e a positive integer, 1 can be implied. No other terms on numerator. No addition etc.
		M1	$\dfrac{f}{2!\times g!}$, f a positive integer, g = 1, 2. No other terms on denominator.
	720	**A1**	
		3	

Question	Answer	Marks	Guidance
7(b)	**Method 1** Number of arrangements with A at each end – Number of arrangements with A at each end and 2 Ds together.		
	$\frac{7!}{2!} - 6!$	**B1**	$\frac{7!}{2!} - e$, ${}^7P_5 - e$, e a positive integer or 0.
		M1	$d - \frac{6!}{r!}$, $d > 720$, $r = 1, 2$.
	= 1800	**A1**	
	Method 2 A ^ ^ ^ ^ ^ A and Ds inserted separately		
	$5! \times \frac{{}^6P_2}{2!}$ or $5! \times \frac{6\times5}{2}$ or $5! \times {}^6C_2$	**B1**	$5! \times s$, s a positive integer, 1 may be implied.
		M1	$t \times \frac{6\times5}{u}$, t a positive integer > 1, u =1, 2.
	= 1800	**A1**	
	Method 3 Number of arrangements with As at each end and Ds placed in different scenarios.		
	Scenario position of first D | | A D ^ ^ ^ ^ ^ ^ A | 5! × 5 | 600 A ^ D ^ ^ ^ ^ ^ A | 5! × 4 | 480 A ^ ^ D ^ ^ ^ ^ A | 5! × 3 | 360 A ^ ^ ^ D ^ ^ ^ A | 5! × 2 | 240 A ^ ^ ^ ^ D ^ ^ A | 5! × 1 | 120	**B1**	Correct outcome/value for 1 identified scenario, accept unsimplified, www.
		M1	Add values of 5 correct scenarios, no incorrect/repeated scenarios.
	[Total =] 1800	**A1**	
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>7(c)</td><td colspan="3">Method 1:</td></tr>
<tr><td></td><td><table><tr><td>Scenarios</td><td></td><td></td></tr><tr><td>A D ^ ^</td><td>$^2C_1 \times {}^2C_1 \times {}^5C_2$</td><td>= 40</td></tr><tr><td>A D D ^</td><td>$^2C_1 \times [\,{}^2C_2 \times]\ {}^5C_1$</td><td>= 10</td></tr></table></td><td>M1</td><td>At least one correct unsimplified expression for an identified scenario.</td></tr>
<tr><td></td><td>[Total =] 40 + 10 or 50 soi</td><td>A1</td><td>www
If M0 scored, SC B1 [total =]50 www.</td></tr>
<tr><td></td><td>[Total number of selections =] $^9C_4$ [= 126]</td><td>B1</td><td>Accept evaluated, accept as denominator of probability expression.
Do not condone $^9C_5$ unless there is a clear explanation for selecting the letters not in the group.</td></tr>
<tr><td></td><td>[Probability =] $\frac{50}{126}$, $\frac{25}{63}$</td><td>B1 FT</td><td>0.396825… to at least 3SF.

FT $\frac{\textit{their}\text{ attempted } 40+10}{126}$. Numerator must be from an attempt to find the 2 appropriate scenarios and must be evaluated.</td></tr>
</table>

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">7(c)</td><td colspan="3">Method 2:</td></tr>
<tr><td rowspan="3"><table><tr><td>Scenarios</td><td></td><td></td></tr><tr><td>A D ^ ^</td><td>$\frac{2}{9}\times\frac{2}{8}\times\frac{5}{7}\times\frac{4}{6}\times {}^4P_2$</td><td>$= \frac{960}{3024}, \frac{20}{63}$</td></tr><tr><td>A D D ^</td><td>$\frac{2}{9}\times\frac{2}{8}\times\frac{1}{7}\times\frac{5}{6}\times\frac{{}^4P_3}{2!}$</td><td>$= \frac{240}{3024}, \frac{5}{63}$</td></tr></table>
[Total Probability =] $\frac{20}{63}+\frac{5}{63}$</td><td>M1</td><td>Numerator for at least one correct unsimplified expression for an identified scenario.
$either\ \frac{2\times2\times5\times4\times12}{a\times b\times c\times d}$ or $\frac{2\times2\times1\times5\times12}{a\times b\times c\times d}$
seen, $6 \leqslant a,b,c,d \leqslant 9$.</td></tr>
<tr><td>A1</td><td>$\frac{2\times2\times5\times4\times12}{a\times b\times c\times d}+\frac{2\times2\times1\times5\times12}{a\times b\times c\times d}$,
$6 \leqslant a,b,c,d \leqslant 9$.
If M0 scored, SC B1 $\frac{1200}{g}$, $g > 1200$, or $\frac{25}{63}$ seen.</td></tr>
<tr><td>B1</td><td>$\frac{p}{9}\times\frac{q}{8}\times\frac{r}{7}\times\frac{s}{6}$ present in all scenarios attempted, accept $\frac{t}{3024}$, $t < 3024$.</td></tr>
<tr><td>$\frac{1200}{3024}, \frac{25}{63}$ oe</td><td>B1 FT</td><td>0.396825… to at least 3SF.

FT $\frac{\textit{their}\ \text{attempted}\ 960+240}{3024}$. Numerator must be from an attempt to find the 2 appropriate scenarios.</td></tr>
</table>

Question	Answer	Marks	Guidance
7(c)	**Method 3: selecting the A and then selecting 3 any letters and removing selections without Ds.**		
	${}^2C_1 \times ({}^7C_3 - {}^5C_3)$ [$= 2 \times (35 - 10)$]	**M1**	$a \times ({}^7C_3 - {}^5C_3)$, $a = 1, 2$.
	[Total =] 50	**A1**	www If M0 scored, **SC B1** [total =]50 www.
	[Total number of selections =] ${}^9C_4$ [= 126]	**B1**	Accept evaluated, accept as denominator of probability expression. Do not condone ${}^9C_5$ unless there is a clear explanation for selecting the letters not in the group.
	[Probability =] $\frac{50}{126}, \frac{25}{63}$	**B1 FT**	0.396825… to at least 3SF. FT $\frac{\textit{their}\text{ attempted }40+10}{126}$. Numerator must be from an attempt to find the 2 appropriate scenarios.
	Method 4: Listing outcomes.		
	Either 10 correct outcomes for ADD^ listed or 40 correct outcomes for AD^^ listed	**M1**	
	50 stated	**A1**	www If M0 scored, **SC B1** [total =]50 www.
	126 stated or correct outcomes listed	**B1**	
	[Probability =] $\frac{50}{126}, \frac{25}{63}$	**B1**	0.396825… to at least 3SF. FT $\frac{\textit{their}\text{ attempted }40+10}{126}$. Numerator must be from an attempt to find the 2 appropriate scenarios.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 5 7 7 5 7 3 6 1 6 3 *

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

1 Becky sometimes works in an office and sometimes works at home. The random variable X denotes the number of days that she works at home in any given week. It is given that

$$P(X = x) = kx(x + 1),$$

where k is a constant and $x = 1, 2, 3$ or 4 only.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

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(b) Find $E(X)$ and $Var(X)$. [3]

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2 The weights of large bags of pasta produced by a company are normally distributed with mean 1.5 kg and standard deviation 0.05 kg.

(a) Find the probability that a randomly chosen large bag of pasta weighs between 1.42 kg and 1.52 kg. [3]

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The weights of small bags of pasta produced by the company are normally distributed with mean 0.75 kg and standard deviation σ kg. It is found that 68% of these small bags have weight less than 0.9 kg.

(b) Find the value of σ. [3]

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3 Tim has two bags of marbles, A and B.

Bag A contains 8 white, 4 red and 3 yellow marbles.

Bag B contains 6 white, 7 red and 2 yellow marbles.

Tim also has an ordinary fair 6-sided dice. He rolls the dice. If he obtains a 1 or 2, he chooses two marbles at random from bag A, without replacement. If he obtains a 3, 4, 5 or 6, he chooses two marbles at random from bag B, without replacement.

(a) Find the probability that both marbles are white. [3]

(b) Find the probability that the two marbles come from bag B given that one is white and one is red. [4]

4 The weights, x kg, of 120 students in a sports college are recorded. The results are summarised in the following table.

Weight (x kg)	$x \leqslant 40$	$x \leqslant 60$	$x \leqslant 65$	$x \leqslant 70$	$x \leqslant 85$	$x \leqslant 100$
Cumulative frequency	0	14	38	60	106	120

(a) Draw a cumulative frequency graph to represent this information. [2]

(b) It is found that 35% of the students weigh more than W kg.

Use your graph to estimate the value of W. [2]

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(c) Calculate estimates for the mean and standard deviation of the weights of the 120 students. [6]

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5 The probability that a driver passes an advanced driving test is 0.3 on any given attempt.

(a) Dipak keeps taking the test until he passes. The random variable X denotes the number of attempts required for Dipak to pass the test.

(i) Find $P(2 \leqslant X \leqslant 6)$. [2]

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(ii) Find $E(X)$. [1]

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Five friends will each take their advanced driving test tomorrow.

(b) Find the probability that at least three of them will pass tomorrow. [3]

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75 people will take their advanced driving test next week.

(c) Use an approximation to find the probability that more than 20 of them will pass next week. [5]

6 Jai and his wife Kaz are having a party. Jai has invited five friends and each friend will bring his wife.

(a) At the beginning of the party, the 12 people will stand in a line for a photograph.

(i) How many different arrangements are there of the 12 people if Jai stands next to Kaz and each friend stands next to his own wife? [3]

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(ii) How many different arrangements are there of the 12 people if Jai and Kaz occupy the two middle positions in the line, with Jai's five friends on one side and the five wives of the friends on the other side? [2]

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(b) For a competition during the party, the 12 people are divided at random into a group of 5, a group of 4 and a group of 3.

Find the probability that Jai and Kaz are in the same group as each other. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Due to a series-specific issue during the live exam series, all candidates were awarded full marks for questions 1 and 4. This published mark scheme for these questions was created alongside the question paper, but has not been used by examiners.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

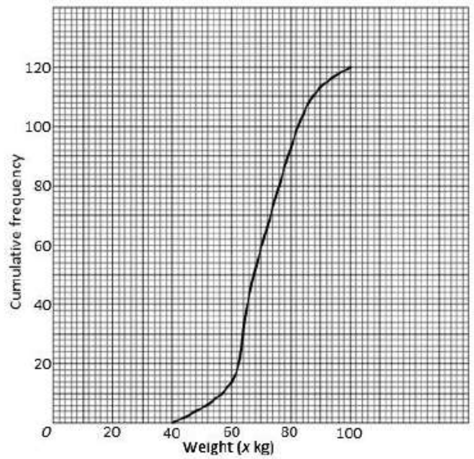
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

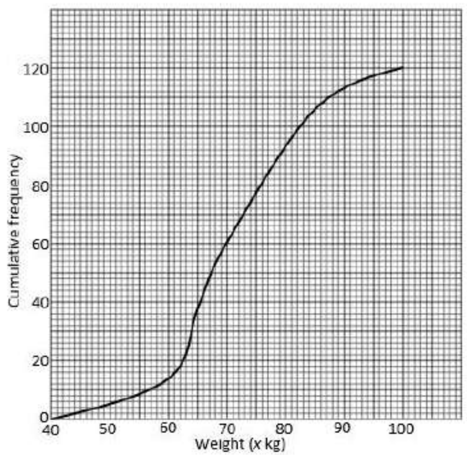
<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">1(a)</td><td>$2k+6k+12k+20k=1,\ \left[k=\frac{1}{40}\right]$</td><td>M1</td><td>Using sum of probabilities = 1 to form an equation in k.
Accept 1×2×k + 2×3×k + 3×4×k + 4×5×k = 1.</td></tr>
<tr><td rowspan="2"><table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>$\frac{2}{40}$</td><td>$\frac{6}{40}$</td><td>$\frac{12}{40}$</td><td>$\frac{20}{40}$</td></tr><tr><td></td><td>0.05</td><td>0.15</td><td>0.3</td><td>0.5</td></tr></table></td><td>M1</td><td><table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>$2k$</td><td>$6k$</td><td>$12k$</td><td>$20k$</td></tr></table>Two correctly linked, accurate probabilities. May be in terms of k.
May not be in a table.</td></tr>
<tr><td>A1</td><td>Table with correct X values and correct probabilities.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="4">1(b)</td><td>$[\mathrm{E}(X)=]\ [\mathrm{E}(X)=\frac{1\times2+2\times6+3\times12+4\times20}{40}]\ \frac{2+12+36+80}{40}$</td><td>M1</td><td>$[\mathrm{E}(X)=1\times2k+2\times6k+3\times12k+4\times20k=130k]$
Accept unsimplified expression. May be calculated in variance. FT their table with 3 or more probabilities summing to 1 (0 < p < 1).
If there are outcomes in the table without probabilities, condone and treat as p = 0.</td></tr>
<tr><td>$\left[\mathrm{Var}(X)=\frac{1^2\times2+2^2\times6+3^2\times12+4^2\times20}{40}-(\textit{their}\ \mathrm{E}(X))^2=\right]$
$\frac{1\times2+4\times6+9\times12+16\times20}{40}-\left(\textit{their}\,\frac{13}{4}\right)^2$
$\left[\frac{2+24+108+320}{40}-\left(\textit{their}\,\frac{13}{4}\right)^2\right]$</td><td>M1</td><td>$[\mathrm{Var}(X)=1^2\times2k+2^2\times6k+3^2\times12k+4^2\times20k-(130k)^2]$
Appropriate variance formula using their $(\mathrm{E}(X))^2$ value. FT their table with 3 or more probabilities (0 < p < 1) which need not sum to 1 and the highlighted calculation (or less simplified) seen,
Note: if table is correct,
$\frac{454}{40}\left(\text{or }\frac{227}{20}\text{ or any calculation}\right)-(\textit{their}\ \mathrm{E}(X))^2$ implies M1.</td></tr>
<tr><td>E(X) = $\frac{13}{4}, 3\frac{1}{4}, 3.05$ Var(X) = $\frac{63}{80}$, 0.7875</td><td>A1</td><td>Answers for E(X) and Var(X) must be identified. E(X) may be identified by correct use in variance. Condone E, V, μ, σ etc.
If A0 earned, SC B1 for identified correct final solutions.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
2(a)	[P(1.42 < X <1.52) =] P($\frac{1.42-1.5}{0.05} < Z < \frac{1.52-1.5}{0.05}$)	**M1**	Use of ± standardisation formula once with 1.5, 0.05 and either 1.42 or 1.52, allow σ^2 or $\sqrt{\sigma}$, no continuity correction.
	[= P($-1.6 < Z < 0.4$) = $\Phi(0.4)+\Phi(1.6)-1$] = 0.6554 + 0.9452 – 1 or 0.6554 – 0.0548	**M1**	Calculating the appropriate probability area (leading to their final answer, expect > 0.5).
	= 0.601	**A1**	$0.6005 < p \leqslant 0.601$ **SC B1** for 0.601 with no standardisation seen.
		3	
2(b)	$\left[P(X<0.9)=P\left(Z<\frac{0.9-0.75}{\sigma}\right)=0.68\right]$ $\frac{0.9-0.75}{\sigma}=0.468$	**B1**	$0.467 < z \leqslant 0.468$ or $-0.468 \leqslant z < -0.467$ seen
		M1	±standardisation formula with 0.9, 0.75, σ equating to a z-value (not 0.32, 0.68, 0.532, 0.7517, 0.2483, 0.6255,) . Condone continuity correct ±0.05, not $\sigma^2, \sqrt{\sigma}$. Condone $\pm\frac{0.15}{\sigma}=0.468$.
	$\sigma=0.321$, $\frac{25}{78}$	**A1**	$0.3205 \leqslant \sigma < 0.3215$ **SC B1** if M0 www.
		3	

Question	Answer	Marks	Guidance
3(a)	[P(WW) = P(AWW) + P(BWW) =] $\frac{2}{6}\times\frac{8}{15}\times\frac{7}{14}+\frac{4}{6}\times\frac{6}{15}\times\frac{5}{14}$	M1	Either $\frac{2}{6}\times\frac{8}{15}\times\frac{7}{14}$ or $\frac{4}{6}\times\frac{6}{15}\times\frac{5}{14}$ seen, accept unsimplified.
		M1	$\frac{q}{6}\times\frac{r}{15}\times\frac{r-1}{14}+\frac{6-q}{6}\times\frac{s}{15}\times\frac{s-1}{14}$ seen, no additional terms, accept unsimplified. Condone $\frac{q}{6}\times\frac{r}{15}\times\frac{r}{15}+\frac{6-q}{6}\times\frac{s}{15}\times\frac{s}{15}$, $1 \leqslant q \leqslant 5$, $1 < r, s < 9$.
	$\left[=\frac{56}{630}+\frac{60}{630}=\frac{4}{45}+\frac{2}{21}\right]=\frac{58}{315}$ or 0.184	A1	**SC B1** for 58/315 if either M mark withheld.
		3	

Question	Answer	Marks	Guidance
3(b)	$\left[\text{P(B}\mid\text{WR or RW)} = \dfrac{\text{P}(\text{W \& R from bag B})}{\text{P}(\text{W and R})} = \right]$ $\dfrac{\frac{4}{6}\times\frac{6}{15}\times\frac{7}{14}+\frac{4}{6}\times\frac{7}{15}\times\frac{6}{14}}{\frac{2}{6}\times\frac{8}{15}\times\frac{4}{14}+\frac{2}{6}\times\frac{4}{15}\times\frac{8}{14}+\frac{4}{6}\times\frac{6}{15}\times\frac{7}{14}+\frac{4}{6}\times\frac{7}{15}\times\frac{6}{14}}$ or $\dfrac{2\times\frac{4}{6}\times\frac{6}{15}\times\frac{7}{14}}{2\times\frac{2}{6}\times\frac{8}{15}\times\frac{4}{14}+2\times\frac{4}{6}\times\frac{6}{15}\times\frac{7}{14}}$	**B1**	P(W & R from bag B) $=\frac{2}{3}\times\frac{6}{15}\times\frac{7}{14}+\frac{2}{3}\times\frac{7}{15}\times\frac{6}{14}$ or $2\times\frac{2}{3}\times\frac{6}{15}\times\frac{7}{14}$ $[=\frac{4}{15}\ or\ 0.267]$ Seen alone or as numerator/denominator of conditional probability.
		M1	P(WR or RW) = P(W & R from bag A) + P(W & R from bag B) $=a\times\frac{2}{6}\times\frac{8}{15}\times\frac{4}{14}+a\times\frac{4}{6}\times\frac{6}{15}\times\frac{7}{14}$ or $=a\times\frac{2}{6}\times\frac{8}{15}\times\frac{4}{14}+\textit{their}\ \text{P(W \& R from bag B)}$. a = 1 or 2. [expect $\frac{116}{315}$ or 0.368] Seen alone or as numerator/denominator of conditional probability.
	$\dfrac{\frac{168}{630}}{\frac{232}{630}} = \dfrac{\frac{4}{15}}{\frac{116}{315}}$	**M1**	$\dfrac{\textit{their}\ \text{identified P}(W\ \&R\ from\ bag\ B)}{\textit{their}\ \text{identified P}(\text{WR or RW})}$ Accept unsimplified.
	$=\frac{168}{232}, \frac{21}{29}$ or 0.724	**A1**	0.7241379 to at least 3SF.
		4	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="3">4(a)</td><td rowspan="2">


</td><td>M1</td><td>At least 3 points plotted accurately at class upper end points (40,0) (60,14) (65,38) (70,60) (85,106) (100,120).
Linear cumulative frequency scale $0 \leqslant cf \leqslant 120$ and linear weight scale $40 \leqslant$ weight(kg) $\leqslant 100$ with at least 3 values identified on each axis.
Condone scale reversed.</td></tr>
<tr><td>A1</td><td>All points plotted correctly, curve drawn (within tolerance) and joined to (40,0).
Axes labelled cumulative frequency (cf), weight (w) and kg (kilograms) – or a suitable title.</td></tr>
<tr><td></td><td>2</td><td></td></tr>
<tr><td rowspan="3">4(b)</td><td>[120 × 0.65 =] 78 seen</td><td>M1</td><td>May be implied by use on graph.</td></tr>
<tr><td>76 [kg]</td><td>A1</td><td>75 < hours < 79. Indication of use of graph required.</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(c)	Frequencies: [0] 14 24 22 46 14	**B1**	At least 5 correct frequencies seen (condone omission of 0).
	Midpoints: 20 50 62.5 67.5 77.5 92.5	**B1**	At least 5 correct midpoints seen (condone omission of 20).
	Mean = $\frac{0\times20+14\times50+24\times62.5+22\times67.5+46\times77.5+14\times92.5}{120}$ $=\frac{[0]+700+1500+1485+3565+12950}{120}\left[=\frac{8545}{120}\right]$	**M1**	Correct formula for mean using *their* midpoints and *their* frequencies, implied by $\frac{8545}{120}$ if correct midpoints & frequencies seen. May be gained in variance calculation. If midpoints not clearly identified, condone midpoints ±0.5.
	= 71.2	**A1**	Accept $\frac{1709}{24}$, $71\frac{5}{24}$ or 71.208333 to at least 3SF. If M0 scored, **SC B1** for $\frac{1709}{24}$, $71\frac{5}{24}$ or 71.208333 to at least 3SF www.
	Variance = $\frac{0\times20^2+14\times50^2+24\times62.5^2+22\times67.5^2+46\times77.5^2+14\times92.5^2}{120}-71.2^2$ $\frac{[0]+35000+93750+100237.5+276287.5+119787.5}{120}-\left(\frac{8545}{120}\right)^2$ [=138.23]	**M1**	Correct formula for variance using *their* midpoints, *their* frequencies and *their* mean. Implied by $\frac{625062.5}{120}-\left(\frac{8545}{120}\right)^2$ if correct midpoints & frequencies seen.
	Standard deviation = 11.8	**A1**	11.757016 to at least 3SF.
		6	

Question	Answer	Marks	Guidance
5(a)(i)	**Method 1**		
	$[\mathrm{P}(2 \leqslant X \leqslant 6) = \mathrm{P}(X \leqslant 6) - \mathrm{P}(X \leqslant 1) =]\ 1 - (0.7)^6 - (1 - 0.7)$	**M1**	$1 - 0.7^n$ seen, $n = 5, 6$.
	= 0.582	**A1**	www 0.582351 to at least 3SF.
	Method 2		
	P(X= 2, 3, 4, 5, 6) $= 0.7 \times 0.3 + 0.7^2 \times 0.3 + 0.7^3 \times 0.3 + 0.7^4 \times 0.3 + 0.7^5 \times 0.3$ = 0.21 + 0.147 + 0.1029 + 0.07203 + 0.050421	**M1**	Sum of first 4 or 5 correct terms – no incorrect terms.
	= 0.582	**A1**	www 0.582351 to at least 3SF.
		2	
5(a)(ii)	$3\frac{1}{3}$	**B1**	Condone 3.33, $3.\dot{3}$ or $\frac{10}{3}$ – NOT $\frac{1}{0.3}$.
		1	

Question	Answer	Marks	Guidance
5(b)	**Method 1**		
	[P(3, 4, 5) =] ${}^5C_3(0.3)^3(0.7)^2 + {}^5C_4(0.3)^4(0.7)^1 + {}^5C_5(0.3)^5(0.7)^0$	**M1**	One term seen ${}^5C_x\,(p)^x(1-p)^{5-x},\ 0<p<1,\ x\neq 0,5$.
	= 0.1323 + 0.02835 + 0.00243	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.163, $\dfrac{4077}{25000}$	**B1**	0.16308 to at least 3SF.
	Method 2		
	[1 – P(0, 1, 2) = $1-({}^5C_0(0.3)^0(0.7)^5 + {}^5C_1(0.3)^1(0.7)^4 + {}^5C_2(0.3)^2(0.7)^3)$	**M1**	One term ${}^5C_x\,(p)^x(1-p)^{5-x},\ 0<p<1,\ x\neq 0,5$.
	= 1 – (0.16807 + 0.36015 + 0.3087)	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.163, $\dfrac{4077}{25000}$	**B1**	0.16308 to at least 3SF.
		3	

Question	Answer	Marks	Guidance
5(c)	[Mean = $75 \times 0.3 =$] 22.5 [Var = $75 \times 0.3 \times 0.7 =$] 15.75	**B1**	22.5, 22½ and 15.75, $15\frac{3}{4}$ seen, allow unsimplified. (σ = $\frac{3\sqrt{7}}{2}$ or 3.9686269… to at least 3SF implies correct variance)
	[$\mathrm{P}(X > 20) =$] $\mathrm{P}\left(Z > \frac{20.5 - 22.5}{\sqrt{15.75}}\right)$	**M1**	Substituting their μ and σ into ±standardisation formula (any number for 20.5), not σ^2 not $\sqrt{\sigma}$.
		M1	Using continuity correction 19.5 or 20.5 in *their* standardisation formula.
	[$\mathrm{P}(Z > -0.504) = \Phi(0.504)$] = 0.693	**M1**	Appropriate area Φ, from final process, must be a probability. Expect final answer > 0.5. Note: correct final answer implies this M1.
		A1	$0.6925 < p \leqslant 0.693$
		5	

Question	Answer	Marks	Guidance
6(a)(i)	**Method 1**		
	$6! \times 2^6$	**M1**	$6! \times a$, a integer > 1.
		M1	$b \times 2^6$, b integer $\geqslant 1$.
	= 46080	**A1**	Accurate answer required. **SC B1** for 46080 if M0 M0 www.
	Alternative method for question 6(a)(i)		
	$12 \times 10 \times 8 \times 6 \times 4 \times 2$	**M1**	$c \times d \times e \times f \times g \times h$ $2 \leqslant c,d,e,f,g,h$ (different integers) $\leqslant 12$
		M1	Correct unsimplified.
	= 46080	**A1**	Accurate answer required. **SC B1** for 46080 if M0 M0 www.
		3	
6(a)(ii)	$5! \times 5! \times 2 \times 2$	**M1**	$5! \times 5! \times k$, k positive integer, 1 may be implied (no adding/subtracting).
	= 57600	**A1**	
		2	

Question	Answer	Marks	Guidance
6(b)	**Method 1 probabilities of J & K being placed:**		
	In the group of 5: $\frac{5}{12}\times\frac{4}{11}$ $\left[=\frac{20}{132},\frac{5}{33}\right]$ In the group of 4: $\frac{4}{12}\times\frac{3}{11}$ $\left[=\frac{12}{132},\frac{1}{11}\right]$ In the group of 3: $\frac{3}{12}\times\frac{2}{11}$ $\left[=\frac{6}{132},\frac{1}{22}\right]$	**B1**	Correct probability for one identified scenario.
		M1	Denominator 12×11 for all probabilities, (1, 2 or 3 scenarios).
		A1	3 correct probabilities, accept unsimplified.
	$\frac{5}{12}\times\frac{4}{11}+\frac{4}{12}\times\frac{3}{11}+\frac{3}{12}\times\frac{2}{11}$	**M1**	Adding probabilities for 3 correct scenarios.
	$\frac{19}{66}$, 0.288	**A1**	0.2878787 to at least 3SF.

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="6">6(b)</td><td colspan="3">Method 2 number of arrangements of J & K being placed:</td></tr>
<tr><td><table><tr><td>In the group of 5</td><td>$^{10}C_3 \times {}^7C_4$</td><td>[= 120 × 35 = 4200]</td></tr><tr><td>In the group of 4</td><td>$^{10}C_2 \times {}^8C_5$</td><td>[= 45 × 56 = 2520]</td></tr><tr><td>In the group of 3</td><td>$^{10}C_1 \times {}^9C_5$</td><td>[= 10 × 126 = 1260]</td></tr></table></td><td>B1

M1</td><td>Correct value of one identified scenario seen, accept unsimplified.

$^{12}C_a \times {}^{12-a}C_b$, $a = 3, 4, 5$; $b = 3, 4, 5$ $(a \neq b)$</td></tr>
<tr><td>[Total number of ways of arranging the 3 groups =]
$^{12}C_5 \times {}^7C_4 = 792 \times 35 = 27720$
or $^{12}C_3 \times {}^9C_4$ or $^{12}C_4 \times {}^8C_5$</td><td>A1</td><td>27720 Seen alone or as denominator of probability –accept unsimplified.
SC B1 if M0.</td></tr>
<tr><td>4200 + 2520 + 1260 = 7980</td><td>M1</td><td>Values of 3 correct scenarios added, accept unsimplified – or correct.</td></tr>
<tr><td>[Probability =] $\frac{7980}{27720}$, $\frac{19}{66}$, 0.288</td><td>A1</td><td>0.2878787 to at least 3SF.</td></tr>
<tr><td></td><td>5</td><td>Note, alternative arrangement calculations possible e.g.<table><tr><td>In the group of 5</td><td>$^{10}C_3 \times {}^7C_4$</td><td>[= 120 × 35 = 4200]</td></tr><tr><td>In the group of 4</td><td>$^{10}C_5 \times {}^5C_2$</td><td>[= 252× 10 = 2520]</td></tr><tr><td>In the group of 3</td><td>$^{10}C_5 \times {}^5C_4$</td><td>[= 252 × 5 = 1260]</td></tr></table></td></tr>
</table>

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (KN) 332579

1 A random variable X has the distribution $N(410, 400)$.

Find the probability that the mean of a random sample of 36 values of X is less than 405. [3]

2 In a survey of 300 randomly chosen adults in Rickton, 134 said that they exercised regularly. This information was used to calculate an $\alpha\%$ confidence interval for the proportion of adults in Rickton who exercise regularly. The upper bound of the confidence interval was found to be 0.487, correct to 3 significant figures.

Find the value of α correct to the nearest integer. [4]

3 A website owner finds that, on average, his website receives 0.3 hits per minute. He believes that the number of hits per minute follows a Poisson distribution.

(a) Assume that the owner is correct.

(i) Find the probability that there will be at least 4 hits during a 10-minute period. [3]

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(ii) Use a suitable approximating distribution to find the probability that there will be fewer than 40 hits during a 3-hour period. [4]

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A friend agrees that the website receives, on average, 0.3 hits per minute. However, she notices that the number of hits during the day-time (9.00 am to 9.00 pm) is usually about twice the number of hits during the night-time (9.00 pm to 9.00 am).

(b) **(i)** Explain why this fact contradicts the owner's belief that the number of hits per minute follows a Poisson distribution. [1]

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(ii) Specify separate Poisson distributions that might be suitable models for the number of hits during the day-time and during the night-time. [2]

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4 The masses, in kilograms, of chemicals A and B produced per day by a factory are modelled by the independent random variables X and Y respectively, where $X \sim N(10.3, 5.76)$ and $Y \sim N(11.4, 9.61)$. The income generated by the chemicals is \$2.50 per kilogram for A and \$3.25 per kilogram for B.

(a) Find the mean and variance of the daily income generated by chemical A. [2]

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(b) Find the probability that, on a randomly chosen day, the income generated by chemical A is greater than the income generated by chemical B. [6]

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5 In the past the number of enquiries per minute at a customer service desk has been modelled by a random variable with distribution Po(0.31). Following a change in the position of the desk, it is expected that the mean number of enquiries per minute will increase. In order to test whether this is the case, the total number of enquiries during a randomly chosen 5-minute period is noted. You should assume that a Poisson model is still appropriate.

Given that the total number of enquiries is 5, carry out the test at the 2.5% significance level. [5]

6 A continuous random variable X takes values from 0 to 6 only and has a probability distribution that is symmetrical.

Two values, a and b, of X are such that $P(a < X < b) = p$ and $P(b < X < 3) = \frac{13}{10}p$, where p is a positive constant.

(a) Show that $p \leqslant \frac{5}{23}$. [1]

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(b) Find $P(b < X < 6 - a)$ in terms of p. [2]

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It is now given that the probability density function of X is f, where

$$f(x) = \begin{cases} \frac{1}{36}(6x - x^2) & 0 \leqslant x \leqslant 6, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $b = 2$ and $p = \frac{5}{27}$, find the value of a. [5]

7 A biologist wishes to test whether the mean concentration μ, in suitable units, of a certain pollutant in a river is below the permitted level of 0.5. She measures the concentration, x, of the pollutant at 50 randomly chosen locations in the river. The results are summarised below.

$$n = 50 \qquad \Sigma x = 23.0 \qquad \Sigma x^2 = 13.02$$

(a) Carry out a test at the 5% significance level of the null hypothesis $\mu = 0.5$ against the alternative hypothesis $\mu < 0.5$. [7]

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Later, a similar test is carried out at the 5% significance level using another sample of size 50 and the same hypotheses as before. You should assume that the standard deviation is unchanged.

(b) Given that, in fact, the value of μ is 0.4, find the probability of a Type II error. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\dfrac{405-410}{\frac{20}{6}}$ [= –1.5]	**M1**	For standardising, must have $\sqrt{36}$. Allow totals method $\dfrac{14580-14760}{\sqrt{14400}}$. No mixed methods.
	ϕ('–1.5') = 1 – ϕ('1.5')	**M1**	For area consistent with their working.
	= 0.0668	**A1**	
		3	

Question	Answer	Marks	Guidance
2	$\dfrac{134}{300} + z\sqrt{\dfrac{\frac{134}{300} \times \frac{166}{300}}{300}} = 0.487$	**M1**	For expression of the correct form.
	z = 1.405	**A1**	Accept 1.404, or anything that rounds to 1.39 to 1.41.
	ϕ^{-1}('1.405') = 0.9199 or 0.92; 1 − 2(1 – 0.92)	**M1**	Attempt area above or below their 1.405 and convert to a confidence level.
	α = 84	**A1**	Allow α = 84%. cwo Note: final answer 0.84 scores A0.
		4	

Question	Answer	Marks	Guidance
3(a)(i)	$\lambda = 3$	**B1**	For mean = 3.
	$1 - e^{-3}(1 + 3 + \frac{3^2}{2} + \frac{3^3}{3!})$ or $1 - e^{-3}(1 + 3 + 4.5 + 4.5)$ or $1 - (0.04979 + 0.14936 + 0.22404 + 0.22404)$	**M1**	Any λ. Allow one end error.
	= 0.353 (3 sf)	**A1**	No working scores B1.
		3	
3(a)(ii)	N(54, 54)	**M1**	soi
	$\frac{39.5-54}{\sqrt{54}}$ (= –1.973)	**M1**	Allow with wrong or no continuity correction. For standardising with their mean and variance.
	$1 - \phi$ ('1.973')	**M1**	For area consistent with their working.
	= 0.0242 (3 sf)	**A1**	Special case: if no working seen, 0.0242 scores **SC B3**, 0.0284 scores **SC B2**.
		4	
3(b)(i)	‘Mean not constant’ or’ ‘number of hits per minute not constant’ or ‘not a constant rate’	**B1**	
		1	
3(b)(ii)	$2p + p = 2 \times 0.3$ [$\Rightarrow p = 0.2$] [where p is the rate per minute for night time]	**M1**	May be implied by answer.
	[During day-time]: Po(0.4). [During night-time]: Po(0.2)	**A1**	Accept Po(24) [per daytime hour], Po(12) [per night time hour]. Accept Po(288) [per day time shift], Po(144)[per night time shift]. Note: Po(432), Po(216) scores M0A0.
		2	

Question	Answer	Marks	Guidance
4(a)	E(*A* income) = [10.3 × 2.50] = 25.75 [$]	**B1**	Accept 3sf.
	Var(*A* income) = [5.76×2.50^2] = 36 [$\$^2$]	**B1**	
		2	
4(b)	*B* income ~ N(37.05, 101.506) or E(*B* income) = 37.05 and Var(*B* income) = 101.51	**B1**	Or N(37.1, 102) soi.
	A income – *B* income ~ N('25.75' – '37.05', '36' + '101.506')	**M1**	Ft their values for *A* and *B*.
	= N(–11.3, 137.506)	**A1**	Accept 3sf.
	$\dfrac{0-(-'11.3')}{\sqrt{'137.506'}}$ [= 0.964]	**M1**	Standardising with their values from attempt at *A* income – *B* income.
	1 – ϕ('0.964')	**M1**	For area consistent with their values.
	= 0.168 or 0.167 (3 sf)	**A1**	cwo
		6	

Question	Answer	Marks	Guidance
5	H_0: Population mean no. enquiries = 1.55 H_1: Population mean no. enquiries > 1.55	**B1**	Or "population mean no. enquiries = 0.31 (per minute)" oe. Allow 'λ = 1.55' or μ = '1.55'.
	$P(X \geqslant 5) = 1 - e^{-1.55}(1 + 1.55 + \frac{1.55^2}{2!} + \frac{1.55^3}{3!} + \frac{1.55^4}{4!})$ or $1 - e^{-1.55}(1 + 1.55 + 1.20125 + 0.62065 + 0.24050)$ or $1 - (0.21225 + 0.32898 + 0.25496 + 0.13173 + 0.05105)$	**M1**	Allow one end error, e.g. extra term: $e^{-1.55} \times \frac{1.55^5}{5!}$.
	= 0.0210 (3 sf)	**A1**	Allow 0.021. **SC B1** no working scores B1 instead of M1A1.
	0.0210 < 0.025	**M1**	For valid comparison.
	[Reject H_0] There is sufficient evidence [at 2.5% level] to suggest that mean no. of enquiries has increased.	**A1 FT**	In context, not definite, e.g., not "Mean no. of enquiries has increased". No contradictions.
		5	Note: $e^{-1.55} \times \frac{1.55^5}{5!} = 0.0158 < 0.025$: scores max B1

Question	Answer	Marks	Guidance
6(a)	$p + \frac{13}{10}p \leqslant \frac{1}{2} \Rightarrow p \leqslant \frac{5}{23}$ AG	**B1**	Allow '=' in working but need an inequality in the answer. Allow $0 < p \leqslant \frac{5}{23}$.
		1	

Question	Answer	Marks	Guidance
6(b)	e.g. $0.5 - 2.3p$, 0 to a; $p + 1.3p$, a to 3; $2 \times 1.3p$, $2 \times$ (b to 3); $2.3p + 1.3p$, a to 3 + b to 3; $2p + 2.6p$, $2 \times$ (a to 3); $0.5 - 1.3p$, 0 to b; $0.5 + 1.3p$, b to 6	**M1**	Any correct expression for the probability of a relevant region.
	$\frac{18}{5}p$ or $3.6p$	**A1**	
		2	

Question	Answer	Marks	Guidance
6(c)	$\frac{1}{36}\int_a^2 (6x - x^2)\,dx = \frac{5}{27}$	**M1**	Attempt to integrate with correct limits and equate to $\frac{5}{27}$, oe. Integrate from 2 to 6 – a and equate $\frac{18}{5}p = \frac{2}{3}$. Integrate from a to 3 and equate to $\frac{23}{54}$. Integrate from 0 to a and equate to $\frac{2}{27}$.
	$\left[\Rightarrow \frac{1}{36}\left[3x^2 - \frac{x^3}{3}\right]_a^2 = \frac{5}{27} \Rightarrow \frac{1}{36}\left(12 - \frac{8}{3} - 3a^2 + \frac{a^3}{3}\right) = \frac{5}{27}\right]$	**M1**	For integrating and substitution of limits to form cubic in a.
	$a^3 - 9a^2 + 8 = 0$	**A1**	Any correct three term cubic equation in a.
	$(a - 1)(a^2 - 8a - 8) = 0$	**M1**	Attempt to factorise their cubic equation.
	$a = \frac{8 \pm \sqrt{96}}{2} = 4 \pm \sqrt{24}$ or –0.899 or 8.90, [not between 0 and 6]		
	$a = 1$ only [other two values rejected]	**A1**	**SC B1** for $a = 1$ only, if no method seen for solving the cubic.
		5	

Question	Answer	Marks	Guidance
7(a)	Est (μ) = 23/50 = 0.46	**B1**	
	Est (σ) = $\sqrt{\frac{50}{49}} \times \sqrt{\frac{13.02}{50} - 0.46^2}$ or Est (σ^2) = $\frac{50}{49} \times \left(\frac{13.02}{50} - 0.46^2\right)$ oe Or estimated unbiased variance = $\frac{1}{49}\left(13.02 - \frac{(23.0)^2}{50}\right)$	**M1**	For an expression of the correct form for unbiased standard deviation or variance.
	Est (σ) = 0.22315 or Est (σ^2) = 0.0497959 = $\left(\frac{61}{1225}\right)$ or 0.0498	**A1**	
	$\frac{0.46 - 0.5}{\frac{'0.22315'}{\sqrt{50}}}$	**M1**	Standardising with their values.
	= –1.268 or -1.267 or = –1.27 (3sf)	**A1**	
	−1.268 > –1.645 or 0.102 to 0.103 > 0.05	**M1**	For a valid comparison.
	[Do not reject Ho] There is insufficient evidence [at 5% level] that the mean concentration is less than 0.5.	**A1 FT**	In context, not definite. E.g., not ‘Mean concentration is not less than 0.5’. No contradictions.
		7	

Question	Answer	Marks	Guidance
7(b)	$\dfrac{cv-0.5}{\frac{'0.22315'}{\sqrt{50}}}=-1.645$	**M1**	
	cv = 0.448(1) or 0.448 (3 sf)	**A1**	
	$\dfrac{'0.448'-\ 0.4}{\frac{'0.22315'}{\sqrt{50}}}$ [=1.521 to 1.524]	**M1**	
	$1-\phi('1.524')$	**M1**	For area consistent with their working.
	= 0.0638 to 0.0642	**A1**	
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7077958469

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 **(a)** A random variable X has the distribution $\mathrm{Po}(25)$.

Use the normal approximation to the Poisson distribution to find $P(X > 30)$. [4]

(b) A random variable Y has the distribution $B(100, p)$ where $p < 0.05$.

Use the Poisson approximation to the binomial distribution to write down an expression, in terms of p, for $P(Y < 3)$. [2]

2 The length, in minutes, of mathematics lectures at a certain college has mean μ and standard deviation 8.3.

(a) The total length of a random sample of 85 lectures was 4590 minutes.

Calculate a 95% confidence interval for μ. [3]

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The length, in minutes, of history lectures at the college has mean m and standard deviation s.

(b) Using a random sample of 100 history lectures, a 95% confidence interval for m was found to have width 2.8 minutes.

Find the value of s. [2]

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3 A researcher read a magazine article which stated that boys aged 1 to 3 prefer green to orange. It claimed that, when offered a green cube and an orange cube to play with, a boy is more likely to choose the green one.

The researcher disagrees with this claim. She believes that boys of this age are equally likely to choose either colour. In order to test her belief, the researcher carried out a hypothesis test at the 5% significance level. She offered a green cube and an orange cube to each of 10 randomly chosen boys aged 1 to 3, and recorded the number, X, of boys who chose the green cube.

Out of the 10 boys, 8 boys chose the green cube.

(a) **(i)** Assuming that the researcher's belief that either colour cube is equally likely to be chosen is valid, a student correctly calculates that $P(X = 8) = 0.0439$, correct to 3 significant figures. He says that, because this value is less than 0.05, the null hypothesis should be rejected.

Explain why this statement is incorrect. [1]

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(ii) Carry out the test on the researcher's claim that either colour cube is equally likely to be chosen. [5]

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(b) Another researcher claims that a Type I error was made in carrying out the test.

Explain why this cannot be true. [1]

A similar test, at the 5% significance level, was carried out later using 10 other randomly chosen boys aged 1 to 3.

(c) Find the probability of a Type I error. [2]

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4 The height H, in metres, of mature trees of a certain variety is normally distributed with standard deviation 0.67. In order to test whether the population mean of H is greater than 4.23, the heights of a random sample of 200 trees are measured.

(a) Write down suitable null and alternative hypotheses for the test. [1]

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The sample mean height, $\bar{h}$ metres, of the 200 trees is found and the test is carried out. The result of the test is to reject the null hypothesis at the 5% significance level.

(b) Find the set of possible values of $\bar{h}$. [3]

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(c) Ajit said, 'In **(b)** we had to assume that $\overline{H}$ is normally distributed, so it was necessary to use the Central Limit Theorem.'

Explain whether you agree with Ajit. [1]

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5 The random variable X has probability density function, f, given by

$$f(x) = \begin{cases} \dfrac{1}{x^2} & a < x < b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are positive constants.

(a) It is given that $E(X) = \ln 2$.

Show that $b = 2a$. [3]

(b) Show that $a = \frac{1}{2}$. [3]

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(c) Find the median of X. [3]

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6 A factory makes loaves of bread in batches. One batch of loaves contains X kilograms of dried yeast and Y kilograms of flour, where X and Y have the independent distributions $\mathrm{N}(0.7,\ 0.02^2)$ and $\mathrm{N}(100.0,\ 3.0^2)$ respectively.

Dried yeast costs \$13.50 per kilogram and flour costs \$0.90 per kilogram. For making one batch of bread the total of all other costs is \$55. The factory sells each batch of bread for \$200.

Find the probability that the profit made on one randomly chosen batch of bread is greater than \$40. [7]

7 A random variable X has the distribution Po(2.4).

(a) Find $P(2 \leqslant X < 4)$. [2]

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(b) Two independent values of X are chosen.

Find the probability that both of these values are greater than 1. [3]

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(c) It is given that $P(X = r) < P(X = r + 1)$.

(i) Find the set of possible values of r. [3]

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(ii) Hence find the value of r for which $P(X = r)$ is greatest. [1]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$X \sim \text{N}(25, 25)$	**B1**	soi
	$\dfrac{30.5-25}{\sqrt{25}}$ [= 1.1]	**M1**	Standardising with their values. Allow with missing or incorrect continuity correction.
	$1 - \phi(\text{'}1.1\text{'})$	**M1**	For area consistent with their working.
	= 0.136 (3 sf)	**A1**	
		4	
1(b)	$e^{-100p}\left(1+100p+\dfrac{(100p)^2}{2!}\right)$	**M1**	For $P_o(100p)$ expression. Accept un-simplified terms (e.g. $p^0/0!$ For M1). Allow one end error (e.g. for correct with extra term $e^{-100p}\times\dfrac{(100p)^3}{3!}$ oe), or brackets omitted.
	$e^{-100p}\left(1+100p+\dfrac{(100p)^2}{2!}\right)$ or $e^{-100p} + e^{-100p} \times 100p + e^{-100p}\times\dfrac{(100p)^2}{2!}$ or $e^{-100p}(1 + 100p + 5000p^2)$ oe	**A1**	Must have brackets. Allow with or without ! sign (but not 0! or p^0). ISW once a fully correct answer seen.
		2	

Question	Answer	Marks	Guidance
2(a)	$\dfrac{4590}{85} \pm z \times \dfrac{8.3}{\sqrt{85}}$	**M1**	For expression of correct form. Any z (but not $\phi(z)$).
	$z = 1.96$	**B1**	
	52.2 to 55.8 (3 sf)	**A1**	Must be an interval.
		3	

Question	Answer	Marks	Guidance
2(b)	$1.96\times\frac{s}{\sqrt{100}}=1.4$ or $2\times1.96\times\frac{s}{\sqrt{100}}=2.8$	**M1**	Equation of correct form (any z). Allow factor of 2 error (i.e. first equation = 2.8).
	s = 7.14 (3 sf) or $\frac{50}{7}$	**A1**	
		2	

Question	Answer	Marks	Guidance
3(a)(i)	Need to find $P(X \geqslant 8)$	**B1**	oe (e.g. invalid because it should be a tail probability compared with 0.05).
		1	
3(a)(ii)	H_0: P(green) = 0.5 H_1: P(green) > 0.5	**B1**	Allow p = 0.5. Allow p > 0.5.
	$P(X \geqslant 8) = 0.0439 + {}^{10}C_9 \times (0.5) \times (0.5)^9 + 0.5^{10}$	**M1**	Attempt 0.0439 + $P(X = 9)$ + $P(X = 10)$. Must see Binomial expressions B(10,0.5).
	= 0.0547 or 0.0546 (3 sf)	**A1**	**SC B1** 0.0547 or 0.0546 with no working.
	0.0547 > 0.05	**M1**	Valid comparison of tail probability with 0.05.
	[Do not reject H_0] 'There is insufficient evidence [at the 5% level] to accept the hypothesis that boys prefer green.' Or 'There is sufficient evidence to support the **researcher's** claim.'	**A1FT**	In context, not definite. No contradictions. Allow 'There is insufficient evidence to reject the hypothesis that boys like green and orange equally'. Not definite, e.g. not 'They don't prefer green" or 'Researchers claim true' 'Magazine's claim untrue'. Any mention of 'claim' must be clear which claim it is.
		5	

Question	Answer	Marks	Guidance
3(b)	H_0 was not rejected	**B1**	Mark independently.
		1	
3(c)	${}^{10}C_9 \times (0.5)\times(0.5)^9 + 0.5^{10}$ or "0.0547" – 0.0439	**M1**	Finding P(9,10) using B(10,0.5). Could be seen in **(a)(ii)**.
	P(Type I error) = 11/1024 or 0.0107 (3 sf) or 0.0108	**A1**	
		2	
4(a)	H_0: population mean [of H] = 4.23 H_1: population mean [of H] > 4.23	**B1**	Allow μ = 4.23 or population mean of h = 4.23 but NOT h = 4.23 or H = 4.23 or $\overline{h}$ = 4.23 or $\overline{H}$ = 4.23.
		1	
4(b)	$\dfrac{\overline{h} - 4.23}{\frac{0.67}{\sqrt{200}}} = 1.645$	**M1**	For standardising and forming an equation. Must have $\sqrt{200}$. Allow $\pm$ 1.645 or $\pm$ 1.96. Accept '>' and '<'. Allow $\overline{H}$ or any letter instead of $\overline{h}$.
	$\overline{h}$ = 4.31 (3 sf)	**A1**	May be implied by $\overline{h}$ > 4.31. Allow $\overline{h}$ < 4.31 for this A1 only, condone 4.15 also seen.
	$\overline{h}$ > 4.31 or $\overline{h} \geqslant 4.31$ (3 sf)	**A1**	Condone any letter instead of $\overline{h}$.
		3	
4(c)	Incorrect, because the population of H is given as normally distributed [with known variance].	**B1**	Allow h instead of H or just 'The population is normal.' Must use 'population' or 'underlying distribution'.
		1	

Question	Answer	Marks	Guidance
5(a)	$\int_a^b \frac{x}{x^2}\,dx$	**M1**	Attempt to integrate $x\mathrm{f}(x)$ from a to b.
	$=[\ln x]_a^b$ or $\ln b - \ln a$	**A1**	
	$\ln\frac{b}{a} = \ln 2$ or $\ln 2a = \ln b$ oe Hence $b = 2a$ (AG)	**A1**	Must see both statements. No errors seen.
		3	
5(b)	$\int_a^b \frac{1}{x^2}\,dx = 1$	**M1**	Attempt to integrate $\mathrm{f}(x)$ and equate to 1. Ignore limits.
	$\left[\left[-\frac{1}{x}\right]_a^b = 1 \text{ or } \frac{1}{a} - \frac{1}{b} = 1\right]$ $\frac{1}{a} - \frac{1}{2a} = 1$	**A1**	Integrate with correct limits and substitute $b = 2a$.
	e.g. $\frac{1}{2a} = 1$ or $2 + (-1) = 2a$ Hence $a =$ $a = \frac{1}{2}$ (AG)	**A1**	Obtain convincingly (at least one step from previous answer), no errors seen (ignore $a = 0$).
		3	

Question	Answer	Marks	Guidance
5(c)	$\int_{0.5}^{m} \frac{1}{x^2}\,dx = \frac{1}{2}$ or $\int_{m}^{1} \frac{1}{x^2}\,dx = \frac{1}{2}$	M1	Attempt integrate f(x), with correct limits, and equate to $\frac{1}{2}$.
	$\left[-\frac{1}{x}\right]_{0.5}^{m} = \frac{1}{2}$ or $\left[-\frac{1}{x}\right]_{m}^{1} = \frac{1}{2}$		
	$2-\frac{1}{m}=\frac{1}{2}$ or $\frac{1}{m}-1=\frac{1}{2}$	A1	oe. Correct equation after substituting limits.
	$m=\frac{2}{3}$ or 0.667 (3 sf)	A1	
		3	

Question	Answer	Marks	Guidance
6	Cost of dried yeast and flour: $\$D$ and $\$F$ $E(D) = 13.5 \times 0.7 = 9.45$ $E(F) = 0.9 \times 100 = 90$	**B1**	One of these soi – can be given at early stage.
	$\text{Var}(D) = 0.02^2 \times 13.50^2 = 0.0729$ $\text{Var}(F) = 3.0^2 \times 0.90^2 = 7.29$	**B1**	One of these soi – can be given at early stage.
	Total cost: $T \sim N(99.45, (0.02^2 \times 13.50^2 + 3.0^2 \times 0.90^2))$	**M1**	Attempt to combine their D and their F with or without 55 and 200 (but variance must not include 55 or 200). Or $N(99.45 + 55, (0.02^2 \times 13.50^2 + 3.0^2 \times 0.90^2))$. Or $N(200 - 55 - 99.45, (0.02^2 \times 13.50^2 + 3.0^2 \times 0.90^2))$. Or $N(99.45 + c, (0.02^2 \times 13.50^2 + 3.0^2 \times 0.90^2))$.
	N(99.45, 7.3629) accept 99.4 or 99.5	**A1**	Or N(154.45,7.3629) or N(45.55, 7.3629). Accept 3sf (accept 45.5 or 45.6).
	[P(profit > $40) = P(T < 105)] $\frac{105 - '99.45'}{\sqrt{'7.3629'}}$ [= 2.045]	**M1**	$160 - 154.45/\sqrt{7.3629}$ or $40 - 45.55/\sqrt{7.3629}$. No mixed methods.
	[P(T < 105) = P(z < 2.045) =] $\phi(2.045)$	**M1**	For area consistent with their working.
	= 0.9795 or 0.9796 or 0.98(0) or 0.979 (3 sf)	**A1**	
		7	

Question	Answer	Marks	Guidance
7(a)	$\left(e^{-2.4}\left(\frac{2.4^2}{2!}+\frac{2.4^3}{3!}\right)=e^{-2.4}(2.88+2.304)=0.2613+0.2090\right.$	**M1**	Allow M1 for $e^{-2.4}\left(\frac{2.4^2}{2!}+\frac{2.4^3}{3!}+\frac{2.4^4}{4!}\right)$. Expression must be seen.
	= 0.47(0)	**A1**	**SC B1** 0.47(0) with no working.
		2	
7(b)	$1 - e^{-2.4}(1 + 2.4)$ [= 0.691558]	**M1**	Allow one end error. Allow any λ.
	$(1 - e^{-2.4}(1 + 2.4))^2$	**M1**	Squaring their probability (λ ≠ 4.8).
	= 0.478 (3 sf)	**A1**	**SC B2** 0.478 with no working.
		3	
7(c)(i)	$e^{-2.4}\times\frac{2.4^r}{r!}<e^{-2.4}\times\frac{2.4^{r+1}}{(r+1)!}$	**M1**	For both expressions seen.
	$r + 1 < 2.4$	**A1***	Or $r < 1.4$ (must have correct inequality).
	Set is $r = 0, 1$	**DA1**	
		3	For trial and error solutions. M1 for substituting one value into correct expression (can be implied by correct values). A1* for [P(0) = 0.0907], P(1) = 0.218, P(2) = 0.261, P(3) = 0.209 (accept 2sf accuracy). **DA1** Set is $r = 0,1$.
7(c)(ii)	$r = 2$	**B1**	
		1	

Cambridge Assessment International Education

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MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 A random variable X has the distribution $N(410, 400)$.

Find the probability that the mean of a random sample of 36 values of X is less than 405. [3]

2 In a survey of 300 randomly chosen adults in Rickton, 134 said that they exercised regularly. This information was used to calculate an $\alpha\%$ confidence interval for the proportion of adults in Rickton who exercise regularly. The upper bound of the confidence interval was found to be 0.487, correct to 3 significant figures.

Find the value of α correct to the nearest integer. [4]

3 A website owner finds that, on average, his website receives 0.3 hits per minute. He believes that the number of hits per minute follows a Poisson distribution.

(a) Assume that the owner is correct.

(i) Find the probability that there will be at least 4 hits during a 10-minute period. [3]

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(ii) Use a suitable approximating distribution to find the probability that there will be fewer than 40 hits during a 3-hour period. [4]

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A friend agrees that the website receives, on average, 0.3 hits per minute. However, she notices that the number of hits during the day-time (9.00 am to 9.00 pm) is usually about twice the number of hits during the night-time (9.00 pm to 9.00 am).

(b) **(i)** Explain why this fact contradicts the owner's belief that the number of hits per minute follows a Poisson distribution. [1]

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(ii) Specify separate Poisson distributions that might be suitable models for the number of hits during the day-time and during the night-time. [2]

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4 The masses, in kilograms, of chemicals A and B produced per day by a factory are modelled by the independent random variables X and Y respectively, where $X \sim \mathrm{N}(10.3,\ 5.76)$ and $Y \sim \mathrm{N}(11.4,\ 9.61)$. The income generated by the chemicals is \$2.50 per kilogram for A and \$3.25 per kilogram for B.

(a) Find the mean and variance of the daily income generated by chemical A. [2]

(b) Find the probability that, on a randomly chosen day, the income generated by chemical A is greater than the income generated by chemical B. [6]

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5 In the past the number of enquiries per minute at a customer service desk has been modelled by a random variable with distribution Po(0.31). Following a change in the position of the desk, it is expected that the mean number of enquiries per minute will increase. In order to test whether this is the case, the total number of enquiries during a randomly chosen 5-minute period is noted. You should assume that a Poisson model is still appropriate.

Given that the total number of enquiries is 5, carry out the test at the 2.5% significance level. [5]

6 A continuous random variable X takes values from 0 to 6 only and has a probability distribution that is symmetrical.

Two values, a and b, of X are such that $P(a < X < b) = p$ and $P(b < X < 3) = \frac{13}{10}p$, where p is a positive constant.

(a) Show that $p \leqslant \frac{5}{23}$. [1]

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(b) Find $P(b < X < 6 - a)$ in terms of p. [2]

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It is now given that the probability density function of X is f, where

$$f(x) = \begin{cases} \frac{1}{36}(6x - x^2) & 0 \leqslant x \leqslant 6, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $b = 2$ and $p = \frac{5}{27}$, find the value of a. [5]

7 A biologist wishes to test whether the mean concentration μ, in suitable units, of a certain pollutant in a river is below the permitted level of 0.5. She measures the concentration, x, of the pollutant at 50 randomly chosen locations in the river. The results are summarised below.

$$n = 50 \qquad \Sigma x = 23.0 \qquad \Sigma x^2 = 13.02$$

(a) Carry out a test at the 5% significance level of the null hypothesis $\mu = 0.5$ against the alternative hypothesis $\mu < 0.5$. [7]

Later, a similar test is carried out at the 5% significance level using another sample of size 50 and the same hypotheses as before. You should assume that the standard deviation is unchanged.

(b) Given that, in fact, the value of μ is 0.4, find the probability of a Type II error. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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