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Toomates Coolección

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Trigonometría , Desigualdades , Números complejos , Funciones

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España: OME OMEFL OMEC OMEA OMEM CDP
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Pizzazz!: Book A Book B Book C Book D Book E Pre-Algebra Algebra
AHSME: Book 1 Book 2 Book 3 Book 4 Book 5 Book 6 Book 7 Book 8 Book 9

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Visita mi **Canal de Youtube**: **https://www.youtube.com/c/GerardRomo**

Versión de este documento: **07/06/2024**

Presentación.

Este documento forma parte del recopilatorio de pruebas IB:

Años 2018-2019: http://www.toomates.net/biblioteca/CompendiumIB2.pdf

Años 2021-2023: http://www.toomates.net/biblioteca/CompendiumIB.pdf

En el año 2020 no se realizó la prueba debido a la pandemia COVID.

Índice.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

12 pages

2218 – 7203

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Let $f(x) = x^4 + px^3 + qx + 5$ where p, q are constants.
The remainder when $f(x)$ is divided by $(x + 1)$ is 7, and the remainder when $f(x)$ is divided by $(x - 2)$ is 1. Find the value of p and the value of q.

2. [Maximum mark: 7]

Let $y = \sin^2\theta,\ 0 \le \theta \le \pi$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}\theta}$. [2]

(b) Hence find the values of θ for which $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 2y$. [5]

3. [Maximum mark: 5]

Two unbiased tetrahedral (four-sided) dice with faces labelled $1, 2, 3, 4$ are thrown and the scores recorded. Let the random variable T be the maximum of these two scores.
The probability distribution of T is given in the following table.

t	1	2	3	4
$\mathrm{P}(T = t)$	$\frac{1}{16}$	a	b	$\frac{7}{16}$

(a) Find the value of a and the value of b. [3]

(b) Find the expected value of T. [2]

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4. [Maximum mark: 6]

Given that $\int_{-2}^{2} f(x)\mathrm{d}x = 10$ and $\int_{0}^{2} f(x)\mathrm{d}x = 12$, find

(a) $\int_{-2}^{0} \left(f(x)+2\right)\mathrm{d}x$; [4]

(b) $\int_{-2}^{0} f(x+2)\,\mathrm{d}x$. [2]

5. [Maximum mark: 6]

Solve $(\ln x)^2 - (\ln 2)(\ln x) < 2(\ln 2)^2$.

6. [Maximum mark: 7]

Use the principle of mathematical induction to prove that

$$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)^3+\ldots+n\left(\frac{1}{2}\right)^{n-1}=4-\frac{n+2}{2^{n-1}}, \text{ where } n\in\mathbb{Z}^+.$$

7. [Maximum mark: 9]

Let $y = \arccos\left(\dfrac{x}{2}\right)$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) Find $\displaystyle\int_0^1 \arccos\left(\dfrac{x}{2}\right)\mathrm{d}x$. [7]

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8. [Maximum mark: 5]

Let $a = \sin b,\ 0 < b < \frac{\pi}{2}$.

Find, in terms of b, the solutions of $\sin 2x = -a,\ 0 \le x \le \pi$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

Let $f(x) = \dfrac{2 - 3x^5}{2x^3}$, $x \in \mathbb{R}, x \neq 0$.

(a) The graph of $y = f(x)$ has a local maximum at A. Find the coordinates of A. [5]

(b) (i) Show that there is exactly one point of inflexion, B, on the graph of $y = f(x)$.

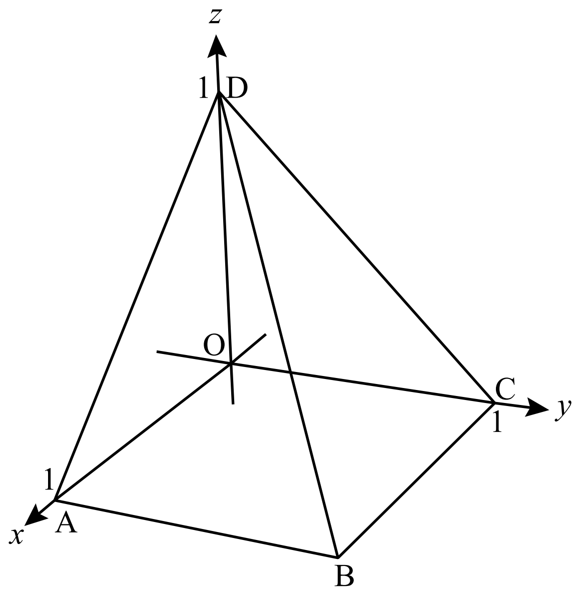
(ii) The coordinates of B can be expressed in the form $\mathrm{B}(2^a, b \times 2^{-3a})$, where $a, b \in \mathbb{Q}$. Find the value of a and the value of b. [8]

(c) Sketch the graph of $y = f(x)$ showing clearly the position of the points A and B. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 19]

The following figure shows a square based pyramid with vertices at $O(0, 0, 0)$, $A(1, 0, 0)$, $B(1, 1, 0)$, $C(0, 1, 0)$ and $D(0, 0, 1)$.



(a) Find the Cartesian equation of the plane Π_1, passing through the points A, B and D. [3]

The Cartesian equation of the plane Π_2, passing through the points B, C and D, is $y + z = 1$.

(b) Find the angle between the faces ABD and BCD. [4]

The plane Π_3 passes through O and is normal to the line BD.

(c) Find the Cartesian equation of Π_3. [3]

Π_3 cuts AD and BD at the points P and Q respectively.

(d) Show that P is the midpoint of AD. [4]

(e) Find the area of the triangle OPQ. [5]

Turn over

Do **not** write solutions on this page.

11. [Maximum mark: 14]

Consider $w = 2\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$.

(a) (i) Express w^2 and w^3 in modulus-argument form.

(ii) Sketch on an Argand diagram the points represented by w^0, w^1, w^2 and w^3. [5]

These four points form the vertices of a quadrilateral, Q.

(b) Show that the area of the quadrilateral Q is $\frac{21\sqrt{3}}{2}$. [3]

Let $z = 2\left(\cos\frac{\pi}{n} + \mathrm{i}\sin\frac{\pi}{n}\right), n \in \mathbb{Z}^+$. The points represented on an Argand diagram by $z^0, z^1, z^2, \ldots, z^n$ form the vertices of a polygon P_n.

(c) Show that the area of the polygon P_n can be expressed in the form $a\left(b^n - 1\right)\sin\frac{\pi}{n}$, where $a, b \in \mathbb{R}$. [6]

Markscheme

May 2018

Mathematics

Higher level

Paper 1

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document "**Mathematics HL: Guidance for e-marking May 2018**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. attempt to substitute $x = -1$ or $x = 2$ or to divide polynomials *(M1)*
$1 - p - q + 5 = 7$, $16 + 8p + 2q + 5 = 1$ or equivalent ***A1A1***
attempt to solve their two equations ***M1***
$p = -3$, $q = 2$ ***A1***

[5 marks]

2. (a) attempt at chain rule or product rule ***(M1)***

$$\frac{dy}{d\theta} = 2\sin\theta\cos\theta$$ ***A1***

[2 marks]

(b) $2\sin\theta\cos\theta = 2\sin^2\theta$

$\sin\theta = 0$ ***(A1)***
$\theta = 0, \pi$ ***A1***
obtaining $\cos\theta = \sin\theta$ ***(M1)***
$\tan\theta = 1$ ***(M1)***
$\theta = \frac{\pi}{4}$ ***A1***

[5 marks]

Total [7 marks]

3. (a) $a = \frac{3}{16}$ and $b = \frac{5}{16}$ ***(M1)A1A1***

[3 marks]

Note: Award ***M1*** for consideration of the possible outcomes when rolling the two dice.

continued

Question 3 continued

(b) $\mathrm{E}(T) = \frac{1+6+15+28}{16} = \frac{25}{8}\left(= 3.125\right)$ ***(M1)A1***

Note: Allow follow through from part (a) even if probabilities do not add up to 1.

[2 marks]

Total [5 marks]

4. (a) $\int_{-2}^{0} f(x)\mathrm{d}x = 10 - 12 = -2$ ***(M1)(A1)***

$\int_{-2}^{0} 2\,\mathrm{d}x = \left[2x\right]_{-2}^{0} = 4$ ***A1***

$\int_{-2}^{0}\left(f(x)+2\right)\mathrm{d}x = 2$ ***A1***

[4 marks]

(b) $\int_{-2}^{0} f(x+2)\mathrm{d}x = \int_{0}^{2} f(x)\mathrm{d}x$ ***(M1)***

$= 12$ ***A1***

[2 marks]

Total [6 marks]

5. $(\ln x)^2 - (\ln 2)(\ln x) - 2(\ln 2)^2 \left(= 0\right)$

EITHER

$\ln x = \frac{\ln 2 \pm \sqrt{(\ln 2)^2 + 8(\ln 2)^2}}{2}$ ***M1***

$= \frac{\ln 2 \pm 3\ln 2}{2}$ ***A1***

OR

$(\ln x - 2\ln 2)(\ln x + \ln 2)\left(= 0\right)$ ***M1A1***

THEN

$\ln x = 2\ln 2$ or $-\ln 2$ ***A1***

$\Rightarrow x = 4$ or $x = \frac{1}{2}$ ***(M1)A1***

Note: ***(M1)*** is for an appropriate use of a log law in either case, dependent on the previous ***M1*** being awarded, ***A1*** for both correct answers.

solution is $\frac{1}{2} < x < 4$ ***A1***

[6 marks]

6. if $n = 1$

$\text{LHS} = 1\,;\text{RHS} = 4 - \dfrac{3}{2^0} = 4 - 3 = 1$ ***M1***

hence true for $n = 1$

assume true for $n = k$ ***M1***

Note: Assumption of truth must be present. Following marks are not dependent on the first two ***M1*** marks.

so $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + k\left(\frac{1}{2}\right)^{k-1} = 4 - \dfrac{k+2}{2^{k-1}}$

if $n = k+1$

$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + k\left(\frac{1}{2}\right)^{k-1} + (k+1)\left(\frac{1}{2}\right)^k$

$= 4 - \dfrac{k+2}{2^{k-1}} + (k+1)\left(\frac{1}{2}\right)^k$ ***M1A1***

finding a common denominator for the two fractions ***M1***

$= 4 - \dfrac{2(k+2)}{2^k} + \dfrac{k+1}{2^k}$

$= 4 - \dfrac{2(k+2)-(k+1)}{2^k} = 4 - \dfrac{k+3}{2^k}\left(= 4 - \dfrac{(k+1)+2}{2^{(k+1)-1}}\right)$ ***A1***

hence if true for $n = k$ then also true for $n = k+1$, as true for $n = 1$, so true (for all $n \in \mathbb{Z}^+$) ***R1***

Note: Award the final ***R1*** only if the first four marks have been awarded.

[7 marks]

7. (a) $y = \arccos\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}} \left(= -\frac{1}{\sqrt{4-x^2}}\right)$ ***M1A1***

Note: ***M1*** is for use of the chain rule.

[2 marks]

(b) attempt at integration by parts ***M1***

$u = \arccos\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{\sqrt{4-x^2}}$

$\frac{\mathrm{d}v}{\mathrm{d}x} = 1 \Rightarrow v = x$ ***(A1)***

$\int_0^1 \arccos\left(\frac{x}{2}\right)\mathrm{d}x = \left[x\arccos\left(\frac{x}{2}\right)\right]_0^1 + \int_0^1 \frac{x}{\sqrt{4-x^2}}\mathrm{d}x$ ***A1***

using integration by substitution or inspection ***(M1)***

$\left[x\arccos\left(\frac{x}{2}\right)\right]_0^1 + \left[-\left(4-x^2\right)^{\frac{1}{2}}\right]_0^1$ ***A1***

Note: Award ***A1*** for $-\left(4-x^2\right)^{\frac{1}{2}}$ or equivalent.

Note: Condone lack of limits to this point.

attempt to substitute limits into their integral ***M1***

$= \frac{\pi}{3} - \sqrt{3} + 2$ ***A1***

[7 marks]

Total [9 marks]

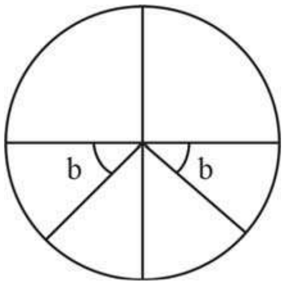
8. $\sin 2x = -\sin b$

EITHER

$\sin 2x = \sin(-b)$ or $\sin 2x = \sin(\pi + b)$ or $\sin 2x = \sin(2\pi - b)\ldots$ ***(M1)(A1)***

Note: Award ***M1*** for any one of the above, ***A1*** for having final two.

OR



(M1)(A1)

Note: Award ***M1*** for one of the angles shown with b clearly labelled, ***A1*** for both angles shown. Do not award ***A1*** if an angle is shown in the second quadrant and subsequent ***A1*** marks not awarded.

THEN

$2x = \pi + b$ or $2x = 2\pi - b$ ***(A1)(A1)***

$x = \frac{\pi}{2} + \frac{b}{2}$, $x = \pi - \frac{b}{2}$ ***A1***

[5 marks]

Section B

9. (a) attempt to differentiate ***(M1)***

$f'(x) = -3x^{-4} - 3x$ ***A1***

Note: Award ***M1*** for using quotient or product rule award ***A1*** if correct derivative seen even in unsimplified form, for example $f'\left(x\right) = \dfrac{-15x^4 \times 2x^3 - 6x^2\left(2 - 3x^5\right)}{\left(2x^3\right)^2}$.

$-\dfrac{3}{x^4} - 3x = 0$ ***M1***

$\Rightarrow x^5 = -1 \Rightarrow x = -1$ ***A1***

$\mathrm{A}\left(-1, -\dfrac{5}{2}\right)$ ***A1***

[5 marks]

(b) (i) $f''\left(x\right) = 0$ ***M1***

$f''(x) = 12x^{-5} - 3\left(= 0\right)$ ***A1***

Note: Award ***A1*** for correct derivative seen even if not simplified.

$\Rightarrow x = \sqrt[5]{4}\left(= 2^{\frac{2}{5}}\right)$ ***A1***

hence (at most) one point of inflexion ***R1***

Note: This mark is independent of the two ***A1*** marks above. If they have shown or stated their equation has only one solution this mark can be awarded.

$f''(x)$ changes sign at $x = \sqrt[5]{4}\left(= 2^{\frac{2}{5}}\right)$ ***R1***

so exactly one point of inflexion

continued

Question 9 continued

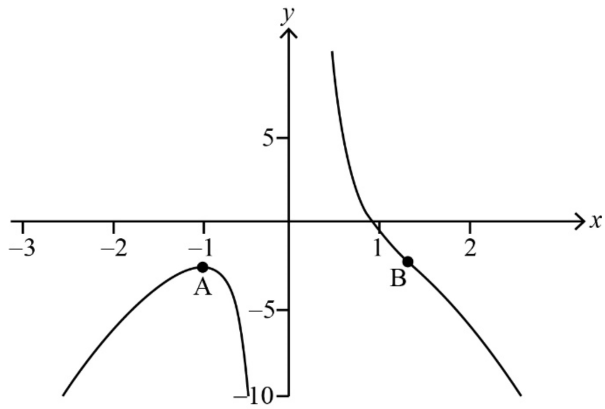
(ii) $x = \sqrt[5]{4} = 2^{\frac{2}{5}} \left(\Rightarrow a = \frac{2}{5}\right)$ ***A1***

$f\left(2^{\frac{2}{5}}\right) = \dfrac{2 - 3 \times 2^2}{2 \times 2^{\frac{6}{5}}} = -5 \times 2^{-\frac{6}{5}} \ (\Rightarrow b = -5)$ ***(M1)A1***

[8 marks]

Note: Award ***M1*** for the substitution of their value for x into $f(x)$.

(c)



A1A1A1A1

A1 for shape for $x < 0$
A1 for shape for $x > 0$
A1 for maximum at A
A1 for POI at B.

Note: Only award last two ***A1***s if A and B are placed in the correct quadrants, allowing for follow through.

[4 marks]

Total [17 marks]

10. (a) recognising normal to plane or attempting to find cross product of two vectors lying in the plane ***(M1)***

for example, $\overrightarrow{AB} \times \overrightarrow{AD} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ***(A1)***

$\Pi_1 : x + z = 1$ ***A1***

[3 marks]

(b) **EITHER**

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 1 = \sqrt{2}\sqrt{2}\cos\theta$ ***M1A1***

OR

$\left| \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right| = \sqrt{3} = \sqrt{2}\sqrt{2}\sin\theta$ ***M1A1***

Note: ***M1*** is for an attempt to find the scalar or vector product of the two normal vectors.

$\Rightarrow \theta = 60^\circ \left(= \frac{\pi}{3} \right)$ ***A1***

angle between faces is $120^\circ \left(= \frac{2\pi}{3} \right)$ ***A1***

[4 marks]

(c) $\overrightarrow{DB} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ or $\overrightarrow{BD} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ ***(A1)***

$\Pi_3 : x + y - z = k$ ***(M1)***

$\Pi_3 : x + y - z = 0$ ***A1***

[3 marks]

continued

Question 10 continued

(d) **METHOD 1**

line $\text{AD}:(\mathbf{r}=)\begin{pmatrix}0\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\-1\end{pmatrix}$ ***M1A1***

intersects Π_3 when $\lambda-(1-\lambda)=0$ ***M1***

so $\lambda=\frac{1}{2}$ ***A1***

hence P is the midpoint of AD ***AG***

METHOD 2

midpoint of AD is $(0.5, 0, 0.5)$ ***(M1)A1***
substitute into $x+y-z=0$ ***M1***
$0.5+0-0.5=0$ ***A1***
hence P is the midpoint of AD ***AG***

[4 marks]

(e) **METHOD 1**

$\text{OP}=\frac{1}{\sqrt{2}}$, $\text{O}\hat{\text{P}}\text{Q}=90^\circ$, $\text{O}\hat{\text{Q}}\text{P}=60^\circ$ ***A1A1A1***

$\text{PQ}=\frac{1}{\sqrt{6}}$ ***A1***

area $=\frac{1}{2\sqrt{12}}=\frac{1}{4\sqrt{3}}=\frac{\sqrt{3}}{12}$ ***A1***

continued

Question 10 continued

METHOD 2

line $\mathrm{BD}:(\mathbf{r}=)\begin{pmatrix}1\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}-1\\-1\\1\end{pmatrix}$

$\Rightarrow \lambda=\frac{2}{3}$ ***(A1)***

$\vec{\mathrm{OQ}}=\begin{pmatrix}\frac{1}{3}\\\frac{1}{3}\\\frac{2}{3}\end{pmatrix}$ ***A1***

area $=\frac{1}{2}\left|\vec{\mathrm{OP}}\times\vec{\mathrm{OQ}}\right|$ ***M1***

$\vec{\mathrm{OP}}=\begin{pmatrix}\frac{1}{2}\\0\\\frac{1}{2}\end{pmatrix}$ ***A1***

Note: This ***A1*** is dependent on ***M1***.

area $=\frac{\sqrt{3}}{12}$ ***A1***

[5 marks]

Total [19 marks]

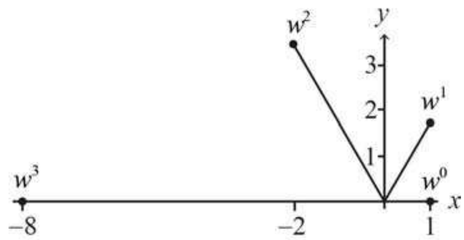
11. (a) (i) $w^2 = 4cis\left(\frac{2\pi}{3}\right)$; $w^3 = 8\text{cis}(\pi)$ ***(M1)A1A1***

Note: Accept Euler form.

Note: ***M1*** can be awarded for either both correct moduli or both correct arguments.

Note: Allow multiplication of correct Cartesian form for ***M1***, final answers must be in modulus-argument form.

(ii)



A1A1

[5 marks]

(b) use of area $= \frac{1}{2}ab\sin C$ ***M1***

$\frac{1}{2}\times 1\times 2\times\sin\frac{\pi}{3}+\frac{1}{2}\times 2\times 4\times\sin\frac{\pi}{3}+\frac{1}{2}\times 4\times 8\times\sin\frac{\pi}{3}$ ***A1A1***

Note: Award ***A1*** for $C = \frac{\pi}{3}$, ***A1*** for correct moduli.

$= \frac{21\sqrt{3}}{2}$ ***AG***

Note: Other methods of splitting the area may receive full marks.

[3 marks]

(c) $\frac{1}{2}\times 2^0\times 2^1\times\sin\frac{\pi}{n}+\frac{1}{2}\times 2^1\times 2^2\times\sin\frac{\pi}{n}+\frac{1}{2}\times 2^2\times 2^3\times\sin\frac{\pi}{n}+\ldots+\frac{1}{2}\times 2^{n-1}\times 2^n\times\sin\frac{\pi}{n}$

M1A1

Note: Award ***M1*** for powers of 2, ***A1*** for any correct expression including both the first and last term.

$= \sin\frac{\pi}{n}\times\left(2^0+2^2+2^4+\ldots+2^{2n-2}\right)$

identifying a geometric series with common ratio $2^2 (=4)$ ***(M1)A1***

$= \frac{1-2^{2n}}{1-4}\times\sin\frac{\pi}{n}$ ***M1***

Note: Award ***M1*** for use of formula for sum of geometric series.

$= \frac{1}{3}\left(4^n-1\right)\sin\frac{\pi}{n}$ ***A1***

[6 marks]

Total [14 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

11 pages

2218 – 7205

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

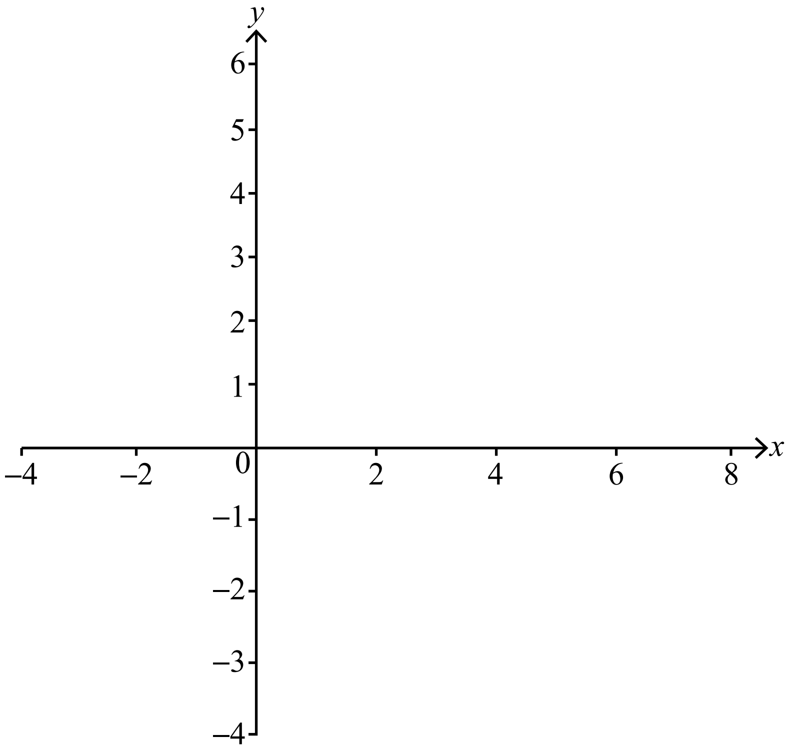
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The acute angle between the vectors $3\boldsymbol{i} - 4\boldsymbol{j} - 5\boldsymbol{k}$ and $5\boldsymbol{i} - 4\boldsymbol{j} + 3\boldsymbol{k}$ is denoted by θ. Find $\cos\theta$.

2. [Maximum mark: 7]

(a) Sketch the graphs of $y = \frac{x}{2} + 1$ and $y = |x - 2|$ on the following axes. [3]



(b) Solve the equation $\frac{x}{2} + 1 = |x - 2|$. [4]

3. [Maximum mark: 6]

The discrete random variable X has the following probability distribution, where p is a constant.

x	0	1	2	3	4
$\mathrm{P}(X=x)$	p	$0.5-p$	0.25	0.125	p^3

(a) Find the value of p. [2]

(b) (i) Find μ, the expected value of X.

(ii) Find $\mathrm{P}(X>\mu)$. [4]

4. [Maximum mark: 6]

Consider the curve $y = \dfrac{1}{1-x} + \dfrac{4}{x-4}$.

Find the x-coordinates of the points on the curve where the gradient is zero.

5. [Maximum mark: 7]

The geometric sequence $u_1, u_2, u_3, \ldots$ has common ratio r.
Consider the sequence $A = \{a_n = \log_2 |u_n| : n \in \mathbb{Z}^+\}$.

(a) Show that A is an arithmetic sequence, stating its common difference d in terms of r. [4]

A particular geometric sequence has $u_1 = 3$ and a sum to infinity of 4.

(b) Find the value of d. [3]

6. [Maximum mark: 7]

Consider the functions f, g, defined for $x \in \mathbb{R}$, given by $f(x) = \mathrm{e}^{-x}\sin x$ and $g(x) = \mathrm{e}^{-x}\cos x$.

(a) Find

(i) $f'(x)$;

(ii) $g'(x)$. [3]

(b) Hence, or otherwise, find $\int_0^{\pi} \mathrm{e}^{-x}\sin x\,\mathrm{d}x$. [4]

7. [Maximum mark: 6]

Consider the distinct complex numbers $z = a + \mathrm{i}b$, $w = c + \mathrm{i}d$, where $a, b, c, d \in \mathbb{R}$.

(a) Find the real part of $\dfrac{z + w}{z - w}$. [4]

(b) Find the value of the real part of $\dfrac{z + w}{z - w}$ when $|z| = |w|$. [2]

8. [Maximum mark: 7]

(a) Use the substitution $u = x^{\frac{1}{2}}$ to find $\int \frac{\mathrm{d}x}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$. [4]

(b) Hence find the value of $\frac{1}{2}\int_1^9 \frac{\mathrm{d}x}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$, expressing your answer in the form $\arctan q$, where $q \in \mathbb{Q}$. [3]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 24]

The points A, B, C and D have position vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{d}$, relative to the origin O.
It is given that $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{DC}}$.

(a) (i) Explain why ABCD is a parallelogram.

(ii) Using vector algebra, show that $\overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{BC}}$. [4]

The position vectors $\overrightarrow{\mathrm{OA}}$, $\overrightarrow{\mathrm{OB}}$, $\overrightarrow{\mathrm{OC}}$ and $\overrightarrow{\mathrm{OD}}$ are given by

$$\begin{aligned}\boldsymbol{a} &= \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}\\ \boldsymbol{b} &= 3\boldsymbol{i} - \boldsymbol{j} + p\boldsymbol{k}\\ \boldsymbol{c} &= q\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}\\ \boldsymbol{d} &= -\boldsymbol{i} + r\boldsymbol{j} - 2\boldsymbol{k}\end{aligned}$$

where p, q and r are constants.

(b) Show that $p = 1$, $q = 1$ and $r = 4$. [5]

(c) Find the area of the parallelogram ABCD. [4]

The point where the diagonals of ABCD intersect is denoted by M.

(d) Find the vector equation of the straight line passing through M and normal to the plane Π containing ABCD. [4]

(e) Find the Cartesian equation of Π. [3]

The plane Π cuts the x, y and z axes at X, Y and Z respectively.

(f) (i) Find the coordinates of X, Y and Z.

(ii) Find YZ. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

The function f is defined by $f(x) = \dfrac{ax + b}{cx + d}$, for $x \in \mathbb{R}, x \neq -\dfrac{d}{c}$.

(a) Find the inverse function f^{-1}, stating its domain. [5]

The function g is defined by $g(x) = \dfrac{2x - 3}{x - 2}$, $x \in \mathbb{R}, x \neq 2$.

(b) (i) Express $g(x)$ in the form $A + \dfrac{B}{x - 2}$ where A, B are constants.

(ii) Sketch the graph of $y = g(x)$. State the equations of any asymptotes and the coordinates of any intercepts with the axes. [5]

The function h is defined by $h(x) = \sqrt{x}$, for $x \geq 0$.

(c) State the domain and range of $h \circ g$. [4]

11. [Maximum mark: 12]

(a) Show that $\log_{r^2} x = \dfrac{1}{2}\log_r x$ where $r, x \in \mathbb{R}^+$. [2]

It is given that $\log_2 y + \log_4 x + \log_4 2x = 0$.

(b) Express y in terms of x. Give your answer in the form $y = px^q$, where p, q are constants. [5]

The region R, is bounded by the graph of the function found in part (b), the x-axis, and the lines $x = 1$ and $x = \alpha$ where $\alpha > 1$. The area of R is $\sqrt{2}$.

(c) Find the value of α. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Markscheme

May 2018

Mathematics

Higher level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document "**Mathematics HL: Guidance for e-marking May 2018**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

Award ***N*** *marks for* ***correct*** *answers where there is* ***no*** *working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets eg (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. $\cos\theta = \dfrac{(3\boldsymbol{i}-4\boldsymbol{j}-5\boldsymbol{k})\bullet(5\boldsymbol{i}-4\boldsymbol{j}+3\boldsymbol{k})}{|3\boldsymbol{i}-4\boldsymbol{j}-5\boldsymbol{k}||5\boldsymbol{i}-4\boldsymbol{j}+3\boldsymbol{k}|}$ ***(M1)***

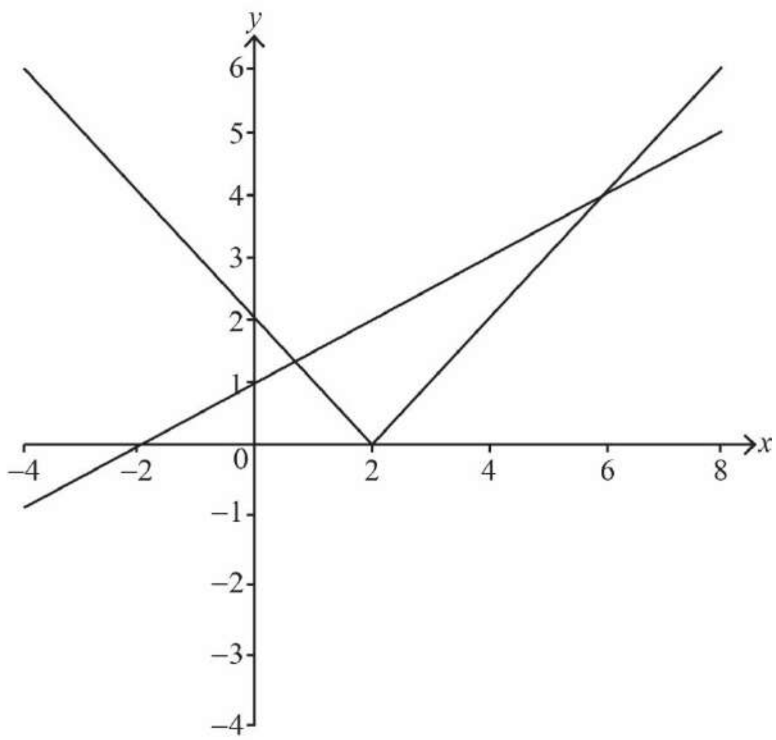
$= \dfrac{16}{\sqrt{50}\sqrt{50}}$ ***A1A1***

Note: ***A1*** for correct numerator and ***A1*** for correct denominator.

$= \dfrac{8}{25}\left(= \dfrac{16}{50} = 0.32\right)$ ***A1***

[4 marks]

2. (a)



straight line graph with correct axis intercepts ***A1***
modulus graph: V shape in upper half plane ***A1***
modulus graph having correct vertex and y-intercept ***A1***

[3 marks]

continued…

Question 2 continued

(b) **METHOD 1**

attempt to solve $\frac{x}{2} + 1 = x - 2$ ***(M1)***

$x = 6$ ***A1***

Note: Accept $x = 6$ using the graph.

attempt to solve (algebraically) $\frac{x}{2} + 1 = 2 - x$ ***M1***

$x = \frac{2}{3}$ ***A1***

[4 marks]

METHOD 2

$\left(\frac{x}{2} + 1\right)^2 = (x - 2)^2$ ***M1***

$\frac{x^2}{4} + x + 1 = x^2 - 4x + 4$

$0 = \frac{3x^2}{4} - 5x + 3$

$3x^2 - 20x + 12 = 0$

attempt to factorise (or equivalent) ***M1***

$(3x - 2)(x - 6) = 0$

$x = \frac{2}{3}$ ***A1***

$x = 6$ ***A1***

[4 marks]

Total [7 marks]

3. (a) equating sum of probabilities to 1 ($p + 0.5 - p + 0.25 + 0.125 + p^3 = 1$) ***M1***

$p^3 = 0.125 = \frac{1}{8}$

$p = 0.5$ ***A1***

[2 marks]

(b) (i) $\mu = 0 \times 0.5 + 1 \times 0 + 2 \times 0.25 + 3 \times 0.125 + 4 \times 0.125$ ***M1***

$= 1.375 \left(= \frac{11}{8}\right)$ ***A1***

continued…

Question 3 continued

(ii) $\mathrm{P}(X > \mu) = \mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \mathrm{P}(X = 4)$ ***(M1)***

$= 0.5$ ***A1***

Note: Do not award follow through ***A*** marks in (b)(i) from an incorrect value of p.

Note: Award ***M*** marks in both (b)(i) and (b)(ii) provided no negative probabilities, and provided a numerical value for μ has been found.

[4 marks]

Total [6 marks]

4. valid attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{(1-x)^2} - \frac{4}{(x-4)^2}$ ***A1A1***

attempt to solve $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ ***M1***

$x = 2,\ x = -2$ ***A1A1***

[6 marks]

5. (a) **METHOD 1**

state that $u_n = u_1 r^{n-1}$ (or equivalent) ***A1***

attempt to consider a_n and use of at least one log rule ***M1***

$\log_2 |u_n| = \log_2 |u_1| + (n-1)\log_2 |r|$ ***A1***

(which is an AP) with $d = \log_2 |r|$ (and 1st term $\log_2 |u_1|$) ***A1***

so A is an arithmetic sequence ***AG***

Note: Condone absence of modulus signs.

Note: The final ***A*** mark may be awarded independently.

Note: Consideration of the first two or three terms only will score ***M0***.

[4 marks]

continued…

Question 5 continued

METHOD 2

consideration of $(d =)\, a_{n+1} - a_n$ ***M1***

$(d) = \log_2 |u_{n+1}| - \log_2 |u_n|$

$(d) = \log_2 \left|\frac{u_{n+1}}{u_n}\right|$ ***M1***

$(d) = \log_2 |r|$ ***A1***

which is constant ***R1***

Note: Condone absence of modulus signs.

Note: the final ***A*** mark may be awarded independently.

Note: Consideration of the first two or three terms only will score ***M0***.

(b) attempting to solve $\frac{3}{1-r} = 4$ ***M1***

$r = \frac{1}{4}$ ***A1***

$d = -2$ ***A1***

[3 marks]

Total [7 marks]

6. (a) (i) attempt at product rule ***M1***

$f'(x) = -e^{-x}\sin x + e^{-x}\cos x$ ***A1***

(ii) $g'(x) = -e^{-x}\cos x - e^{-x}\sin x$ ***A1***

[3 marks]

(b) **METHOD 1**

Attempt to add $f'(x)$ and $g'(x)$ ***(M1)***

$f'(x) + g'(x) = -2e^{-x}\sin x$ ***A1***

$\int_0^{\pi} e^{-x}\sin x\,dx = \left[-\frac{e^{-x}}{2}(\sin x + \cos x)\right]_0^{\pi}$ (or equivalent) ***A1***

Note: Condone absence of limits.

$= \frac{1}{2}(1 + e^{-\pi})$ ***A1***

continued…

Question 6 continued

METHOD 2

$I = \int e^{-x} \sin x dx$

$= -e^{-x} \cos x - \int e^{-x} \cos x dx$ **OR** $= -e^{-x} \sin x + \int e^{-x} \cos x dx$ ***M1A1***

$= -e^{-x} \sin x - e^{-x} \cos x - \int e^{-x} \sin x dx$

$I = -\frac{1}{2} e^{-x} (\sin x + \cos x)$ ***A1***

$\int_0^{\pi} e^{-x} \sin x dx = \frac{1}{2}\left(1 + e^{-\pi}\right)$ ***A1***

[4 marks]

Total [7 marks]

7. (a) $\frac{z + w}{z - w} = \frac{(a + c) + i(b + d)}{(a - c) + i(b - d)}$

$= \frac{(a + c) + i(b + d)}{(a - c) + i(b - d)} \times \frac{(a - c) - i(b - d)}{(a - c) - i(b - d)}$ ***M1A1***

real part $= \frac{(a+c)(a-c) + (b+d)(b-d)}{(a - c)^2 + (b - d)^2} \left(= \frac{a^2 - c^2 + b^2 - d^2}{(a - c)^2 + (b - d)^2} \right)$ ***A1A1***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

[4 marks]

(b) $|z| = |w| \Rightarrow a^2 + b^2 = c^2 + d^2$ ***R1***

hence real part $= 0$ ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [6 marks]

8. (a) $\frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$ (accept $du = \frac{1}{2} x^{-\frac{1}{2}} dx$ or equivalent) ***A1***

substitution, leading to an integrand in terms of u ***M1***

$\int \frac{2u du}{u^3 + u}$ or equivalent ***A1***

$= 2 \arctan(\sqrt{x})(+c)$ ***A1***

[4 marks]

continued…

Question 8 continued

(b) $\frac{1}{2}\int_1^9 \frac{\mathrm{d}x}{x^{\frac{3}{2}} + x^{\frac{1}{2}}} = \arctan 3 - \arctan 1$ ***A1***

$\tan(\arctan 3 - \arctan 1) = \frac{3-1}{1+3\times 1}$ ***(M1)***

$\tan(\arctan 3 - \arctan 1) = \frac{1}{2}$

$\arctan 3 - \arctan 1 = \arctan \frac{1}{2}$ ***A1***

[3 marks]

Total [7 marks]

Section B

9. (a) (i) a pair of opposite sides have equal length and are parallel ***R1***
hence ABCD is a parallelogram ***AG***

(ii) attempt to rewrite the given information in vector form ***M1***
$\boldsymbol{b}-\boldsymbol{a}=\boldsymbol{c}-\boldsymbol{d}$ ***A1***
rearranging $\boldsymbol{d}-\boldsymbol{a}=\boldsymbol{c}-\boldsymbol{b}$ ***M1***
hence $\overrightarrow{\mathrm{AD}}=\overrightarrow{\mathrm{BC}}$ ***AG***

Note: Candidates may correctly answer part i) by answering part ii) correctly and then deducing there are two pairs of parallel sides.

[4 marks]

(b) **EITHER**

use of $\overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{DC}}$ ***(M1)***

$$\begin{pmatrix} 2 \\ -3 \\ p+3 \end{pmatrix}=\begin{pmatrix} q+1 \\ 1-r \\ 4 \end{pmatrix}$$ ***A1A1***

OR

use of $\overrightarrow{\mathrm{AD}}=\overrightarrow{\mathrm{BC}}$ ***(M1)***

$$\begin{pmatrix} -2 \\ r-2 \\ 1 \end{pmatrix}=\begin{pmatrix} q-3 \\ 2 \\ 2-p \end{pmatrix}$$ ***A1A1***

THEN

attempt to compare coefficients of $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ in their equation or statement to that effect ***M1***
clear demonstration that the given values satisfy their equation ***A1***
$p=1,\ q=1,\ r=4$ ***AG***

[5 marks]

(c) attempt at computing $\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AD}}$ (or equivalent) ***M1***

$$\begin{pmatrix} -11 \\ -10 \\ -2 \end{pmatrix}$$ ***A1***

area $=\left|\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AD}}\right|\left(=\sqrt{225}\right)$ ***(M1)***

$=15$ ***A1***

[4 marks]

continued…

Question 9 continued

(d) valid attempt to find $\overrightarrow{\mathrm{OM}}\left(=\frac{1}{2}(\boldsymbol{a}+\boldsymbol{c})\right)$ ***(M1)***

$\begin{pmatrix} 1 \\ \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix}$ ***A1***

the equation is

$\boldsymbol{r}=\begin{pmatrix} 1 \\ \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix}+t\begin{pmatrix} 11 \\ 10 \\ 2 \end{pmatrix}$ or equivalent ***M1A1***

Note: Award maximum ***M1A0*** if '$\mathbf{r}=\ldots$' (or equivalent) is not seen.

[4 marks]

(e) attempt to obtain the equation of the plane in the form $ax+by+cz=d$ ***M1***

$11x+10y+2z=25$ ***A1A1***

Note: ***A1*** for right hand side, ***A1*** for left hand side.

[3 marks]

(f) (i) putting two coordinates equal to zero ***(M1)***

$\mathrm{X}\left(\frac{25}{11},0,0\right),\ \mathrm{Y}\left(0,\frac{5}{2},0\right),\ \mathrm{Z}\left(0,0,\frac{25}{2}\right)$ ***A1***

(ii) $\mathrm{YZ}=\sqrt{\left(\frac{5}{2}\right)^2+\left(\frac{25}{2}\right)^2}$ ***M1***

$=\sqrt{\frac{325}{2}}\left(=\frac{5\sqrt{104}}{4}=\frac{5\sqrt{26}}{2}\right)$ ***A1***

[4 marks]

Total [24 marks]

10. (a) attempt to make x the subject of $y = \frac{ax+b}{cx+d}$ ***M1***

$y(cx+d) = ax + b$ ***A1***

$x = \frac{dy-b}{a-cy}$ ***A1***

$f^{-1}(x) = \frac{dx-b}{a-cx}$, ***A1***

Note: Do not allow $y =$ in place of $f^{-1}(x)$.

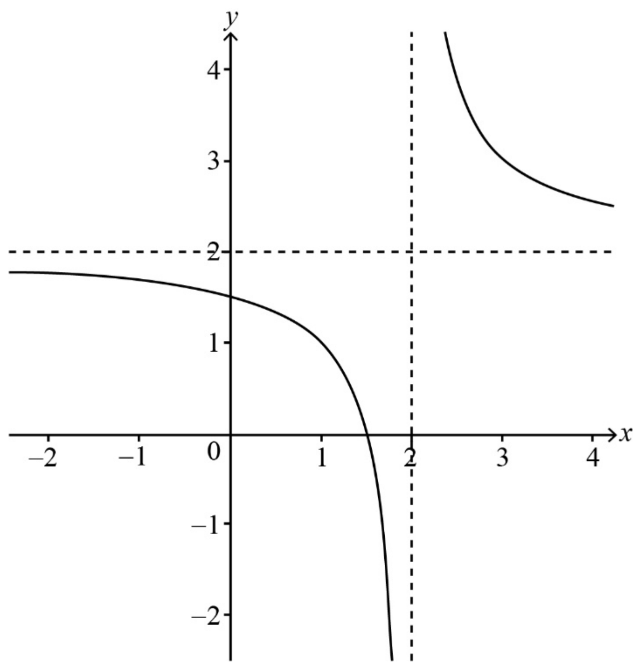
$x \neq \frac{a}{c}$, $(x \in \mathbb{R})$ ***A1***

Note: The final ***A*** mark is independent.

[5 marks]

(b) (i) $g(x) = 2 + \frac{1}{x-2}$ ***A1A1***

(ii)



hyperbola shape, with single curves in second and fourth quadrants and third quadrant blank, including vertical asymptote $x = 2$ ***A1***

horizontal asymptote $y = 2$ ***A1***

intercepts $\left(\frac{3}{2}, 0\right), \left(0, \frac{3}{2}\right)$ ***A1***

[5 marks]

continued…

Question 10 continued

(c) the domain of $h \circ g$ is $x \le \frac{3}{2},\ x > 2$ ***A1A1***

the range of $h \circ g$ is $y \ge 0,\ y \ne \sqrt{2}$ ***A1A1***

[4 marks]

Total [14 marks]

11. (a) **METHOD 1**

$\log_{r^2} x = \frac{\log_r x}{\log_r r^2} \left(= \frac{\log_r x}{2\log_r r}\right)$ ***M1A1***

$= \frac{\log_r x}{2}$ ***AG***

[2 marks]

METHOD 2

$\log_{r^2} x = \frac{1}{\log_x r^2}$ ***M1***

$= \frac{1}{2\log_x r}$ ***A1***

$= \frac{\log_r x}{2}$ ***AG***

[2 marks]

(b) **METHOD 1**

$\log_2 y + \log_4 x + \log_4 2x = 0$

$\log_2 y + \log_4 2x^2 = 0$ ***M1***

$\log_2 y + \frac{1}{2}\log_2 2x^2 = 0$ ***M1***

$\log_2 y = -\frac{1}{2}\log_2 2x^2$

$\log_2 y = \log_2\left(\frac{1}{\sqrt{2}x}\right)$ ***M1A1***

$y = \frac{1}{\sqrt{2}} x^{-1}$ ***A1***

Note: For the final ***A*** mark, y must be expressed in the form px^q.

[5 marks]

continued…

Question 11 continued

METHOD 2

$\log_2 y + \log_4 x + \log_4 2x = 0$

$\log_2 y + \frac{1}{2}\log_2 x + \frac{1}{2}\log_2 2x = 0$ ***M1***

$\log_2 y + \log_2 x^{\frac{1}{2}} + \log_2 (2x)^{\frac{1}{2}} = 0$ ***M1***

$\log_2 \left(\sqrt{2}xy\right) = 0$ ***M1***

$\sqrt{2}xy = 1$ ***A1***

$y = \frac{1}{\sqrt{2}}x^{-1}$ ***A1***

Note: For the final ***A*** mark, y must be expressed in the form px^q.

[5 marks]

(c) the area of R is $\int_1^{\alpha} \frac{1}{\sqrt{2}}x^{-1}\mathrm{d}x$ ***M1***

$= \left[\frac{1}{\sqrt{2}}\ln x\right]_1^{\alpha}$ ***A1***

$= \frac{1}{\sqrt{2}}\ln \alpha$ ***A1***

$\frac{1}{\sqrt{2}}\ln \alpha = \sqrt{2}$ ***M1***

$\alpha = \mathrm{e}^2$ ***A1***

Note: Only follow through from part (b) if y is in the form $y = px^q$.

[5 marks]

Total [12 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel superior
Prueba 1

Miércoles 2 de mayo de 2018 (tarde)

Número de convocatoria del alumno

2 horas

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba no se permite el uso de ninguna calculadora.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[100 puntos]**.

2218 – 7225

11 páginas

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

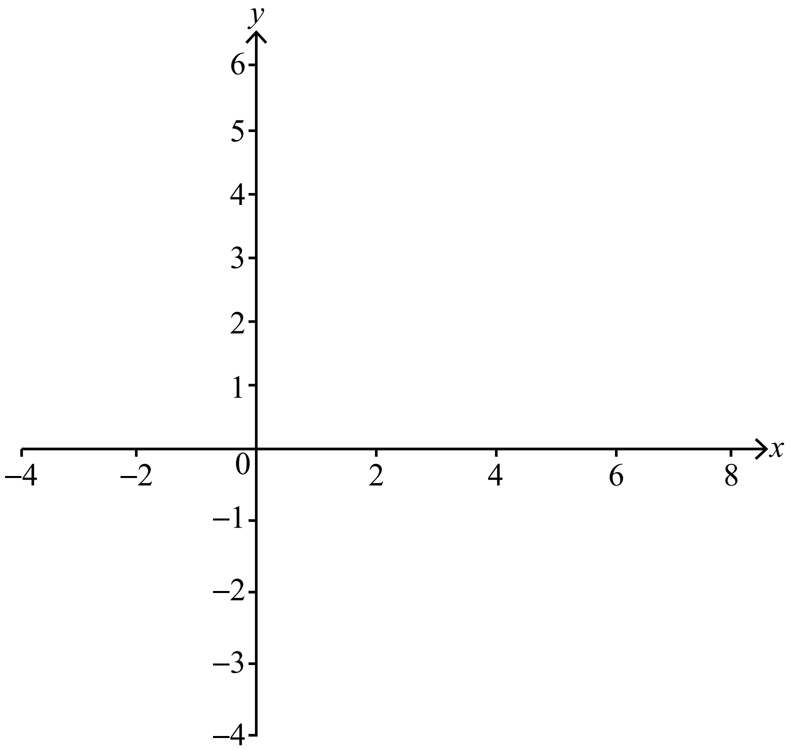
Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 4]

El ángulo agudo que forman los vectores $3\boldsymbol{i} - 4\boldsymbol{j} - 5\boldsymbol{k}$ y $5\boldsymbol{i} - 4\boldsymbol{j} + 3\boldsymbol{k}$ se denomina θ.
Halle $\cos\theta$.

2. [Puntuación máxima: 7]

(a) Dibuje aproximadamente los gráficos de $y = \frac{x}{2} + 1$ y de $y = |x - 2|$ en los siguientes ejes de coordenadas. [3]



(b) Resuelva la ecuación $\frac{x}{2} + 1 = |x - 2|$. [4]

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3. [Puntuación máxima: 6]

La variable aleatoria discreta X tiene la siguiente distribución de probabilidad, donde p es una constante.

x	0	1	2	3	4
$\mathrm{P}(X=x)$	p	$0{,}5-p$	$0{,}25$	$0{,}125$	p^3

(a) Halle el valor de p. [2]

(b) (i) Halle μ, el valor esperado de X.

(ii) Halle $\mathrm{P}(X>\mu)$. [4]

4. [Puntuación máxima: 6]

Considere la curva $y = \frac{1}{1-x} + \frac{4}{x-4}$.

Halle las coordenadas x de los puntos de la curva donde la pendiente es cero.

5. [Puntuación máxima: 7]

La progresión geométrica $u_1, u_2, u_3, \ldots$ tiene razón común r.
Considere la progresión $A = \{a_n = \log_2|u_n| : n \in \mathbb{Z}^+\}$.

(a) Muestre que A es una progresión aritmética, e indique cuál es la diferencia común, d, en función de r. [4]

En una progresión geométrica en particular, $u_1 = 3$ y la suma de los infinitos términos es igual a 4.

(b) Halle el valor de d. [3]

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6. [Puntuación máxima: 7]

Considere las funciones f, g, definidas para $x \in \mathbb{R}$, mediante $f(x) = \mathrm{e}^{-x} \operatorname{sen} x$ y $g(x) = \mathrm{e}^{-x} \cos x$.

(a) Halle

(i) $f'(x)$;

(ii) $g'(x)$. [3]

(b) A partir de lo anterior o de cualquier otro modo, halle $\int_0^{\pi} \mathrm{e}^{-x} \operatorname{sen} x \, \mathrm{d}x$. [4]

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7. [Puntuación máxima: 6]

Considere los números complejos distintos $z = a + \mathrm{i}b$, $w = c + \mathrm{i}d$, donde $a, b, c, d \in \mathbb{R}$.

(a) Halle la parte real de $\dfrac{z + w}{z - w}$. [4]

(b) Halle el valor de la parte real de $\dfrac{z + w}{z - w}$ cuando $|z| = |w|$. [2]

8. [Puntuación máxima: 7]

(a) Utilice la sustitución $u = x^{\frac{1}{2}}$ para hallar $\int \frac{\mathrm{d}x}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$. [4]

(b) A partir de lo anterior, halle el valor de $\frac{1}{2}\int_{1}^{9} \frac{\mathrm{d}x}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$, en la forma $\arctan q$, donde $q \in \mathbb{Q}$. [3]

No escriba soluciones en esta página.

Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

9. [Puntuación máxima: 24]

Sean $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ y $\boldsymbol{d}$ los vectores de posición con respecto al origen de coordenadas O de los puntos A, B, C y D, respectivamente.

Se sabe que $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{DC}}$.

(a) (i) Explique por qué ABCD es un paralelogramo.

(ii) Utilizando el álgebra de vectores, muestre que $\overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{BC}}$. [4]

Los vectores de posición $\overrightarrow{\mathrm{OA}}$, $\overrightarrow{\mathrm{OB}}$, $\overrightarrow{\mathrm{OC}}$ y $\overrightarrow{\mathrm{OD}}$ vienen dados por

$$\begin{aligned}\boldsymbol{a} &= \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}\\ \boldsymbol{b} &= 3\boldsymbol{i} - \boldsymbol{j} + p\boldsymbol{k}\\ \boldsymbol{c} &= q\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}\\ \boldsymbol{d} &= -\boldsymbol{i} + r\boldsymbol{j} - 2\boldsymbol{k}\end{aligned}$$

donde p, q y r son constantes.

(b) Muestre que $p = 1$, $q = 1$ y $r = 4$. [5]

(c) Halle el área del paralelogramo ABCD. [4]

El punto en el que se cortan las diagonales de ABCD se denomina M.

(d) Halle la ecuación vectorial de la recta que pasa por M y es normal al plano Π que contiene a ABCD. [4]

(e) Halle la ecuación cartesiana de Π. [3]

El plano Π corta a los ejes x, y y z en X, Y y Z, respectivamente.

(f) (i) Halle las coordenadas de X, Y y Z.

(ii) Halle YZ. [4]

No escriba soluciones en esta página.

10. [Puntuación máxima: 14]

La función f se define mediante $f(x) = \dfrac{ax+b}{cx+d}$, para $x \in \mathbb{R}$, $x \neq -\dfrac{d}{c}$.

(a) Halle la función inversa f^{-1} e indique su dominio. [5]

La función g se define mediante $g(x) = \dfrac{2x-3}{x-2}$, $x \in \mathbb{R}, x \neq 2$.

(b) (i) Exprese $g(x)$ en la forma $A + \dfrac{B}{x-2}$, donde A y B son constantes.

(ii) Dibuje aproximadamente el gráfico de $y = g(x)$. Indique la ecuación de cada una de las asíntotas y las coordenadas de todos los puntos de corte con los ejes. [5]

La función h se define mediante $h(x) = \sqrt{x}$, para $x \geq 0$.

(c) Indique el dominio y el recorrido de $h \circ g$. [4]

11. [Puntuación máxima: 12]

(a) Muestre que $\log_{r^2} x = \dfrac{1}{2}\log_r x$, donde $r, x \in \mathbb{R}^+$. [2]

Se sabe que $\log_2 y + \log_4 x + \log_4 2x = 0$.

(b) Exprese y en función de x. Dé la respuesta en la forma $y = px^q$, donde p, q son constantes. [5]

La región R está delimitada por el gráfico de la función hallada en el apartado (b), por el eje x y por las rectas $x = 1$ y $x = \alpha$, donde $\alpha > 1$. El área de R es igual a $\sqrt{2}$.

(c) Halle el valor de α. [5]

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathématiques
Niveau supérieur
Épreuve 1

Mercredi 2 mai 2018 (après-midi)

2 heures

Numéro de session du candidat

Instructions destinées aux candidats

- Écrivez votre numéro de session dans les cases ci-dessus.
- N'ouvrez pas cette épreuve avant d'y être autorisé(e).
- Aucune calculatrice n'est autorisée pour cette épreuve.
- Section A : répondez à toutes les questions. Rédigez vos réponses dans les cases prévues à cet effet.
- Section B : répondez à toutes les questions sur le livret de réponses prévu à cet effet. Écrivez votre numéro de session sur la première page du livret de réponses, et attachez ce livret à cette épreuve d'examen et à votre page de couverture en utilisant l'attache fournie.
- Sauf indication contraire dans l'intitulé de la question, toutes les réponses numériques devront être exactes ou correctes à trois chiffres significatifs près.
- Un exemplaire non annoté du **livret de formules pour les cours de mathématiques NS et de mathématiques complémentaires NS** est nécessaire pour cette épreuve.
- Le nombre maximum de points pour cette épreuve d'examen est de **[100 points]**.

2218 – 7219

Le total des points ne sera pas nécessairement attribué pour une réponse correcte si le raisonnement n'a pas été indiqué. Les réponses doivent être appuyées par un raisonnement et/ou des explications. Lorsque la réponse est fausse, certains points peuvent être attribués si la méthode utilisée est correcte, pour autant que le raisonnement soit indiqué par écrit. On vous recommande donc de montrer tout votre raisonnement.

Section A

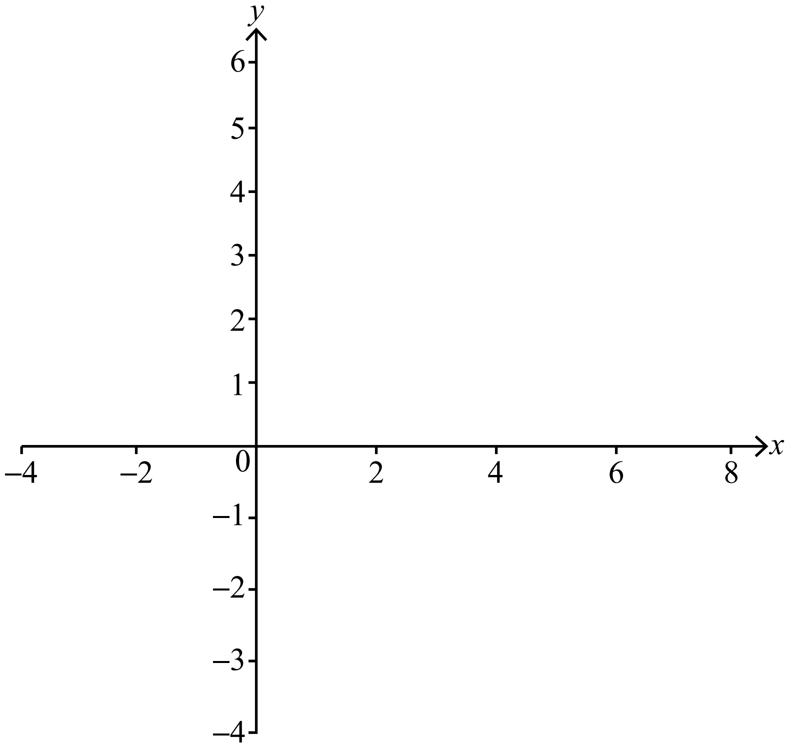
Répondez à **toutes** les questions. Rédigez vos réponses dans les cases prévues à cet effet. Si cela est nécessaire, vous pouvez poursuivre votre raisonnement en dessous des lignes.

1. [Note maximale : 4]

L'angle aigu entre les vecteurs $3\boldsymbol{i} - 4\boldsymbol{j} - 5\boldsymbol{k}$ et $5\boldsymbol{i} - 4\boldsymbol{j} + 3\boldsymbol{k}$ est désigné par θ.
Trouvez $\cos\theta$.

2. [Note maximale : 7]

(a) Esquissez les représentations graphiques de $y = \frac{x}{2} + 1$ et $y = |x - 2|$ sur le système d'axes suivant. [3]



(b) Résolvez l'équation $\frac{x}{2} + 1 = |x - 2|$. [4]

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3. [Note maximale : 6]

La variable aléatoire discrète X possède la distribution de probabilité suivante, où p est une constante.

x	0	1	2	3	4
$\mathrm{P}(X=x)$	p	$0{,}5-p$	$0{,}25$	$0{,}125$	p^3

(a) Trouvez la valeur de p. [2]

(b) (i) Trouvez μ, l'espérance mathématique de X.

(ii) Trouvez $\mathrm{P}(X>\mu)$. [4]

4. [Note maximale : 6]

Considérez la courbe $y = \dfrac{1}{1-x} + \dfrac{4}{x-4}$.

Trouvez les abscisses des points sur la courbe où la pente est égale à zéro.

5. [Note maximale : 7]

Soit $u_1, u_2, u_3, \ldots$ la suite géométrique de raison r.
Considérez la suite $A = \{a_n = \log_2 |u_n| : n \in \mathbb{Z}^+\}$.

(a) Montrez que A est une suite arithmétique, en indiquant sa raison d en fonction de r. [4]

Une suite géométrique particulière est définie par $u_1 = 3$ et $S_\infty = 4$.

(b) Trouvez la valeur de d. [3]

6. [Note maximale : 7]

Considérez les fonctions f et g, définies pour $x \in \mathbb{R}$ et données par $f(x) = e^{-x} \sin x$ et $g(x) = e^{-x} \cos x$.

(a) Trouvez

(i) $f'(x)$;

(ii) $g'(x)$. [3]

(b) À partir de là ou par toute autre méthode, trouvez $\int_0^{\pi} e^{-x} \sin x \, dx$. [4]

7. [Note maximale : 6]

Considérez les nombres complexes distincts $z = a + \mathrm{i}b$, $w = c + \mathrm{i}d$, où $a, b, c, d \in \mathbb{R}$.

(a) Trouvez la partie réelle de $\dfrac{z + w}{z - w}$. [4]

(b) Trouvez la valeur de la partie réelle de $\dfrac{z + w}{z - w}$ lorsque $|z| = |w|$. [2]

8. [Note maximale : 7]

(a) Utilisez le changement de variable $u = x^{\frac{1}{2}}$ pour trouver $\int \frac{\mathrm{d}x}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$. [4]

(b) À partir de là, trouvez la valeur de $\frac{1}{2}\int_1^9 \frac{\mathrm{d}x}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$, en exprimant votre réponse sous la forme $\arctan q$, où $q \in \mathbb{Q}$. [3]

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N’écrivez **pas** vos solutions sur cette page.

Section B

Répondez à **toutes** les questions sur le livret de réponses fourni. Veuillez répondre à chaque question sur une nouvelle page.

9. [Note maximale : 24]

Les points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ et D ont comme vecteurs-position $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ et $\boldsymbol{d}$, par rapport à l’origine O.

On donne $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{DC}}$.

(a) (i) Expliquez pourquoi ABCD est un parallélogramme.

(ii) En utilisant l’algèbre vectorielle, montrez que $\overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{BC}}$. [4]

Les vecteurs-position $\overrightarrow{\mathrm{OA}}$, $\overrightarrow{\mathrm{OB}}$, $\overrightarrow{\mathrm{OC}}$ et $\overrightarrow{\mathrm{OD}}$ sont donnés par

$$\begin{aligned}\boldsymbol{a} &= \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}\\ \boldsymbol{b} &= 3\boldsymbol{i} - \boldsymbol{j} + p\boldsymbol{k}\\ \boldsymbol{c} &= q\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}\\ \boldsymbol{d} &= -\boldsymbol{i} + r\boldsymbol{j} - 2\boldsymbol{k}\end{aligned}$$

où p, q et r sont des constantes.

(b) Montrez que $p = 1$, $q = 1$ et $r = 4$. [5]

(c) Trouvez l’aire du parallélogramme ABCD. [4]

Le point d’intersection des diagonales de ABCD est désigné par M.

(d) Trouvez l’équation vectorielle de la droite passant par M et normale au plan Π contenant ABCD. [4]

(e) Trouvez l’équation cartésienne de Π. [3]

Le plan Π coupe l’axe des abscisses Ox, l’axe des ordonnées Oy et l’axe des cotes Oz respectivement en X, Y et Z.

(f) (i) Trouvez les coordonnées de X, Y et Z.

(ii) Trouvez YZ. [4]

N'écrivez **pas** vos solutions sur cette page.

10. [Note maximale : 14]

La fonction f est définie par $f(x) = \dfrac{ax + b}{cx + d}$, pour $x \in \mathbb{R}, x \neq -\dfrac{d}{c}$.

(a) Trouvez la fonction réciproque f^{-1}, en indiquant son domaine. [5]

La fonction g est définie par $g(x) = \dfrac{2x - 3}{x - 2}$, $x \in \mathbb{R}, x \neq 2$.

(b) (i) Exprimez $g(x)$ sous la forme $A + \dfrac{B}{x - 2}$, où A et B sont des constantes.

(ii) Esquissez la représentation graphique de $y = g(x)$. Indiquez les équations de toute asymptote et les coordonnées de tout point d'intersection avec les axes. [5]

La fonction h est définie par $h(x) = \sqrt{x}$, pour $x \geq 0$.

(c) Indiquez le domaine et l'image de $h \circ g$. [4]

11. [Note maximale : 12]

(a) Montrez que $\log_{r^2} x = \dfrac{1}{2}\log_r x$, où $r, x \in \mathbb{R}^+$. [2]

On donne $\log_2 y + \log_4 x + \log_4 2x = 0$.

(b) Exprimez y en fonction de x. Donnez votre réponse sous la forme $y = px^q$, où p et q sont des constantes. [5]

La région R est délimitée par la représentation graphique de la fonction trouvée dans la partie (b), l'axe des abscisses Ox, et les droites $x = 1$ et $x = \alpha$, où $\alpha > 1$. L'aire de R est $\sqrt{2}$.

(c) Trouvez la valeur de α. [5]

Veuillez ne **pas** écrire sur cette page.

Les réponses rédigées sur cette page
ne seront pas corrigées.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2218 – 7204

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

The 3rd term of an arithmetic sequence is 1407 and the 10th term is 1183.

(a) Find the first term and the common difference of the sequence. [4]

(b) Calculate the number of positive terms in the sequence. [3]

2. [Maximum mark: 6]

The equation $x^2 - 5x - 7 = 0$ has roots α and β. The equation $x^2 + px + q = 0$ has roots $\alpha + 1$ and $\beta + 1$. Find the value of p and the value of q.

3. [Maximum mark: 5]

Let $f(x) = \tan(x + \pi)\cos\left(x - \frac{\pi}{2}\right)$ where $0 < x < \frac{\pi}{2}$.

Express $f(x)$ in terms of $\sin x$ and $\cos x$.

4. [Maximum mark: 5]

The age, L, in years, of a wolf can be modelled by the normal distribution $L \sim \mathrm{N}(8\,,\,5)$.

(a) Find the probability that a wolf selected at random is at least 5 years old. [2]

Eight wolves are independently selected at random and their ages recorded.

(b) Find the probability that more than six of these wolves are at least 5 years old. [3]

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5. [Maximum mark: 7]

(a) Given that $2x^3 - 3x + 1$ can be expressed in the form $Ax(x^2 + 1) + Bx + C$, find the values of the constants A, B and C. [2]

(b) Hence find $\int \frac{2x^3 - 3x + 1}{x^2 + 1}\,\mathrm{d}x$. [5]

6. [Maximum mark: 5]

The mean number of squirrels in a certain area is known to be 3.2 squirrels per hectare of woodland. Within this area, there is a 56 hectare woodland nature reserve. It is known that there are currently at least 168 squirrels in this reserve.

Assuming the population of squirrels follow a Poisson distribution, calculate the probability that there are more than 190 squirrels in the reserve.

7. [Maximum mark: 8]

It is known that the number of fish in a given lake will decrease by 7% each year unless some new fish are added. At the end of each year, 250 new fish are added to the lake. At the start of 2018, there are 2500 fish in the lake.

(a) Show that there will be approximately 2645 fish in the lake at the start of 2020. [3]

(b) Find the approximate number of fish in the lake at the start of 2042. [5]

8. [Maximum mark: 7]

Each of the 25 students in a class are asked how many pets they own. Two students own three pets and no students own more than three pets. The mean and standard deviation of the number of pets owned by students in the class are $\frac{18}{25}$ and $\frac{24}{25}$ respectively.

Find the number of students in the class who do not own a pet.

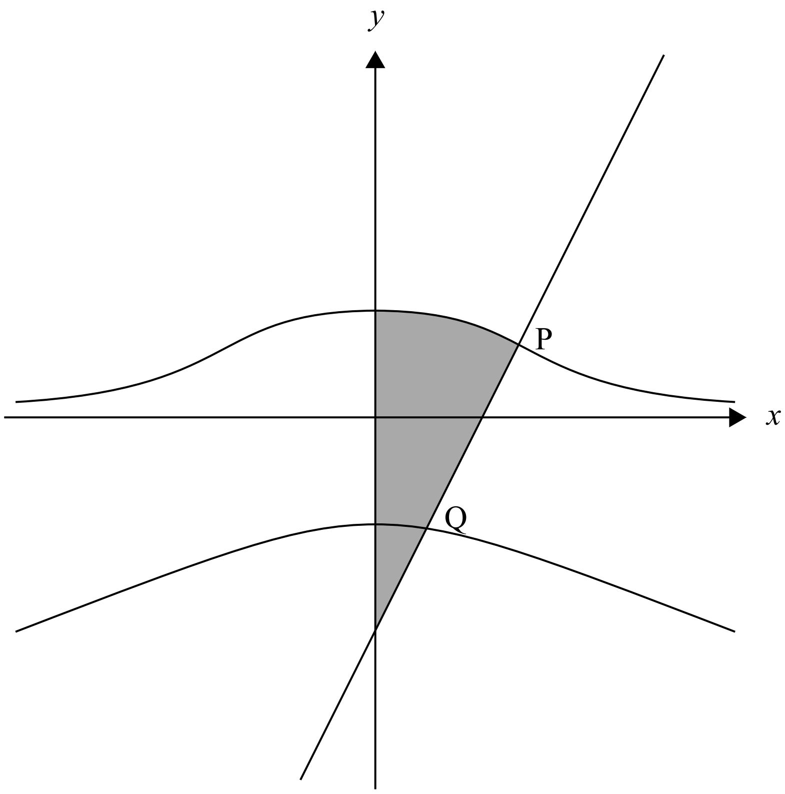
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 22]

The following graph shows the two parts of the curve defined by the equation $x^2y = 5 - y^4$, and the normal to the curve at the point $\mathrm{P}(2, 1)$.



(a) Show that there are exactly two points on the curve where the gradient is zero. [7]

(b) Find the equation of the normal to the curve at the point P. [5]

(c) The normal at P cuts the curve again at the point Q. Find the x-coordinate of Q. [3]

(d) The shaded region is rotated by 2π about the y-axis. Find the volume of the solid formed. [7]

Do **not** write solutions on this page.

10. [Maximum mark: 13]

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 3ax & , \quad 0 \le x < 0.5 \\ a(2-x) & , \quad 0.5 \le x < 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

(a) Show that $a = \frac{2}{3}$. [3]

(b) Find $\mathrm{P}(X < 1)$. [3]

(c) Given that $\mathrm{P}(s < X < 0.8) = 2 \times \mathrm{P}(2s < X < 0.8)$, and that $0.25 < s < 0.4$, find the value of s. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 15]

Two submarines A and B have their routes planned so that their positions at time t hours, $0 \le t < 20$, would be defined by the position vectors $\boldsymbol{r}_A = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 1 \\ -0.15 \end{pmatrix}$ and $\boldsymbol{r}_B = \begin{pmatrix} 0 \\ 3.2 \\ -2 \end{pmatrix} + t\begin{pmatrix} -0.5 \\ 1.2 \\ 0.1 \end{pmatrix}$ relative to a fixed point on the surface of the ocean (all lengths are in kilometres).

(a) Show that the two submarines would collide at a point P and write down the coordinates of P. [4]

To avoid the collision submarine B adjusts its velocity so that its position vector is now given by

$$\boldsymbol{r}_B = \begin{pmatrix} 0 \\ 3.2 \\ -2 \end{pmatrix} + t\begin{pmatrix} -0.45 \\ 1.08 \\ 0.09 \end{pmatrix}.$$

(b) (i) Show that submarine B travels in the same direction as originally planned.

(ii) Find the value of t when submarine B passes through P. [3]

(c) (i) Find an expression for the distance between the two submarines in terms of t.

(ii) Find the value of t when the two submarines are closest together.

(iii) Find the distance between the two submarines at this time. [8]

Markscheme

May 2018

Mathematics

Higher level

Paper 2

14 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document **"Mathematics HL: Guidance for e-marking May 2018"**. It is essential that you read this document before you start marking. In particular, please note the following.

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, for example, ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (for example, substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets**, for example, **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(= 10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT***.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. (a) $u_1 + 2d = 1407$, $u_1 + 9d = 1183$ ***(M1)(A1)***

$u_1 = 1471$, $d = -32$ ***A1A1***

[4 marks]

(b) $1471 + (n-1)(-32) > 0$ ***(M1)***

$\Rightarrow n < \dfrac{1471}{32} + 1$

$n < 46.96\ldots$ ***(A1)***

so 46 positive terms ***A1***

[3 marks]

Total [7 marks]

2. **METHOD 1**

$\alpha + \beta = 5$, $\alpha\beta = -7$ ***(M1)(A1)***

Note: Award ***M1A0*** if only one equation obtained.

$(\alpha + 1) + (\beta + 1) = 5 + 2 = 7$ ***A1***

$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$ ***(M1)***

$= -7 + 5 + 1 = -1$

$p = -7$, $q = -1$ ***A1A1***

METHOD 2

$\alpha = \dfrac{5 + \sqrt{53}}{2} = 6.1\ldots$; $\beta = \dfrac{5 - \sqrt{53}}{2} = -1.1\ldots$ ***(M1)(A1)***

$\alpha + 1 = \dfrac{7 + \sqrt{53}}{2} = 7.1\ldots$; $\beta + 1 = \dfrac{7 - \sqrt{53}}{2} = -0.1\ldots$ ***A1***

$(x - 7.14\ldots)(x + 0.14\ldots) = x^2 - 7x - 1$ ***(M1)***

$p = -7$, $q = -1$ ***A1A1***

Note: Exact answers only.

[6 marks]

3. $\tan(x + \pi) = \tan x \left(= \dfrac{\sin x}{\cos x} \right)$ ***(M1)A1***

$\cos\left(x - \dfrac{\pi}{2} \right) = \sin x$ ***(M1)A1***

Note: The two ***M1***'s can be awarded for observation or for expanding.

$\tan(x + \pi)\cos\left(x - \dfrac{\pi}{2} \right) = \dfrac{\sin^2 x}{\cos x}$ ***A1***

[5 marks]

4. (a) $\mathrm{P}(L \geq 5) = 0.910$ ***(M1)A1***

[2 marks]

(b) X is the number of wolves found to be at least 5 years old
recognising binomial distribution ***M1***
$X \sim \mathrm{B}(8,\ 0.910\ldots)$
$\mathrm{P}(X > 6) = 1 - \mathrm{P}(X \leq 6)$ ***(M1)***
$= 0.843$ ***A1***

Note: Award ***M1A0*** for finding $\mathrm{P}(X \geq 6)$.

[3 marks]

Total [5 marks]

5. (a) $2x^3 - 3x + 1 = Ax\left(x^2 + 1\right) + Bx + C$
$A = 2,\ C = 1,$ ***A1***
$A + B = -3 \Rightarrow B = -5$ ***A1***

[2 marks]

(b) $\int \frac{2x^3 - 3x + 1}{x^2 + 1}\,\mathrm{d}x = \int \left(2x - \frac{5x}{x^2+1} + \frac{1}{x^2+1}\right)\mathrm{d}x$ ***M1M1***

Note: Award ***M1*** for dividing by (x^2+1) to get $2x$, ***M1*** for separating the $5x$ and 1.

$= x^2 - \frac{5}{2}\ln\left(x^2+1\right) + \arctan x (+c)$ ***(M1)A1A1***

Note: Award ***(M1)A1*** for integrating $\frac{5x}{x^2+1}$, ***A1*** for the other two terms.

[5 marks]

Total [7 marks]

6. X is number of squirrels in reserve
$X \sim \mathrm{Po}(179.2)$ ***A1***

Note: Award ***A1*** if 179.2 or 56×3.2 seen or implicit in future calculations.

recognising conditional probability ***M1***
$\mathrm{P}\left(X > 190 \mid X \geq 168\right)$
$= \frac{\mathrm{P}(X > 190)}{\mathrm{P}(X \geq 168)}\left(= \frac{0.19827\ldots}{0.80817\cdots}\right)$ ***(A1)(A1)***
$= 0.245$ ***A1***

[5 marks]

7. (a) **EITHER**

2019: $2500 \times 0.93 + 250 = 2575$ ***(M1)A1***

2020: $2575 \times 0.93 + 250$ ***M1***

OR

2020: $2500 \times 0.93^2 + 250(0.93 + 1)$ ***M1M1A1***

Note: Award ***M1*** for starting with 2500, ***M1*** for multiplying by 0.93 and adding 250 twice. ***A1*** for correct expression. Can be shown in recursive form.

THEN

$(= 2644.75) = 2645$ ***AG***

[3 marks]

(b) 2020: $2500 \times 0.93^2 + 250(0.93 + 1)$

2042: $2500 \times 0.93^{24} + 250\left(0.93^{23} + 0.93^{22} + \ldots + 1\right)$ ***(M1)(A1)***

$= 2500 \times 0.93^{24} + 250\dfrac{\left(0.93^{24} - 1\right)}{(0.93 - 1)}$ ***(M1)(A1)***

$= 3384$ ***A1***

Note: If recursive formula used, award ***M1*** for $u_n = 0.93\, u_{n-1} + 250$ **and** u_0 or u_1 seen (can be awarded if seen in part (a)). Then award ***M1A1*** for attempt to find u_{24} or u_{25} respectively (different term if other than 2500 used) (***M1A0*** if incorrect term is being found) and ***A2*** for correct answer.

Note: Accept all answers that round to 3380.

[5 marks]

Total [8 marks]

8. **METHOD 1**

let p have no pets, q have one pet and r have two pets ***(M1)***

$p + q + r + 2 = 25$ ***(A1)***

$0p + 1q + 2r + 6 = 18$ ***A1***

Note: Accept a statement that there are a total of 12 pets.

attempt to use variance equation, or evidence of trial and error ***(M1)***

$\dfrac{0p + 1q + 4r + 18}{25} - \left(\dfrac{18}{25}\right)^2 = \left(\dfrac{24}{25}\right)^2$ ***(A1)***

attempt to solve a system of linear equations ***(M1)***

$p = 14$ ***A1***

continued…

Question 8 continued

METHOD 2

x	0	1	2	3
$\mathrm{P}(X = x)$	p	q	r	$\frac{2}{25}$

(M1)

$p+q+r+\frac{2}{25}=1$ ***(A1)***

$q+2r+\frac{6}{25}=\frac{18}{25}\left(\Rightarrow q+2r=\frac{12}{25}\right)$ ***A1***

$q+4r+\frac{18}{25}-\left(\frac{18}{25}\right)^2=\frac{576}{625}\left(\Rightarrow q+4r=\frac{18}{25}\right)$ ***(M1)(A1)***

$q=\frac{6}{25}$, $r=\frac{3}{25}$ ***(M1)***

$p=\frac{14}{25}$ ***A1***

so 14 have no pets

[7 marks]

Section B

9. (a) differentiating implicitly: ***M1***

$$2xy + x^2 \frac{\mathrm{d}y}{\mathrm{d}x} = -4y^3 \frac{\mathrm{d}y}{\mathrm{d}x}$$ ***A1A1***

Note: Award ***A1*** for each side.

if $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ then either $x = 0$ or $y = 0$ ***M1A1***

$x = 0 \Rightarrow$ two solutions for $y\left(y = \pm\sqrt[4]{5}\right)$ ***R1***

$y = 0$ not possible (as $0 \neq 5$) ***R1***

hence exactly two points ***AG***

Note: For a solution that only refers to the graph giving two solutions at $x = 0$ and no solutions for $y = 0$ award ***R1*** only.

[7 marks]

(b) at $(2, 1)$ $4 + 4\frac{\mathrm{d}y}{\mathrm{d}x} = -4\frac{\mathrm{d}y}{\mathrm{d}x}$ ***M1***

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}$$ ***(A1)***

gradient of normal is 2 ***M1***

$1 = 4 + c$ ***(M1)***

equation of normal is $y = 2x - 3$ ***A1***

[5 marks]

(c) substituting ***(M1)***

$x^2(2x - 3) = 5 - (2x - 3)^4$ or $\left(\frac{y+3}{2}\right)^2 y = 5 - y^4$ ***(A1)***

$x = 0.724$ ***A1***

[3 marks]

continued…

Question 9 continued

(d) recognition of two volumes ***(M1)***

volume 1 $= \pi\int_1^{\sqrt[4]{5}} \frac{5-y^4}{y}\,\mathrm{d}y (= 1.01\pi = 3.178\ldots)$ ***M1A1A1***

Note: Award ***M1*** for attempt to use $\pi\int x^2 \mathrm{d}y$, ***A1*** for limits, ***A1*** for $\frac{5-y^4}{y}$. Condone omission of π at this stage.

volume 2

EITHER

$= \frac{1}{3}\pi \times 2^2 \times 4 (= 16.75\ldots)$ ***(M1)(A1)***

OR

$= \pi\int_{-3}^{1}\left(\frac{y+3}{2}\right)^2 \mathrm{d}y \left(= \frac{16\pi}{3} = 16.75...\right)$ ***(M1)(A1)***

THEN

total volume $= 19.9$ ***A1***

[7 marks]

Total [22 marks]

10. (a) $a\left[\int_0^{0.5} 3x\,\mathrm{d}x + \int_{0.5}^{2} (2-x)\mathrm{d}x\right] = 1$ ***M1***

Note: Award the ***M1*** for the total integral equalling 1, or equivalent.

$a\left(\frac{3}{2}\right) = 1$ ***(M1)A1***

$a = \frac{2}{3}$ ***AG***

[3 marks]

continued…

Question 10 continued

(b) **EITHER**

$\int_0^{0.5} 2x \, dx + \frac{2}{3}\int_{0.5}^{1} (2-x) \, dx$ ***(M1)(A1)***

$= \frac{2}{3}$ ***A1***

OR

$\frac{2}{3}\int_1^2 (2-x) \, dx = \frac{1}{3}$ ***(M1)***

so $\mathrm{P}(X < 1) = \frac{2}{3}$ ***(M1)A1***

[3 marks]

(c) $\mathrm{P}(s < X < 0.8) = \int_s^{0.5} 2x \, dx + \frac{2}{3}\int_{0.5}^{0.8} (2-x) \, dx$ ***M1A1***

$= \left[x^2\right]_s^{0.5} + 0.27$

$0.25 - s^2 + 0.27$ ***(A1)***

$\mathrm{P}(2s < X < 0.8) = \frac{2}{3}\int_{2s}^{0.8} (2-x) \, dx$ ***A1***

$= \frac{2}{3}\left[2x - \frac{x^2}{2}\right]_{2s}^{0.8}$

$\frac{2}{3}\left(1.28 - (4s - 2s^2)\right)$

equating

$0.25 - s^2 + 0.27 = \frac{4}{3}\left(1.28 - (4s - 2s^2)\right)$ ***(A1)***

attempt to solve for s ***(M1)***

$s = 0.274$ ***A1***

[7 marks]

Total [13 marks]

11. (a) $\boldsymbol{r}_A = \boldsymbol{r}_B$ ***(M1)***

$2 - t = -0.5t \Rightarrow t = 4$ ***A1***

checking $t = 4$ satisfies $4 + t = 3.2 + 1.2t$ and $-1 - 0.15t = -2 + 0.1t$ ***R1***

$\mathrm{P}(-2, 8, -1.6)$ ***A1***

Note: Do not award final ***A1*** if answer given as column vector.

[4 marks]

(b) (i) $0.9 \times \begin{pmatrix} -0.5 \\ 1.2 \\ 0.1 \end{pmatrix} = \begin{pmatrix} -0.45 \\ 1.08 \\ 0.09 \end{pmatrix}$ ***A1***

Note: Accept use of cross product equalling zero.

hence in the same direction ***AG***

(ii) $\begin{pmatrix} -0.45t \\ 3.2 + 1.08t \\ -2 + 0.09t \end{pmatrix} = \begin{pmatrix} -2 \\ 8 \\ -1.6 \end{pmatrix}$ ***M1***

Note: The ***M1*** can be awarded for any one of the resultant equations.

$\Rightarrow t = \dfrac{40}{9} = 4.44\ldots$ ***A1***

[3 marks]

(c) (i) $\boldsymbol{r}_A - \boldsymbol{r}_B = \begin{pmatrix} 2 - t \\ 4 + t \\ -1 - 0.15t \end{pmatrix} - \begin{pmatrix} -0.45t \\ 3.2 + 1.08t \\ -2 + 0.09t \end{pmatrix}$ ***(M1)(A1)***

$= \begin{pmatrix} 2 - 0.55t \\ 0.8 - 0.08t \\ 1 - 0.24t \end{pmatrix}$ ***(A1)***

Note: Accept $\boldsymbol{r}_B - \boldsymbol{r}_A$.

distance $D = \sqrt{(2 - 0.55t)^2 + (0.8 - 0.08t)^2 + (1 - 0.24t)^2}$ ***M1A1***

$(= \sqrt{8.64 - 2.688t + 0.317t^2})$

(ii) minimum when $\dfrac{\mathrm{d}D}{\mathrm{d}t} = 0$ ***(M1)***

$t = 3.83$ ***A1***

(iii) 0.511 (km) ***A1***

[8 marks]

Total [15 marks]

Mathematics
Higher level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

12 pages

2218 – 7206

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the complex number $z = \dfrac{2 + 7\mathrm{i}}{6 + 2\mathrm{i}}$.

(a) Express z in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{Q}$. [2]

(b) Find the exact value of the modulus of z. [2]

(c) Find the argument of z, giving your answer to 4 decimal places. [2]

2. [Maximum mark: 5]

The polynomial $x^4 + px^3 + qx^2 + rx + 6$ is exactly divisible by each of $(x-1)$, $(x-2)$ and $(x-3)$.

Find the values of p, q and r.

3. [Maximum mark: 6]

The random variable X has a normal distribution with mean $\mu = 50$ and variance $\sigma^2 = 16$.

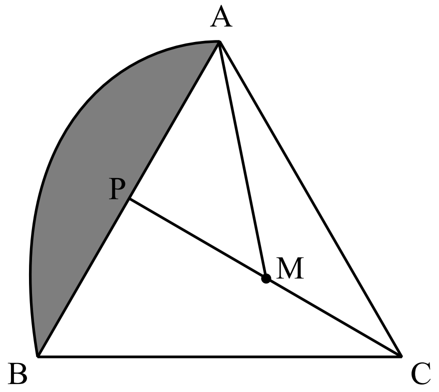
(a) Sketch the probability density function for X, and shade the region representing $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(b) Find the value of $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(c) Find the value of k for which $\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) = 0.5$. [2]

4. [Maximum mark: 8]

Consider the following diagram.



The sides of the equilateral triangle ABC have lengths $1\,\mathrm{m}$. The midpoint of $[\mathrm{AB}]$ is denoted by P. The circular arc AB has centre, M, the midpoint of $[\mathrm{CP}]$.

(a) (i) Find AM.

(ii) Find $\mathrm{A\hat{M}P}$ in radians. [5]

(b) Find the area of the shaded region. [3]

5. [Maximum mark: 6]

(a) Express the binomial coefficient $\binom{3n+1}{3n-2}$ as a polynomial in n. [3]

(b) Hence find the least value of n for which $\binom{3n+1}{3n-2} > 10^6$. [3]

6. [Maximum mark: 7]

Use mathematical induction to prove that $(1-a)^n > 1-na$ for $\{n : n \in \mathbb{Z}^+, n \geq 2\}$ where $0 < a < 1$.

7. [Maximum mark: 5]

A point P moves in a straight line with velocity $v\ \mathrm{ms}^{-1}$ given by $v(t) = \mathrm{e}^{-t} - 8t^2\mathrm{e}^{-2t}$ at time t seconds, where $t \geq 0$.

(a) Determine the first time t_1 at which P has zero velocity. [2]

(b) (i) Find an expression for the acceleration of P at time t.

(ii) Find the value of the acceleration of P at time t_1. [3]

8. [Maximum mark: 7]

The random variable X has a binomial distribution with parameters n and p.
It is given that $\mathrm{E}(X) = 3.5$.

(a) Find the least possible value of n. [2]

It is further given that $\mathrm{P}(X \leq 1) = 0.09478$ correct to 4 significant figures.

(b) Determine the value of n and the value of p. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 13]

The number of taxis arriving at Cardiff Central railway station can be modelled by a Poisson distribution. During busy periods of the day, taxis arrive at a mean rate of 5.3 taxis every 10 minutes. Let T represent a random 10 minute busy period.

(a) (i) Find the probability that exactly 4 taxis arrive during T.

(ii) Find the most likely number of taxis that would arrive during T.

(iii) Given that more than 5 taxis arrive during T, find the probability that exactly 7 taxis arrive during T. [7]

During quiet periods of the day, taxis arrive at a mean rate of 1.3 taxis every 10 minutes.

(b) Find the probability that during a period of 15 minutes, of which the first 10 minutes is busy and the next 5 minutes is quiet, that exactly 2 taxis arrive. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 18]

Consider the expression $f(x) = \tan\left(x + \frac{\pi}{4}\right)\cot\left(\frac{\pi}{4} - x\right)$.

(a) (i) Sketch the graph of $y = f(x)$ for $-\frac{5\pi}{8} \le x \le \frac{\pi}{8}$.

(ii) With reference to your graph, explain why f is a function on the given domain.

(iii) Explain why f has no inverse on the given domain.

(iv) Explain why f is not a function for $-\frac{3\pi}{4} \le x \le \frac{\pi}{4}$. [5]

The expression $f(x)$ can be written as $g(t)$ where $t = \tan x$.

(b) Show that $g(t) = \left(\frac{1+t}{1-t}\right)^2$. [3]

(c) Sketch the graph of $y = g(t)$ for $t \le 0$. Give the coordinates of any intercepts and the equations of any asymptotes. [3]

(d) Let α, β be the roots of $g(t) = k$, where $0 < k < 1$.

(i) Find α and β in terms of k.

(ii) Show that $\alpha + \beta < -2$. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 19]

A curve C is given by the implicit equation $x + y - \cos(xy) = 0$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(\frac{1 + y\sin(xy)}{1 + x\sin(xy)}\right)$. [5]

(b) The curve $xy = -\frac{\pi}{2}$ intersects C at P and Q.

(i) Find the coordinates of P and Q.

(ii) Given that the gradients of the tangents to C at P and Q are m_1 and m_2 respectively, show that $m_1 \times m_2 = 1$. [7]

(c) Find the coordinates of the three points on C, nearest the origin, where the tangent is parallel to the line $y = -x$. [7]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Mathematics

Higher level

Paper 2

18 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document **"Mathematics HL: Guidance for e-marking May 2018"**. It is essential that you read this document before you start marking. In particular, please note the following.

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, for example, ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (for example, substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets**, for example, **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(= 10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Note: Accept answers that round to the correct 2sf unless otherwise stated in the markscheme.

Section A

1. (a) $z = \frac{(2+7\text{i})}{(6+2\text{i})} \times \frac{(6-2\text{i})}{(6-2\text{i})}$ ***(M1)***

$= \frac{26+38\text{i}}{40} \left(= \frac{13+19\text{i}}{20} = 0.65 + 0.95\text{i} \right)$ ***A1***

[2 marks]

(b) attempt to use $|z| = \sqrt{a^2+b^2}$ ***(M1)***

$|z| = \sqrt{\frac{53}{40}} \left(= \frac{\sqrt{530}}{20} \right)$ or equivalent ***A1***

Note: ***A1*** is only awarded for the correct exact value.

[2 marks]

(c) **EITHER**

$\arg z = \arg(2+7\text{i}) - \arg(6+2\text{i})$ ***(M1)***

OR

$\arg z = \arctan\left(\frac{19}{13}\right)$ ***(M1)***

THEN

$\arg z = 0.9707$ (radians) (= 55.6197 degrees) ***A1***

Note: Only award the last ***A1*** if 4 decimal places are given.

[2 marks]

Total [6 marks]

2. **METHOD 1**

substitute each of $x = 1, 2$ and 3 into the quartic and equate to zero ***(M1)***

$p+q+r = -7$

$4p+2q+r = -11$ or equivalent ***(A2)***

$9p+3q+r = -29$

Note: Award ***A2*** for all three equations correct, ***A1*** for two correct.

attempting to solve the system of equations ***(M1)***

$p = -7,\ q = 17,\ r = -17$ ***A1***

Note: Only award ***M1*** when some numerical values are found when solving algebraically or using GDC.

continued…

Question 2 continued

METHOD 2

attempt to find fourth factor **(M1)**
$(x-1)$ **A1**

attempt to expand $(x-1)^2(x-2)(x-3)$ **M1**

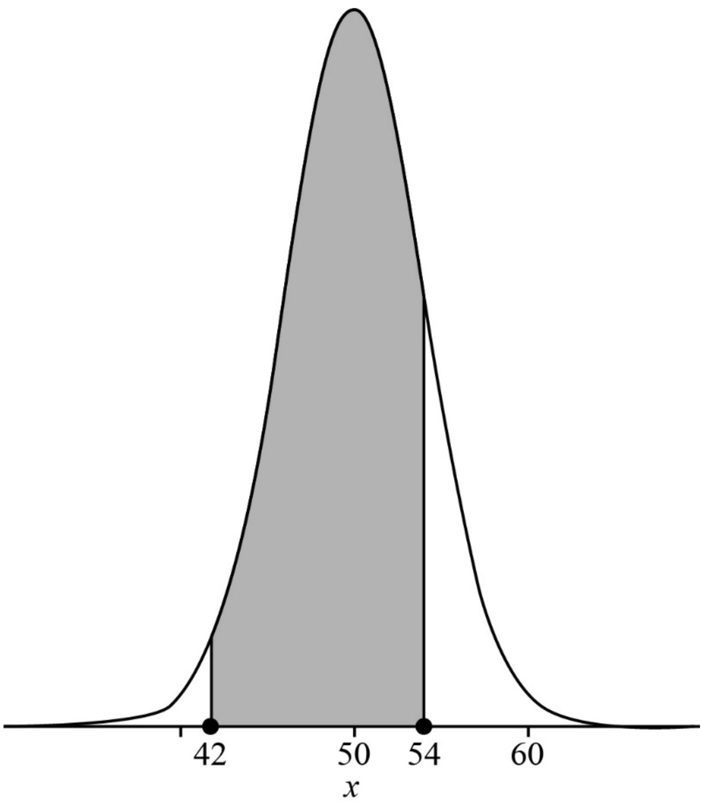
$= x^4 - 7x^3 + 17x^2 - 17x + 6 \left(p = -7,\ q = 17,\ r = -17\right)$ **A2**

Note: Award ***A2*** for all three values correct, ***A1*** for two correct.

Note: Accept long / synthetic division.

[5 marks]

3. (a)



normal curve centred on 50 **A1**
vertical lines at $x = 42$ and $x = 54$, with shading in between **A1**

[2 marks]

(b) $\mathrm{P}(42 < X < 54)$ $\left(= \mathrm{P}(-2 < Z < 1)\right)$ **(M1)**
$= 0.819$ **A1**

[2 marks]

continued…

Question 3 continued

(c) $\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) = 0.5 \Rightarrow \mathrm{P}(X < \mu + k\sigma) = 0.75$ ***(M1)***

$k = 0.674$ ***A1***

Note: Award ***M1A0*** for $k = -0.674$.

[2 marks]

Total [6 marks]

4. (a) (i) **METHOD 1**

$\mathrm{PC} = \dfrac{\sqrt{3}}{2}$ or 0.8660 ***(M1)***

$\mathrm{PM} = \dfrac{1}{2}\mathrm{PC} = \dfrac{\sqrt{3}}{4}$ or 0.4330 ***(A1)***

$\mathrm{AM} = \sqrt{\dfrac{1}{4} + \dfrac{3}{16}}$

$= \dfrac{\sqrt{7}}{4}$ or 0.661 (m) ***A1***

Note: Award ***M1*** for attempting to solve triangle AMP.

METHOD 2

using the cosine rule

$\mathrm{AM}^2 = 1^2 + \left(\dfrac{\sqrt{3}}{4}\right)^2 - 2 \times \dfrac{\sqrt{3}}{4} \times \cos(30°)$ ***M1A1***

$\mathrm{AM} = \dfrac{\sqrt{7}}{4}$ or 0.661 (m) ***A1***

(ii) $\tan\left(\mathrm{A\hat{M}P}\right) = \dfrac{2}{\sqrt{3}}$ or equivalent ***(M1)***

$= 0.857$ ***A1***

[5 marks]

continued…

Question 4 continued

(b) **EITHER**

$\frac{1}{2}\text{AM}^2\left(2\text{A}\hat{\text{M}}\text{P} - \sin(2\text{A}\hat{\text{M}}\text{P})\right)$ ***(M1)A1***

OR

$\frac{1}{2}\text{AM}^2 \times 2\text{A}\hat{\text{M}}\text{P} - \frac{\sqrt{3}}{8}$ ***(M1)A1***

$= 0.158(\text{m}^2)$ ***A1***

Note: Award ***M1*** for attempting to calculate area of a sector minus area of a triangle.

[3 marks]

Total [8 marks]

5. (a) $\binom{3n+1}{3n-2} = \frac{(3n+1)!}{(3n-2)!3!}$ ***(M1)***

$= \frac{(3n+1)3n(3n-1)}{3!}$ ***A1***

$= \frac{9}{2}n^3 - \frac{1}{2}n$ or equivalent ***A1***

[3 marks]

(b) attempt to solve $\frac{9}{2}n^3 - \frac{1}{2}n > 10^6$ ***(M1)***

$n > 60.57\ldots$ ***(A1)***

Note: Allow equality.

$\Rightarrow n = 61$ ***A1***

[3 marks]

Total [6 marks]

6. let P_n be the statement: $(1-a)^n > 1-na$ for some $n \in \mathbb{Z}^+, n \geq 2,$ where $0 < a < 1$

consider the case $n = 2$: $(1-a)^2 = 1-2a+a^2$ ***M1***

$> 1-2a$ because $a^2 > 0$. Therefore P_2 is true ***R1***

assume P_n is true for some $n = k$

$(1-a)^k > 1-ka$ ***M1***

Note: Assumption of truth must be present. Following marks are not dependent on this ***M1***.

EITHER

consider $(1-a)^{k+1} = (1-a)(1-a)^k$ ***M1***

$> 1-(k+1)a+ka^2$ ***A1***

$> 1-(k+1)a \Rightarrow P_{k+1}$ is true (as $ka^2 > 0$) ***R1***

OR

multiply both sides by $(1-a)$ (which is positive) ***M1***

$(1-a)^{k+1} > (1-ka)(1-a)$

$(1-a)^{k+1} > 1-(k+1)a+ka^2$ ***A1***

$(1-a)^{k+1} > 1-(k+1)a \Rightarrow P_{k+1}$ is true (as $ka^2 > 0$) ***R1***

THEN

P_2 is true and P_k is true $\Rightarrow P_{k+1}$ is true so P_n true for all $n \geq 2$ (or equivalent) ***R1***

Note: Only award the last ***R1*** if at least four of the previous marks are gained including the ***A1***.

[7 marks]

7. (a) attempt to solve $v(t) = 0$ for t or equivalent ***(M1)***

$t_1 = 0.441\text{(s)}$ ***A1***

[2 marks]

(b) (i) $a(t) = \dfrac{\mathrm{d}v}{\mathrm{d}t} = -\mathrm{e}^{-t} - 16t\mathrm{e}^{-2t} + 16t^2\mathrm{e}^{-2t}$ ***M1A1***

Note: Award ***M1*** for attempting to differentiate using the product rule.

(ii) $a(t_1) = -2.28\left(\text{ms}^{-2}\right)$ ***A1***

[3 marks]

Total [5 marks]

8. (a) $np = 3.5$ ***(A1)***

$p \leq 1 \Rightarrow$ least $n = 4$ ***A1***

[2 marks]

continued…

Question 8 continued

(b) $(1-p)^n + np(1-p)^{n-1} = 0.09478$ ***M1A1***

attempt to solve above equation with $np = 3.5$ ***(M1)***

$n = 12,\ p = \dfrac{7}{24}\ (= 0.292)$ ***A1A1***

Note: Do not accept n as a decimal.

[5 marks]

Total [7 marks]

Section B

9. (a) (i) $X \sim \mathrm{Po}(5.3)$

$\mathrm{P}(X = 4) = \mathrm{e}^{-5.3}\dfrac{5.3^4}{4!}$ ***(M1)***

$= 0.164$ ***A1***

(ii) **METHOD 1**

listing probabilities (table or graph) ***M1***
mode $X = 5$ (with probability 0.174) ***A1***

Note: Award ***M0A0*** for 5 (taxis) or mode $= 5$ with no justification.

METHOD 2

mode is the integer part of mean ***R1***
$E(X) = 5.3 \Rightarrow \text{mode} = 5$ ***A1***

Note: Do not allow ***R0A1***.

(iii) attempt at conditional probability ***(M1)***

$\dfrac{\mathrm{P}(X = 7)}{\mathrm{P}(X \geq 6)}$ or equivalent $\left(= \dfrac{0.1163...}{0.4365...}\right)$ ***A1***

$= 0.267$ ***A1***

[7 marks]

(b) **METHOD 1**

the possible arrivals are $(2,0)$, $(1,1)$, $(0,2)$ ***(A1)***
$Y \sim \mathrm{Po}(0.65)$ ***A1***
attempt to compute, using sum and product rule, ***(M1)***
$0.070106\ldots \times 0.52204\ldots + 0.026455\ldots \times 0.33932\ldots + 0.0049916\ldots \times 0.11028\ldots$

(A1)(A1)

Note: Award ***A1*** for one correct product and ***A1*** for two other correct products.

$= 0.0461$ ***A1***

[6 marks]

continued…

Question 9 continued

METHOD 2

recognising a sum of 2 independent Poisson variables *eg* $Z = X + Y$ ***R1***

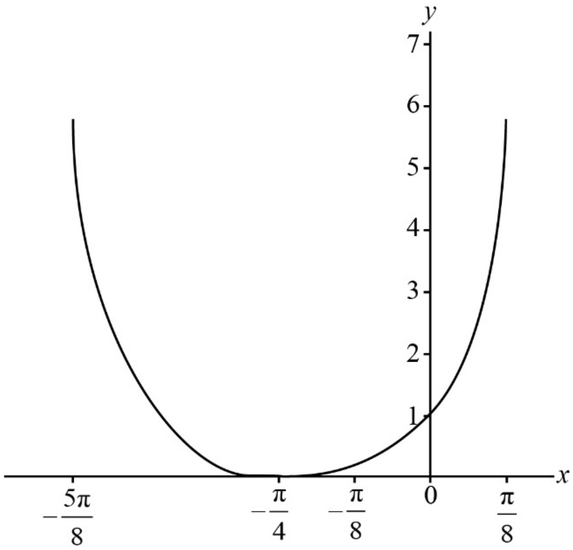
$\lambda = 5.3 + \frac{1.3}{2}$ ***A1***

$\text{P}(Z = 2) = 0.0461$ ***(M1)A3***

[6 marks]

Total [13 marks]

10. (a) (i)



A1A1

A1 for correct concavity, many to one graph, symmetrical about the midpoint of the domain and with two axes intercepts.

Note: Axes intercepts and scales not required.

A1 for correct domain

(ii) for each value of x there is a unique value of $f(x)$ ***A1***

Note: Accept "passes the vertical line test" or equivalent.

(iii) no inverse because the function fails the horizontal line test or equivalent ***R1***

Note: No **FT** if the graph is in degrees (one-to-one).

(iv) the expression is not valid at either of $x = \frac{\pi}{4}$ $\left(\text{or } -\frac{3\pi}{4}\right)$ ***R1***

[5 marks]

continued…

Question 10 continued

(b) **METHOD 1**

$$f(x) = \frac{\tan\left(x + \frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4} - x\right)}$$ ***M1***

$$= \frac{\dfrac{\tan x + \tan\frac{\pi}{4}}{1 - \tan x \tan\frac{\pi}{4}}}{\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}}$$ ***M1A1***

$$= \left(\frac{1+t}{1-t}\right)^2$$ ***AG***

METHOD 2

$$f(x) = \tan\left(x + \frac{\pi}{4}\right)\tan\left(\frac{\pi}{2} - \frac{\pi}{4} + x\right)$$ ***(M1)***

$$= \tan^2\left(x + \frac{\pi}{4}\right)$$ ***A1***

$$g(t) = \left(\frac{\tan x + \tan\frac{\pi}{4}}{1 - \tan x \tan\frac{\pi}{4}}\right)^2$$ ***A1***

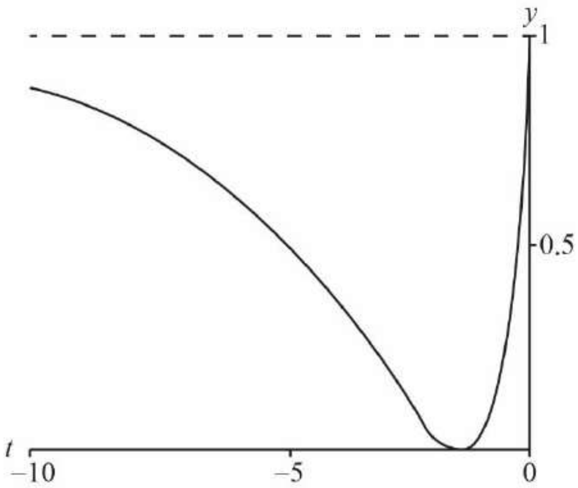
$$= \left(\frac{1+t}{1-t}\right)^2$$ ***AG***

[3 marks]

continued…

Question 10 continued

(c)



for $t \le 0$, correct concavity with two axes intercepts and with asymptote $y = 1$ ***A1***

t intercept at $(-1, 0)$ ***A1***

y intercept at $(0, 1)$ ***A1***

[3 marks]

(d) (i) **METHOD 1**

α, β satisfy $\frac{(1+t)^2}{(1-t)^2} = k$ ***M1***

$1 + t^2 + 2t = k(1 + t^2 - 2t)$ ***A1***

$(k-1)t^2 - 2(k+1)t + (k-1) = 0$ ***A1***

attempt at using quadratic formula ***M1***

$\alpha, \beta = \frac{k + 1 \pm 2\sqrt{k}}{k-1}$ or equivalent ***A1***

METHOD 2

α, β satisfy $\frac{1+t}{1-t} = (\pm)\sqrt{k}$ ***M1***

$t + \sqrt{k}t = \sqrt{k} - 1$ ***M1***

$t = \frac{\sqrt{k}-1}{\sqrt{k}+1}$ (or equivalent) ***A1***

$t - \sqrt{k}t = -\left(\sqrt{k}+1\right)$ ***M1***

$t = \frac{\sqrt{k}+1}{\sqrt{k}-1}$ (or equivalent) ***A1***

so for *eg*, $\alpha = \frac{\sqrt{k}-1}{\sqrt{k}+1}$, $\beta = \frac{\sqrt{k}+1}{\sqrt{k}-1}$

continued…

Question 10 continued

(ii) $\alpha + \beta = 2\frac{(k+1)}{(k-1)} \left(= -2\frac{(1+k)}{(1-k)} \right)$ ***A1***

since $1 + k > 1 - k$ ***R1***

$\alpha + \beta < -2$ ***AG***

Note: Accept a valid graphical reasoning.

[7 marks]

Total [18 marks]

11. (a) attempt at implicit differentiation ***M1***

$1 + \frac{\mathrm{d}y}{\mathrm{d}x} + (y + x\frac{\mathrm{d}y}{\mathrm{d}x})\sin(xy) = 0$ ***A1M1A1***

Note: Award ***A1*** for first two terms. Award ***M1*** for an attempt at chain rule ***A1*** for last term.

$(1 + x\sin(xy))\frac{\mathrm{d}y}{\mathrm{d}x} = -1 - y\sin(xy)$ or equivalent ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(\frac{1 + y\sin(xy)}{1 + x\sin(xy)}\right)$ ***AG***

[5 marks]

(b) (i) **EITHER**

when $xy = -\frac{\pi}{2}$, $\cos xy = 0$ ***M1***

$\Rightarrow x + y = 0$ ***(A1)***

OR

$x - \frac{\pi}{2x} - \cos\left(\frac{-\pi}{2}\right) = 0$ or equivalent ***M1***

$x - \frac{\pi}{2x} = 0$ ***(A1)***

THEN

therefore $x^2 = \frac{\pi}{2}$ $\left(x = \pm\sqrt{\frac{\pi}{2}}\right)(x = \pm 1.25)$ ***A1***

$\mathrm{P}\left(\sqrt{\frac{\pi}{2}}, -\sqrt{\frac{\pi}{2}}\right), \mathrm{Q}\left(-\sqrt{\frac{\pi}{2}}, \sqrt{\frac{\pi}{2}}\right)$ or $P(1.25, -1.25)$, $Q(-1.25, 1.25)$ ***A1***

continued…

Question 11 continued

(ii) $m_1 = -\left(\dfrac{1-\sqrt{\dfrac{\pi}{2}}\times -1}{1+\sqrt{\dfrac{\pi}{2}}\times -1}\right)$ ***M1A1***

$m_2 = -\left(\dfrac{1+\sqrt{\dfrac{\pi}{2}}\times -1}{1-\sqrt{\dfrac{\pi}{2}}\times -1}\right)$ ***A1***

$m_1\, m_2 = 1$ ***AG***

Note: Award ***M1A0A0*** if decimal approximations are used.

Note: No ***FT*** applies.

[7 marks]

(c) equate derivative to -1 ***M1***

$(y - x)\sin(xy) = 0$ ***(A1)***

$y = x$, $\sin(xy) = 0$ ***R1***

in the first case, attempt to solve $2x = \cos\left(x^2\right)$ ***M1***

$(0.486,0.486)$ ***A1***

in the second case, $\sin(xy) = 0 \Rightarrow xy = 0$ and $x + y = 1$ ***(M1)***

$(0,1)$, $(1,0)$ ***A1***

[7 marks]

Total [19 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel superior
Prueba 2

Jueves 3 de mayo de 2018 (mañana)

Número de convocatoria del alumno

2 horas

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[100 puntos]**.

2218 – 7226

12 páginas

No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 6]

Considere el número complejo $z = \dfrac{2 + 7\mathrm{i}}{6 + 2\mathrm{i}}$.

(a) Exprese z en la forma $a + \mathrm{i}b$, donde $a, b \in \mathbb{Q}$. [2]

(b) Halle el valor exacto del módulo de z. [2]

(c) Halle el argumento de z, con una aproximación de 4 lugares decimales. [2]

2. [Puntuación máxima: 5]

El polinomio $x^4 + px^3 + qx^2 + rx + 6$ es exactamente divisible entre $(x - 1)$, entre $(x - 2)$ y entre $(x - 3)$.

Halle los valores de p, q y r.

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3. [Puntuación máxima: 6]

La variable aleatoria X sigue una distribución normal de media $\mu = 50$ y varianza $\sigma^2 = 16$.

(a) Dibuje aproximadamente la función densidad de probabilidad correspondiente a X, y sombree la región que representa $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(b) Halle el valor de $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(c) Halle el valor de k para el cual $\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) = 0{,}5$. [2]

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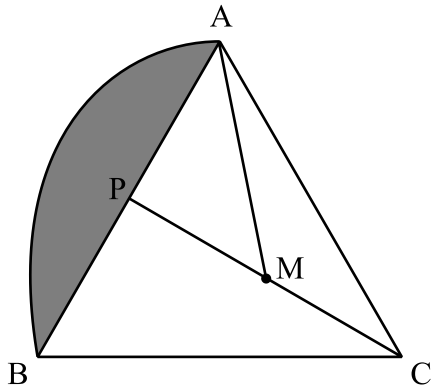
4. [Puntuación máxima: 8]

Considere la siguiente figura.



Los lados del triángulo equilátero ABC tienen longitudes de $1\,\mathrm{m}$. P es el punto medio de $[\mathrm{AB}]$. El arco de circunferencia AB tiene por centro M, el punto medio de $[\mathrm{CP}]$.

(a) (i) Halle AM.

(ii) Halle $\mathrm{A\hat{M}P}$ en radianes. [5]

(b) Halle el área de la región sombreada. [3]

5. [Puntuación máxima: 6]

(a) Exprese el coeficiente binomial $\binom{3n+1}{3n-2}$ como un polinomio en n. [3]

(b) A partir de lo anterior, halle el menor valor de n para el cual $\binom{3n+1}{3n-2} > 10^6$. [3]

6. [Puntuación máxima: 7]

Utilice la inducción matemática para demostrar que $(1-a)^n > 1-na$ para $\{n : n \in \mathbb{Z}^+, n \geq 2\}$, donde $0 < a < 1$.

7. [Puntuación máxima: 5]

Un punto P se mueve en línea recta. Su velocidad $v \text{ ms}^{-1}$ en el instante t segundos viene dada por $v(t) = \mathrm{e}^{-t} - 8t^2\mathrm{e}^{-2t}$, donde $t \geq 0$.

(a) Determine el primer instante t_1 en el que P tiene velocidad cero. [2]

(b) (i) Halle una expresión para la aceleración de P en el instante t.

(ii) Halle el valor de la aceleración de P en el instante t_1. [3]

8. [Puntuación máxima: 7]

La variable aleatoria X sigue una distribución binomial de parámetros n y p.
Se sabe que $\mathrm{E}(X) = 3{,}5$.

(a) Halle el menor valor posible de n. [2]

Se sabe además que $\mathrm{P}(X \leq 1) = 0{,}09478$ con una aproximación de 4 cifras significativas.

(b) Determine el valor de n y el valor de p. [5]

No escriba soluciones en esta página.

Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

9. [Puntuación máxima: 13]

El número de taxis que llegan a la estación de trenes de Cardiff Central se puede modelizar por una distribución de Poisson. Durante las horas del día de mayor afluencia de viajeros los taxis llegan a razón media de 5,3 taxis cada 10 minutos. Sea T un período aleatorio de 10 minutos perteneciente a esas horas de mayor afluencia.

(a) (i) Halle la probabilidad de que durante T lleguen exactamente 4 taxis.

(ii) Halle el número más probable de taxis que llegarían durante T.

(iii) Sabiendo que durante T llegan más de 5 taxis, halle la probabilidad de que durante T lleguen exactamente 7 taxis. [7]

Durante las horas tranquilas del día los taxis llegan a razón media de 1,3 taxis cada 10 minutos.

(b) Halle la probabilidad de que durante un período de 15 minutos —en el cual los primeros 10 minutos son de mayor afluencia de viajeros y los siguientes 5 minutos son tranquilos— lleguen exactamente 2 taxis. [6]

No escriba soluciones en esta página.

10. [Puntuación máxima: 18]

Considere la expresión $f(x) = \tan\left(x + \frac{\pi}{4}\right)\text{cotan}\left(\frac{\pi}{4} - x\right)$.

(a) (i) Dibuje aproximadamente el gráfico de $y = f(x)$ para $-\frac{5\pi}{8} \le x \le \frac{\pi}{8}$.

(ii) Haciendo referencia al gráfico anterior, explique por qué f es una función en el dominio dado.

(iii) Explique por qué f no tiene inversa en el dominio dado.

(iv) Explique por qué f no es una función para $-\frac{3\pi}{4} \le x \le \frac{\pi}{4}$. [5]

La expresión de $f(x)$ se puede escribir como $g(t)$, donde $t = \tan x$.

(b) Muestre que $g(t) = \left(\frac{1+t}{1-t}\right)^2$. [3]

(c) Dibuje aproximadamente el gráfico de $y = g(t)$ para $t \le 0$. Dé las coordenadas de todos los puntos de corte con los ejes y las ecuaciones de todas las asíntotas. [3]

(d) Sean α, β las raíces de $g(t) = k$, donde $0 < k < 1$.

(i) Halle α y β en función de k.

(ii) Muestre que $\alpha + \beta < -2$. [7]

No escriba soluciones en esta página.

11. [Puntuación máxima: 19]

Una curva C viene dada por la ecuación implícita $x + y - \cos(xy) = 0$.

(a) Muestre que $\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(\frac{1 + y\,\mathrm{sen}(xy)}{1 + x\,\mathrm{sen}(xy)}\right)$. [5]

(b) La curva $xy = -\frac{\pi}{2}$ y C se cortan en P y en Q.

(i) Halle las coordenadas de P y de Q.

(ii) Sabiendo que las pendientes de las tangentes a C en P y en Q son m_1 y m_2, respectivamente, muestre que $m_1 \times m_2 = 1$. [7]

(c) Halle las coordenadas de los tres puntos de C más próximos al origen de coordenadas en los que la tangente es paralela a la recta $y = -x$. [7]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathématiques
Niveau supérieur
Épreuve 2

Jeudi 3 mai 2018 (matin)

Numéro de session du candidat

2 heures

Instructions destinées aux candidats

- Écrivez votre numéro de session dans les cases ci-dessus.
- N'ouvrez pas cette épreuve avant d'y être autorisé(e).
- Une calculatrice à écran graphique est nécessaire pour cette épreuve.
- Section A : répondez à toutes les questions. Rédigez vos réponses dans les cases prévues à cet effet.
- Section B : répondez à toutes les questions sur le livret de réponses prévu à cet effet. Écrivez votre numéro de session sur la première page du livret de réponses, et attachez ce livret à cette épreuve d'examen et à votre page de couverture en utilisant l'attache fournie.
- Sauf indication contraire dans l'intitulé de la question, toutes les réponses numériques devront être exactes ou correctes à trois chiffres significatifs près.
- Un exemplaire non annoté du **livret de formules pour les cours de mathématiques NS et de mathématiques complémentaires NS** est nécessaire pour cette épreuve.
- Le nombre maximum de points pour cette épreuve d'examen est de **[100 points]**.

12 pages

2218 – 7220

Le total des points ne sera pas nécessairement attribué pour une réponse correcte si le raisonnement n'a pas été indiqué. Les réponses doivent être appuyées par un raisonnement et/ou des explications. En particulier, les solutions obtenues à l'aide d'une calculatrice à écran graphique doivent être accompagnées d'un raisonnement adéquat. Par exemple, si des représentations graphiques sont utilisées pour trouver la solution, veuillez inclure une esquisse de ces représentations graphiques dans votre réponse. Lorsque la réponse est fausse, certains points peuvent être attribués si la méthode utilisée est correcte, pour autant que le raisonnement soit indiqué par écrit. On vous recommande donc de montrer tout votre raisonnement.

Section A

Répondez à **toutes** les questions. Rédigez vos réponses dans les cases prévues à cet effet. Si cela est nécessaire, vous pouvez poursuivre votre raisonnement en dessous des lignes.

1. [Note maximale : 6]

Considérez le nombre complexe $z = \dfrac{2 + 7\mathrm{i}}{6 + 2\mathrm{i}}$.

(a) Exprimez z sous la forme $a + \mathrm{i}b$, où $a, b \in \mathbb{Q}$. [2]

(b) Trouvez la valeur exacte du module de z. [2]

(c) Trouvez l'argument de z, en donnant votre réponse avec une précision de 4 décimales. [2]

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2. [Note maximale : 5]

Le polynôme $x^4 + px^3 + qx^2 + rx + 6$ est divisible sans reste par $(x - 1)$, par $(x - 2)$ et par $(x - 3)$.

Trouvez les valeurs de p, q et r.

3. [Note maximale : 6]

La variable aléatoire X suit une distribution normale de moyenne $\mu = 50$ et de variance $\sigma^2 = 16$.

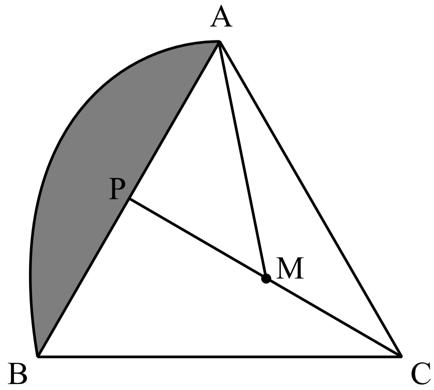
(a) Esquissez la fonction de densité pour X, et colorez la région correspondant à $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(b) Trouvez la valeur de $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(c) Trouvez la valeur de k pour laquelle $\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) = 0{,}5$. [2]

4. [Note maximale : 8]

Considérez le diagramme suivant.



Les côtés du triangle équilatéral ABC sont de longueur égale à $1\,\mathrm{m}$. Le milieu de $[\mathrm{AB}]$ est désigné par P. L’arc de cercle AB est centré en M, le milieu de $[\mathrm{CP}]$.

(a) (i) Trouvez AM.

(ii) Trouvez $\mathrm{A\hat{M}P}$ en radians. [5]

(b) Trouvez l’aire de la région grisée. [3]

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5. [Note maximale : 6]

(a) Exprimez le coefficient binomial $\binom{3n+1}{3n-2}$ comme un polynôme en n. [3]

(b) À partir de là, trouvez la plus petite valeur de n pour laquelle $\binom{3n+1}{3n-2} > 10^6$. [3]

6. [Note maximale : 7]

Utilisez la récurrence pour prouver que $(1-a)^n > 1 - na$ pour $\{n : n \in \mathbb{Z}^+, n \geq 2\}$, où $0 < a < 1$.

7. [Note maximale : 5]

Un point P se déplace en ligne droite avec une vitesse de $v\ \mathrm{ms}^{-1}$, donnée par $v(t) = \mathrm{e}^{-t} - 8t^2\mathrm{e}^{-2t}$ à l'instant t secondes, où $t \geq 0$.

(a) Déterminez la première fois, t_1, où P a une vitesse nulle. [2]

(b) (i) Trouvez une expression pour l'accélération de P à l'instant t.

(ii) Trouvez la valeur de l'accélération de P à l'instant t_1. [3]

8. [Note maximale : 7]

La variable aléatoire X suit une distribution binomiale de paramètres n et p.
On donne $\mathrm{E}(X) = 3{,}5$.

(a) Trouvez la plus petite valeur possible de n. [2]

On donne également $\mathrm{P}(X \le 1) = 0{,}09478$ avec une précision de 4 chiffres significatifs.

(b) Déterminez la valeur de n et la valeur de p. [5]

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N'écrivez **pas** vos solutions sur cette page.

Section B

Répondez à **toutes** les questions sur le livret de réponses fourni. Veuillez répondre à chaque question sur une nouvelle page.

9. [Note maximale : 13]

Le nombre de taxis arrivant à la gare centrale de Cardiff peut être modélisé par une distribution de Poisson. Lors des périodes chargées de la journée, les taxis arrivent à un taux moyen de 5,3 taxis par période de 10 minutes. Soit T une période chargée de 10 minutes choisie au hasard.

(a) (i) Trouvez la probabilité qu'exactement 4 taxis arrivent au cours de T.

(ii) Trouvez le nombre le plus probable de taxis qui arriveraient au cours de T.

(iii) Étant donné que plus de 5 taxis arrivent au cours de T, trouvez la probabilité qu'exactement 7 taxis arrivent au cours de T. [7]

Lors des périodes calmes de la journée, les taxis arrivent à un taux moyen de 1,3 taxi par période de 10 minutes.

(b) Trouvez la probabilité qu'exactement 2 taxis arrivent au cours d'une période de 15 minutes, dont les 10 premières minutes sont chargées et les 5 minutes restantes sont calmes. [6]

N'écrivez **pas** vos solutions sur cette page.

10. [Note maximale : 18]

Considérez l'expression $f(x) = \tan\left(x + \frac{\pi}{4}\right)\cot\left(\frac{\pi}{4} - x\right)$.

(a) (i) Esquissez la représentation graphique de $y = f(x)$ pour $-\frac{5\pi}{8} \le x \le \frac{\pi}{8}$.

(ii) En faisant référence à votre représentation graphique, expliquez pourquoi f est une fonction sur le domaine donné.

(iii) Expliquez pourquoi f n'a pas de réciproque sur le domaine donné.

(iv) Expliquez pourquoi f n'est pas une fonction pour $-\frac{3\pi}{4} \le x \le \frac{\pi}{4}$. [5]

L'expression $f(x)$ peut être écrite comme $g(t)$, où $t = \tan x$.

(b) Montrez que $g(t) = \left(\frac{1+t}{1-t}\right)^2$. [3]

(c) Esquissez la représentation graphique de $y = g(t)$ pour $t \le 0$. Donnez les coordonnées de tout point d'intersection avec les axes et les équations de toute asymptote. [3]

(d) Soit α, β les racines de $g(t) = k$, où $0 < k < 1$.

(i) Trouvez α et β en fonction de k.

(ii) Montrez que $\alpha + \beta < -2$. [7]

N’écrivez **pas** vos solutions sur cette page.

11. [Note maximale : 19]

Une courbe C est donnée par l’équation implicite $x + y - \cos(xy) = 0$.

(a) Montrez que $\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(\frac{1 + y\sin(xy)}{1 + x\sin(xy)}\right)$. [5]

(b) La courbe $xy = -\frac{\pi}{2}$ coupe C en P et en Q.

(i) Trouvez les coordonnées de P et Q.

(ii) Étant donné que les pentes des tangentes à C en P et Q sont respectivement m_1 et m_2, montrez que $m_1 \times m_2 = 1$. [7]

(c) Trouvez les coordonnées des trois points de C les plus près de l’origine, où la tangente est parallèle à la droite $y = -x$. [7]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – calculus

Wednesday 9 May 2018 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

2218 – 7208

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 10]

(a) Given that $n > \ln n$ for $n > 0$, use the comparison test to show that the series $\sum_{n=0}^{\infty} \frac{1}{\ln(n+2)}$ is divergent. [3]

(b) Find the interval of convergence for $\sum_{n=0}^{\infty} \frac{(3x)^n}{\ln(n+2)}$. [7]

2. [Maximum mark: 6]

The function f is defined by

$$f(x) = \begin{cases} |x-2| + 1 & x < 2 \\ ax^2 + bx & x \geq 2 \end{cases}$$

where a and b are real constants.

Given that both f and its derivative are continuous at $x = 2$, find the value of a and the value of b.

3. [Maximum mark: 11]

(a) Find the value of $\int_4^{\infty} \frac{1}{x^3}\,dx$. [3]

(b) Illustrate graphically the inequality $\sum_{n=5}^{\infty} \frac{1}{n^3} < \int_4^{\infty} \frac{1}{x^3}\,dx < \sum_{n=4}^{\infty} \frac{1}{n^3}$. [4]

(c) Hence write down a lower bound for $\sum_{n=4}^{\infty} \frac{1}{n^3}$. [1]

(d) Find an upper bound for $\sum_{n=4}^{\infty} \frac{1}{n^3}$. [3]

4. [Maximum mark: 11]

The function f is defined by $f(x) = (\arcsin x)^2$, $-1 \le x \le 1$.

(a) Show that $f'(0) = 0$. [2]

The function f satisfies the equation $\left(1 - x^2\right) f''(x) - xf'(x) - 2 = 0$.

(b) By differentiating the above equation twice, show that

$$\left(1 - x^2\right) f^{(4)}(x) - 5xf^{(3)}(x) - 4f''(x) = 0$$

where $f^{(3)}(x)$ and $f^{(4)}(x)$ denote the 3rd and 4th derivative of $f(x)$ respectively. [4]

(c) Hence show that the Maclaurin series for $f(x)$ up to and including the term in x^4 is $x^2 + \frac{1}{3}x^4$. [3]

(d) Use this series approximation for $f(x)$ with $x = \frac{1}{2}$ to find an approximate value for π^2. [2]

5. [Maximum mark: 12]

Consider the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^p + 1$ where $x \in \mathbb{R}, x \ne 0$ and p is a positive integer, $p > 1$.

(a) Solve the differential equation given that $y = -1$ when $x = 1$. Give your answer in the form $y = f(x)$. [8]

(b) (i) Show that the x-coordinate(s) of the points on the curve $y = f(x)$ where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ satisfy the equation $x^{p-1} = \frac{1}{p}$.

(ii) Deduce the set of values for p such that there are two points on the curve $y = f(x)$ where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Give a reason for your answer. [4]

M18/5/MATHL/HP3/ENG/TZ0/SE/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Calculus

Higher level

Paper 3

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685\ldots$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a - b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets** eg **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x - 3)$, the markscheme gives:

$$f'(x) = \big(2\cos(5x-3)\big)5 \quad \big(=10\cos(5x-3)\big)$$ ***A1***

Award ***A1*** for $\big(2\cos(5x-3)\big)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT***.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) **METHOD 1**

$\ln(n+2) < n+2$ ***(A1)***

$\Rightarrow \dfrac{1}{\ln(n+2)} > \dfrac{1}{n+2}$ (for $n \geq 0$) ***A1***

> **Note:** Award ***A0*** for statements such as $\sum_{n=0}^{\infty}\dfrac{1}{\ln(n+2)} > \sum_{n=0}^{\infty}\dfrac{1}{n+2}$.
> However condone such a statement if the above ***A1*** has already been awarded.

$\sum_{n=0}^{\infty}\dfrac{1}{n+2}$ (is a harmonic series which) diverges ***R1***

> **Note:** The ***R1*** is independent of the ***A1***s.
> Award ***R0*** for statements such as "$\dfrac{1}{n+2}$ diverges".

so $\sum_{n=0}^{\infty}\dfrac{1}{\ln(n+2)}$ diverges by the comparison test ***AG***

METHOD 2

$\dfrac{1}{\ln n} > \dfrac{1}{n}$ (for $n \geq 2$) ***A1***

> **Note:** Award ***A0*** for statements such as $\sum_{n=2}^{\infty}\dfrac{1}{\ln n} > \sum_{n=2}^{\infty}\dfrac{1}{n}$.
> However condone such a statement if the above ***A1*** has already been awarded.

a correct statement linking n and $n+2$ *eg*,

$\sum_{n=0}^{\infty}\dfrac{1}{\ln(n+2)} = \sum_{n=2}^{\infty}\dfrac{1}{\ln n}$ or $\sum_{n=0}^{\infty}\dfrac{1}{n+2} = \sum_{n=2}^{\infty}\dfrac{1}{n}$ ***A1***

> **Note:** Award ***A0*** for $\sum_{n=0}^{\infty}\dfrac{1}{n}$.

$\sum_{n=2}^{\infty}\dfrac{1}{n}$ (is a harmonic series which) diverges

(which implies that $\sum_{n=2}^{\infty}\dfrac{1}{\ln n}$ diverges by the comparison test) ***R1***

> **Note:** The ***R1*** is independent of the ***A1***s.
> Award ***R0*** for statements such as $\sum_{n=0}^{\infty}\dfrac{1}{n}$ diverges and "$\dfrac{1}{n}$ diverges".
> Award ***A1A0R1*** for arguments based on $\sum_{n=1}^{\infty}\dfrac{1}{n}$.

so $\sum_{n=0}^{\infty}\dfrac{1}{\ln(n+2)}$ diverges by the comparison test ***AG***

continued…

Question 1 continued

(b) applying the ratio test $\lim_{n\to\infty}\left|\frac{(3x)^{n+1}}{\ln(n+3)}\times\frac{\ln(n+2)}{(3x)^n}\right|$ ***M1***

$=|3x|$ (as $\lim_{n\to\infty}\left|\frac{\ln(n+2)}{\ln(n+3)}\right|=1$) ***A1***

Note: Condone the absence of limits and modulus signs.

Note: Award ***M1A0*** for $3x^n$. Subsequent marks can be awarded.

series converges for $-\frac{1}{3}<x<\frac{1}{3}$

considering $x=-\frac{1}{3}$ and $x=\frac{1}{3}$ ***M1***

Note: Award ***M1*** to candidates who consider one endpoint.

when $x=\frac{1}{3}$, series is $\sum_{n=0}^{\infty}\frac{1}{\ln(n+2)}$ which is divergent (from (a)) ***A1***

Note: Award this ***A1*** if $\sum_{n=0}^{\infty}\frac{1}{\ln(n+2)}$ is not stated but reference to part (a) is.

when $x=-\frac{1}{3}$, series is $\sum_{n=0}^{\infty}\frac{(-1)^n}{\ln(n+2)}$ ***A1***

$\sum_{n=0}^{\infty}\frac{(-1)^n}{\ln(n+2)}$ converges (conditionally) by the alternating series test ***R1***

(strictly alternating, $|u_n|>|u_{n+1}|$ for $n\geq 0$ and $\lim_{n\to\infty}(u_n)=0$)

so the interval of convergence of S is $-\frac{1}{3}\leq x<\frac{1}{3}$ ***A1***

Note: The final ***A1*** is dependent on previous ***A1***s – *ie*, considering correct series when $x=-\frac{1}{3}$ and $x=\frac{1}{3}$ and on the final ***R1***.

Award as above to candidates who firstly consider $x=-\frac{1}{3}$ and then state conditional convergence implies divergence at $x=\frac{1}{3}$.

[7 marks]

Total [10 marks]

2. considering continuity at $x = 2$

$\lim\limits_{x \to 2^-} f(x) = 1$ and $\lim\limits_{x \to 2^+} f(x) = 4a + 2b$ ***(M1)***

$4a + 2b = 1$ ***A1***

considering differentiability at $x = 2$

$$f'(x) = \begin{cases} -1 & x < 2 \\ 2ax + b & x \geq 2 \end{cases}$$ ***(M1)***

$\lim\limits_{x \to 2^-} f'(x) = -1$ and $\lim\limits_{x \to 2^+} f'(x) = 4a + b$ ***(M1)***

Note:	The above ***M1*** is for attempting to find the left and right limit of their derived piecewise function at $x = 2$.

$4a + b = -1$ ***A1***

$a = -\frac{3}{4}$ and $b = 2$ ***A1***

[6 marks]

3. (a) $\int_4^\infty \frac{1}{x^3}\,\mathrm{d}x = \lim\limits_{R \to \infty} \int_4^R \frac{1}{x^3}\,\mathrm{d}x$ ***(A1)***

Note:	The above ***A1*** for using a limit can be awarded at any stage. Condone the use of $\lim\limits_{x \to \infty}$. Do not award this mark to candidates who use ∞ as the upper limit throughout.

$= \lim\limits_{R \to \infty} \left[-\frac{1}{2}x^{-2}\right]_4^R \left(= \left[-\frac{1}{2}x^{-2}\right]_4^\infty\right)$ ***M1***

$= \lim\limits_{R \to \infty} \left(-\frac{1}{2}\left(R^{-2} - 4^{-2}\right)\right)$

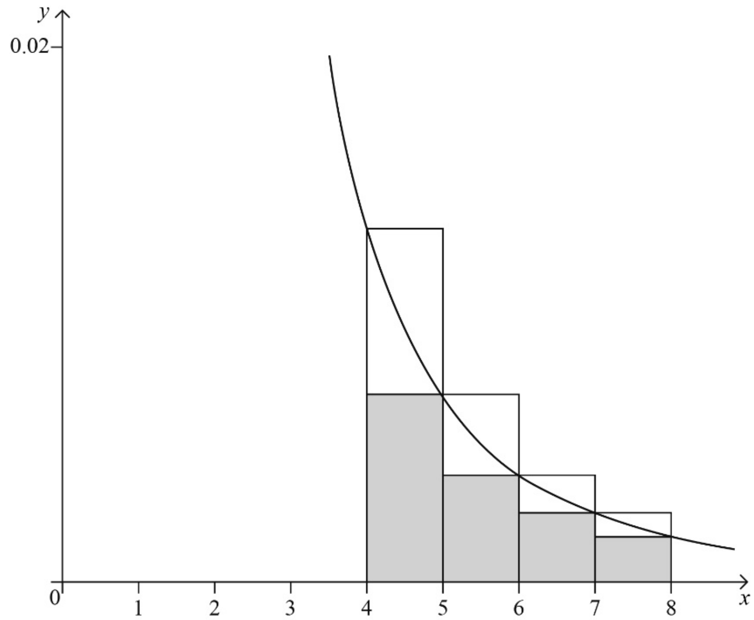
$= \frac{1}{32}$ ***A1***

[3 marks]

continued…

Question 3 continued

(b)



A1A1A1A1

A1 for the curve
A1 for rectangles starting at $x = 4$
A1 for at least three upper rectangles
A1 for at least three lower rectangles

Note: Award ***A0A1*** for two upper rectangles and two lower rectangles.

sum of areas of the lower rectangles $<$ the area under the curve $<$ the sum of the areas of the upper rectangles so

$$\sum_{n=5}^{\infty}\frac{1}{n^3} < \int_4^{\infty}\frac{1}{x^3}\,dx < \sum_{n=4}^{\infty}\frac{1}{n^3}$$ ***AG***

[4 marks]

(c) a lower bound is $\frac{1}{32}$ ***A1***

Note: Allow ***FT*** from part (a).

[1 mark]

(d) **METHOD 1**

$$\sum_{n=5}^{\infty}\frac{1}{n^3} < \frac{1}{32}$$ ***(M1)***

$$\frac{1}{64} + \sum_{n=5}^{\infty}\frac{1}{n^3} < \frac{1}{32} + \frac{1}{64}$$ ***(M1)***

$\sum_{n=4}^{\infty}\frac{1}{n^3} < \frac{3}{64}$, an upper bound ***A1***

Note: Allow ***FT*** from part (a).

continued…

Question 3 continued

METHOD 2

changing the lower limit in the inequality in part (b) gives

$$\sum_{n=4}^{\infty}\frac{1}{n^3} < \int_3^{\infty}\frac{1}{x^3}\,\mathrm{d}x\left(<\sum_{n=3}^{\infty}\frac{1}{n^3}\right)$$ ***(A1)***

$$\sum_{n=4}^{\infty}\frac{1}{n^3} < \lim_{R\to\infty}\left[-\frac{1}{2}x^{-2}\right]_3^R$$ ***(M1)***

$\sum_{n=4}^{\infty}\frac{1}{n^3} < \frac{1}{18}$, an upper bound ***A1***

Note: Condone candidates who do not use a limit.

[3 marks]

Total [11 marks]

4. (a) $f'(x)=\frac{2\arcsin(x)}{\sqrt{1-x^2}}$ ***M1A1***

Note: Award ***M1*** for an attempt at chain rule differentiation.
Award ***M0A0*** for $f'(x)=2\arcsin(x)$.

$f'(0)=0$ ***AG***

[2 marks]

(b) differentiating gives $\left(1-x^2\right)f^{(3)}(x)-2xf''(x)-f'(x)-xf''(x)(=0)$ ***M1A1***

differentiating again gives $\left(1-x^2\right)f^{(4)}(x)-2xf^{(3)}(x)-3f''(x)-3xf^{(3)}(x)-f''(x)(=0)$

M1A1

Note: Award ***M1*** for an attempt at product rule differentiation of at least one product in each of the above two lines.
Do not penalise candidates who use poor notation.

$\left(1-x^2\right)f^{(4)}(x)-5xf^{(3)}(x)-4f''(x)=0$ ***AG***

[4 marks]

continued…

Question 4 continued

(c) attempting to find **one of** $f''(0)$, $f^{(3)}(0)$ or $f^{(4)}(0)$ by substituting $x=0$ into relevant differential equation(s) ***(M1)***

Note: Condone $f''(0)$ found by calculating $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{2\arcsin(x)}{\sqrt{1-x^2}}\right)$ at $x=0$.

$\left(f(0)=0,\ f'(0)=0\right)$

$f''(0)=2$ and $f^{(4)}(0)-4f''(0)=0 \Rightarrow f^{(4)}(0)=8$ ***A1***

$f^{(3)}(0)=0$ and so $\frac{2}{2!}x^2+\frac{8}{4!}x^4$ ***A1***

Note: Only award the above ***A1***, for correct first differentiation in part (b) leading to $f^{(3)}(0)=0$ stated or $f^{(3)}(0)=0$ seen from use of the general Maclaurin series. **Special Case:** Award ***(M1)A0A1*** if $f^{(4)}(0)=8$ is stated without justification or found by working backwards from the general Maclaurin series.

so the Maclaurin series for $f(x)$ up to and including the term in x^4 is $x^2+\frac{1}{3}x^4$ ***AG***

[3 marks]

(d) substituting $x=\frac{1}{2}$ into $x^2+\frac{1}{3}x^4$ ***M1***

the series approximation gives a value of $\frac{13}{48}$

so $\pi^2 \simeq \frac{13}{48}\times 36$

$\simeq 9.75\left(\simeq \frac{39}{4}\right)$ ***A1***

Note: Accept 9.76.

[2 marks]

Total [11 marks]

5. (a) **METHOD 1**

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{y}{x} = x^{p-1} + \frac{1}{x}$$ ***(M1)***

integrating factor $= \mathrm{e}^{\int -\frac{1}{x}\mathrm{d}x}$ ***M1***

$= \mathrm{e}^{-\ln x}$ ***(A1)***

$= \frac{1}{x}$ ***A1***

$$\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{y}{x^2} = x^{p-2} + \frac{1}{x^2}$$ ***(M1)***

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y}{x}\right) = x^{p-2} + \frac{1}{x^2}$$

$$\frac{y}{x} = \frac{1}{p-1}x^{p-1} - \frac{1}{x} + C$$ ***A1***

Note: Condone the absence of C.

$$y = \frac{1}{p-1}x^p + Cx - 1$$

substituting $x = 1$, $y = -1 \Rightarrow C = -\frac{1}{p-1}$ ***M1***

Note: Award ***M1*** for attempting to find their value of C.

$$y = \frac{1}{p-1}\left(x^p - x\right) - 1$$ ***A1***

[8 marks]

continued…

Question 5 continued

METHOD 2

put $y = vx$ so that $\frac{\mathrm{d}y}{\mathrm{d}x} = v + x\frac{\mathrm{d}v}{\mathrm{d}x}$ ***M1(A1)***

substituting, ***M1***

$x\left(v + x\frac{\mathrm{d}v}{\mathrm{d}x}\right) - vx = x^p + 1$ ***(A1)***

$x\frac{\mathrm{d}v}{\mathrm{d}x} = x^{p-1} + \frac{1}{x}$ ***M1***

$\frac{\mathrm{d}v}{\mathrm{d}x} = x^{p-2} + \frac{1}{x^2}$

$v = \frac{1}{p-1}x^{p-1} - \frac{1}{x} + C$ ***A1***

Note: Condone the absence of C.

$y = \frac{1}{p-1}x^p + Cx - 1$

substituting $x = 1$, $y = -1 \Rightarrow C = -\frac{1}{p-1}$ ***M1***

Note: Award ***M1*** for attempting to find their value of C.

$y = \frac{1}{p-1}\left(x^p - x\right) - 1$ ***A1***

[8 marks]

(b) (i) **METHOD 1**

find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and solve $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ for x

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{p-1}\left(px^{p-1} - 1\right)$ ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow px^{p-1} - 1 = 0$ ***A1***

$px^{p-1} = 1$

Note: Award a maximum of ***M1A0*** if a candidate's answer to part (a) is incorrect.

$x^{p-1} = \frac{1}{p}$ ***AG***

continued…

Question 5 continued

METHOD 2

substitute $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and their y into the differential equation and solve for x

$\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow -\left(\frac{x^p - x}{p-1}\right)+1 = x^p + 1$ ***M1***

$x^p - x = x^p - px^p$ ***A1***

$px^{p-1}=1$

Note: Award a maximum of ***M1A0*** if a candidate's answer to part (a) is incorrect.

$x^{p-1}=\frac{1}{p}$ ***AG***

(ii) there are two solutions for x when p is odd (and $p>1$) ***A1***

if $p-1$ is even there are two solutions (to $x^{p-1}=\frac{1}{p}$)

and if $p-1$ is odd there is only one solution (to $x^{p-1}=\frac{1}{p}$) ***R1***

Note: Only award the ***R1*** if both cases are considered.

[4 marks]

Total [12 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel superior
Prueba 3 – Análisis

Miércoles 9 de mayo de 2018 (tarde)

1 hora

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

2218 – 7228

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Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 10]

(a) Sabiendo que $n > \ln n$ para $n > 0$, utilice el criterio de comparación para mostrar que la serie $\sum_{n=0}^{\infty} \frac{1}{\ln(n+2)}$ es divergente. [3]

(b) Halle el intervalo de convergencia de $\sum_{n=0}^{\infty} \frac{(3x)^n}{\ln(n+2)}$. [7]

2. [Puntuación máxima: 6]

La función f se define mediante

$$f(x) = \begin{cases} |x-2|+1 & x < 2 \\ ax^2 + bx & x \geq 2 \end{cases}$$

donde a y b son constantes reales.

Sabiendo que tanto f como su derivada son continuas en $x = 2$, halle el valor de a y el valor de b.

3. [Puntuación máxima: 11]

(a) Halle el valor de $\int_4^{\infty} \frac{1}{x^3}\,\mathrm{d}x$. [3]

(b) Represente gráficamente la inecuación $\sum_{n=5}^{\infty} \frac{1}{n^3} < \int_4^{\infty} \frac{1}{x^3}\,\mathrm{d}x < \sum_{n=4}^{\infty} \frac{1}{n^3}$. [4]

(c) A partir de lo anterior, escriba un límite inferior para $\sum_{n=4}^{\infty} \frac{1}{n^3}$. [1]

(d) Halle un límite superior para $\sum_{n=4}^{\infty} \frac{1}{n^3}$. [3]

4. [Puntuación máxima: 11]

La función f viene dada por $f(x) = (\operatorname{arcsen} x)^2$, $-1 \le x \le 1$.

(a) Muestre que $f'(0) = 0$. [2]

La función f satisface la ecuación $\left(1 - x^2\right) f''(x) - x f'(x) - 2 = 0$.

(b) Derive la ecuación anterior dos veces para mostrar que

$$\left(1 - x^2\right) f^{(4)}(x) - 5x f^{(3)}(x) - 4 f''(x) = 0,$$

donde $f^{(3)}(x)$ y $f^{(4)}(x)$ son, respectivamente, la derivada 3.ª y la derivada 4.ª de $f(x)$. [4]

(c) A partir de lo anterior, muestre que la serie de Maclaurin de $f(x)$ hasta el término en x^4 inclusive es $x^2 + \frac{1}{3}x^4$. [3]

(d) Utilice esta aproximación mediante una serie para $f(x)$, con $x = \frac{1}{2}$, para hallar un valor aproximado para π^2. [2]

5. [Puntuación máxima: 12]

Considere la ecuación diferencial $x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^p + 1$, donde $x \in \mathbb{R}$, $x \neq 0$ y p es un número entero positivo, $p > 1$.

(a) Resuelva la ecuación diferencial sabiendo que para $x = 1$, $y = -1$. Dé la respuesta en la forma $y = f(x)$. [8]

(b) (i) Muestre que las coordenadas x de los puntos de la curva $y = f(x)$ en donde $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ satisfacen la ecuación $x^{p-1} = \frac{1}{p}$.

(ii) Deduzca el conjunto de valores de p para los cuales existen dos puntos de la curva $y = f(x)$ donde $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Dé una razón que justifique su respuesta. [4]

Mathématiques
Niveau supérieur
Épreuve 3 – analyse

Mercredi 9 mai 2018 (après-midi)

1 heure

Instructions destinées aux candidats

- N'ouvrez pas cette épreuve avant d'y être autorisé(e).
- Répondez à toutes les questions.
- Sauf indication contraire dans l'intitulé de la question, toutes les réponses numériques devront être exactes ou correctes à trois chiffres significatifs près.
- Une calculatrice à écran graphique est nécessaire pour cette épreuve.
- Un exemplaire non annoté du **livret de formules pour les cours de mathématiques NS et de mathématiques complémentaires NS** est nécessaire pour cette épreuve.
- Le nombre maximum de points pour cette épreuve d'examen est de **[50 points]**.

2218 – 7222

3 pages

Veuillez répondre à chaque question sur une nouvelle page. Le total des points ne sera pas nécessairement attribué pour une réponse correcte si le raisonnement n'a pas été indiqué. Les réponses doivent être appuyées par un raisonnement et/ou des explications. En particulier, les solutions obtenues à l'aide d'une calculatrice à écran graphique doivent être accompagnées d'un raisonnement adéquat. Par exemple, si des représentations graphiques sont utilisées pour trouver la solution, veuillez inclure une esquisse de ces représentations graphiques dans votre réponse. Lorsque la réponse est fausse, certains points peuvent être attribués si la méthode utilisée est correcte, pour autant que le raisonnement soit indiqué par écrit. On vous recommande donc de montrer tout votre raisonnement.

1. [Note maximale : 10]

(a) Étant donné que $n > \ln n$ pour $n > 0$, utilisez le critère de comparaison pour montrer que la série $\sum_{n=0}^{\infty} \frac{1}{\ln(n+2)}$ est divergente. [3]

(b) Trouvez l'intervalle de convergence pour $\sum_{n=0}^{\infty} \frac{(3x)^n}{\ln(n+2)}$. [7]

2. [Note maximale : 6]

La fonction f est définie par

$$f(x) = \begin{cases} |x-2|+1 & x < 2 \\ ax^2 + bx & x \geq 2 \end{cases}$$

où a et b sont des constantes réelles.

Étant donné que f et sa dérivée sont toutes deux continues en $x = 2$, trouvez la valeur de a et la valeur de b.

3. [Note maximale : 11]

(a) Trouvez la valeur de $\int_4^{\infty} \frac{1}{x^3}\,\mathrm{d}x$. [3]

(b) Illustrez graphiquement l'inégalité $\sum_{n=5}^{\infty} \frac{1}{n^3} < \int_4^{\infty} \frac{1}{x^3}\,\mathrm{d}x < \sum_{n=4}^{\infty} \frac{1}{n^3}$. [4]

(c) À partir de là, écrivez une borne inférieure pour $\sum_{n=4}^{\infty} \frac{1}{n^3}$. [1]

(d) Trouvez une borne supérieure pour $\sum_{n=4}^{\infty} \frac{1}{n^3}$. [3]

4. [Note maximale : 11]

La fonction f est définie par $f(x) = (\arcsin x)^2$, $-1 \le x \le 1$.

(a) Montrez que $f'(0) = 0$. [2]

La fonction f satisfait l'équation $\left(1 - x^2\right) f''(x) - x f'(x) - 2 = 0$.

(b) En trouvant deux fois la dérivée de l'équation ci-dessus, montrez que

$$\left(1 - x^2\right) f^{(4)}(x) - 5x f^{(3)}(x) - 4 f''(x) = 0$$

où $f^{(3)}(x)$ et $f^{(4)}(x)$ désignent respectivement la 3e et la 4e dérivée de $f(x)$. [4]

(c) À partir de là, montrez que la série de Maclaurin pour $f(x)$, jusqu'au terme en x^4 inclusivement, est $x^2 + \frac{1}{3}x^4$. [3]

(d) Utilisez l'approximation de cette série pour $f(x)$ avec $x = \frac{1}{2}$ pour trouver une valeur approchée de π^2. [2]

5. [Note maximale : 12]

Considérez l'équation différentielle $x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^p + 1$, où $x \in \mathbb{R}, x \ne 0$ et p est un entier strictement positif, $p > 1$.

(a) Résolvez l'équation différentielle, étant donné que $y = -1$ lorsque $x = 1$. Donnez votre réponse sous la forme $y = f(x)$. [8]

(b) (i) Montrez que la ou les abscisses des points sur la courbe $y = f(x)$ pour lesquels $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ satisfont l'équation $x^{p-1} = \frac{1}{p}$.

(ii) Déduisez l'ensemble des valeurs de p pour lesquelles il y a deux points sur la courbe $y = f(x)$ où $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Donnez une raison pour votre réponse. [4]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – discrete mathematics

Wednesday 9 May 2018 (afternoon)

1 hour

Instructions to candidates

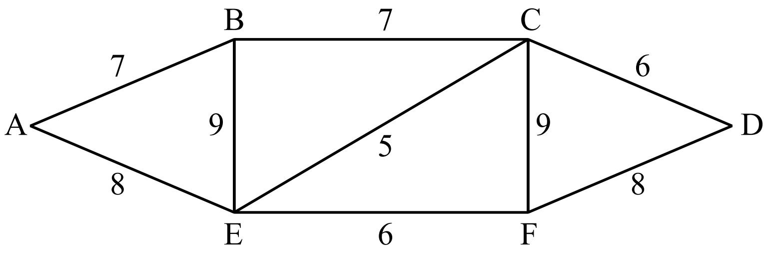
- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

2218 – 7207

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 10]

Consider the following weighted graph G.



(a) State what feature of G ensures that

(i) G has an Eulerian trail;

(ii) G does not have an Eulerian circuit. [2]

(b) Write down an Eulerian trail in G. [2]

(c) (i) State the Chinese postman problem.

(ii) Starting and finishing at B, find a solution to the Chinese postman problem for G.

(iii) Calculate the total weight of the solution. [6]

2. [Maximum mark: 8]

(a) State Fermat's little theorem. [2]

(b) Consider the linear congruence $ax \equiv b(\text{mod } p)$ where $a, b, p, x \in \mathbb{Z}^+$, p is prime and a is not a multiple of p.

(i) Use Fermat's little theorem to show that $x \equiv a^{p-2}b(\text{mod } p)$.

(ii) Hence solve the linear congruence $5x \equiv 7(\text{mod } 13)$. [6]

3. [Maximum mark: 11]

Consider the complete bipartite graph $\kappa_{3,3}$.

(a) (i) Draw $\kappa_{3,3}$.

(ii) Show that $\kappa_{3,3}$ has a Hamiltonian cycle.

(iii) Draw $\kappa_{3,2}$ and explain why it does not have a Hamiltonian cycle. [4]

(b) (i) In the context of graph theory, state the handshaking lemma.

(ii) Hence show that a graph G with degree sequence 2, 3, 3, 4, 4, 5 cannot exist. [3]

Let T be a tree with v vertices where $v \geq 2$.

(c) Use the handshaking lemma to prove that T has at least two vertices of degree one. [4]

4. [Maximum mark: 6]

(a) Show that $\gcd(4k+2\,,\,3k+1) = \gcd(k-1\,,\,2)$, where $k \in \mathbb{Z}^+$, $k > 1$. [4]

(b) State the value of $\gcd(4k+2\,,\,3k+1)$ for

(i) odd positive integers k ;

(ii) even positive integers k. [2]

5. [Maximum mark: 15]

The Fibonacci sequence can be described by the recurrence relation $f_{n+2} = f_{n+1} + f_n$ where $f_0 = 0$, $f_1 = 1$.

(a) Write down the auxiliary equation and use it to find an expression for f_n in terms of n. [7]

It is known that $\alpha^2 = \alpha + 1$ where $\alpha = \dfrac{1+\sqrt{5}}{2}$.

(b) For integers $n \geq 3$, use strong induction on the recurrence relation $f_{n+2} = f_{n+1} + f_n$ to prove that $f_n > \alpha^{n-2}$. [8]

Markscheme

May 2018

Discrete mathematics

Higher level

Paper 3

11 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685\ldots$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 ***N*** marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets** eg **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT***.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) (i) G has an Eulerian trail because it has (exactly) two vertices (B and F) of odd degree ***R1***

(ii) G does not have an Eulerian circuit because not all vertices are of even degree ***R1***

[2 marks]

(b) for example $\mathrm{BAEBCEFCDF}$ ***A1A1***

Note: Award ***A1*** for start/finish at B/F, ***A1*** for the middle vertices.

[2 marks]

(c) (i) to determine the shortest route (walk) around a weighted graph ***A1***
using each edge (at least once, returning to the starting vertex) ***A1***

Note: Correct terminology must be seen. Do not accept trail, path, cycle or circuit.

(ii) we require the Eulerian trail in (b), (weight $= 65$) ***(M1)***
and the minimum walk $\mathrm{FEB}\,(15)$ ***A1***
for example $\mathrm{BAEBCEFCDFEB}$ ***A1***

Note: Accept EB added to the end or FE added to the start of their answer in (b) in particular for follow through.

(iii) total weight is $(65+15=)80$ ***A1***

[6 marks]

Total [10 marks]

2. (a) **EITHER**

if p is prime (and a is any integer) then $a^p \equiv a \pmod p$ ***A1A1***

Note: Award ***A1*** for p prime and ***A1*** for the congruence or for stating that $p \mid a^p - a$.

OR

if p is prime (and $a \not\equiv 0 \pmod p$) then $a^{p-1} \equiv 1 \pmod p$ ***A1A1***

Note: Award ***A1*** for p prime and ***A1*** for the congruence or for stating that $p \mid a^{p-1} - 1$.

Note: Condone use of equals sign provided $(\mathrm{mod}\ p)$ is seen.

[2 marks]

(b) (i) multiplying both sides of the linear congruence by a^{p-2} ***(M1)***

$a^{p-1}x \equiv a^{p-2}b \pmod p$ ***A1***

as $a^{p-1} \equiv 1 \pmod p$ ***R1***

$x \equiv a^{p-2}b \pmod p$ ***AG***

continued…

Question 2 continued

(ii) $x \equiv 5^{11} \times 7 \pmod{13}$ ***M1***

$\equiv 341796875 \pmod{13}$ ***(A1)***

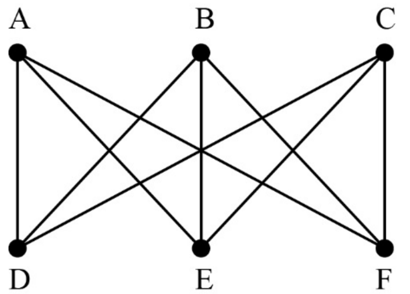
Note: Accept equivalent calculation *eg*, using $5^2 \equiv -1 \bmod 13$.

$\equiv 4 \pmod{13}$ ***A1***

[6 marks]

Total [8 marks]

3. (a) (i)



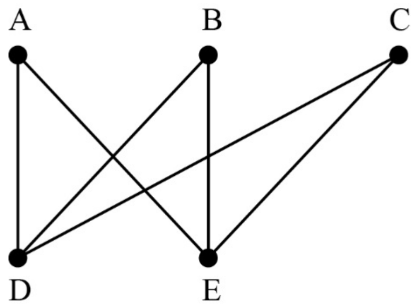
A1

(ii) for example ADBECFA ***A1***

Note: Accept drawing the cycle on their diagram.

Note: Accept Dirac's theorem (although it is not on the syllabus for (a)(ii). There is no converse that could be applied for (a)(iii).

(iii)



A1

a Hamiltonian cycle would have to alternate between the two vertex subsets which is impossible as $2 \neq 3$ ***R1***

Note: Award ***R1*** for an attempt to construct a Hamiltonian cycle and an explanation of why it fails, *eg*, ADBEC but there is no route from C to A without re-using D or E so no cycle. There are other proofs *eg*, have to go in and out of A, similarly B and C giving all edges leading to a contradiction.

[4 marks]

(b) (i) the sum of the vertex degrees is twice the number of edges ***A1***

continued…

Question 3 continued

(ii) assume G exists
the sum $2+3+3+4+4+5=21$ ***A1***
this is odd (not even) ***R1***
this contradicts the handshaking lemma
so G does not exist ***AG***

[3 marks]

(c) T has $v-1$ edges ***A1***

EITHER

if k vertices have degree 1 then $v-k$ vertices have degree ≥ 2 ***R1***
by the handshaking lemma
$2v-2 \geq 1 \times k + 2(v-k)(=2v-k)$ ***M1***
this gives $k \geq 2$ ***A1***

OR

let S be the sum of vertex degrees
consider T having either no or one vertex of degree 1 ***R1***
case 1 suppose T has no vertices of degree 1 (*eg*, all vertices have degrees ≥ 2)
by the handshaking lemma
$S \geq 2v \neq 2(v-1)$ (not possible) ***A1***
case 2 suppose T has one vertex of degree 1 (*eg*, $v-1$ vertices have degrees ≥ 2)
by the handshaking lemma
$S \geq 2(v-1)+1 \neq 2(v-1)$ (not possible) ***A1***

THEN

so T has at least two vertices of degree 1 ***AG***

[4 marks]

Total [11 marks]

4. (a) **METHOD 1**

attempting to use the Euclidean algorithm ***M1***
$4k+2=1(3k+1)+(k+1)$ ***A1***
$3k+1=2(k+1)+(k-1)$ ***A1***
$k+1=(k-1)+2$ ***A1***
$=\gcd(k-1,\ 2)$ ***AG***

continued…

Question 4 continued

METHOD 2

$\gcd(4k+2,\ 3k+1)$

$= \gcd(4k+2-(3k+1),\ 3k+1)$ ***M1***

$= \gcd(3k+1,\ k+1)\ \left(= \gcd(k+1,\ 3k+1)\right)$ ***A1***

$= \gcd(3k+1-2(k+1),\ k+1)\ \left(= \gcd(k-1,\ k+1)\right)$ ***A1***

$= \gcd(k+1-(k-1),\ k-1)\ \left(= \gcd(2,\ k-1)\right)$ ***A1***

$= \gcd(k-1,\ 2)$ ***AG***

[4 marks]

(b) (i) (for k odd), $\gcd(4k+2,\ 3k+1) = 2$ ***A1***

(ii) (for k even), $\gcd(4k+2,\ 3k+1) = 1$ ***A1***

[2 marks]

Total [6 marks]

5. (a) attempt to find the auxiliary equation ($\lambda^2 - \lambda - 1 = 0$) ***M1***

$\lambda = \frac{1 \pm \sqrt{5}}{2}$ ***(A1)***

the general solution is $f_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$ ***(M1)***

imposing initial conditions (substituting $n = 0,1$) ***M1***

$A + B = 0$ and $A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right) = 1$ ***A1***

$A = \frac{1}{\sqrt{5}},\ B = -\frac{1}{\sqrt{5}}$ ***A1***

$f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$ ***A1***

[7 marks]

Note: Condone use of decimal numbers rather than exact answers.

continued…

Question 5 continued

(b) let $\mathrm{P}(n)$ be $f_n > \alpha^{n-2}$ for integers $n \ge 3$

consideration of two consecutive values of f ***R1***

$f_3 = 2$ and $\alpha^{3-2} = \frac{1+\sqrt{5}}{2}(1.618\ldots) \Rightarrow \mathrm{P}(3)$ is true ***A1***

$f_4 = 3$ and $\alpha^{4-2} = \frac{3+\sqrt{5}}{2}(2.618\ldots) \Rightarrow \mathrm{P}(4)$ is true ***A1***

Note: Do not award ***A*** marks for values of n other than $n = 3$ and $n = 4$.

(for $k \ge 4$), assume that $\mathrm{P}(k)$ and $\mathrm{P}(k-1)$ are true ***M1***

required to prove that $\mathrm{P}(k+1)$ is true

Note: Accept equivalent notation. Needs to start with 2 general consecutive integers and then prove for the next integer. This will affect the powers of the alphas.

$f_{k+1} = f_k + f_{k-1}$ (and $f_k > \alpha^{k-2}$, $f_{k-1} > \alpha^{k-3}$) ***M1***

$f_{k+1} > \alpha^{k-2} + \alpha^{k-3} = \alpha^{k-3}(\alpha + 1)$ ***A1***

$= \alpha^{k-3}\alpha^{2} = \alpha^{k-1} = \alpha^{(k+1)-2}$ ***A1***

as $\mathrm{P}(3)$ and $\mathrm{P}(4)$ are true, and $\mathrm{P}(k)$, $\mathrm{P}(k-1)$ true $\Rightarrow \mathrm{P}(k+1)$ true

then $\mathrm{P}(k)$ is true for $k \ge 3$ by strong induction ***R1***

Note: To obtain the final ***R1***, at least five of the previous marks must have been awarded.

[8 marks]

Total [15 marks]

Matemáticas
Nivel superior
Prueba 3 – Matemáticas discretas

Miércoles 9 de mayo de 2018 (tarde)

1 hora

Instrucciones para los alumnos

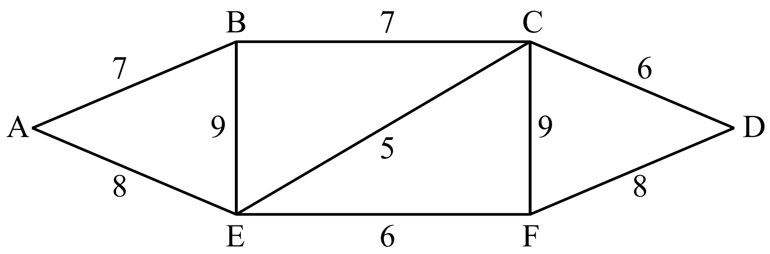
- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

2218 – 7227

Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 10]

Considere el siguiente grafo ponderado G.



(a) Indique qué característica de G garantiza que

(i) G contenga un sendero euleriano;

(ii) G no contenga un circuito euleriano. [2]

(b) Escriba un sendero euleriano que haya en G. [2]

(c) (i) Indique cuál es el problema del "cartero chino".

(ii) Empezando y acabando en B, halle una solución para el problema del "cartero chino" en G.

(iii) Calcule el peso total de la solución. [6]

2. [Puntuación máxima: 8]

(a) Indique qué dice el pequeño teorema de Fermat. [2]

(b) Considere la congruencia lineal $ax \equiv b(\text{mod}\, p)$, donde $a, b, p, x \in \mathbb{Z}^+$, p es primo y a no es múltiplo de p.

(i) Utilice el pequeño teorema de Fermat para mostrar que $x \equiv a^{p-2}b(\text{mod}\, p)$.

(ii) A partir de lo anterior, resuelva la congruencia lineal $5x \equiv 7(\text{mod } 13)$. [6]

3. [Puntuación máxima: 11]

Considere el grafo bipartito completo $\kappa_{3,3}$.

(a) (i) Dibuje $\kappa_{3,3}$.

(ii) Muestre que $\kappa_{3,3}$ contiene un ciclo hamiltoniano.

(iii) Dibuje $\kappa_{3,2}$ y explique por qué no contiene un ciclo hamiltoniano. [4]

(b) (i) En el contexto de la teoría de grafos, indique qué dice el lema del apretón de manos.

(ii) A partir de lo anterior, muestre que no puede existir un grafo G que tenga una secuencia de grados $2, 3, 3, 4, 4, 5$. [3]

Sea T un árbol con v vértices, donde $v \geq 2$.

(c) Utilice el lema del apretón de manos para demostrar que T tiene al menos dos vértices de grado uno. [4]

4. [Puntuación máxima: 6]

(a) Muestre que $\text{mcd}(4k+2, 3k+1) = \text{mcd}(k-1, 2)$, donde $k \in \mathbb{Z}^+, k > 1$. [4]

(b) Indique el valor de $\text{mcd}(4k+2, 3k+1)$ para

(i) valores de k enteros, positivos e impares;

(ii) valores de k enteros, positivos y pares. [2]

5. [Puntuación máxima: 15]

La serie de Fibonacci se puede describir mediante la relación de recurrencia $f_{n+2} = f_{n+1} + f_n$ donde $f_0 = 0$, $f_1 = 1$.

(a) Escriba la ecuación auxiliar y utilícela para hallar una expresión para f_n en función de n. [7]

Se sabe que $\alpha^2 = \alpha + 1$ donde $\alpha = \frac{1+\sqrt{5}}{2}$.

(b) Para números enteros $n \geq 3$, utilice la inducción fuerte aplicada a la relación de recurrencia $f_{n+2} = f_{n+1} + f_n$ para demostrar que $f_n > \alpha^{n-2}$. [8]

Mathématiques
Niveau supérieur
Épreuve 3 – mathématiques discrètes

Mercredi 9 mai 2018 (après-midi)

1 heure

Instructions destinées aux candidats

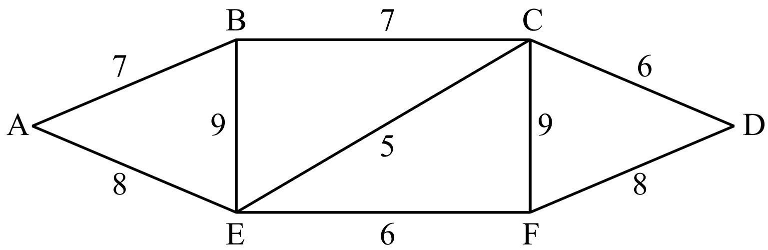
- N'ouvrez pas cette épreuve avant d'y être autorisé(e).
- Répondez à toutes les questions.
- Sauf indication contraire dans l'intitulé de la question, toutes les réponses numériques devront être exactes ou correctes à trois chiffres significatifs près.
- Une calculatrice à écran graphique est nécessaire pour cette épreuve.
- Un exemplaire non annoté du **livret de formules pour les cours de mathématiques NS et de mathématiques complémentaires NS** est nécessaire pour cette épreuve.
- Le nombre maximum de points pour cette épreuve d'examen est de **[50 points]**.

2218 – 7221

Veuillez répondre à chaque question sur une nouvelle page. Le total des points ne sera pas nécessairement attribué pour une réponse correcte si le raisonnement n'a pas été indiqué. Les réponses doivent être appuyées par un raisonnement et/ou des explications. En particulier, les solutions obtenues à l'aide d'une calculatrice à écran graphique doivent être accompagnées d'un raisonnement adéquat. Par exemple, si des représentations graphiques sont utilisées pour trouver la solution, veuillez inclure une esquisse de ces représentations graphiques dans votre réponse. Lorsque la réponse est fausse, certains points peuvent être attribués si la méthode utilisée est correcte, pour autant que le raisonnement soit indiqué par écrit. On vous recommande donc de montrer tout votre raisonnement.

1. [Note maximale : 10]

Considérez le graphe pondéré G suivant.



(a) Indiquez quelle caractéristique de G garantit que

(i) G possède une chaîne eulérienne ;

(ii) G ne possède pas un circuit eulérien. [2]

(b) Écrivez une chaîne eulérienne dans G. [2]

(c) (i) Indiquez le problème du facteur chinois.

(ii) En commençant et en finissant en B, trouvez une solution au problème du facteur chinois pour G.

(iii) Calculez le poids total de la solution. [6]

2. [Note maximale : 8]

(a) Indiquez le petit théorème de Fermat. [2]

(b) Considérez la congruence linéaire $ax \equiv b(\bmod p)$, où a, b, p, $x \in \mathbb{Z}^+$, p est premier et a n'est pas un multiple de p.

(i) Utilisez le petit théorème de Fermat pour montrer que $x \equiv a^{p-2}b(\bmod p)$.

(ii) À partir de là, résolvez la congruence linéaire $5x \equiv 7(\bmod 13)$. [6]

3. [Note maximale : 11]

Considérons le graphe biparti complet $\kappa_{3,3}$.

(a) (i) Dessinez $\kappa_{3,3}$.

(ii) Montrez que $\kappa_{3,3}$ possède un cycle hamiltonien.

(iii) Dessinez $\kappa_{3,2}$ et expliquez pourquoi il ne possède pas de cycle hamiltonien. [4]

(b) (i) Dans le contexte de la théorie des graphes, indiquez le lemme des poignées de main.

(ii) À partir de là, montrez qu'un graphe G ayant la suite de degrés 2, 3, 3, 4, 4, 5 ne peut pas exister. [3]

Soit T un arbre ayant v sommets, où $v \geq 2$.

(c) Utilisez le lemme des poignées de main pour prouver que T possède au moins deux sommets de degré un. [4]

4. [Note maximale : 6]

(a) Montrez que $\text{pgcd}(4k+2, 3k+1) = \text{pgcd}(k-1, 2)$, où $k \in \mathbb{Z}^+$, $k > 1$. [4]

(b) Indiquez la valeur du $\text{pgcd}(4k+2, 3k+1)$ pour

(i) des entiers impairs strictement positifs k;

(ii) des entiers pairs strictement positifs k. [2]

5. [Note maximale : 15]

La suite de Fibonacci peut être décrite par la relation de récurrence $f_{n+2} = f_{n+1} + f_n$, où $f_0 = 0$, $f_1 = 1$.

(a) Écrivez l'équation caractéristique et utilisez-la pour trouver une expression pour f_n en fonction de n. [7]

On sait que $\alpha^2 = \alpha + 1$, où $\alpha = \dfrac{1+\sqrt{5}}{2}$.

(b) Pour des entiers $n \geq 3$, utilisez la récurrence forte sur la relation de récurrence $f_{n+2} = f_{n+1} + f_n$ pour prouver que $f_n > \alpha^{n-2}$. [8]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – sets, relations and groups

Wednesday 9 May 2018 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

2218 – 7209

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 13]

The binary operation multiplication modulo 10, denoted by $\times_{10}$, is defined on the set $T = \{2, 4, 6, 8\}$ and represented in the following Cayley table.

$\times_{10}$	**2**	**4**	**6**	**8**
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

(a) Show that $\{T, \times_{10}\}$ is a group. (You may assume associativity.) [4]

(b) By making reference to the Cayley table, explain why T is Abelian. [1]

(c) (i) Find the order of each element of $\{T, \times_{10}\}$.

(ii) Hence show that $\{T, \times_{10}\}$ is cyclic and write down all its generators. [6]

The binary operation multiplication modulo 10, denoted by $\times_{10}$, is defined on the set $V = \{1, 3, 5, 7, 9\}$.

(d) Show that $\{V, \times_{10}\}$ is not a group. [2]

2. [Maximum mark: 8]

(a) Consider the sets $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7, 11\}$ and $C = \{1, 3, 7, 15, 31\}$.

(i) Find $(A \cup B) \cap (A \cup C)$.

(ii) Verify that $A \setminus C \neq C \setminus A$. [5]

Let S be a set containing n elements where $n \in \mathbb{N}$.

(b) Show that S has 2^n subsets. [3]

3. [Maximum mark: 8]

The relation R is defined such that xRy if and only if $|x|+|y|=|x+y|$ for $x, y \in \mathbb{R}$.

(a) Show that R is

(i) reflexive;

(ii) symmetric. [4]

(b) Show, by means of an example, that R is not transitive. [4]

4. [Maximum mark: 12]

The set of all permutations of the list of the integers $1, 2, 3, 4$ is a group, S_4, under the operation of function composition.

(a) Determine the order of S_4. [2]

In the group S_4 let $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ and $p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$.

(b) Find the proper subgroup H of order 6 containing p_1, p_2 and their compositions. Express each element of H in cycle form. [5]

Let $f: S_4 \to S_4$ be defined by $f(p) = p \circ p$ for $p \in S_4$.

(c) Using p_1 and p_2, explain why f is not a homomorphism. [5]

5. [Maximum mark: 9]

The function $f: \mathbb{Z} \to \mathbb{Z}$ is defined by $f(n) = n + (-1)^n$.

(a) Prove that $f \circ f$ is the identity function. [6]

(b) Show that

(i) f is injective;

(ii) f is surjective. [3]

Markscheme

May 2018

Sets, relations and groups

Higher level

Paper 3

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685\ldots$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a - b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets** eg **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(= 10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT***.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) closure: there are no new elements in the table ***A1***
identity: 6 is the identity element ***A1***
inverse: every element has an inverse because there is a 6 in every row and column ($2^{-1} = 8,\ 4^{-1} = 4,\ 6^{-1} = 6,\ 8^{-1} = 2$) ***A1***
we are given that (modulo) multiplication is associative ***R1***
so $\{T, \times_{10}\}$ is a group ***AG***

[4 marks]

(b) the Cayley table is symmetric (about the main diagonal) ***R1***
so T is Abelian ***AG***

[1 mark]

(c) (i) considering powers of elements ***(M1)***

elements	order
2	4
4	2
6	1
8	4

A2

Note: Award ***A2*** for all correct and ***A1*** for one error.

(ii) **EITHER**

$\{T, \times_{10}\}$ is cyclic because there is an element of order 4 ***R1***

Note: Accept "there are elements of order 4".

OR

$\{T, \times_{10}\}$ is cyclic because there is a generator ***R1***

Note: Accept "because there are generators".

THEN

2 and 8 are generators ***A1A1***

[6 marks]

continued…

Question 1 continued

(d) **EITHER**

considering singular elements *(M1)*

5 has no inverse ($5 \times_{10} a = 1,\ a \in V$ has no solution) ***R1***

OR

considering Cayley table for $\{V, \times_{10}\}$

$\times_{10}$	1	3	5	7	9
1	1	3	5	7	9
3	3	9	5	1	7
5	5	5	5	5	5
7	7	1	5	9	3
9	9	7	5	3	1

(M1)

the Cayley table is not a Latin square (or equivalent) ***R1***

OR

considering cancellation law

eg, $5 \times_{10} 9 = 5 \times_{10} 1 = 5$ ***M1***

if $\{V, \times_{10}\}$ is a group the cancellation law gives $9 = 1$ ***R1***

OR

considering order of subgroups

eg, $\{1, 9\}$ is a subgroup ***M1***

it is not possible to have a subgroup of order 2 for a group of order 5 (Lagrange's theorem) ***R1***

THEN

so $\{V, \times_{10}\}$ is not a group ***AG***

[2 marks]

Total [13 marks]

2. (a) (i) **EITHER**

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 5, 7, 9, 11\} \cap \{1, 3, 5, 7, 9, 15, 31\}$ ***M1A1***

OR

$A \cup (B \cap C) = \{1, 3, 5, 7, 9\} \cup \{3, 7\}$ ***M1A1***

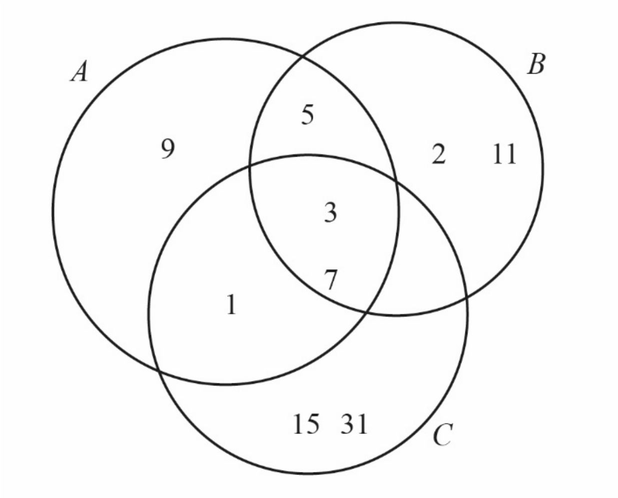
OR

$B \cap C$ is contained within A ***(M1)A1***

THEN

$= \{1, 3, 5, 7, 9\} (= A)$ ***A1***

Note: Accept a Venn diagram representation.



(ii) $A \setminus C = \{5, 9\}$ ***A1***

$C \setminus A = \{15, 31\}$ ***A1***

so $A \setminus C \neq C \setminus A$ ***AG***

Note: Accept a Venn diagram representation.

[5 marks]

continued…

Question 2 continued

(b) **METHOD 1**

if $S = \varnothing$ then $n = 0$ and the number of subsets of S is given by $2^0 = 1$ ***A1***
if $n > 0$
for every subset of S , there are 2 possibilities for each element $x \in S$
either x will be in the subset or it will not ***R1***
so for all n elements there are $(2 \times 2 \times ... \times 2 =) 2^n$ different choices
in forming a subset of S ***R1***
so S has 2^n subsets ***AG***

Note: If candidates attempt induction, award ***A1*** for case $n = 0$, ***R1*** for setting up the induction method (assume $P(k)$ and consider $P(k+1)$) and ***R1*** for showing how the $P(k)$ true implies $P(k+1)$ true).

METHOD 2

$\sum_{k=0}^{n} \binom{n}{k}$ is the number of subsets of S (of all possible sizes from 0 to n) ***R1***

$(1+1)^n = \sum_{k=0}^{n} \binom{n}{k} (1^k)(1^{n-k})$ ***M1***

$2^n = \sum_{k=0}^{n} \binom{n}{k}$ (= number of subsets of S) ***A1***

so S has 2^n subsets ***AG***

[3 marks]

Total [8 marks]

3. (a) (i) (for $x \in \mathbb{R}$), $|x| + |x| = 2|x|$ ***A1***
and $|x + x| = |2x| = 2|x|$ ***A1***
hence xRx
so R is reflexive ***AG***

Note: Award ***A1*** for correct verification of identity for $x > 0$; ***A1*** for correct verification for $x \le 0$.

(ii) if $xRy \Rightarrow |x| + |y| = |x + y|$
$|x| + |y| = |y| + |x|$ ***A1***
$|x + y| = |y + x|$ ***A1***
hence yRx
so R is symmetric ***AG***

[4 marks]

continued…

Question 3 continued

(b) recognising a condition where transitivity does not hold **(M1)**
(*eg*, $x > 0$, $y = 0$ and $z < 0$)
for example, $1R0$ and $0R(-1)$ **A1**
however $|1|+|-1| \neq |1+-1|$ **A1**
so $1R(-1)$ (for example) is not true **R1**
hence R is not transitive **AG**

[4 marks]

Total [8 marks]

4. (a) number of possible permutations is $4 \times 3 \times 2 \times 1$ **(M1)**
$= 24 (= 4!)$ **A1**

[2 marks]

(b) attempting to find one of $p_1 \circ p_1$, $p_1 \circ p_2$ or $p_2 \circ p_1$ **M1**
$p_1 \circ p_1 = (132)$ or equivalent (*eg*, $p_1^{-1} = (132)$) **A1**
$p_1 \circ p_2 = (13)$ or equivalent (*eg*, $p_2 \circ p_1 \circ p_1 = (13)$) **A1**
$p_2 \circ p_1 = (23)$ or equivalent (*eg*, $p_1 \circ p_1 \circ p_2 = (23)$) **A1**

Note: Award ***A1A0A0*** for one correct permutation in any form; ***A1A1A0*** for two correct permutations in any form.

$e = (1)$, $p_1 = (123)$ and $p_2 = (12)$ **A1**

Note: Condone omission of identity in cycle form as long as it is clear it is considered one of the elements of H.

[5 marks]

(c) **METHOD 1**

if f is a homomorphism $f(p_1 \circ p_2) = f(p_1) \circ f(p_2)$
attempting to express one of $f(p_1 \circ p_2)$ or $f(p_1) \circ f(p_2)$ in terms of p_1 and p_2 **M1**
$f(p_1 \circ p_2) = p_1 \circ p_2 \circ p_1 \circ p_2$ **A1**
$f(p_1) \circ f(p_2) = p_1 \circ p_1 \circ p_2 \circ p_2$ **A1**
$\Rightarrow p_2 \circ p_1 = p_1 \circ p_2$ **A1**
but $p_1 \circ p_2 \neq p_2 \circ p_1$ **R1**
so f is not a homomorphism **AG**

Note: Award ***R1*** only if ***M1*** is awarded.

Note: Award marks only if p_1 and p_2 are used; cycle form is not required.

continued…

Question 4 continued

METHOD 2

if f is a homomorphism $f(p_1 \circ p_2) = f(p_1) \circ f(p_2)$

attempting to find one of $f(p_1 \circ p_2)$ or $f(p_1) \circ f(p_2)$ ***M1***

$f(p_1 \circ p_2) = e$ ***A1***

$f(p_1) \circ f(p_2) = (132)$ ***(M1)A1***

so $f(p_1 \circ p_2) \neq f(p_1) \circ f(p_2)$ ***R1***

so f is not a homomorphism ***AG***

Note: Award ***R1*** only if both ***M1***s are awarded.

Note: Award marks only if p_1 and p_2 are used; cycle form is not required.

[5 marks]

Total [12 marks]

5. (a) **METHOD 1**

$(f \circ f)(n) = n + (-1)^n + (-1)^{n+(-1)^n}$ ***M1A1***

$= n + (-1)^n + (-1)^n \times (-1)^{(-1)^n}$ ***(A1)***

considering $(-1)^n$ for even and odd n ***M1***

if n is odd, $(-1)^n = -1$ and if n is even, $(-1)^n = 1$ and so $(-1)^{\pm 1} = -1$ ***A1***

$= n + (-1)^n - (-1)^n$ ***A1***

$= n$ and so $f \circ f$ is the identity function ***AG***

METHOD 2

$(f \circ f)(n) = n + (-1)^n + (-1)^{n+(-1)^n}$ ***M1A1***

$= n + (-1)^n + (-1)^n \times (-1)^{(-1)^n}$ ***(A1)***

$= n + (-1)^n \times \left(1 + (-1)^{(-1)^n}\right)$ ***M1***

$(-1)^{\pm 1} = -1$ ***R1***

$1 + (-1)^{(-1)^n} = 0$ ***A1***

$(f \circ f)(n) = n$ and so $f \circ f$ is the identity function ***AG***

continued…

Question 5 continued

METHOD 3

$\left(f \circ f\right)(n) = f\left(n + (-1)^n\right)$ ***M1***

considering even and odd n ***M1***

if n is even, $f\left(n\right) = n + 1$ which is odd ***A1***

so $\left(f \circ f\right)(n) = f\left(n+1\right) = \left(n+1\right) - 1 = n$ ***A1***

if n is odd, $f\left(n\right) = n - 1$ which is even ***A1***

so $\left(f \circ f\right)(n) = f\left(n-1\right) = \left(n-1\right) + 1 = n$ ***A1***

$\left(f \circ f\right)(n) = n$ in both cases

hence $f \circ f$ is the identity function ***AG***

[6 marks]

(b) (i) suppose $f(n) = f(m)$ ***M1***

applying f to both sides $\Rightarrow n = m$ ***R1***

hence f is injective ***AG***

(ii) $m = f(n)$ has solution $n = f(m)$ ***R1***

hence surjective ***AG***

[3 marks]

Total [9 marks]

Matemáticas
Nivel superior
Prueba 3 – Conjuntos, relaciones y grupos

Miércoles 9 de mayo de 2018 (tarde)

1 hora

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

2218 – 7229

4 páginas

Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 13]

La operación binaria multiplicación módulo 10, denotada mediante $\times_{10}$, se define sobre el conjunto $T = \{2, 4, 6, 8\}$ y aparece representada en la siguiente tabla de Cayley.

$\times_{10}$	**2**	**4**	**6**	**8**
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

(a) Muestre que $\{T, \times_{10}\}$ es un grupo. (Puede dar por supuesto que se cumple la propiedad asociativa.) [4]

(b) Haciendo referencia a la tabla de Cayley, explique por qué T es abeliano. [1]

(c) (i) Halle el orden de cada elemento de $\{T, \times_{10}\}$.

(ii) A partir de lo anterior, muestre que $\{T, \times_{10}\}$ es cíclico y escriba todos sus generadores. [6]

La operación binaria multiplicación módulo 10, denotada mediante $\times_{10}$, se define sobre el conjunto $V = \{1, 3, 5, 7, 9\}$.

(d) Muestre que $\{V, \times_{10}\}$ no es un grupo. [2]

2. [Puntuación máxima: 8]

(a) Considere los conjuntos $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7, 11\}$ y $C = \{1, 3, 7, 15, 31\}$.

(i) Halle $(A \cup B) \cap (A \cup C)$.

(ii) Verifique que $A \setminus C \neq C \setminus A$. [5]

Sea S un conjunto que contiene n elementos, donde $n \in \mathbb{N}$.

(b) Muestre que S tiene 2^n subconjuntos. [3]

3. [Puntuación máxima: 8]

La relación R se define de modo tal que xRy si y solo si $|x| + |y| = |x + y|$ para $x, y \in \mathbb{R}$.

(a) Muestre que R es

(i) reflexiva;

(ii) simétrica. [4]

(b) Muestre mediante un ejemplo que R no es transitiva. [4]

4. [Puntuación máxima: 12]

El conjunto de todas las permutaciones de la lista de números enteros $1, 2, 3, 4$ forma un grupo, S_4, con respecto a la operación de composición de funciones.

(a) Determine el orden de S_4. [2]

En el grupo S_4 sean $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ y $p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$.

(b) Halle el subgrupo propio H de orden 6 que contiene a p_1, p_2 y a sus composiciones. Exprese cada elemento de H en forma de ciclo. [5]

Sea $f: S_4 \to S_4$ la función definida mediante $f(p) = p \circ p$ para $p \in S_4$.

(c) Utilizando p_1 y p_2, explique por qué f no es un homomorfismo. [5]

5. [Puntuación máxima: 9]

La función $f : \mathbb{Z} \to \mathbb{Z}$ se define mediante $f(n) = n + (-1)^n$.

(a) Demuestre que $f \circ f$ es la función identidad. [6]

(b) Muestre que

(i) f es inyectiva;

(ii) f es sobreyectiva. [3]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – statistics and probability

Wednesday 9 May 2018 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

2218 – 7210

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 11]

The weights, X kg, of the males of a species of bird may be assumed to be normally distributed with mean 4.8 kg and standard deviation 0.2 kg.

(a) Find the probability that a randomly chosen male bird weighs between 4.75 kg and 4.85 kg. [1]

The weights, Y kg, of female birds of the same species may be assumed to be normally distributed with mean 2.7 kg and standard deviation 0.15 kg.

(b) Find the probability that the weight of a randomly chosen male bird is more than twice the weight of a randomly chosen female bird. [6]

(c) Two randomly chosen male birds and three randomly chosen female birds are placed on a weighing machine that has a weight limit of 18 kg. Find the probability that the total weight of these five birds is greater than the weight limit. [4]

2. [Maximum mark: 8]

Consider an unbiased tetrahedral (four-sided) die with faces labelled 1, 2, 3 and 4 respectively.
The random variable X represents the number of throws required to obtain a 1.

(a) State the distribution of X. [1]

(b) Show that the probability generating function, $G(t)$, for X is given by $G(t) = \dfrac{t}{4 - 3t}$. [4]

(c) Find $G'(t)$. [2]

(d) Determine the mean number of throws required to obtain a 1. [1]

3. [Maximum mark: 12]

A smartphone's battery life is defined as the number of hours a fully charged battery can be used before the smartphone stops working. A company claims that the battery life of a model of smartphone is, on average, 9.5 hours. To test this claim, an experiment is conducted on a random sample of 20 smartphones of this model. For each smartphone, the battery life, b hours, is measured and the sample mean, $\bar{b}$, calculated. It can be assumed the battery lives are normally distributed with standard deviation 0.4 hours.

(a) State suitable hypotheses for a two-tailed test. [1]

(b) Find the critical region for testing $\bar{b}$ at the 5% significance level. [4]

It is then found that this model of smartphone has an average battery life of 9.8 hours.

(c) Find the probability of making a Type II error. [3]

Another model of smartphone whose battery life may be assumed to be normally distributed with mean μ hours and standard deviation 1.2 hours is tested. A researcher measures the battery life of six of these smartphones and calculates a confidence interval of $[10.2, 11.4]$ for μ.

(d) Calculate the confidence level of this interval. [4]

4. [Maximum mark: 11]

The random variables X, Y follow a bivariate normal distribution with product moment correlation coefficient ρ.

(a) State suitable hypotheses to investigate whether or not a negative linear association exists between X and Y. [1]

A random sample of 11 observations on X, Y was obtained and the value of the sample product moment correlation coefficient, r, was calculated to be -0.708.

(b) (i) Determine the p-value.

(ii) State your conclusion at the 1% significance level. [4]

The covariance of the random variables U, V is defined by

$$\text{Cov}(U, V) = \text{E}\big((U - \text{E}(U))(V - \text{E}(V))\big).$$

(c) (i) Show that $\text{Cov}(U, V) = \text{E}(UV) - \text{E}(U)\text{E}(V)$.

(ii) Hence show that if U, V are independent random variables then the population product moment correlation coefficient, ρ, is zero. [6]

5. [Maximum mark: 8]

The random variable X has a binomial distribution with parameters n and p.

(a) Show that $P = \frac{X}{n}$ is an unbiased estimator of p. [2]

Let $U = nP(1 - P)$.

(b) (i) Show that $\mathrm{E}(U) = (n - 1)p(1 - p)$.

(ii) Hence write down an unbiased estimator of $\mathrm{Var}(X)$. [6]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Statistics and probability

Higher level

Paper 3

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685\ldots$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a - b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets** eg **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x - 3)$, the markscheme gives:

$$f'(x) = \big(2\cos(5x-3)\big)5 \quad \big(= 10\cos(5x-3)\big) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\big(2\cos(5x-3)\big)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT***.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

Note: In question 1, accept answers that round correctly to 2 significant figures.

1. (a) $P(4.75 < X < 4.85) = 0.197$ ***A1***

[1 mark]

(b) consider the random variable $X - 2Y$ ***(M1)***

$E(X - 2Y) = -0.6$ ***(A1)***

$\text{Var}(X - 2Y) = \text{Var}(X) + 4\text{Var}(Y)$ ***(M1)***

$= 0.13$ ***(A1)***

$X - 2Y \sim N(-0.6, 0.13)$

$P(X - 2Y > 0)$ ***(M1)***

$= 0.0480$ ***A1***

[6 marks]

(c) let $W = X_1 + X_2 + Y_1 + Y_2 + Y_3$ be the total weight

$E(W) = 17.7$ ***(A1)***

$\text{Var}(W) = 2\text{Var}(X) + 3\text{Var}(Y) = 0.1475$ ***(M1)(A1)***

$W \sim N(17.7, 0.1475)$

$P(W > 18) = 0.217$ ***A1***

[4 marks]

Total [11 marks]

2. (a) X is geometric (or negative binomial) ***A1***

[1 mark]

(b) $G(t) = \frac{1}{4}t + \frac{1}{4}\left(\frac{3}{4}\right)t^2 + \frac{1}{4}\left(\frac{3}{4}\right)^2 t^3 + \ldots$ ***M1A1***

recognition of GP $\left(u_1 = \frac{1}{4}t,\ r = \frac{3}{4}t\right)$ ***(M1)***

$= \dfrac{\frac{1}{4}t}{1 - \frac{3}{4}t}$ ***A1***

leading to $G(t) = \dfrac{t}{4 - 3t}$ ***AG***

[4 marks]

(c) attempt to use product or quotient rule ***M1***

$G'(t) = \dfrac{4}{(4 - 3t)^2}$ ***A1***

[2 marks]

continued…

Question 2 continued

(d) 4 ***A1***

Note: Award ***A1FT*** to a candidate that correctly calculates the value of $G'(1)$ from their $G'(t)$.

[1 mark]

Total [8 marks]

Note: In question 3, accept answers that round correctly to 2 significant figures.

3. (a) $\mathrm{H}_0 : \mu = 9.5$; $\mathrm{H}_1 : \mu \neq 9.5$ ***A1***

[1 mark]

(b) the critical values are $9.5 \pm 1.95996... \times \frac{0.4}{\sqrt{20}}$ ***(M1)(A1)***

i.e. $9.3247\ldots,\ 9.6753\ldots$

the critical region is $\bar{b} < 9.32$, $\bar{b} > 9.68$ ***A1A1***

Note: Award ***A1*** for correct inequalities, ***A1*** for correct values.

Note: Award ***M0*** if t-distribution used, note that $t(19)_{97.5} = 2.093\ldots$

[4 marks]

(c) $\bar{B} \sim \mathrm{N}\left(9.8, \left(\frac{0.4}{\sqrt{20}}\right)^2\right)$ ***(A1)***

$\mathrm{P}\left(9.3247\ldots < \bar{B} < 9.6753\ldots\right)$ ***(M1)***

$= 0.0816$ ***A1***

Note: FT the critical values from (b). Note that critical values of 9.32 and 9.68 give 0.0899.

[3 marks]

(d) **METHOD 1**

$X \sim \mathrm{N}\left(10.8, \frac{1.2^2}{6}\right)$ ***(M1)(A1)***

$\mathrm{P}(10.2 < X < 11.4) = 0.7793\ldots$ ***(A1)***

confidence level is 77.9% ***A1***

Note: Accept 78%.

METHOD 2

$11.4 - 10.2 = 2z \times \frac{1.2}{\sqrt{6}}$ ***(M1)***

$z = 1.224...$ ***(A1)***

$\mathrm{P}(-1.224\ldots < Z < 1.224\ldots) = 0.7793\ldots$ ***(A1)***

confidence level is 77.9% ***A1***

Note: Accept 78%.

[4 marks]

Total [12 marks]

4. (a) $H_0 : \rho = 0$; $H_1 : \rho < 0$ ***A1***

[1 mark]

(b) (i) $t = -0.708\sqrt{\dfrac{11-2}{1-(-0.708)^2}}$ $(= -3.0075...)$ ***(M1)***

degrees of freedom $= 9$ ***(A1)***

$P(T < -3.0075...) = 0.00739$ ***A1***

Note: Accept any answer that rounds to 0.0074.

(ii) reject H_0 or equivalent statement ***R1***

Note: Apply follow through on the candidate's p-value.

[4 marks]

(c) (i) $\text{Cov}(U, V) = \text{E}\big((U - \text{E}(U))(V - \text{E}(V))\big)$

$= \text{E}\big(UV - \text{E}(U)V - \text{E}(V)U + \text{E}(U)\text{E}(V)\big)$ ***M1***

$= \text{E}(UV) - \text{E}\big(\text{E}(U)V\big) - \text{E}\big(\text{E}(V)U\big) + \text{E}\big(\text{E}(U)\text{E}(V)\big)$ ***(A1)***

$= \text{E}(UV) - \text{E}(U)\text{E}(V) - \text{E}(V)\text{E}(U) + \text{E}(U)\text{E}(V)$ ***A1***

$\text{Cov}(U, V) = \text{E}(UV) - \text{E}(U)\text{E}(V)$ ***AG***

(ii) $\text{E}(UV) = \text{E}(U)\text{E}(V)$ (independent random variables) ***R1***

$\Rightarrow \text{Cov}(U, V) = \text{E}(U)\text{E}(V) - \text{E}(U)\text{E}(V) = 0$ ***A1***

hence, $\rho = \dfrac{\text{Cov}(U, V)}{\sqrt{\text{Var}(U)\text{Var}(V)}} = 0$ ***A1AG***

Note: Accept the statement that $\text{Cov}(U,V)$ is the numerator of the formula for ρ.

Note: Only award the first ***A1*** if the ***R1*** is awarded.

[6 marks]

Total [11 marks]

5. (a) $\text{E}(P) = \text{E}\left(\dfrac{X}{n}\right) = \dfrac{1}{n}\text{E}(X)$ ***M1***

$= \dfrac{1}{n}(np) = p$ ***A1***

so P is an unbiased estimator of p ***AG***

[2 marks]

continued…

Question 5 continued

(b) (i) $\mathrm{E}\left(nP(1-P)\right)=\mathrm{E}\left(n\left(\frac{X}{n}\right)\left(1-\frac{X}{n}\right)\right)$

$=\mathrm{E}(X)-\frac{1}{n}\mathrm{E}\left(X^2\right)$ ***M1A1***

use of $\mathrm{E}\left(X^2\right)=\mathrm{Var}(X)+\left(\mathrm{E}(X)\right)^2$ ***M1***

Note: Allow candidates to work with P rather than X for the above 3 marks.

$=np-\frac{1}{n}\left(np(1-p)+(np)^2\right)$ ***A1***

$=np-p(1-p)-np^2$

$=np(1-p)-p(1-p)$ ***A1***

Note: Award ***A1*** for the factor of $(1-p)$.

$=(n-1)p(1-p)$ ***AG***

(ii) an unbiased estimator is $\frac{n^2P(1-P)}{n-1}\left(=\frac{nU}{n-1}\right)$ ***A1***

[6 marks]

Total [8 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel superior
Prueba 3 – Estadística y probabilidad

Miércoles 9 de mayo de 2018 (tarde)

1 hora

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

2218 – 7230

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 11]

Los pesos, X kg, de los machos de una determinada especie de ave se puede suponer que siguen una distribución normal de media 4,8 kg y desviación típica 0,2 kg.

(a) Halle la probabilidad de que un macho de esta especie elegido al azar pese entre 4,75 kg y 4,85 kg. [1]

Los pesos, Y kg, de las hembras de esa misma especie de ave se puede suponer que siguen una distribución normal de media 2,7 kg y desviación típica 0,15 kg.

(b) Halle la probabilidad de que el peso de un macho elegido al azar sea más del doble del peso de una hembra elegida al azar. [6]

(c) Dos machos elegidos al azar y tres hembras elegidas al azar se colocan sobre una báscula cuyo límite de peso es 18 kg. Halle la probabilidad de que el peso total de estas cinco aves sea mayor que el límite de peso. [4]

2. [Puntuación máxima: 8]

Considere un dado tetraédrico (de cuatro lados) equilibrado, cuyas caras están rotuladas con un 1, un 2, un 3 y un 4 respectivamente.
La variable aleatoria X representa el número de veces que hay que tirar el dado para obtener un 1.

(a) Indique la distribución de X. [1]

(b) Muestre que $G(t)$, la función generatriz de probabilidad para X, viene dada por $G(t) = \dfrac{t}{4 - 3t}$. [4]

(c) Halle $G'(t)$. [2]

(d) Determine la media del número de veces que hay que tirar el dado para obtener un 1. [1]

3. [Puntuación máxima: 12]

La duración de la batería de un teléfono móvil se define como el número de horas que se puede utilizar una batería completamente cargada antes de que el teléfono deje de funcionar. Una empresa afirma que, para uno de sus modelos de teléfono móvil, la duración promedio de la batería es de 9,5 horas. Para contrastar esta afirmación, se realiza un experimento con una muestra aleatoria compuesta por 20 teléfonos móviles de este modelo. Se mide la duración de la batería, b horas, de cada teléfono móvil y luego se calcula la media muestral, $\bar{b}$. Se puede suponer que las duraciones de las baterías siguen una distribución normal, con una desviación típica igual a 0,4 horas.

(a) Indique hipótesis apropiadas para un contraste de dos colas. [1]

(b) Halle la región crítica para contrastar $\bar{b}$ a un nivel de significación del 5%. [4]

Más tarde se halla que la duración promedio de la batería para este modelo de teléfono móvil es de 9,8 horas.

(c) Halle la probabilidad de cometer un error de tipo II. [3]

Se decide analizar otro modelo de teléfono móvil, cuya duración de batería se puede suponer que sigue una distribución normal de media μ horas y desviación típica 1,2 horas. Un investigador mide la duración de la batería de seis de estos teléfonos móviles y calcula para μ un intervalo de confianza de $[10{,}2\,;\,11{,}4]$.

(d) Calcule el nivel de confianza de este intervalo. [4]

4. [Puntuación máxima: 11]

Las variables aleatorias X, Y siguen una distribución normal bidimensional con coeficiente de correlación momento-producto ρ.

(a) Indique hipótesis apropiadas para investigar si existe o no una relación lineal negativa entre X e Y. [1]

Se obtuvo una muestra aleatoria compuesta por 11 observaciones de X, Y, se calculó el coeficiente de correlación momento-producto, r, y el resultado fue $-0{,}708$.

(b) (i) Determine el valor del parámetro p.

(ii) Indique su conclusión, a un nivel de significación del 1%. [4]

La covarianza de las variables aleatorias U, V se define mediante

$$\mathrm{Cov}(U, V) = \mathrm{E}\big((U - \mathrm{E}(U))(V - \mathrm{E}(V))\big).$$

(c) (i) Muestre que $\mathrm{Cov}(U, V) = \mathrm{E}(UV) - \mathrm{E}(U)\mathrm{E}(V)$.

(ii) A partir de lo anterior, muestre que, si U, V son variables aleatorias independientes, entonces el coeficiente de correlación momento-producto de la población, ρ, es igual a cero. [6]

5. [Puntuación máxima: 8]

La variable aleatoria X sigue una distribución binomial con parámetros n y p.

(a) Muestre que $P = \dfrac{X}{n}$ es un estimador sin sesgo de p. [2]

Sea $U = nP(1 - P)$.

(b) (i) Muestre que $\mathrm{E}(U) = (n - 1)p(1 - p)$.

(ii) A partir de lo anterior, escriba un estimador sin sesgo de $\mathrm{Var}(X)$. [6]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2218 – 7303

11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

Let $f(x)=\sqrt{x+2}$ for $x \geq -2$ and $g(x)=3x-7$ for $x \in \mathbb{R}$.

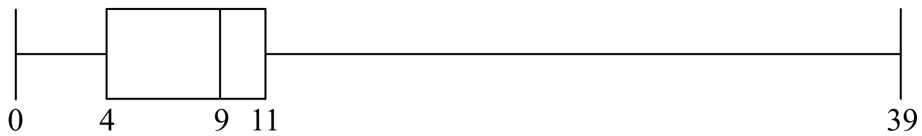
(a) Write down $f(14)$. [1]

(b) Find $(g \circ f)(14)$. [2]

(c) Find $g^{-1}(x)$. [3]

2. [Maximum mark: 6]

The following box-and-whisker plot shows the number of text messages sent by students in a school on a particular day.

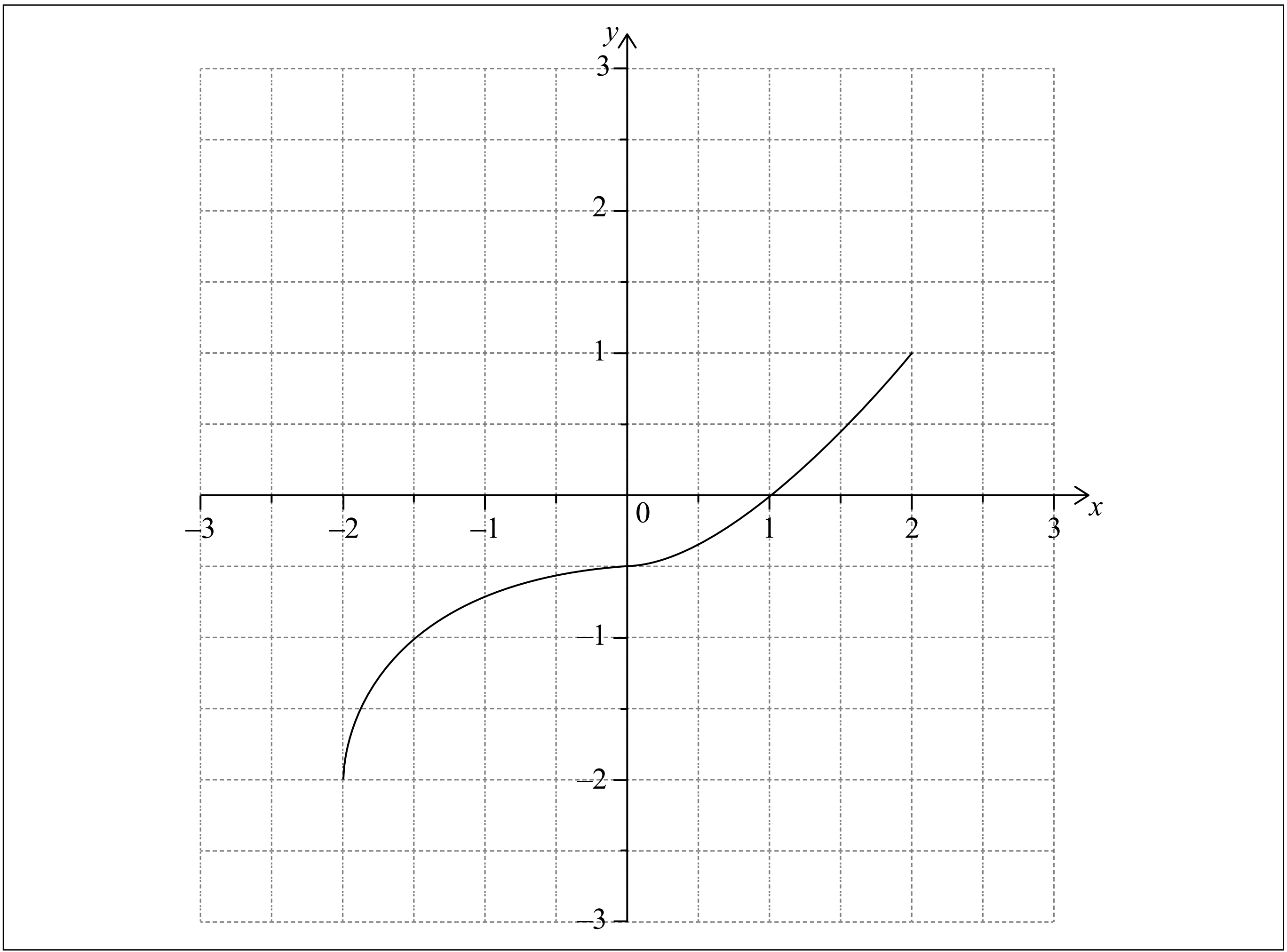


(a) Find the value of the interquartile range. [2]

(b) One student sent k text messages, where $k > 11$. Given that k is an outlier, find the least value of k. [4]

3. [Maximum mark: 7]

Consider a function $f(x)$, for $-2 \le x \le 2$. The following diagram shows the graph of f.



(a) Write down the value of

(i) $f(0)$;

(ii) $f^{-1}(1)$. [2]

(b) Write down the range of f^{-1}. [1]

(c) On the grid above, sketch the graph of f^{-1}. [4]

(This question continues on the following page)

(Question 3 continued)

4. [Maximum mark: 7]

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L, intersects the graph of f at $x = -1$ and $x = 3$.

(a) (i) The equation of the axis of symmetry is $x = p$. Find p.

(ii) Hence, show that $a = 2$. [4]

(b) The equation of L is $y = 5$. Find the value of c. [3]

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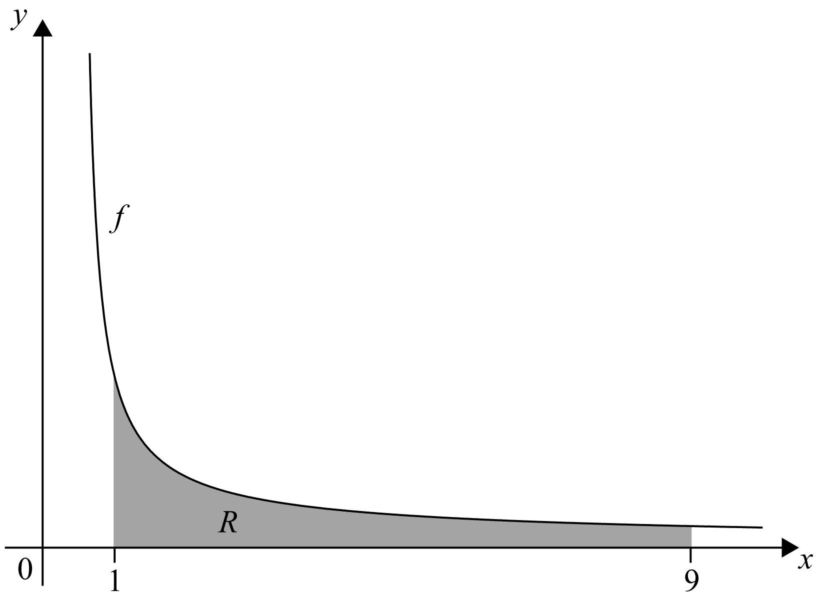
5. [Maximum mark: 7]

Let $f(x)=\frac{1}{\sqrt{2x-1}}$, for $x>\frac{1}{2}$.

(a) Find $\int\left(f(x)\right)^2\mathrm{d}x$. [3]

(b) Part of the graph of f is shown in the following diagram.

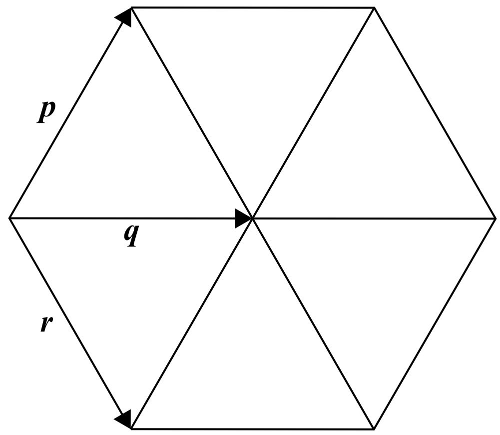


The shaded region R is enclosed by the graph of f, the x-axis, and the lines $x=1$ and $x=9$. Find the volume of the solid formed when R is revolved 360° about the x-axis. [4]

6. [Maximum mark: 6]

Six equilateral triangles, each with side length $3\,\text{cm}$, are arranged to form a hexagon. This is shown in the following diagram.

diagram not to scale



The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are shown on the diagram.

Find $\boldsymbol{p}\bullet(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r})$.

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7. [Maximum mark: 7]

Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in \mathbb{R}$ where $h(x) = (f \circ g)(x)$.

Given that $g(3) = 7$, $g'(3) = 4$ and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

A function $f(x)$ has derivative $f'(x) = 3x^2 + 18x$. The graph of f has an x-intercept at $x = -1$.

(a) Find $f(x)$. [6]

(b) The graph of f has a point of inflexion at $x = p$. Find p. [4]

(c) Find the values of x for which the graph of f is concave-down. [3]

9. [Maximum mark: 16]

Point A has coordinates $(-4, -12, 1)$ and point B has coordinates $(2, -4, -4)$.

(a) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$. [1]

(b) The line L passes through A and B.

(i) Find a vector equation for L.

(ii) Point $\mathrm{C}(k, 12, -k)$ is on L. Show that $k = 14$. [6]

(c) (i) Find $\overrightarrow{\mathrm{OB}} \bullet \overrightarrow{\mathrm{AB}}$

(ii) Write down the value of angle OBA. [3]

Point D is also on L and has coordinates $(8, 4, -9)$.

(d) Find the area of triangle OCD. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

(a) (i) Find an expression for r in terms of θ.

(ii) Find the possible values of r. [5]

(b) Show that the sum of the infinite sequence is $\dfrac{54}{2+\cos(2\theta)}$. [4]

(c) Find the values of θ which give the greatest value of the sum. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Mathematics

Standard level

Paper 1

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*
- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*
- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) $f(14) = 4$ ***A1*** ***N1***

[1 mark]

(b) attempt to substitute ***(M1)***

eg $g(4)$, $3 \times 4 - 7$

5 ***A1*** ***N2***

[2 marks]

(c) interchanging x and y (seen anywhere) ***(M1)***

eg $x = 3y - 7$

evidence of correct manipulation ***(A1)***

eg $x + 7 = 3y$

$g^{-1}(x) = \frac{x+7}{3}$ ***A1*** ***N3***

[3 marks]

Total [6 marks]

2. (a) recognizing Q_1 or Q_3 (seen anywhere) ***(M1)***

eg $4, 11$, indicated on diagram

$\text{IQR} = 7$ ***A1*** ***N2***

[2 marks]

(b) recognizing the need to find $1.5\ \text{IQR}$ ***(M1)***

eg $1.5 \times \text{IQR}$, 1.5×7

valid approach to find k ***(M1)***

eg $10.5 + 11$, $1.5 \times \text{IQR} + Q_3$

21.5 ***(A1)***

$k = 22$ ***A1*** ***N3***

Note: If no working shown, award ***N2*** for an answer of 21.5.

[4 marks]

Total [6 marks]

3. (a) (i) $f(0) = -\frac{1}{2}$ ***A1*** ***N1***

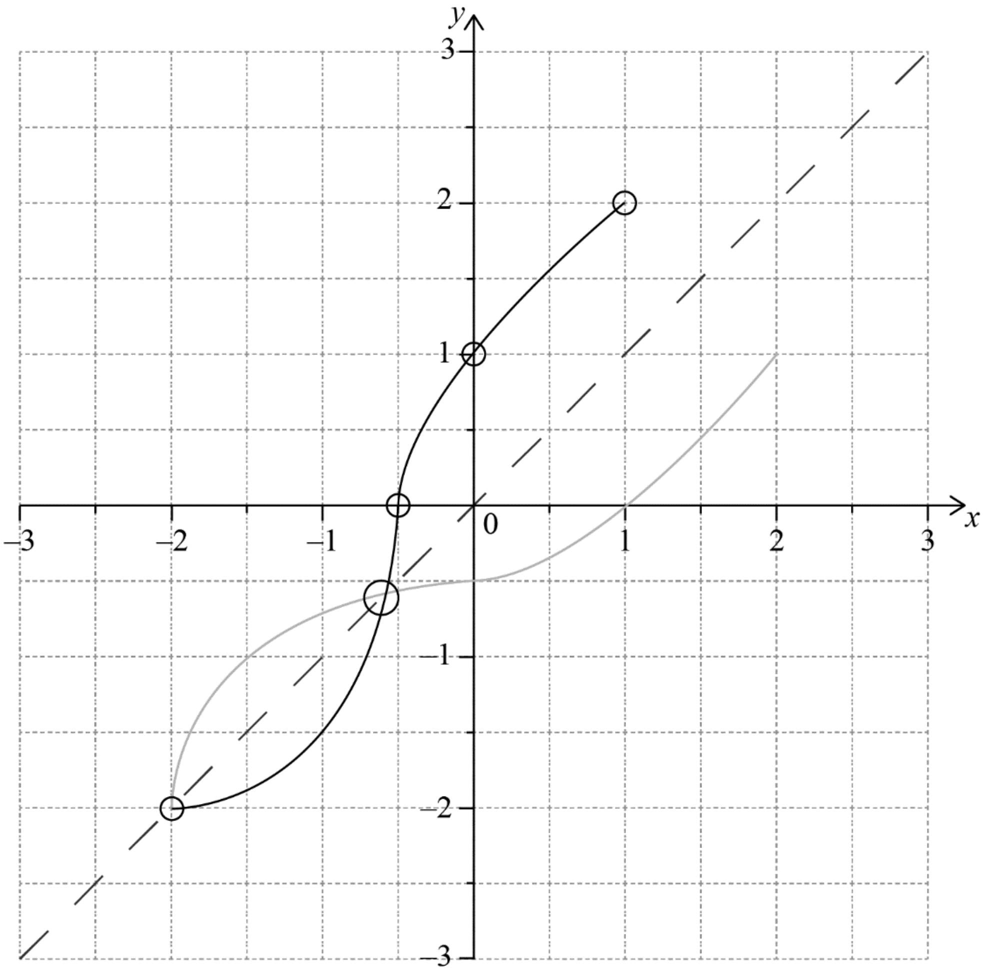
(ii) $f^{-1}(1) = 2$ ***A1*** ***N1***

[2 marks]

(b) $-2 \le y \le 2$, $y \in [-2, 2]$ (accept $-2 \le x \le 2$) ***A1*** ***N1***

[1 mark]

(c)



A1

A1A1A1 ***N4***

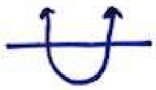
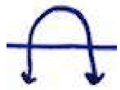
Note:	Award ***A1*** for evidence of approximately correct reflection in $y = x$ with correct curvature. ($y = x$ does not need to be explicitly seen) Only if this mark is awarded, award marks as follows: ***A1*** for both correct invariant points in circles, ***A1*** for the three other points in circles, ***A1*** for correct domain.

[4 marks]

Total [7 marks]

4. (a) **METHOD 1** (using symmetry to find p)

(i) valid approach **_(M1)_**

eg $\frac{-1+3}{2}$,  , 

$p=1$ **_A1_** **_N2_**

Note: Award no marks if they work backwards by substituting $a=2$ into $-\frac{b}{2a}$ to find p. Do not accept $p=\frac{2}{a}$.

(ii) valid approach **_M1_**

eg $-\frac{b}{2a}$, $\frac{4}{2a}$ (might be seen in (i)), $f'(1)=0$

correct equation **_A1_**

eg $\frac{4}{2a}=1$, $2a(1)-4=0$

$a=2$ **_AG_** **_N0_**

METHOD 2 (calculating a first)

(i) & (ii) valid approach to calculate a **_M1_**

eg $a+4-c=a(3^2)-4(3)-c$, $f(-1)=f(3)$

correct working **_A1_**

eg $8a=16$

$a=2$ **_AG_** **_N0_**

valid approach to find p **_(M1)_**

eg $-\frac{b}{2a}$, $\frac{4}{2(2)}$

$p=1$ **_A1_** **_N2_**

[4 marks]

(b) valid approach **_(M1)_**

eg $f(-1)=5$, $f(3)=5$

correct working **_(A1)_**

eg $2+4-c=5$, $18-12-c=5$

$c=1$ **_A1_** **_N2_**

[3 marks]

Total [7 marks]

5. (a) correct working ***(A1)***

eg $\int \frac{1}{2x-1}\mathrm{d}x$, $\int (2x-1)^{-1}$, $\frac{1}{2x-1}$, $\int \left(\frac{1}{\sqrt{u}}\right)^2 \frac{\mathrm{d}u}{2}$

$\int (f(x))^2\,\mathrm{d}x = \frac{1}{2}\ln(2x-1) + c$ ***A2*** ***N3***

Note: Award ***A1*** for $\frac{1}{2}\ln(2x-1)$.

[3 marks]

(b) attempt to substitute either limits or the function into formula involving f^2 (accept absence of π / $\mathrm{d}x$) ***(M1)***

eg $\int_1^9 y^2\mathrm{d}x$, $\pi\int\left(\frac{1}{\sqrt{2x-1}}\right)^2 \mathrm{d}x$, $\left[\frac{1}{2}\ln(2x-1)\right]_1^9$

substituting limits into **their** integral and subtracting (in any order) ***(M1)***

eg $\frac{\pi}{2}(\ln(17)-\ln(1))$, $\pi\left(0-\frac{1}{2}\ln(2\times 9-1)\right)$

correct working involving calculating a log value or using log law ***(A1)***

eg $\ln(1)=0$, $\ln\left(\frac{17}{1}\right)$

$\frac{\pi}{2}\ln 17$ (accept $\pi\ln\sqrt{17}$) ***A1*** ***N3***

Note: Full ***FT*** may be awarded as normal, from their incorrect answer in part (a), however, do not award the final two ***A*** marks unless they involve logarithms.

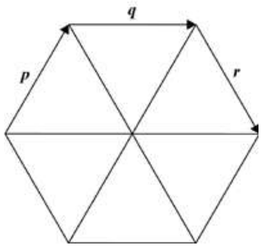
[4 marks]

Total [7 marks]

6. **METHOD 1** (using $|\boldsymbol{p}||2\boldsymbol{q}|\cos\theta$)

finding $\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}$ ***(A1)***

eg $2\boldsymbol{q}$,



$|\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}| = 2\times 3 \ (=6)$ (seen anywhere) ***A1***

correct angle between $\boldsymbol{p}$ and $\boldsymbol{q}$ (seen anywhere) ***(A1)***

$\frac{\pi}{3}$ (accept 60°)

substitution of **their** values ***(M1)***

eg $3\times 6\times\cos\left(\frac{\pi}{3}\right)$

correct value for $\cos\left(\frac{\pi}{3}\right)$ (seen anywhere) ***(A1)***

eg $\frac{1}{2}$, $3\times 6\times\frac{1}{2}$

$\boldsymbol{p}\bullet(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}) = 9$ ***A1*** ***N3***

METHOD 2 (scalar product using distributive law)

correct expression for scalar distribution ***(A1)***

eg $\boldsymbol{p}\bullet\boldsymbol{p}+\boldsymbol{p}\bullet\boldsymbol{q}+\boldsymbol{p}\bullet\boldsymbol{r}$

three correct angles between the vector pairs (seen anywhere) ***(A2)***

eg 0° between $\boldsymbol{p}$ and $\boldsymbol{p}$, $\frac{\pi}{3}$ between $\boldsymbol{p}$ and $\boldsymbol{q}$, $\frac{2\pi}{3}$ between $\boldsymbol{p}$ and $\boldsymbol{r}$

Note: Award ***A1*** for only two correct angles.

substitution of **their** values ***(M1)***

eg $3.3.\cos 0 + 3.3.\cos\frac{\pi}{3} + 3.3.\cos 120$

one correct value for $\cos 0$, $\cos\left(\frac{\pi}{3}\right)$ or $\cos\left(\frac{2\pi}{3}\right)$ (seen anywhere) ***A1***

eg $\frac{1}{2}$, $3\times 6\times\frac{1}{2}$

$\boldsymbol{p}\bullet(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}) = 9$ ***A1*** ***N3***

continued…

Question 6 continued

METHOD 3 (scalar product using relative position vectors)

valid attempt to find one component of $\boldsymbol{p}$ or $\boldsymbol{r}$ ***(M1)***

eg $\sin 60 = \frac{x}{3}$, $\cos 60 = \frac{x}{3}$, one correct value $\frac{3}{2}, \frac{3\sqrt{3}}{2}, \frac{-3\sqrt{3}}{2}$

one correct vector (two or three dimensions) (seen anywhere) ***A1***

eg $p = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix}$, $q = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, $r = \begin{pmatrix} \frac{3}{2} \\ -\frac{3\sqrt{3}}{2} \\ 0 \end{pmatrix}$

three correct vectors or $\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r} = 2\boldsymbol{q}$ ***(A1)***

$\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ (seen anywhere, including scalar product) ***(A1)***

correct working ***(A1)***

eg $\left(\frac{3}{2}\times 6\right)+\left(\frac{3\sqrt{3}}{2}\times 0\right)$, $9+0+0$

$\boldsymbol{p}\bullet(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}) = 9$ ***A1 N3***

Total [6 marks]

7. recognizing the need to find h' ***(M1)***

recognizing the need to find $h'(3)$ (seen anywhere) ***(M1)***

evidence of choosing chain rule ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$, $f'\left(g(3)\right) \times g'(3)$, $f'(g) \times g'$

correct working ***(A1)***

eg $f'(7) \times 4$, -5×4

$h'(3) = -20$ ***(A1)***

evidence of taking **their** negative reciprocal for normal ***(M1)***

eg $-\frac{1}{h'(3)}$, $m_1 m_2 = -1$

gradient of normal is $\frac{1}{20}$ ***A1*** ***N4***

Total [7 marks]

Section B

8. (a) evidence of integration *(M1)*

eg $\int f'(x)$

correct integration (accept absence of C) *(A1)(A1)*

eg $x^3 + \frac{18}{2}x^2 + C$, $x^3 + 9x^2$

attempt to substitute $x = -1$ into **their** $f = 0$ (must have C) ***M1***

eg $(-1)^3 + 9(-1)^2 + C = 0$, $-1 + 9 + C = 0$

Note: Award ***M0*** if they substitute into original or differentiated function.

correct working *(A1)*

eg $8 + C = 0$, $C = -8$

$f(x) = x^3 + 9x^2 - 8$ ***A1*** ***N5***

[6 marks]

(b) **METHOD 1** (using 2^{nd} derivative)

recognizing that $f'' = 0$ (seen anywhere) ***M1***

correct expression for f'' *(A1)*

eg $6x + 18$, $6p + 18$

correct working *(A1)*

$6p + 18 = 0$

$p = -3$ ***A1*** ***N3***

METHOD 2 (using 1^{st} derivative)

recognizing the vertex of f' is needed *(M2)*

eg $-\frac{b}{2a}$ (must be clear this is for f')

correct substitution *(A1)*

eg $\frac{-18}{2 \times 3}$

$p = -3$ ***A1*** ***N3***

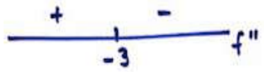
[4 marks]

continued…

Question 8 continued

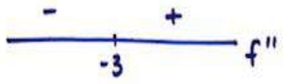
(c) valid attempt to use $f''(x)$ to determine concavity **(M1)**

eg $f''(x) < 0$, $f''(-2)$, $f''(-4)$, $6x + 18 \leq 0$,



correct working **(A1)**

eg $6x + 18 < 0$, $f''(-2) = 6$, $f''(-4) = -6$,



f concave down for $x < -3$ (do not accept $x \leq -3$) ***A1*** ***N2***

[3 marks]

Total [13 marks]

9. (a) correct approach **_A1_**

eg $\vec{AO}+\vec{OB}$, $B-A$, $\begin{pmatrix}2\\-4\\-4\end{pmatrix}-\begin{pmatrix}-4\\-12\\1\end{pmatrix}$

$\vec{AB}=\begin{pmatrix}6\\8\\-5\end{pmatrix}$ **_AG_** **_N0_**

[1 mark]

(b) (i) any correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (any parameter for t) **_A2_** **_N2_**

where $\boldsymbol{a}$ is $\begin{pmatrix}2\\-4\\-4\end{pmatrix}$ or $\begin{pmatrix}-4\\-12\\1\end{pmatrix}$ and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}6\\8\\-5\end{pmatrix}$

eg $\boldsymbol{r}=\begin{pmatrix}-4\\-12\\1\end{pmatrix}+t\begin{pmatrix}6\\8\\-5\end{pmatrix}$, $(x,\ y,\ z)=(2,\ -4,\ -4)+t(6,\ 8,\ -5)$, $\boldsymbol{r}=\begin{pmatrix}-4+6t\\-12+8t\\1-5t\end{pmatrix}$

Note:	Award ***A1*** for the form $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for the form $\boldsymbol{L}=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

(ii) **METHOD 1** (solving for t)

valid approach **_(M1)_**

eg $\begin{pmatrix}k\\12\\-k\end{pmatrix}=\begin{pmatrix}2\\-4\\-4\end{pmatrix}+t\begin{pmatrix}6\\8\\-5\end{pmatrix}$, $\begin{pmatrix}k\\12\\-k\end{pmatrix}=\begin{pmatrix}-4\\-12\\1\end{pmatrix}+t\begin{pmatrix}6\\8\\-5\end{pmatrix}$

one correct equation **_A1_**

eg $-4+8t=12$, $-12+8t=12$

correct value for t **_(A1)_**

eg $t=2$ or 3

correct substitution **_A1_**

eg $2+6(2)$, $-4+6(3)$, $-[1+3(-5)]$

$k=14$ **_AG_** **_N0_**

continued…

Question 9 continued

METHOD 2 (solving simultaneously)

valid approach ***(M1)***

eg $\begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}, \begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

two correct equations in ***A1***

eg $k = -4 + 6t, -k = 1 - 5t$

EITHER (eliminating k)

correct value for t ***(A1)***

eg $t = 2$ or 3

correct substitution ***A1***

eg $2 + 6(2), -4 + 6(3)$

OR (eliminating t)

correct equation(s) ***(A1)***

eg $5k + 20 = 30t$ **and** $-6k - 6 = -30t$, $-k = 1 - 5\left(\frac{k+4}{6}\right)$

correct working clearly leading to $k = 14$ ***A1***

eg $-k + 14 = 0$, $-6k = 6 - 5k - 20$, $5k = -20 + 6(1 + k)$

THEN

$k = 14$ ***AG*** ***N0***

[6 marks]

continued…

Question 9 continued

(c) (i) correct substitution into scalar product **A1**

eg $(2)(6)-(4)(8)-(4)(-5)$, $12-32+20$

$\vec{OB} \bullet \vec{AB} = 0$ **A1** **N0**

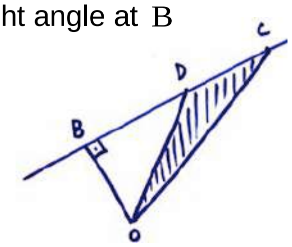
(ii) $\mathrm{O\hat{B}A} = \frac{\pi}{2},\ 90^\circ$ $\left(\text{accept } \frac{3\pi}{2},\ 270^\circ\right)$ **A1** **N1**

[3 marks]

(d) **METHOD 1** ($\frac{1}{2} \times \text{height} \times \text{CD}$)

recognizing that OB is altitude of triangle with base CD (seen anywhere) **M1**

eg $\frac{1}{2} \times \left|\vec{OB}\right| \times \left|\vec{CD}\right|$, $\text{OB} \perp \text{CD}$, sketch showing right angle at B



$\vec{CD} = \begin{pmatrix} -6 \\ -8 \\ 5 \end{pmatrix}$ or $\vec{DC} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$ (seen anywhere) ***(A1)***

correct magnitudes (seen anywhere) ***(A1)(A1)***

$\left|\vec{OB}\right| = \sqrt{(2)^2 + (-4)^2 + (-4)^2} \left(= \sqrt{36}\right)$

$.\left|\vec{CD}\right| = \sqrt{(-6)^2 + (-8)^2 + (5)^2} \left(= \sqrt{125}\right)$

correct substitution into $\frac{1}{2}bh$ **A1**

eg $\frac{1}{2} \times 6 \times \sqrt{125}$

area $= 3\sqrt{125}$, $15\sqrt{5}$ **A1** **N3**

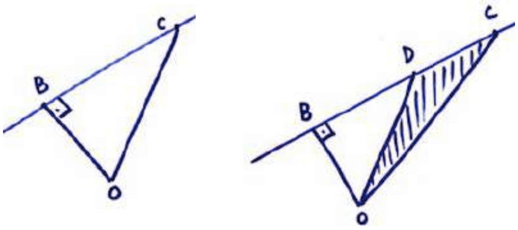
continued…

Question 9 continued

METHOD 2 (subtracting triangles)

recognizing that OB is altitude of either $\Delta\mathrm{OBD}$ or $\Delta\mathrm{OBC}$ (seen anywhere) ***M1***

eg $\frac{1}{2}\times\left|\overrightarrow{\mathrm{OB}}\right|\times\left|\overrightarrow{\mathrm{BD}}\right|$, $\mathrm{OB}\perp\mathrm{BC}$, sketch of triangle showing right angle at B



one correct vector $\overrightarrow{\mathrm{BD}}$ or $\overrightarrow{\mathrm{DB}}$ or $\overrightarrow{\mathrm{BC}}$ or $\overrightarrow{\mathrm{CB}}$ (seen anywhere) ***(A1)***

eg $\overrightarrow{\mathrm{BD}}=\begin{pmatrix}6\\8\\-5\end{pmatrix}$, $\overrightarrow{\mathrm{CB}}=\begin{pmatrix}-12\\-16\\10\end{pmatrix}$

$\left|\overrightarrow{\mathrm{OB}}\right|=\sqrt{(2)^2+(-4)^2+(-4)^2}\left(=\sqrt{36}\right)$ (seen anywhere) ***(A1)***

one correct magnitude of a base (seen anywhere) ***(A1)***

$\left|\overrightarrow{\mathrm{BD}}\right|=\sqrt{(6)^2+(8)^2+(5)^2}\left(=\sqrt{125}\right)$, $\left|\overrightarrow{\mathrm{BC}}\right|=\sqrt{144+256+100}\left(=\sqrt{500}\right)$

correct working ***A1***

eg $\frac{1}{2}\times6\times\sqrt{500}-\frac{1}{2}\times6\times5\sqrt{5}$, $\frac{1}{2}\times6\times\sqrt{500}\times\sin90-\frac{1}{2}\times6\times5\sqrt{5}\times\sin90$

area $=3\sqrt{125}$, $15\sqrt{5}$ ***A1*** ***N3***

continued…

Question 9 continued

METHOD 3 (using $\frac{1}{2}ab\sin C$ with ΔOCD)

two correct side lengths (seen anywhere) ***(A1)(A1)***

$$\left|\vec{\text{OD}}\right| = \sqrt{(8)^2+(4)^2+(-9)^2}\left(=\sqrt{161}\right),\ \left|\vec{\text{CD}}\right| = \sqrt{(-6)^2+(-8)^2+(5)^2}\left(=\sqrt{125}\right),$$

$$\left|\vec{\text{OC}}\right| = \sqrt{(14)^2+(12)^2+(-14)^2}\left(=\sqrt{536}\right)$$

attempt to find cosine ratio (seen anywhere) ***M1***

eg $\dfrac{536-286}{-2\sqrt{161}\sqrt{125}}$, $\dfrac{\text{OD}\bullet\text{DC}}{|\text{OD}||\text{DC}|}$

correct working for sine ratio ***A1***

eg $\dfrac{(125)^2}{161\times 125}+\sin^2 D = 1$

correct substitution into $\frac{1}{2}ab\sin C$ ***A1***

eg $0.5\times\sqrt{161}\times\sqrt{125}\times\dfrac{6}{\sqrt{161}}$

area $=3\sqrt{125}$, $15\sqrt{5}$ ***A1*** ***N3***

[6 marks]

Total [16 marks]

10. (a) (i) valid approach *(M1)*

eg $\frac{u_2}{u_1}, \frac{u_1}{u_2}$

$r = \frac{12\sin^2\theta}{18}\left(=\frac{2\sin^2\theta}{3}\right)$ ***A1*** ***N2***

(ii) recognizing that $\sin\theta$ is bounded *(M1)*

eg $0 \le \sin^2\theta \le 1$, $-1 \le \sin\theta \le 1$, $-1 < \sin\theta < 1$

$0 < r \le \frac{2}{3}$ ***A2*** ***N3***

Note:	If working shown, award ***M1A1*** for correct values with incorrect inequality sign(s). If no working shown, award ***N1*** for correct values with incorrect inequality sign(s).

[5 marks]

(b) correct substitution into formula for infinite sum ***A1***

eg $\frac{18}{1-\frac{2\sin^2\theta}{3}}$

evidence of choosing an appropriate rule for $\cos 2\theta$ (seen anywhere) *(M1)*

eg $\cos 2\theta = 1 - 2\sin^2\theta$

correct substitution of identity/working (seen anywhere) *(A1)*

eg $\frac{18}{1-\frac{2}{3}\left(\frac{1-\cos 2\theta}{2}\right)}$, $\frac{54}{3-2\left(\frac{1-\cos 2\theta}{2}\right)}$, $\frac{18}{\frac{3-2\sin^2\theta}{3}}$

correct working that clearly leads to the given answer ***A1***

eg $\frac{18\times 3}{2+(1-2\sin^2\theta)}$, $\frac{54}{3-(1-\cos 2\theta)}$

$\frac{54}{2+\cos(2\theta)}$ ***AG*** ***N0***

[4 marks]

continued…

Question 10 continued

(c) **METHOD 1** (using differentiation)

recognizing $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta} = 0$ (seen anywhere) ***(M1)***

finding any correct expression for $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta}$ ***(A1)***

eg $\frac{0 - 54 \times (-2\sin 2\theta)}{(2 + \cos 2\theta)^2}$, $-54(2 + \cos 2\theta)^{-2}(-2\sin 2\theta)$

correct working ***(A1)***
eg $\sin 2\theta = 0$

any correct value for $\sin^{-1}(0)$ (seen anywhere) ***(A1)***
eg $0,\ \pi,\ \ldots$, sketch of sine curve with x-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***
$2\theta = \pi,\ 3\pi$ (accept values in degrees)

both correct answers $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ ***A1*** ***N4***

Note:	Award ***A0*** if either or both correct answers are given in degrees. Award ***A0*** if additional values are given.

continued…

Question 10 continued

METHOD 2 (using denominator)

recognizing when S_∞ is greatest ***(M1)***

eg $2+\cos 2\theta$ is a minimum, $1-r$ is smallest

correct working ***(A1)***

eg minimum value of $2+\cos 2\theta$ is 1, minimum $r=\frac{2}{3}$

correct working ***(A1)***

eg $\cos 2\theta=-1$, $\frac{2}{3}\sin^2\theta=\frac{2}{3}$, $\sin^2\theta=1$

EITHER (using $\cos 2\theta$)

any correct value for $\cos^{-1}(-1)$ (seen anywhere) ***(A1)***

eg $\pi, 3\pi, \ldots$ (accept values in degrees), sketch of cosine curve with x-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta=\pi, 3\pi$ (accept values in degrees)

OR (using $\sin\theta$**)**

$\sin\theta=\pm 1$ ***(A1)***

$\sin^{-1}(1)=\frac{\pi}{2}$ (accept values in degrees) (seen anywhere) ***A1***

THEN

both correct answers $\theta=\frac{\pi}{2}, \frac{3\pi}{2}$ ***A1*** ***N4***

Note:	Award ***A0*** if either or both correct answers are given in degrees. Award ***A0*** if additional values are given.

[6 marks]

Total [15 marks]

Mathematics
Standard level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2218 – 7305

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Let $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, where O is the origin. L_1 is the line that passes through A and B.

(a) Find a vector equation for L_1. [2]

(b) The vector $\begin{pmatrix} 2 \\ p \\ 0 \end{pmatrix}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$. Find the value of p. [3]

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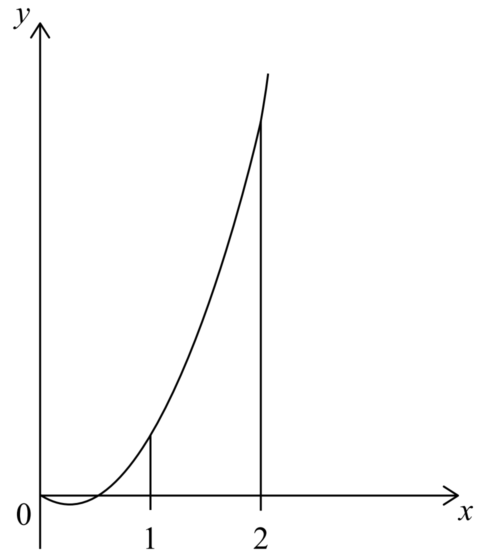
2. [Maximum mark: 6]

Let $f(x) = 6x^2 - 3x$. The graph of f is shown in the following diagram.



(a) Find $\int (6x^2 - 3x)\,\mathrm{d}x$. [2]

(b) Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$. [4]

Turn over

3. [Maximum mark: 6]

A data set has n items. The sum of the items is 800 and the mean is 20.

(a) Find n. [2]

The standard deviation of this data set is 3. Each value in the set is multiplied by 10.

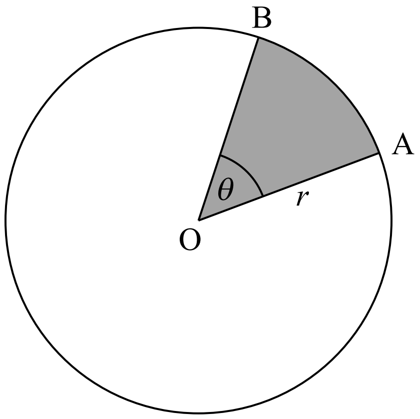
(b) (i) Write down the value of the new mean.

(ii) Find the value of the new variance. [4]

4. [Maximum mark: 7]

The following diagram shows a circle with centre O and radius r cm.

diagram not to scale

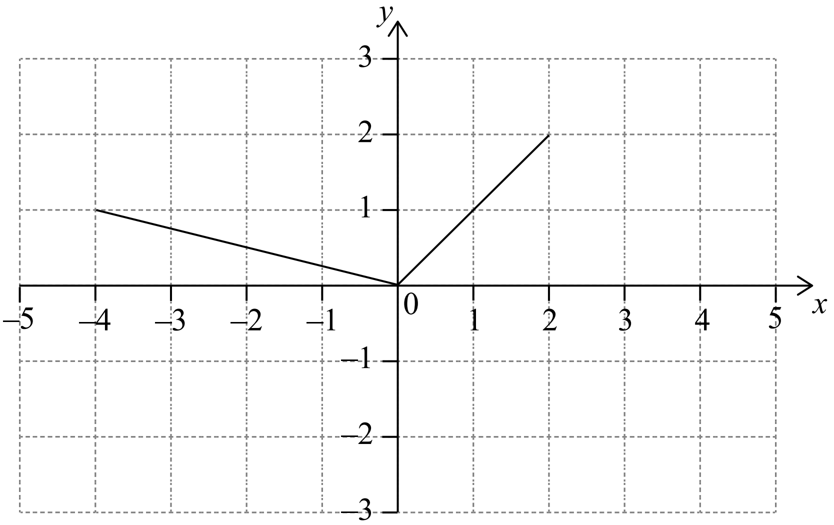


The points A and B lie on the circumference of the circle, and $\text{A}\hat{\text{O}}\text{B} = \theta$. The area of the shaded sector AOB is $12\,\text{cm}^2$ and the length of arc AB is $6\,\text{cm}$.

Find the value of r.

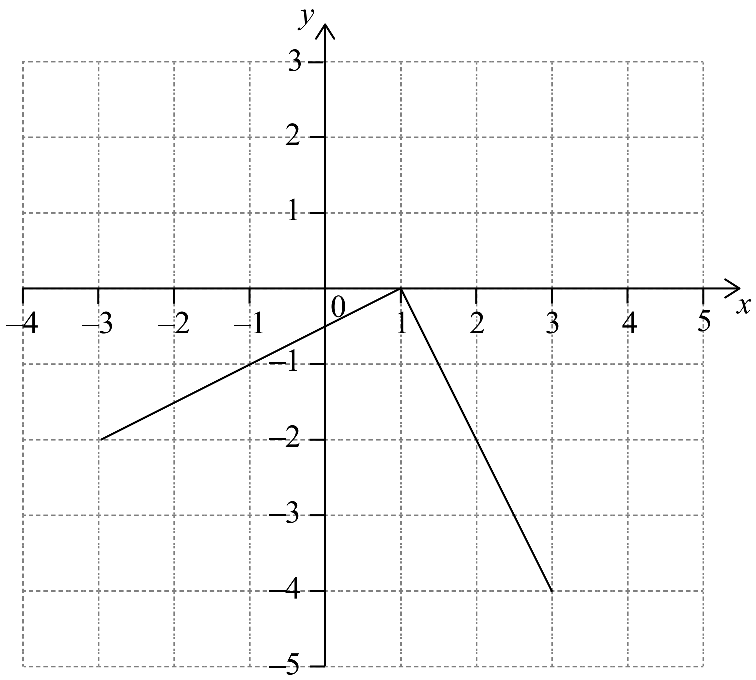
5. [Maximum mark: 6]

The following diagram shows the graph of a function f, for $-4 \le x \le 2$.



(a) On the same axes, sketch the graph of $f(-x)$. [2]

(b) Another function, g, can be written in the form $g(x) = a \times f(x + b)$. The following diagram shows the graph of g.



Write down the value of a and of b. [4]

(This question continues on the following page)

(Question 5 continued)

6. [Maximum mark: 7]

Let $f(x) = px^2 + qx - 4p$, where $p \neq 0$. Find the number of roots for the equation $f(x) = 0$. Justify your answer.

7. [Maximum mark: 8]

An arithmetic sequence has $u_1 = \log_c(p)$ and $u_2 = \log_c(pq)$, where $c > 1$ and $p, q > 0$.

(a) Show that $d = \log_c(q)$. [2]

(b) Let $p = c^2$ and $q = c^3$. Find the value of $\sum_{n=1}^{20} u_n$. [6]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

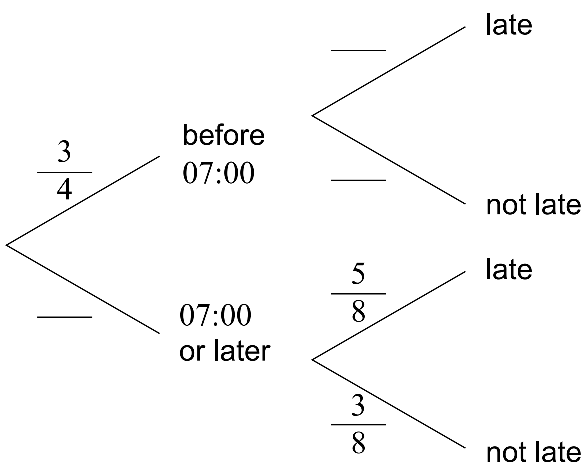
8. [Maximum mark: 14]

Pablo drives to work. The probability that he leaves home before $07{:}00$ is $\frac{3}{4}$.

If he leaves home before $07{:}00$ the probability he will be late for work is $\frac{1}{8}$.

If he leaves home at $07{:}00$ or later the probability he will be late for work is $\frac{5}{8}$.

(a) **Copy** and complete the following tree diagram. [3]



(b) Find the probability that Pablo leaves home before $07{:}00$ and is late for work. [2]

(c) Find the probability that Pablo is late for work. [3]

(d) Given that Pablo is late for work, find the probability that he left home before $07{:}00$. [3]

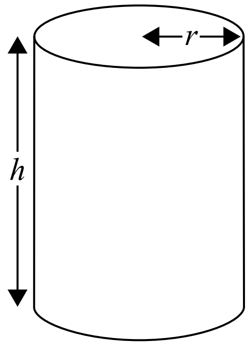
(e) Two days next week Pablo will drive to work. Find the probability that he will be late at least once. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A closed cylindrical can with radius r centimetres and height h centimetres has a volume of $20\pi \text{ cm}^3$.

diagram not to scale



(a) Express h in terms of r. [2]

The material for the base and top of the can costs 10 cents per cm^2 and the material for the curved side costs 8 cents per cm^2. The total cost of the material, in cents, is C.

(b) Show that $C = 20\pi r^2 + \dfrac{320\pi}{r}$. [4]

(c) Given that there is a minimum value for C, find this minimum value in terms of π. [9]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

Consider a function f. The line L_1 with equation $y = 3x + 1$ is a tangent to the graph of f when $x = 2$.

(a) (i) Write down $f'(2)$.

(ii) Find $f(2)$. [4]

Let $g(x) = f(x^2 + 1)$ and P be the point on the graph of g where $x = 1$.

(b) Show that the graph of g has a gradient of 6 at P. [5]

(c) Let L_2 be the tangent to the graph of g at P. L_1 intersects L_2 at the point Q. Find the y-coordinate of Q. [7]

Markscheme

May 2018

Mathematics

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ ***A2*** ***N2***

eg $\boldsymbol{r} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, $\boldsymbol{r} = 2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k} + s(\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for the form $L = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

(b) **METHOD 1**

correct scalar product ***(A1)***

eg $(1 \times 2) + (3 \times p) + (1 \times 0)$, $2 + 3p$

evidence of equating **their** scalar product to zero ***(M1)***

eg $\boldsymbol{a} \bullet \boldsymbol{b} = 0$, $2 + 3p = 0$, $3p = -2$

$p = -\frac{2}{3}$ ***A1*** ***N3***

METHOD 2

valid attempt to find angle between vectors ***(M1)***

correct substitution into numerator and/or angle ***(A1)***

eg $\cos\theta = \frac{(1 \times 2) + (3 \times p) + (1 \times 0)}{|a||b|}$, $\cos\theta = 0$

$p = -\frac{2}{3}$ ***A1*** ***N3***

[3 marks]

[Total: 5 marks]

2. (a) $2x^3 - \dfrac{3x^2}{2} + c$ $\left(\text{accept } \dfrac{6x^3}{3} - \dfrac{3x^2}{2} + c\right)$ ***A1A1*** ***N2***

Notes:	Award ***A1A0*** for both correct terms if $+c$ is omitted. Award ***A1A0*** for one correct term *eg* $2x^3 + c$. Award ***A1A0*** if both terms are correct, but candidate attempts further working to solve for c.

[2 marks]

(b) substitution of limits or function ***(A1)***

eg $\int_1^2 f(x)\,dx$, $\left[2x^3 - \dfrac{3x^2}{2}\right]_1^2$

substituting limits into their integrated function and subtracting ***(M1)***

eg $\dfrac{6\times 2^3}{3} - \dfrac{3\times 2^2}{2} - \left(\dfrac{6\times 1^3}{3} - \dfrac{3\times 1^2}{2}\right)$

Note: Award ***M0*** if substituted into original function.

correct working ***(A1)***

eg $\dfrac{6\times 8}{3} - \dfrac{3\times 4}{2} - \dfrac{6\times 1}{3} + \dfrac{3\times 1}{2}$, $(16-6) - \left(2 - \dfrac{3}{2}\right)$

$\dfrac{19}{2}$ ***A1*** ***N3***

[4 marks]

[Total: 6 marks]

3. (a) correct approach *(A1)*

eg $\frac{800}{n}=20$

40 ***A1*** ***N2***

[2 marks]

(b) (i) 200 ***A1*** ***N1***

(ii) **METHOD 1**

recognizing variance $=\sigma^2$ *(M1)*

eg $3^2=9$

correct working to find new variance *(A1)*

eg $\sigma^2\times 10^2,\ 9\times 100$

900 ***A1*** ***N3***

METHOD 2

new standard deviation is 30 *(A1)*

recognizing variance $=\sigma^2$ *(M1)*

eg $3^2=9,\ 30^2$

900 ***A1*** ***N3***

[4 marks]

[Total: 6 marks]

4. evidence of correctly substituting into circle formula (may be seen later) ***A1A1***

eg $\frac{1}{2}\theta r^2=12,\ r\theta=6$

attempt to eliminate one variable *(M1)*

eg $r=\frac{6}{\theta},\ \theta=\frac{l}{r},\ \frac{\frac{1}{2}\theta r^2}{r\theta}=\frac{12}{6}$

correct elimination *(A1)*

eg $\frac{1}{2}\times\frac{6}{r}\times r^2=12,\ \frac{1}{2}\theta\times\left(\frac{6}{\theta}\right)^2=12,\ A=\frac{1}{2}\times r^2\times\frac{l}{r},\ \frac{r^2}{2r}=2$

correct equation *(A1)*

eg $\frac{1}{2}\times 6r=12,\ \frac{1}{2}\times\frac{36}{\theta}=12,\ 12=\frac{1}{2}\times r^2\times\frac{6}{r}$

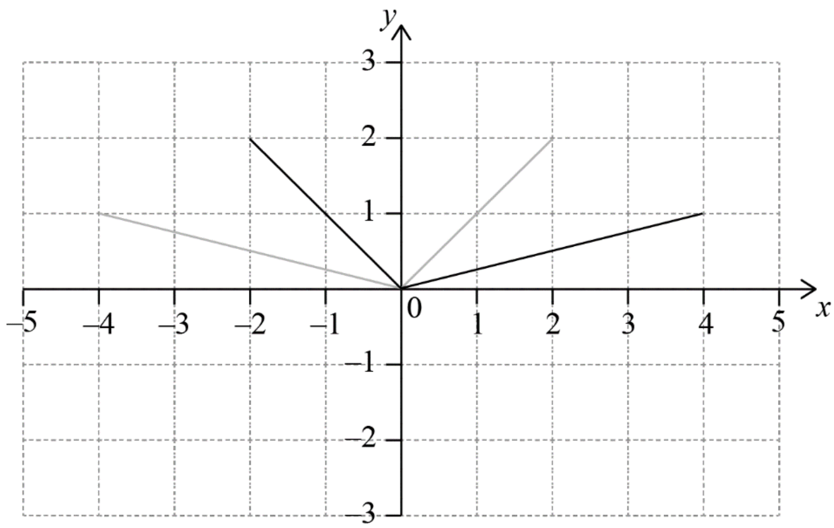
correct working *(A1)*

eg $3r=12,\ \frac{18}{\theta}=12,\ \frac{r}{2}=2,\ 24=6r$

$r=4$ (cm) ***A1*** ***N2***

[7 marks]

5. (a)



A2 ***N2***

[2 marks]

(b) recognizing horizontal shift/translation of 1 unit ***(M1)***

eg $b = 1$, moved 1 right

recognizing vertical stretch/dilation with scale factor 2 ***(M1)***

eg $a = 2$, $y \times (-2)$

$a = -2$, $b = -1$ ***A1A1*** ***N2N2***

[4 marks]

[Total: 6 marks]

6. **METHOD 1**

evidence of discriminant *(M1)*
eg $b^2 - 4ac$, Δ

correct substitution into discriminant *(A1)*
eg $q^2 - 4p(-4p)$

correct discriminant *A1*
eg $q^2 + 16p^2$

$16p^2 > 0$ (accept $p^2 > 0$) *A1*
$q^2 \geq 0$ (do not accept $q^2 > 0$) *A1*

$q^2 + 16p^2 > 0$ *A1*

f has 2 roots *A1* *N0*

METHOD 2

y-intercept $= -4p$ (seen anywhere) *A1*

if p is positive, then the y-intercept will be negative *A1*

an upward-opening parabola with a negative y-intercept *R1*
eg sketch that must indicate $p > 0$.

if p is negative, then the y-intercept will be positive *A1*

a downward-opening parabola with a positive y-intercept *R1*
eg sketch that must indicate $p < 0$.

f has 2 roots *A2* *N0*

[7 marks]

7. (a) valid approach involving addition or subtraction *M1*

eg $u_2 = \log_c p + d$, $u_1 - u_2$

correct application of log law *A1*

eg $\log_c(pq) = \log_c p + \log_c q$, $\log_c\left(\frac{pq}{p}\right)$

$d = \log_c q$ *AG* *N0*

[2 marks]

(b) **METHOD 1** (finding u_1 and d)

recognizing $\sum = S_{20}$ (seen anywhere) *(A1)*

attempt to find u_1 or d using $\log_c c^k = k$ *(M1)*

eg $2\log_c c$, $3\log_c c$, correct value of u_1 or d

$u_1 = 2$, $d = 3$ (seen anywhere) *(A1)(A1)*

correct working *(A1)*

eg $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$, $S_{20} = \frac{20}{2}(2 + 59)$, $10(61)$

$\sum_{n=1}^{20} u_n = 610$ *A1* *N2*

METHOD 2 (expressing S in terms of c)

recognizing $\sum = S_{20}$ (seen anywhere) *(A1)*

correct expression for S in terms of c *(A1)*

eg $10(2\log_c c^2 + 19\log_c c^3)$

$\log_c c^2 = 2$, $\log_c c^3 = 3$ (seen anywhere) *(A1)(A1)*

correct working *(A1)*

eg $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$, $S_{20} = \frac{20}{2}(2 + 59)$, $10(61)$

$\sum_{n=1}^{20} u_n = 610$ *A1* *N2*

continued…

Question 7 continued

METHOD 3 (expressing S in terms of c)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

correct expression for S in terms of c ***(A1)***

eg $10(2\log_c c^2 + 19\log_c c^3)$

correct application of log law ***(A1)***

eg $2\log_c c^2 = \log_c c^4$, $19\log_c c^3 = \log_c c^{57}$, $10\left(\log_c \left(c^2\right)^2 + \log_c \left(c^3\right)^{19}\right)$,

$10\left(\log_c c^4 + \log_c c^{57}\right)$, $10\left(\log_c c^{61}\right)$

correct application of definition of log ***(A1)***

eg $\log_c c^{61} = 61$, $\log_c c^4 = 4$, $\log_c c^{57} = 57$

correct working ***(A1)***

eg $S_{20} = \frac{20}{2}(4+57)$, $10(61)$

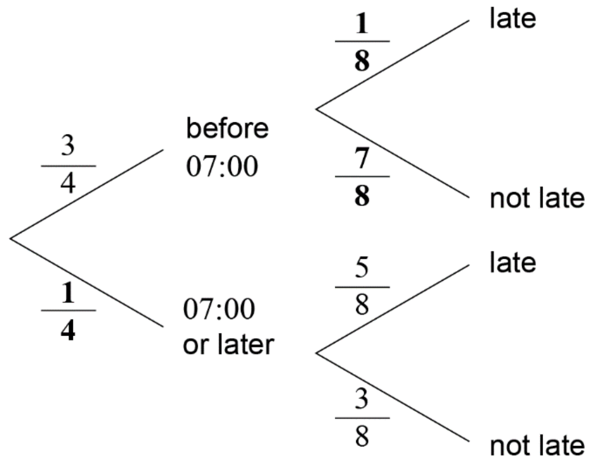
$\sum_{n=1}^{20} u_n = 610$ ***A1*** ***N2***

[6 marks]

[Total: 8 marks]

Section B

8. (a)



A1A1A1 ***N3***

Note: Award ***A1*** for each bold fraction.

[3 marks]

(b) multiplying along correct branches ***(A1)***

eg $\frac{3}{4}\times\frac{1}{8}$

$\mathrm{P}(\text{leaves before } 07{:}00 \cap \text{late}) = \frac{3}{32}$ ***A1*** ***N2***

[2 marks]

(c) multiplying along other "late" branch ***(M1)***

eg $\frac{1}{4}\times\frac{5}{8}$

adding probabilities of two mutually exclusive late paths ***(A1)***

eg $\left(\frac{3}{4}\times\frac{1}{8}\right)+\left(\frac{1}{4}\times\frac{5}{8}\right)$, $\frac{3}{32}+\frac{5}{32}$

$\mathrm{P}(L)=\frac{8}{32}\ \left(=\frac{1}{4}\right)$ ***A1*** ***N2***

[3 marks]

continued…

Question 8 continued

(d) recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P}(A|\,B)$, $\mathrm{P}(\text{before } 7|\text{late})$

correct substitution of **their** values into formula ***(A1)***

eg $\dfrac{\frac{3}{32}}{\frac{1}{4}}$

$\mathrm{P}(\text{left before } 07{:}00|\text{ late}) = \dfrac{3}{8}$ ***A1 N2***

[3 marks]

(e) valid approach ***(M1)***

eg $1-\mathrm{P}(\text{not late twice})$, $\mathrm{P}(\text{late once})+\mathrm{P}(\text{late twice})$

correct working ***(A1)***

eg $1-\left(\dfrac{3}{4}\times\dfrac{3}{4}\right)$, $2\times\dfrac{1}{4}\times\dfrac{3}{4}+\dfrac{1}{4}\times\dfrac{1}{4}$

$\dfrac{7}{16}$ ***A1 N2***

[3 marks]

[Total: 14 marks]

9. (a) correct equation for volume *(A1)*

eg $\pi r^2 h = 20\pi$

$h = \frac{20}{r^2}$ ***A1*** ***N2***

[2 marks]

(b) attempt to find formula for cost of parts *(M1)*

eg $10 \times$ two circles, $8 \times$ curved side

correct expression for cost of two circles in terms of r (seen anywhere) ***A1***

eg $2\pi r^2 \times 10$

correct expression for cost of curved side (seen anywhere) *(A1)*

eg $2\pi r \times h \times 8$

correct expression for cost of curved side in terms of r ***A1***

eg $8 \times 2\pi r \times \frac{20}{r^2}$, $\frac{320\pi r}{r^2}$

$C = 20\pi r^2 + \frac{320\pi}{r}$ ***AG*** ***N0***

[4 marks]

(c) recognize $C' = 0$ at minimum *(R1)*

eg $C' = 0$, $\frac{\mathrm{d}C}{\mathrm{d}r} = 0$

correct differentiation (may be seen in equation)

$C' = 40\pi r - \frac{320\pi}{r^2}$ ***A1A1***

correct equation ***A1***

eg $40\pi r - \frac{320\pi}{r^2} = 0$, $40\pi r = \frac{320\pi}{r^2}$

correct working *(A1)*

eg $40r^3 = 320$, $r^3 = 8$

$r = 2$ (m) ***A1***

attempt to substitute **their** value of r into C

eg $20\pi \times 4 + 320 \times \frac{\pi}{2}$ *(M1)*

correct working

eg $80\pi + 160\pi$ *(A1)*

240π (cents) ***A1*** ***N3***

Note: Do not accept 753.6, 753.98 or 754, even if 240π is seen.

[9 marks]

[Total: 15 marks]

10. (a) (i) recognize that $f'(x)$ is the gradient of the tangent at x ***(M1)***
eg $f'(x) = m$

$f'(2) = 3$ (accept $m = 3$) ***A1*** ***N2***

(ii) recognize that $f(2) = y(2)$ ***(M1)***
eg $f(2) = 3 \times 2 + 1$

$f(2) = 7$ ***A1*** ***N2***

[4 marks]

(b) recognize that the gradient of the graph of g is $g'(x)$ ***(M1)***

choosing chain rule to find $g'(x)$ ***(M1)***
eg $\frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$, $u = x^2 + 1$, $u' = 2x$

$g'(x) = f'(x^2 + 1) \times 2x$ ***A2***

$g'(1) = 3 \times 2$ ***A1***

$g'(1) = 6$ ***AG*** ***N0***

[5 marks]

(c) at Q, $L_1 = L_2$ (seen anywhere) ***(M1)***

recognize that the gradient of L_2 is $g'(1)$ (seen anywhere) ***(M1)***
eg $m = 6$

finding $g(1)$ (seen anywhere) ***(A1)***
eg $g(1) = f(2)$, $g(1) = 7$

attempt to substitute gradient and/or coordinates
into equation of a straight line ***M1***
eg $y - g(1) = 6(x - 1)$, $y - 1 = g'(1)(x - 7)$, $7 = 6(1) + \mathrm{b}$

correct equation for L_2
eg $y - 7 = 6(x - 1)$, $y = 6x + 1$ ***A1***

correct working to find Q ***(A1)***
eg same y-intercept, $3x = 0$

$y = 1$ ***A1*** ***N2***

[7 marks]

[Total: 16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel medio
Prueba 1

Miércoles 2 de mayo de 2018 (tarde)

Número de convocatoria del alumno

1 hora 30 minutos

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba no se permite el uso de ninguna calculadora.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NM** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

2218 – 7309

No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 5]

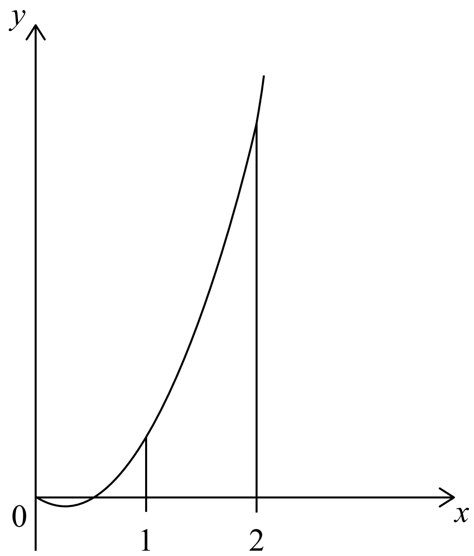
Sean $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ y $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, donde O es el origen. L_1 es la recta que pasa por A y por B.

(a) Halle una ecuación vectorial para L_1. [2]

(b) El vector $\begin{pmatrix} 2 \\ p \\ 0 \end{pmatrix}$ es perpendicular a $\overrightarrow{\mathrm{AB}}$. Halle el valor de p. [3]

2. [Puntuación máxima: 6]

Sea $f(x) = 6x^2 - 3x$. La siguiente figura muestra el gráfico de f.



(a) Halle $\int (6x^2 - 3x)\,\mathrm{d}x$. [2]

(b) Halle el área de la región delimitada por el gráfico de f, el eje x y las rectas $x = 1$ y $x = 2$. [4]

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3. [Puntuación máxima: 6]

Un conjunto de datos consta de n elementos. La suma de esos elementos es igual a 800 y la media es 20.

(a) Halle n. [2]

La desviación típica de este conjunto de datos es igual a 3. Cada uno de los valores del conjunto se multiplica por 10.

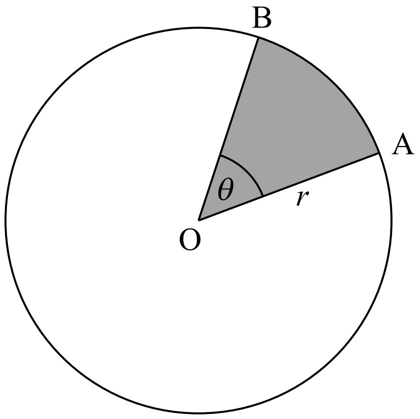
(b) (i) Escriba el nuevo valor de la media.

(ii) Halle el nuevo valor de la varianza. [4]

4. [Puntuación máxima: 7]

La siguiente figura muestra un círculo de centro O y radio r cm.

la figura no está dibujada a escala

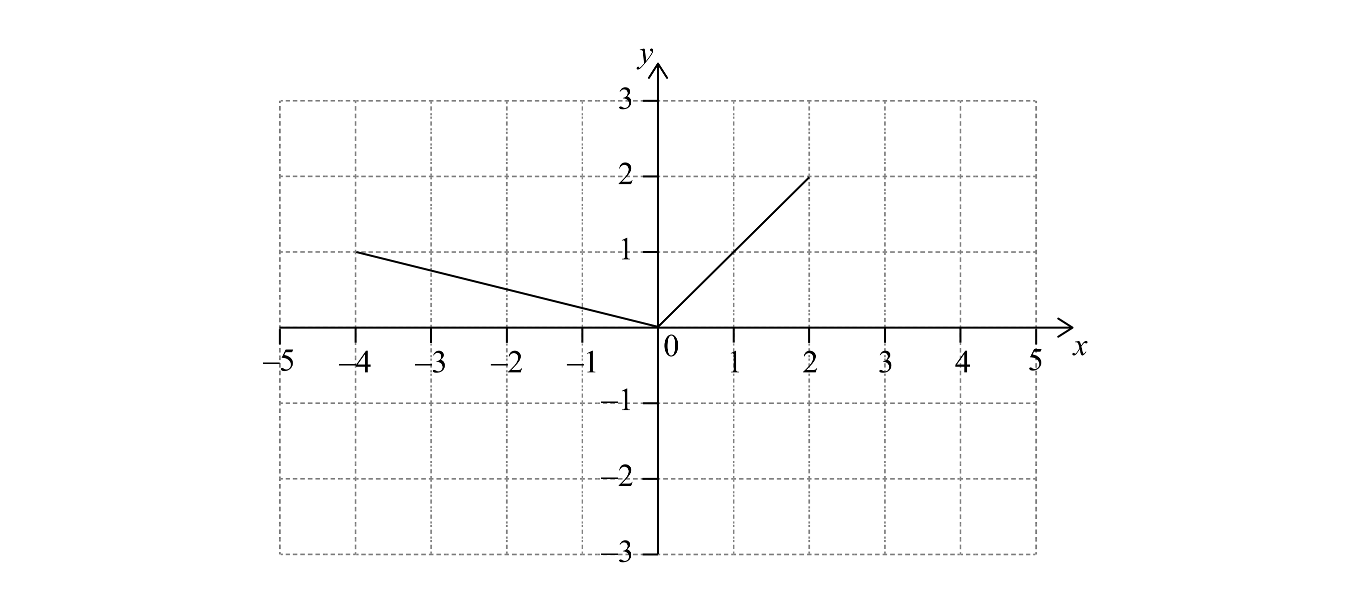


Los puntos A y B pertenecen a la circunferencia del círculo, y $\mathrm{A\hat{O}B} = \theta$. El área del sector circular sombreado AOB es igual a $12\,\mathrm{cm}^2$ y la longitud del arco AB es igual a $6\,\mathrm{cm}$.

Halle el valor de r.

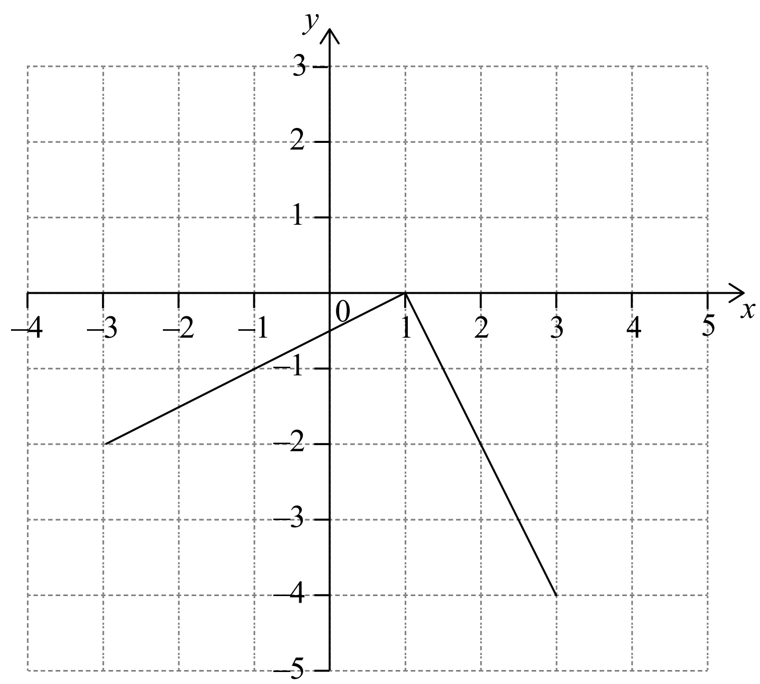
5. [Puntuación máxima: 6]

La siguiente figura muestra el gráfico de una función f, para $-4 \leq x \leq 2$.



(a) Sobre esos mismos ejes de coordenadas, dibuje aproximadamente el gráfico de $f(-x)$. [2]

(b) Otra función, g, se puede escribir de la forma $g(x) = a \times f(x+b)$.
La siguiente figura muestra el gráfico de g.



Escriba el valor de a y el de b. [4]

(Esta pregunta continúa en la página siguiente)

(Pregunta 5: continuación)

6. [Puntuación máxima: 7]

Sea $f(x) = px^2 + qx - 4p$, donde $p \neq 0$. Halle el número de raíces para la ecuación $f(x) = 0$. Justifique su respuesta.

7. [Puntuación máxima: 8]

Una progresión aritmética es tal que $u_1 = \log_c(p)$ y $u_2 = \log_c(pq)$, donde $c > 1$ y $p, q > 0$.

(a) Muestre que $d = \log_c(q)$. [2]

(b) Sean $p = c^2$ y $q = c^3$. Halle el valor de $\sum_{n=1}^{20} u_n$. [6]

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No escriba soluciones en esta página.

Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

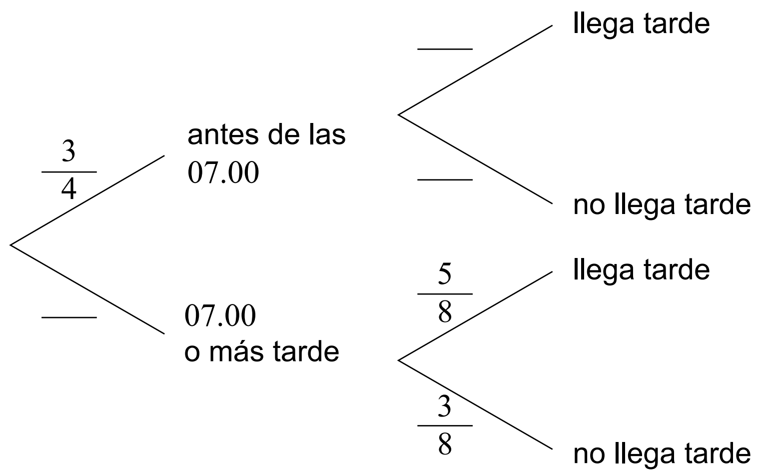
8. [Puntuación máxima: 14]

Pablo va al trabajo en coche. La probabilidad de que salga de casa antes de las 07.00 es igual a $\frac{3}{4}$.

Si sale de casa antes de las 07.00, la probabilidad de que llegue tarde al trabajo es igual a $\frac{1}{8}$.

Si sale de casa a las 07.00 o más tarde, la probabilidad de que llegue tarde al trabajo es igual a $\frac{5}{8}$.

(a) **Copie** y complete el siguiente diagrama de árbol. [3]



(b) Halle la probabilidad de que Pablo salga de casa antes de las 07.00 y llegue tarde al trabajo. [2]

(c) Halle la probabilidad de que Pablo llegue tarde al trabajo. [3]

(d) Sabiendo que Pablo ha llegado tarde al trabajo, halle la probabilidad de que haya salido de casa antes de las 07.00. [3]

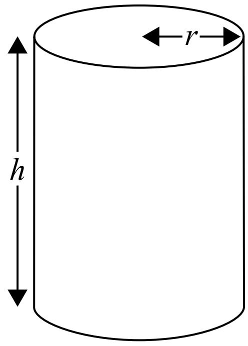
(e) La próxima semana habrá dos días en los que Pablo irá al trabajo en coche. Halle la probabilidad de que llegue tarde al menos una vez. [3]

No escriba soluciones en esta página.

9. [Puntuación máxima: 15]

Una lata cilíndrica cerrada de radio r centímetros y altura h centímetros tiene un volumen de $20\pi\,\text{cm}^3$.

la figura no está dibujada a escala



(a) Exprese h en función de r. [2]

El material del que están hechas la base y la parte superior de la lata cuesta 10 céntimos por cm^2 y el material del lado curvo cuesta 8 céntimos por cm^2. El coste total del material, en céntimos, es igual a C.

(b) Muestre que $C = 20\pi r^2 + \dfrac{320\pi}{r}$. [4]

(c) Sabiendo que existe un valor mínimo para C, halle dicho valor mínimo en función de π. [9]

No escriba soluciones en esta página.

10. [Puntuación máxima: 16]

Considere una función f. La recta L_1, cuya ecuación es $y = 3x + 1$, es tangente al gráfico de f en $x = 2$.

(a) (i) Escriba $f'(2)$.

(ii) Halle $f(2)$. [4]

Sea $g(x) = f(x^2 + 1)$ y sea P el punto del gráfico de g para $x = 1$.

(b) Muestre que la pendiente del gráfico de g en P es igual a 6. [5]

(c) Sea L_2 la tangente al gráfico de g en P. L_1 y L_2 se cortan en el punto Q. Halle la coordenada y de Q. [7]

Mathématiques
Niveau moyen
Épreuve 1

Mercredi 2 mai 2018 (après-midi)

Numéro de session du candidat

1 heure 30 minutes

Instructions destinées aux candidats

- Écrivez votre numéro de session dans les cases ci-dessus.
- N'ouvrez pas cette épreuve avant d'y être autorisé(e).
- Aucune calculatrice n'est autorisée pour cette épreuve.
- Section A : répondez à toutes les questions. Rédigez vos réponses dans les cases prévues à cet effet.
- Section B : répondez à toutes les questions sur le livret de réponses prévu à cet effet. Écrivez votre numéro de session sur la première page du livret de réponses, et attachez ce livret à cette épreuve d'examen et à votre page de couverture en utilisant l'attache fournie.
- Sauf indication contraire dans l'intitulé de la question, toutes les réponses numériques devront être exactes ou correctes à trois chiffres significatifs près.
- Un exemplaire non annoté du **livret de formules pour le cours de mathématiques NM** est nécessaire pour cette épreuve.
- Le nombre maximum de points pour cette épreuve d'examen est de **[90 points]**.

2218 – 7307

Le total des points ne sera pas nécessairement attribué pour une réponse correcte si le raisonnement n'a pas été indiqué. Les réponses doivent être appuyées par un raisonnement et/ou des explications. Lorsque la réponse est fausse, certains points peuvent être attribués si la méthode utilisée est correcte, pour autant que le raisonnement soit indiqué par écrit. On vous recommande donc de montrer tout votre raisonnement.

Section A

Répondez à **toutes** les questions. Rédigez vos réponses dans les cases prévues à cet effet. Si cela est nécessaire, vous pouvez poursuivre votre raisonnement en dessous des lignes.

1. [Note maximale : 5]

Soit $\vec{\mathrm{OA}} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ et $\vec{\mathrm{AB}} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, où O est l'origine. L_1 est la droite passant par A et B.

(a) Trouvez une équation vectorielle de L_1. [2]

(b) Le vecteur $\begin{pmatrix} 2 \\ p \\ 0 \end{pmatrix}$ est perpendiculaire à $\vec{\mathrm{AB}}$. Trouvez la valeur de p. [3]

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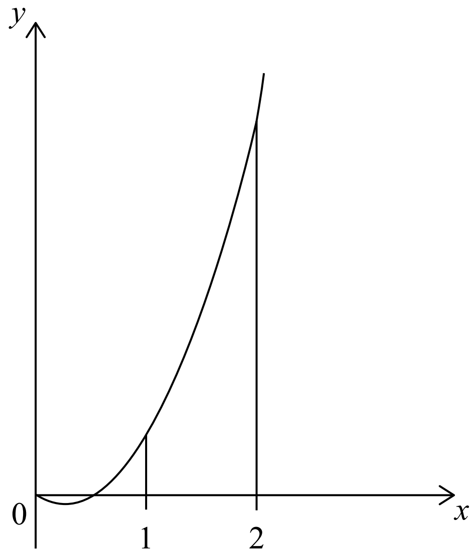
2. [Note maximale : 6]

Soit $f(x) = 6x^2 - 3x$. Le diagramme suivant montre la représentation graphique de f.



(a) Trouvez $\int (6x^2 - 3x)\,\mathrm{d}x$. [2]

(b) Trouvez l'aire de la région délimitée par la représentation graphique de f, l'axe des abscisses et les droites $x = 1$ et $x = 2$. [4]

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3. [Note maximale : 6]

Un ensemble de données comprend n valeurs. La somme des valeurs est 800 et la moyenne est 20.

(a) Trouvez n. [2]

L'écart type de cet ensemble de données est 3. Chaque valeur de l'ensemble est multipliée par 10.

(b) (i) Écrivez la valeur de la nouvelle moyenne.

(ii) Trouvez la valeur de la nouvelle variance. [4]

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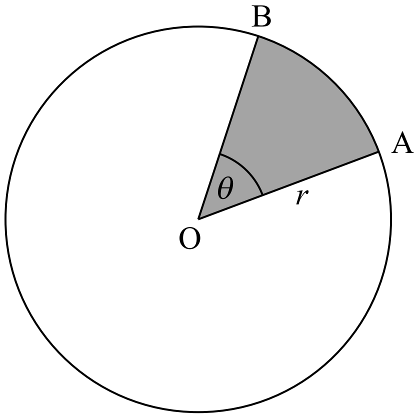
4. [Note maximale : 7]

Le diagramme suivant montre un cercle de centre O et de rayon égal à r cm.

la figure n’est pas à l’échelle

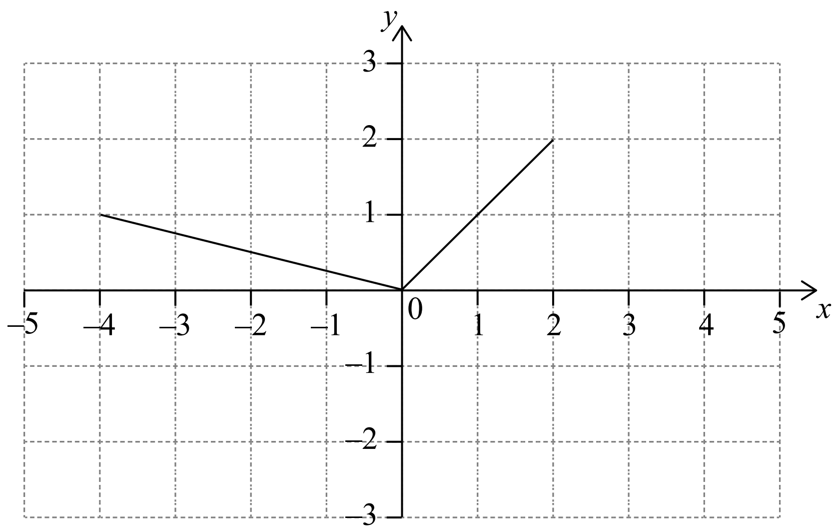


Les points A et B sont situés sur la circonférence du cercle et $\text{A}\hat{\text{O}}\text{B} = \theta$. L’aire du secteur grisé AOB est $12\,\text{cm}^2$ et la longueur de l’arc AB est $6\,\text{cm}$.

Trouvez la valeur de r.

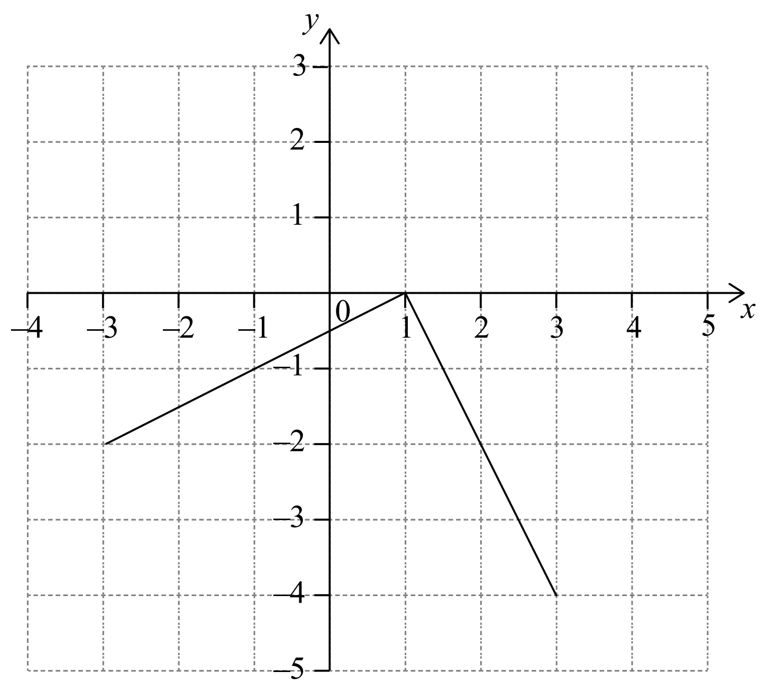
5. [Note maximale : 6]

Le diagramme suivant montre la représentation graphique d'une fonction f, pour $-4 \le x \le 2$.



(a) Sur ce même système d'axes, esquissez la représentation graphique de $f(-x)$. [2]

(b) Une autre fonction, g, peut s'écrire sous la forme $g(x) = a \times f(x+b)$. Le diagramme suivant montre la représentation graphique de g.



Écrivez la valeur de a et celle de b. [4]

(Suite de la question à la page suivante)

(Suite de la question 5)

6. [Note maximale : 7]

Soit $f(x) = px^2 + qx - 4p$, où $p \neq 0$. Trouvez le nombre de racines de l'équation $f(x) = 0$. Justifiez votre réponse.

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7. [Note maximale : 8]

Une suite arithmétique est telle que $u_1 = \log_c(p)$ et $u_2 = \log_c(pq)$, où $c > 1$ et $p, q > 0$.

(a) Montrez que $d = \log_c(q)$. [2]

(b) Soit $p = c^2$ et $q = c^3$. Trouvez la valeur de $\sum_{n=1}^{20} u_n$. [6]

N’écrivez **pas** vos solutions sur cette page.

Section B

Répondez à **toutes** les questions sur le livret de réponses fourni. Veuillez répondre à chaque question sur une nouvelle page.

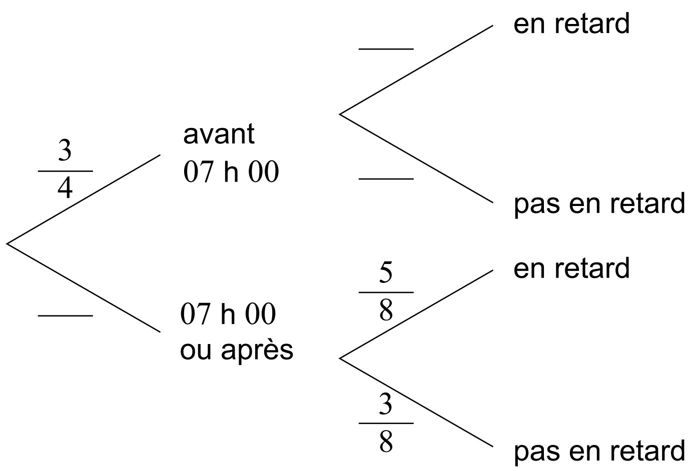
8. [Note maximale : 14]

Pablo se rend au travail en voiture. La probabilité qu’il quitte la maison avant $07 \text{ h } 00$ est de $\frac{3}{4}$.

S’il quitte la maison avant $07 \text{ h } 00$, la probabilité qu’il soit en retard au travail est de $\frac{1}{8}$.

S’il quitte la maison à $07 \text{ h } 00$ ou après, la probabilité qu’il soit en retard au travail est de $\frac{5}{8}$.

(a) **Recopiez** et complétez le diagramme en arbre suivant. [3]



(b) Trouvez la probabilité que Pablo quitte la maison avant $07 \text{ h } 00$ et qu’il soit en retard au travail. [2]

(c) Trouvez la probabilité que Pablo soit en retard au travail. [3]

(d) Sachant que Pablo est en retard au travail, trouvez la probabilité qu’il ait quitté la maison avant $07 \text{ h } 00$. [3]

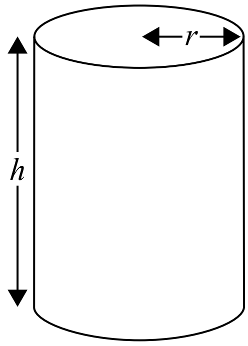
(e) Au cours de la semaine prochaine, Pablo se rendra en voiture au travail deux jours. Trouvez la probabilité qu’il soit au moins une fois en retard. [3]

N'écrivez **pas** vos solutions sur cette page.

9. [Note maximale : 15]

Une boîte en métal, cylindrique et fermée, de rayon égal à r centimètres et de hauteur égale à h centimètres possède un volume de $20\pi\ \text{cm}^3$.

la figure n'est pas à l'échelle



(a) Exprimez h en fonction de r. [2]

Le métal pour la base et le couvercle de la boîte coûte 10 cents le cm^2 et le métal pour le côté incurvé coûte 8 cents le cm^2. Le coût total du métal, en cents, est C.

(b) Montrez que $C = 20\pi r^2 + \dfrac{320\pi}{r}$. [4]

(c) Sachant qu'il existe une valeur minimale pour C, trouvez cette valeur minimale en fonction de π. [9]

N'écrivez **pas** vos solutions sur cette page.

10. [Note maximale : 16]

Considérez une fonction f. La droite L_1 d'équation $y = 3x + 1$ est une tangente à la représentation graphique de f lorsque $x = 2$.

(a) (i) Écrivez $f'(2)$.

(ii) Trouvez $f(2)$. [4]

Soit $g(x) = f(x^2 + 1)$ et P le point sur la représentation graphique de g où $x = 1$.

(b) Montrez que la représentation graphique de g a une pente de 6 au point P. [5]

(c) Soit L_2 la tangente à la représentation graphique de g au point P. L_1 coupe L_2 au point Q. Trouvez l'ordonnée de Q. [7]

Mathematics
Standard level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2218 – 7304

15 pages

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Let $f(x) = \ln x - 5x$, for $x > 0$.

(a) Find $f'(x)$. [2]

(b) Find $f''(x)$. [1]

(c) Solve $f'(x) = f''(x)$. [2]

2. [Maximum mark: 6]

A biased four-sided die is rolled. The following table gives the probability of each score.

Score	1	2	3	4
Probability	0.28	k	0.15	0.3

(a) Find the value of k. [2]

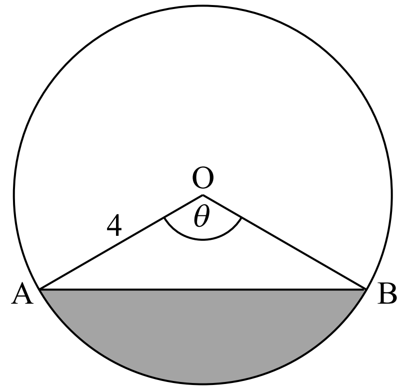
(b) Calculate the expected value of the score. [2]

(c) The die is rolled 80 times. On how many rolls would you expect to obtain a three? [2]

3. [Maximum mark: 6]

The diagram shows a circle, centre O, with radius $4\,\text{cm}$. Points A and B lie on the circumference of the circle and $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 \le \theta \le \pi$.

diagram not to scale



(a) Find the area of the shaded region, in terms of θ. [3]

(b) The area of the shaded region is $12\,\text{cm}^2$. Find the value of θ. [3]

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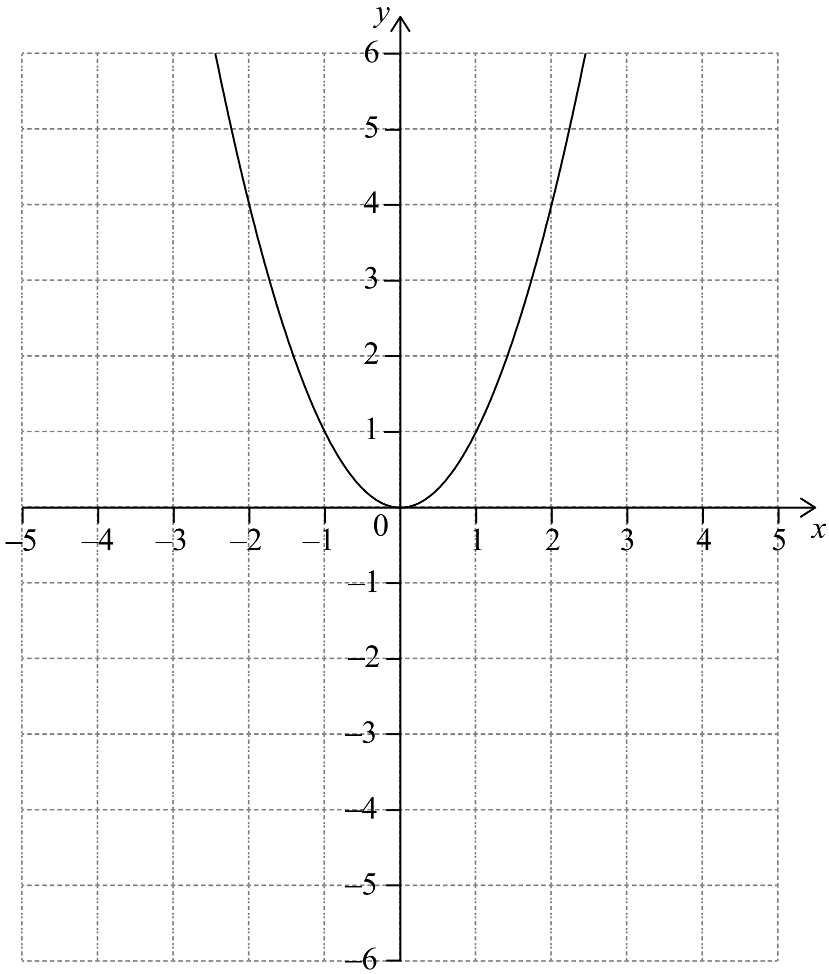
4. [Maximum mark: 7]

Let $g(x) = -(x-1)^2 + 5$.

(a) Write down the coordinates of the vertex of the graph of g. [1]

Let $f(x) = x^2$. The following diagram shows part of the graph of f.



The graph of g intersects the graph of f at $x = -1$ and $x = 2$.

(b) On the grid above, sketch the graph of g for $-2 \le x \le 4$. [3]

(c) Find the area of the region enclosed by the graphs of f and g. [3]

(This question continues on the following page)

(Question 4 continued)

Please **do not** write on this page.

Answers written on this page
will not be marked.

5. [Maximum mark: 6]

Two events A and B are such that $\mathrm{P}(A) = 0.62$ and $\mathrm{P}(A \cap B) = 0.18$.

(a) Find $\mathrm{P}(A \cap B')$. [2]

(b) Given that $\mathrm{P}\big((A \cup B)'\big) = 0.19$, find $\mathrm{P}(A \mid B')$. [4]

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6. [Maximum mark: 7]

Triangle ABC has $a = 8.1\ \text{cm}$, $b = 12.3\ \text{cm}$ and area $15\ \text{cm}^2$. Find the largest possible perimeter of triangle ABC.

7. [Maximum mark: 8]

Let $f(x) = e^{2\sin\left(\frac{\pi x}{2}\right)}$, for $x > 0$.

The kth maximum point on the graph of f has x-coordinate x_k where $k \in \mathbb{Z}^+$.

(a) Given that $x_{k+1} = x_k + a$, find a. [4]

(b) Hence find the value of n such that $\sum_{k=1}^{n} x_k = 861$. [4]

Turn over

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

The following table shows values of $\ln x$ and $\ln y$.

$\ln x$	1.10	2.08	4.30	6.03
$\ln y$	5.63	5.22	4.18	3.41

The relationship between $\ln x$ and $\ln y$ can be modelled by the regression equation $\ln y = a \ln x + b$.

(a) Find the value of a and of b. [3]

(b) Use the regression equation to estimate the value of y when $x = 3.57$. [3]

The relationship between x and y can be modelled using the formula $y = kx^n$, where $k \neq 0$, $n \neq 0$, $n \neq 1$.

(c) By expressing $\ln y$ in terms of $\ln x$, find the value of n and of k. [7]

Do **not** write solutions on this page.

9. [Maximum mark: 17]

The weights, in grams, of oranges grown in an orchard, are normally distributed with a mean of $297\,\text{g}$. It is known that $79\,\%$ of the oranges weigh more than $289\,\text{g}$ and $9.5\,\%$ of the oranges weigh more than $310\,\text{g}$.

(a) Find the probability that an orange weighs between $289\,\text{g}$ and $310\,\text{g}$. [2]

The weights of the oranges have a standard deviation of σ.

(b) (i) Find the standardized value for $289\,\text{g}$.

(ii) Hence, find the value of σ. [5]

The grocer at a local grocery store will buy the oranges whose weights exceed the 35th percentile.

(c) To the nearest gram, find the minimum weight of an orange that the grocer will buy. [3]

The orchard packs oranges in boxes of 36.

(d) Find the probability that the grocer buys more than half the oranges in a box selected at random. [5]

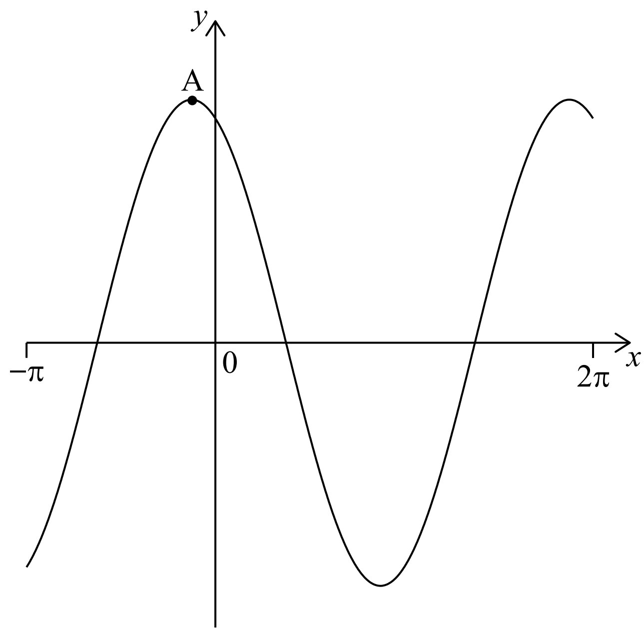
The grocer selects two boxes at random.

(e) Find the probability that the grocer buys more than half the oranges in each box. [2]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

Let $f(x) = 12\cos x - 5\sin x$, $-\pi \le x \le 2\pi$, be a periodic function with $f(x) = f(x + 2\pi)$.
The following diagram shows the graph of f.



There is a maximum point at A. The minimum value of f is -13.

(a) Find the coordinates of A. [2]

(b) For the graph of f, write down

(i) the amplitude;

(ii) the period. [2]

(c) Hence, write $f(x)$ in the form $p\cos(x + r)$. [3]

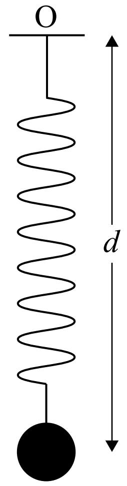
(This question continues on the following page)

Do **not** write solutions on this page.

(Question 10 continued)

A ball on a spring is attached to a fixed point O. The ball is then pulled down and released, so that it moves back and forth vertically.

diagram not to scale



The distance, d centimetres, of the centre of the ball from O at time t seconds, is given by

$$d(t) = f(t) + 17, \ 0 \le t \le 5.$$

(d) Find the maximum speed of the ball. [3]

(e) Find the first time when the ball's speed is changing at a rate of $2\,\text{cm}\,\text{s}^{-2}$. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Mathematics

Standard level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

Implied marks appear in ***brackets*** *eg* ***(M1)****.*
- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1)***.

Must be seen marks appear without ***brackets*** *eg* ***M1***.
- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (****FT****) marks are awarded where an incorrect answer (final or intermediate) from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s) or subpart(s). Usually, to award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the* ***final*** *answer, then* ***FT*** *marks should be awarded if appropriate. Examiners are expected to check student work in order to award* ***FT*** *marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r>1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have *another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

- a truncated 6 sf value
- the exact value if applicable, the correct 3 sf answer

Units will appear in brackets at the end.

Section A

1. (a) $f'(x) = \frac{1}{x} - 5$ ***A1A1*** ***N2***

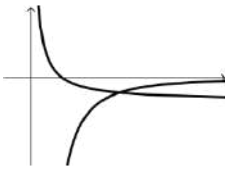
[2 marks]

(b) $f''(x) = -x^{-2}$ ***A1*** ***N1***

[1 mark]

(c) **METHOD 1 (using GDC)**

valid approach ***(M1)***

eg  , -0.358

0.558257

$x = 0.558$ ***A1*** ***N2***

Note: Do not award ***A1*** if additional answers given.

METHOD 2 (analytical)

attempt to solve their equation $f'(x) = f''(x)$ $\left(\text{do not accept } \frac{1}{x} - 5 = -\frac{1}{x^2}\right)$ ***(M1)***

eg $5x^2 - x - 1 = 0$, $\frac{1 \pm \sqrt{21}}{10}$, $\frac{1}{x} = \frac{-1 \pm \sqrt{21}}{2}$, -0.358

0.558257

$x = 0.558$ ***A1*** ***N2***

Note: Do not award ***A1*** if additional answers given.

[2 marks]

Total [5 marks]

2. (a) evidence of summing to 1 ***(M1)***

eg $0.28+k+0.15+0.3=1$, $0.73+k=1$

$k=0.27$ ***A1*** ***N2***

[2 marks]

(b) correct substitution into formula for $\mathrm{E}(X)$ ***(A1)***

eg $1\times 0.28+2\times k+3\times 0.15+4\times 0.3$

$\mathrm{E}(X)=2.47$ (exact) ***A1*** ***N2***

[2 marks]

(c) valid approach ***(M1)***

eg np, 80×0.15

12 ***A1*** ***N2***

[2 marks]

Total [6 marks]

3. (a) valid approach to find area of segment ***(M1)***

eg area of sector – area of triangle, $\frac{1}{2}r^2(\theta-\sin\theta)$

correct substitution ***(A1)***

eg $\frac{1}{2}(4)^2\theta-\frac{1}{2}(4)^2\sin\theta$, $\frac{1}{2}\times 16[\theta-\sin\theta]$

area $=8\theta-8\sin\theta$, $8(\theta-\sin\theta)$ ***A1*** ***N2***

[3 marks]

(b) setting **their** area expression equal to 12 ***(M1)***

eg $12=8(\theta-\sin\theta)$

2.26717

$\theta=2.27$ (do not accept an answer in degrees) ***A2*** ***N3***

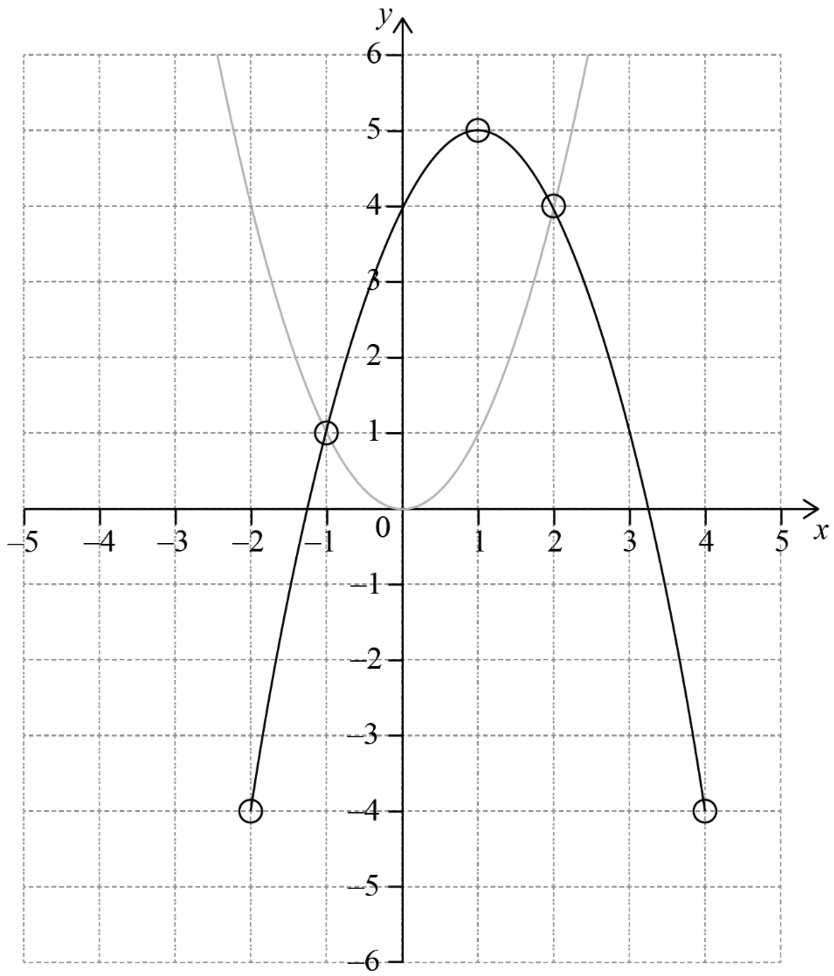
[3 marks]

Total [6 marks]

4. (a) $(1, 5)$ (exact) ***A1*** ***N1***

[1 mark]

(b)



A1A1A1 ***N3***

Note: The shape must be a concave-down parabola.
Only if the shape is correct, award the following for points in circles:
A1 for vertex,
A1 for correct intersection points,
A1 for correct endpoints.

[3 marks]

(c) integrating and subtracting functions (in any order) ***(M1)***

eg $\int f - g$

correct substitution of limits or functions (accept missing $\mathrm{d}x$, but do not accept any errors, including extra bits) ***(A1)***

eg $\int_{-1}^{2} g - f$, $\int -(x-1)^2 + 5 - x^2$

area $= 9$ (exact) ***A1*** ***N2***

[3 marks]

Total [7 marks]

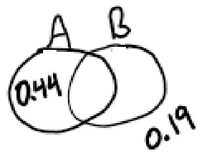
5. (a) valid approach

eg Venn diagram, $\mathrm{P}(A)-\mathrm{P}(A\cap B)$, $0.62-0.18$ ***(M1)***

$\mathrm{P}(A\cap B')=0.44$ ***A1*** ***N2***

[2 marks]

(b) valid approach to find either $\mathrm{P}(B')$ or $\mathrm{P}(B)$ ***(M1)***

eg 

(seen anywhere), $1-\mathrm{P}(A\cap B')-\mathrm{P}\left((A\cup B)'\right)$

correct calculation for $\mathrm{P}(B')$ or $\mathrm{P}(B)$ ***(A1)***

eg $0.44+0.19$, $0.81-0.62+0.18$

correct substitution into $\dfrac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}$ ***(A1)***

eg $\dfrac{0.44}{0.19+0.44}$, $\dfrac{0.44}{1-0.37}$

0.698412

$\mathrm{P}(A\,|\,B')=\dfrac{44}{63}$ (exact), 0.698 ***A1*** ***N3***

[4 marks]

Total [6 marks]

6. correct substitution into the formula for area of a triangle ***(A1)***

eg $15 = \frac{1}{2} \times 8.1 \times 12.3 \times \sin C$

correct working for angle C ***(A1)***

eg $\sin C = 0.301114$, $17.5245\ldots$, 0.305860

recognizing that obtuse angle needed ***(M1)***

eg 162.475, 2.83573, $\cos C < 0$

evidence of choosing the cosine rule ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc\cos(A)$

correct substitution into cosine rule to find c ***(A1)***

eg $c^2 = (8.1)^2 + (12.3)^2 - 2(8.1)(12.3)\cos C$

$c = 20.1720$ ***(A1)***

$8.1 + 12.3 + 20.1720 = 40.5720$

perimeter $= 40.6$ ***A1*** ***N4***

Total [7 marks]

7. (a) valid approach to find maxima ***(M1)***

eg one correct value of x_k, sketch of f

any two correct consecutive values of x_k ***(A1)(A1)***

eg $x_1 = 1$, $x_2 = 5$

$a = 4$ ***A1*** ***N3***

[4 marks]

(b) recognizing the sequence x_1, x_2, x_3, ..., x_n is arithmetic ***(M1)***

eg $d = 4$

correct expression for sum ***(A1)***

eg $\frac{n}{2}\left(2(1) + 4(n-1)\right)$

valid attempt to solve for n ***(M1)***

eg graph, $2n^2 - n - 861 = 0$

$n = 21$ ***A1*** ***N2***

[4 marks]

Total [8 marks]

Section B

8. (a) valid approach *(M1)*

eg one correct value

-0.453620, 6.14210

$a = -0.454$, $b = 6.14$ ***A1A1*** ***N3***

[3 marks]

(b) correct substitution *(A1)*

eg $-0.454 \ln 3.57 + 6.14$

correct working *(A1)*

eg $\ln y = 5.56484$

261.083 (260.409 from 3 sf)

$y = 261$, ($y = 260$ from 3sf) ***A1*** ***N3***

Note: If no working shown, award ***N1*** for 5.56484. If no working shown, award ***N2*** for $\ln y = 5.56484$.

[3 marks]

(c) **METHOD 1**

valid approach for expressing $\ln y$ in terms of $\ln x$ *(M1)*

eg $\ln y = \ln(kx^n)$, $\ln(kx^n) = a \ln x + b$

correct application of addition rule for logs *(A1)*

eg $\ln k + \ln(x^n)$

correct application of exponent rule for logs ***A1***

eg $\ln k + n \ln x$

comparing one term with regression equation (check ***FT***) *(M1)*

eg $n = a$, $b = \ln k$

correct working for k *(A1)*

eg $\ln k = 6.14210$, $k = \mathrm{e}^{6.14210}$

465.030

$n = -0.454$, $k = 465$ (464 from 3sf) ***A1A1*** ***N2N2***

continued…

Question 8(c) continued

METHOD 2

valid approach **(M1)**

eg $\mathrm{e}^{\ln y} = \mathrm{e}^{a \ln x + b}$

correct use of exponent laws for $\mathrm{e}^{a \ln x + b}$ **(A1)**

eg $\mathrm{e}^{a \ln x} \times \mathrm{e}^{b}$

correct application of exponent rule for $a \ln x$ **(A1)**

eg $\ln x^{a}$

correct equation in *y* **A1**

eg $y = x^{a} \times \mathrm{e}^{b}$

comparing one term with equation of model (check ***FT***) **(M1)**

eg $k = \mathrm{e}^{b}$, $n = a$

465.030

$n = -0.454$, $k = 465$ $(464$ from 3sf) **A1A1** **N2N2**

METHOD 3

valid approach for expressing $\ln y$ in terms of $\ln x$ (seen anywhere) **(M1)**

eg $\ln y = \ln(kx^{n})$, $\ln(kx^{n}) = a \ln x + b$

correct application of exponent rule for logs (seen anywhere) **(A1)**

eg $\ln(x^{a}) + b$

correct working for b (seen anywhere) **(A1)**

eg $b = \ln(\mathrm{e}^{b})$

correct application of addition rule for logs **A1**

eg $\ln(\mathrm{e}^{b} x^{a})$

comparing one term with equation of model (check ***FT***) **(M1)**

eg $k = \mathrm{e}^{b}$, $n = a$

465.030

$n = -0.454$, $k = 465$ $(464$ from 3sf) **A1A1** **N2N2**

[7 marks]

Total [13 marks]

9. (a) correct approach indicating subtraction *(A1)*

eg $0.79 - 0.095$, appropriate shading in diagram

$\mathrm{P}(289 < w < 310) = 0.695$ (exact), 69.5% ***A1*** ***N2***

[2 marks]

(b) **METHOD 1**

(i) valid approach *(M1)*

eg $1 - p$, 21

-0.806421

$z = -0.806$ ***A1*** ***N2***

(ii) attempt to standardize *(M1)*

eg $\sigma = \frac{289 - 297}{z}$, $\frac{289 - 297}{\sigma}$

correct substitution with their z (do not accept a probability) ***A1***

eg $-0.806 = \frac{289 - 297}{\sigma}$, $\frac{289 - 297}{-0.806}$

9.92037

$\sigma = 9.92$ ***A1*** ***N2***

METHOD 2

(i) & (ii)

correct expression for z (seen anywhere) *(A1)*

eg $\frac{289 - \mu}{\sigma}$

valid approach *(M1)*

eg $1 - p$, 21

-0.806421

$z = -0.806$ (seen anywhere) ***A1*** ***N2***

valid attempt to set up an equation with **their** z
(do not accept a probability) *(M1)*

eg $-0.806 = \frac{289 - 297}{\sigma}$, $\frac{289 - 297}{-0.806}$

9.92037

$\sigma = 9.92$ ***A1*** ***N2***

[5 marks]

continued…

Question 9 continued

(c) valid approach ***(M1)***
eg $\mathrm{P}(W < w) = 0.35$, -0.385320 (accept 0.385320), diagram showing values in a standard normal distribution

correct score at the 35th percentile ***(A1)***
eg 293.177

294 (g) ***A1*** ***N2***

> **Note:** If working shown, award ***(M1)(A1)A0*** for 293.
> If no working shown, award ***N1*** for 293.177, ***N1*** for 293.
>
> Exception to the ***FT*** rule: If the score is incorrect, and working shown, award ***A1FT*** for correctly finding their minimum weight (by rounding up)

[3 marks]

(d) evidence of recognizing binomial (seen anywhere) ***(M1)***
eg $X \sim \mathrm{B}(36,\ p)$, ${}_n\mathrm{C}_a \times p^a \times q^{n-a}$

correct probability (seen anywhere) ***(A1)***
eg 0.65

EITHER

finding $\mathrm{P}(X \le 18)$ from GDC ***(A1)***
eg 0.045720

evidence of using complement ***(M1)***
eg $1 - \mathrm{P}(X \le 18)$

0.954279

$\mathrm{P}\left(X > 18\right) = 0.954$ ***A1*** ***N2***

OR

recognizing $\mathrm{P}(X > 18) = \mathrm{P}(X \ge 19)$ ***(M1)***
summing terms from 19 to 36 ***(A1)***
eg $\mathrm{P}(X = 19) + \mathrm{P}(X = 20) + \ldots + \mathrm{P}(X = 36)$

0.954279

$\mathrm{P}(X > 18) = 0.954$ ***A1*** ***N2***
[5 marks]

(e) correct calculation ***(A1)***
0.954^2, $\begin{pmatrix} 2 \\ 2 \end{pmatrix} 0.954^2 (1 - 0.954)^0$

0.910650

0.911 ***A1*** ***N2***
[2 marks]

Total [17 marks]

10. (a) $-0.394791, 13$

$\mathrm{A}(-0.395, 13)$ ***A1A1*** ***N2***

[2 marks]

(b) (i) 13 ***A1*** ***N1***

(ii) 2π, 6.28 ***A1*** ***N1***

[2 marks]

(c) valid approach ***(M1)***

eg recognizing that amplitude is p or shift is r

$f(x) = 13\cos(x + 0.395)$ (accept $p = 13$, $r = 0.395$) ***A1A1*** ***N3***

Note: Accept any value of r of the form $0.395 + 2\pi k$, $k \in \mathbb{Z}$

[3 marks]

(d) recognizing need for $d'(t)$ ***(M1)***

eg $-12\sin(t) - 5\cos(t)$

correct approach (accept any variable for t) ***(A1)***

eg $-13\sin(t + 0.395)$, sketch of d', $(1.18, -13)$, $t = 4.32$

maximum speed $= 13\ (\mathrm{cm\,s^{-1}})$ ***A1*** ***N2***

[3 marks]

(e) recognizing that acceleration is needed ***(M1)***

eg $a(t)$, $d''(t)$

correct equation (accept any variable for t) ***(A1)***

eg $a(t) = -2$, $\left|\frac{\mathrm{d}}{\mathrm{d}t}\left(d'(t)\right)\right| = 2$, $-12\cos(t) + 5\sin(t) = -2$

valid attempt to solve **their** equation ***(M1)***

eg sketch, 1.33

1.02154

1.02 ***A2*** ***N3***

[5 marks]

Total [15 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2218 – 7306

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following table shows the mean weight, y kg, of children who are x years old.

Age (x years)	1.25	2.25	3.5	4.4	5.85
Weight (y kg)	10	13	14	17	19

The relationship between the variables is modelled by the regression line with equation $y = ax + b$.

(a) (i) Find the value of a and of b.

(ii) Write down the correlation coefficient. [4]

(b) Use your equation to estimate the mean weight of a child that is 1.95 years old. [2]

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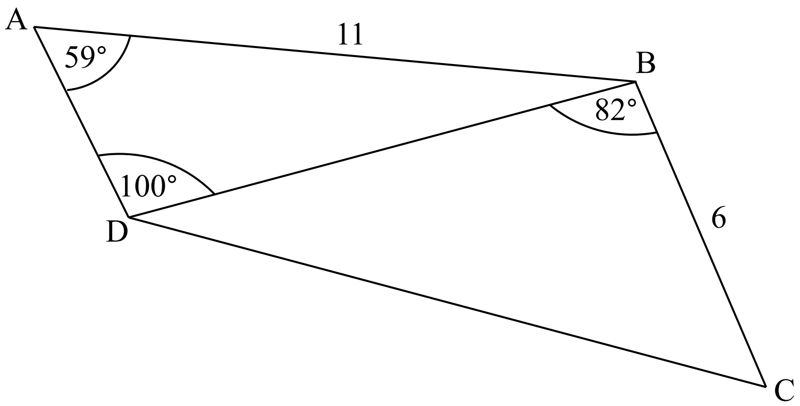
2. [Maximum mark: 6]

The following diagram shows quadrilateral ABCD.

diagram not to scale



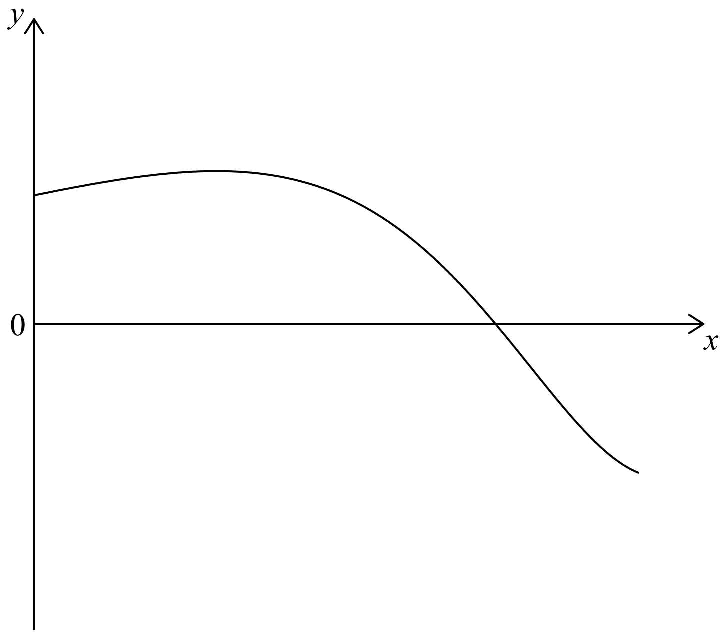
$AB = 11\text{ cm}$, $BC = 6\text{ cm}$, $B\hat{A}D = 59^\circ$, $A\hat{D}B = 100^\circ$, and $C\hat{B}D = 82^\circ$

(a) Find DB. [3]

(b) Find DC. [3]

3. [Maximum mark: 5]

Let $f(x) = \sin(e^x)$ for $0 \le x \le 1.5$. The following diagram shows the graph of f.



(a) Find the x-intercept of the graph of f. [2]

(b) The region enclosed by the graph of f, the y-axis and the x-axis is rotated 360° about the x-axis.

Find the volume of the solid formed. [3]

4. [Maximum mark: 7]

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

(a) Find the common ratio. [2]

(b) Find the sum of the first 8 terms. [2]

(c) Find the least value of n for which $S_n > 163$. [3]

Please **do not** write on this page.

Answers written on this page
will not be marked.

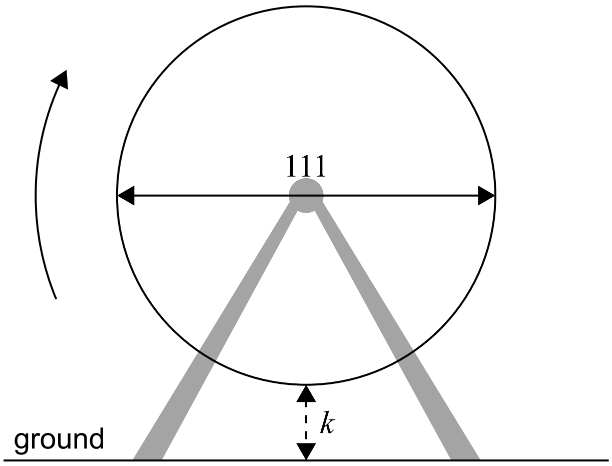
5. [Maximum mark: 6]

Consider the expansion of $\left(2x+\frac{k}{x}\right)^9$, where $k>0$. The coefficient of the term in x^3 is equal to the coefficient of the term in x^5. Find k.

6. [Maximum mark: 8]

At an amusement park, a Ferris wheel with diameter 111 metres rotates at a constant speed. The bottom of the wheel is k metres above the ground. A seat starts at the bottom of the wheel.

diagram not to scale



The wheel completes one revolution in 16 minutes.

(a) After 8 minutes, the seat is $117\,\mathrm{m}$ above the ground. Find k. [2]

After t minutes, the height of the seat above ground is given by $h(t) = 61.5 + a\cos\left(\frac{\pi}{8}t\right)$, for $0 \le t \le 32$.

(b) Find the value of a. [3]

(c) Find when the seat is $30\,\mathrm{m}$ above the ground for the third time. [3]

(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 7]

Let $f(x)=\dfrac{8x-5}{cx+6}$ for $x\neq-\dfrac{6}{c}$, $c\neq0$.

(a) The line $x=3$ is a vertical asymptote to the graph of f. Find the value of c. [2]

(b) Write down the equation of the horizontal asymptote to the graph of f. [2]

(c) The line $y=k$, where $k\in\mathbb{R}$ intersects the graph of $\left|f(x)\right|$ at exactly one point. Find the possible values of k. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

Two points P and Q have coordinates $(3, 2, 5)$ and $(7, 4, 9)$ respectively.

(a) (i) Find $\overrightarrow{\mathrm{PQ}}$.

(ii) Find $\left|\overrightarrow{\mathrm{PQ}}\right|$. [4]

Let $\overrightarrow{\mathrm{PR}} = 6\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$.

(b) Find the angle between PQ and PR. [4]

(c) Find the area of triangle PQR. [2]

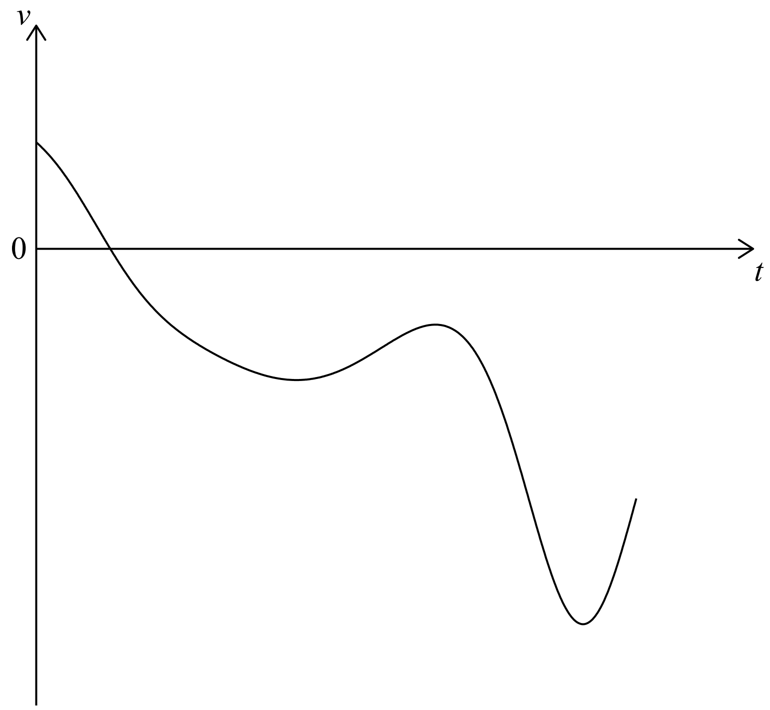
(d) Hence or otherwise find the shortest distance from R to the line through P and Q. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A particle P moves along a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of P after t seconds is given by $v(t) = 7\cos t - 5t^{\cos t}$, for $0 \le t \le 7$.

The following diagram shows the graph of v.



(a) Find the initial velocity of P. [2]

(b) Find the maximum speed of P. [3]

(c) Write down the number of times that the acceleration of P is $0\,\mathrm{m\,s^{-2}}$. [3]

(d) Find the acceleration of P when it changes direction. [4]

(e) Find the total distance travelled by P. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 17]

The mass M of apples in grams is normally distributed with mean μ. The following table shows probabilities for values of M.

Values of M	$M < 93$	$93 \le M \le 119$	$M > 119$
$\mathrm{P}(X)$	k	0.98	0.01

(a) (i) Write down the value of k.

(ii) Show that $\mu = 106$. [4]

(b) Find $\mathrm{P}(M < 95)$. [5]

The apples are packed in bags of ten.

Any apples with a mass less than $95\,\mathrm{g}$ are classified as small.

(c) Find the probability that a bag of apples selected at random contains at most one small apple. [3]

(d) A crate contains 50 bags of apples. A crate is selected at random.

(i) Find the expected number of bags in this crate that contain at most one small apple.

(ii) Find the probability that at least 48 bags in this crate contain at most one small apple. [5]

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will not be marked.

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will not be marked.

Markscheme

May 2018

Mathematics

Standard level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r>1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find *the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value

the exact value if applicable, the correct 3 sf answer

Units will appear in brackets at the end.

Section A

1. (a) (i) valid approach *(M1)*

eg correct value for a or b (or for r seen in (ii))

$a = 1.91966$ $b = 7.97717$

$a = 1.92$, $b = 7.98$ ***A1A1*** ***N3***

(ii) 0.984674

$r = 0.985$ ***A1*** ***N1***

[4 marks]

(b) correct substitution into their equation *(A1)*

eg $1.92 \times 1.95 + 7.98$

11.7205

11.7 (kg) ***A1*** ***N2***

[2 marks]

[Total: 6 marks]

2. (a) evidence of choosing sine rule *(M1)*

eg $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

correct substitution *(A1)*

eg $\dfrac{\mathrm{DB}}{\sin 59^\circ} = \dfrac{11}{\sin 100^\circ}$

9.57429

$\mathrm{DB} = 9.57$ (cm) ***A1*** ***N2***

[3 marks]

(b) evidence of choosing cosine rule *(M1)*

eg $a^2 = b^2 + c^2 - 2bc\cos(A)$, $\mathrm{DC}^2 = \mathrm{DB}^2 + \mathrm{BC}^2 - 2\mathrm{DB} \times \mathrm{BC} \times \cos(\mathrm{D\hat{B}C})$

correct substitution into RHS *(A1)*

eg $9.57^2 + 6^2 - 2 \times 9.57 \times 6 \times \cos 82^\circ$, 111.677

10.5677

$\mathrm{DC} = 10.6$ (cm) ***A1*** ***N2***

[3 marks]

[Total: 6 marks]

3. (a) valid approach *(M1)*

eg $f(x) = 0$, $e^x = 180$ or $0 \ldots$

1.14472

$x = \ln \pi$ (exact), 1.14 ***A1*** ***N2***

[2 marks]

(b) attempt to substitute either **their** limits or the function into formula involving f^2 *(M1)*

eg $\int_0^{1.14} f^2$, $\pi \int \left(\sin(e^x) \right)^2 dx$, 0.795135

2.49799

volume $= 2.50$ ***A2*** ***N3***

[3 marks]

[Total: 5 marks]

4. (a) correct substitution into infinite sum ***(A1)***

eg $200 = \frac{4}{1-r}$

$r = 0.98$ (exact) ***A1*** ***N2***

[2 marks]

(b) correct substitution ***(A1)***

$\frac{4(1-0.98^8)}{1-0.98}$

29.8473

29.8 ***A1*** ***N2***

[2 marks]

(c) attempt to set up inequality (accept equation) ***(M1)***

eg $\frac{4(1-0.98^n)}{1-0.98} > 163, \ \frac{4(1-0.98^n)}{1-0.98} = 163$

correct inequality for n (accept equation) or crossover values ***(A1)***

eg $n > 83.5234, \ n = 83.5234, \ S_{83} = 162.606$ **and** $S_{84} = 163.354$

$n = 84$ ***A1*** ***N1***

[3 marks]

[Total: 7 marks]

5. valid approach to find one of the required terms (must have correct substitution for parameters but accept " r " or an incorrect value for r) ***(M1)***

eg $\binom{9}{r}(2x)^{9-r}\left(\frac{k}{x}\right)^r$, $\binom{9}{6}(2x)^6\left(\frac{k}{x}\right)^3$, $\binom{9}{0}(2x)^0\left(\frac{k}{x}\right)^9+\binom{9}{1}(2x)^1\left(\frac{k}{x}\right)^8+\ldots$, Pascal's triangle to 9th row

Note: Award ***M0*** if there is clear evidence of adding instead of multiplying.

identifying correct terms (must be clearly indicated if only seen in expansion) ***(A1)(A1)***

eg for x^3 term: $r=3$, $r=6$, 7th term, $\binom{9}{6},\binom{9}{3}$, $(2x)^6\left(\frac{k}{x}\right)^3$, $5376k^3$

for x^5 term: $r=2$, $r=7$, 8th term, $\binom{9}{7},\binom{9}{2}$, $(2x)^7\left(\frac{k}{x}\right)^2$, $4608k^2$

correct equation (may include powers of x) ***A1***

eg $\binom{9}{3}(2x)^6\left(\frac{k}{x}\right)^3=\binom{9}{2}(2x)^7\left(\frac{k}{x}\right)^2$

valid attempt to solve their equation in terms of k only ***(M1)***

eg sketch, $84\times 64k^3-36\times 128k^2=0$, $5376k-4608=0$, $\binom{9}{3}2^6k^3=\binom{9}{2}2^7k^2$

0.857142

$k=\frac{4608}{5376}$ $\left(=\frac{6}{7}\right)$ (exact), 0.857 ***A1*** ***N4***

[6 marks]

6. (a) valid approach to find k ***(M1)***

eg 8 minutes is half a turn, $k+$ diameter, $k+111=117$

$k=6$ ***A1 N2***

[2 marks]

(b) **METHOD 1**

valid approach ***(M1)***

eg $\frac{\max - \min}{2}$, $a=$ radius

$|a|=\frac{117-6}{2}$, 55.5 ***(A1)***

$a=-55.5$ ***A1 N2***

METHOD 2

attempt to substitute valid point into equation for f ***(M1)***

eg $h(0)=6$, $h(8)=117$

correct equation ***(A1)***

eg $6=61.5+a\cos\left(\frac{\pi}{8}\times 0\right)$, $117=61.5+a\cos\left(\frac{\pi}{8}\times 8\right)$, $6=61.5+a$

$a=-55.5$ ***A1 N2***

[3 marks]

(c) valid approach ***(M1)***

eg sketch of h and $y=30$, $h=30$, $61.5-55.5\cos\left(\frac{\pi}{8}t\right)=30$, $t=2.46307$, $t=13.5369$

18.4630

$t=18.5$ (minutes) ***A2 N3***

[3 marks]

[Total: 8 marks]

7. (a) valid approach ***(M1)***

eg $cx+6=0, -\frac{6}{c}=3$

$c=-2$ ***A1*** ***N2***

[2 marks]

(b) valid approach ***(M1)***

eg $\lim_{x\to\infty} f(x),\ y=\frac{8}{c}$

$y=-4$ (must be an equation) ***A1*** ***N2***

[2 marks]

(c) valid approach to analyze modulus function ***(M1)***

eg sketch, horizontal asymptote at $y=4,\ y=0$

$k=4,\ k=0$ ***A2*** ***N3***

[3 marks]

[Total: 7 marks]

Section B

8. (a) (i) valid approach *(M1)*

eg $(7, 4, 9)-(3, 2, 5)$, $A-B$

$\overrightarrow{\mathrm{PQ}}=4\boldsymbol{i}+2\boldsymbol{j}+4\boldsymbol{k}\ \left(=\begin{pmatrix}4\\2\\4\end{pmatrix}\right)$ ***A1*** ***N2***

(ii) correct substitution into magnitude formula *(A1)*

eg $\sqrt{4^2+2^2+4^2}$

$\left|\overrightarrow{\mathrm{PQ}}\right|=6$ ***A1*** ***N2***

[4 marks]

(b) finding scalar product and magnitudes *(A1)(A1)*

scalar product $=(4\times 6)+\left(2\times(-1)\right)+(4\times 3)\ (=34)$

magnitude of $\mathrm{PR}=\sqrt{36+1+9}\ (=6.782)$

correct substitution of **their** values to find cos $\mathrm{Q\hat{P}R}$ ***M1***

eg $\cos\mathrm{Q\hat{P}R}=\dfrac{24-2+12}{(6)\times\left(\sqrt{46}\right)}$, 0.8355

0.581746

$\mathrm{Q\hat{P}R}=0.582$ radians or $\mathrm{Q\hat{P}R}=33.3^{\circ}$ ***A1*** ***N3***

[4 marks]

(c) correct substitution *(A1)*

eg $\dfrac{1}{2}\times\left|\overrightarrow{\mathrm{PQ}}\right|\times\left|\overrightarrow{\mathrm{PR}}\right|\times\sin P$, $\dfrac{1}{2}\times 6\times\sqrt{46}\times\sin 0.582$

11.1803

area is 11.2 (sq. units) ***A1*** ***N2***

[2 marks]

(d) recognizing shortest distance is perpendicular distance from R to line through P and Q *(M1)*

eg sketch, height of triangle with base $[\mathrm{PQ}]$, $\dfrac{1}{2}\times 6\times h$, $\sin 33.3^{\circ}=\dfrac{h}{\sqrt{46}}$

correct working *(A1)*

eg $\dfrac{1}{2}\times 6\times d=11.2$, $\left|\overrightarrow{\mathrm{PR}}\right|\times\sin P$, $\sqrt{46}\sin 33.3^{\circ}$

3.72677

distance $=3.73$ (units) ***A1*** ***N2***

[3 marks]

[Total 13 marks]

9. (a) initial velocity when $t = 0$ ***(M1)***

eg $v(0)$

$v = 7 \ (\text{m}\,\text{s}^{-1})$ ***A1*** ***N2***

[2 marks]

(b) recognizing maximum speed when $|v|$ is greatest ***(M1)***

eg minimum, maximum, $v' = 0$

one correct coordinate for minimum ***(A1)***

eg $6.37896, -24.6571$

$24.7 \ (\text{m}\,\text{s}^{-1})$ ***A1*** ***N2***

[3 marks]

(c) recognizing $a = v'$ ***(M1)***

eg $a = \dfrac{\text{d}v}{\text{d}t}$, correct derivative of first term

identifying when $a = 0$ ***(M1)***

eg turning points of v, t-intercepts of v'

3 ***A1*** ***N3***

[3 marks]

(d) recognizing P changes direction when $v = 0$ ***(M1)***

$t = 0.863851$ ***(A1)***

-9.24689

$a = -9.25 \, (\text{m}\,\text{s}^{-2})$ ***A2*** ***N3***

[4 marks]

(e) correct substitution of limits or function into formula ***(A1)***

eg $\int_0^7 |v|, \int_0^{0.8638} v\,\text{d}t - \int_{0.8638}^7 v\,\text{d}t, \int \left| 7\cos x - 5x^{\cos x} \right| \text{d}x, \ 3.32 + 60.6$

63.8874

63.9 (metres) ***A2*** ***N3***

[3 marks]

[Total: 15 Marks]

10. (a) (i) evidence of using $\sum p_i = 1$ *(M1)*

eg $k + 0.98 + 0.01 = 1$

$k = 0.01$ ***A1*** ***N2***

(ii) recognizing that 93 and 119 are symmetrical about μ *(M1)*

eg μ is midpoint of 93 and 119

correct working to find μ ***A1***

$\frac{119+93}{2}$

$\mu = 106$ ***AG*** ***N0***

[4 marks]

(b) finding standardized value for 93 or 119 *(A1)*

eg $z = -2.32634,\ z = 2.32634$

correct substitution using **their** z value *(A1)*

eg $\frac{93-106}{\sigma} = -2.32634$, $\frac{119-106}{2.32634} = \sigma$

$\sigma = 5.58815$ *(A1)*

0.024508

$\mathrm{P}(X < 95) = 0.0245$ ***A2*** ***N3***

[5 marks]

(c) evidence of recognizing binomial *(M1)*

eg ${}_nC_a \times p^a \times q^{n-a}$, $n = 10$ and $p = 0.0245$, $\mathrm{B}(n,\ p)$

valid approach *(M1)*

eg $\mathrm{P}(X \le 1)$, $\mathrm{P}(X = 0) + \mathrm{P}(X = 1)$

0.976285

0.976 ***A1*** ***N2***

[3 marks]

continued…

Question 10 continued

(d) (i) recognizing **new** binomial probability ***(M1)***

eg $\mathrm{B}(50,\ 0.976)$

correct substitution ***(A1)***

eg $\mathrm{E}(X)=50(0.976285)$

48.81425

48.8 ***A1*** ***N2***

(ii) valid approach ***(M1)***

eg $\mathrm{P}(\mathrm{X}\geq 48),\ 1-\mathrm{P}(\mathrm{X}\leq 47)$

0.884688

0.885 ***A1*** ***N2***

[5 marks]

[Total: 17 marks]

Matemáticas
Nivel medio
Prueba 2

Jueves 3 de mayo de 2018 (mañana)

Número de convocatoria del alumno

1 hora 30 minutos

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NM** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

2218 – 7310

No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente la misma como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 6]

La siguiente tabla muestra el peso medio, y kg, de niños que tienen x años.

Edad (x años)	1,25	2,25	3,5	4,4	5,85
Peso (y kg)	10	13	14	17	19

La relación que existe entre estas variables está modelizada por la recta de regresión cuya ecuación es $y = ax + b$.

(a) (i) Halle el valor de a y el de b.

(ii) Escriba el coeficiente de correlación. [4]

(b) Utilice esa ecuación para estimar el peso medio de un niño que tiene $1,95$ años. [2]

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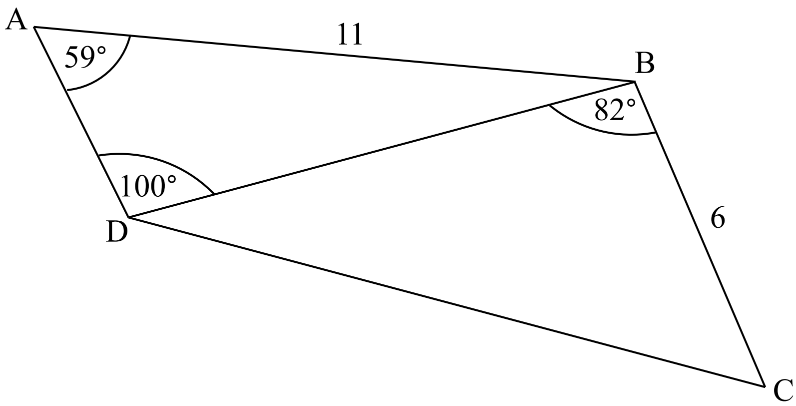
2. [Puntuación máxima: 6]

La siguiente figura muestra el cuadrilátero ABCD.

la figura no está dibujada a escala



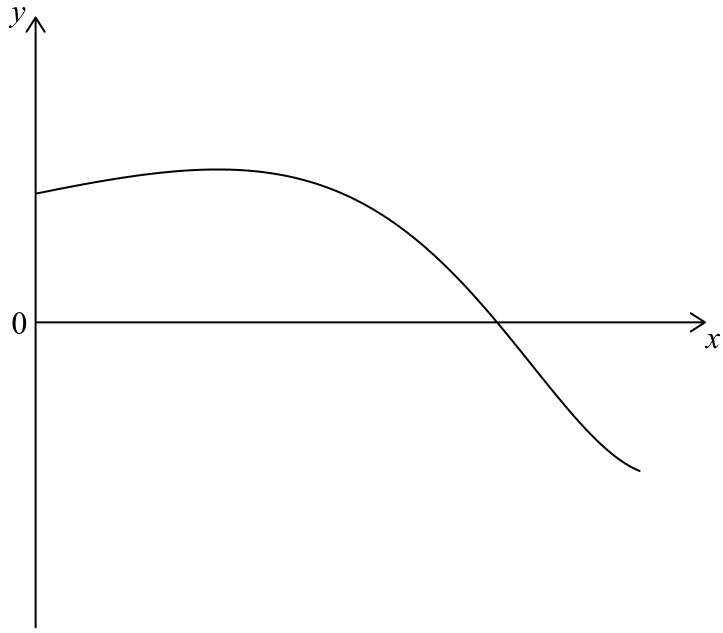
$\text{AB} = 11\text{ cm}$, $\text{BC} = 6\text{ cm}$, $\text{B}\hat{\text{A}}\text{D} = 59^\circ$, $\text{A}\hat{\text{D}}\text{B} = 100^\circ$, y $\text{C}\hat{\text{B}}\text{D} = 82^\circ$

(a) Halle DB. [3]

(b) Halle DC. [3]

3. [Puntuación máxima: 5]

Sea $f(x) = \operatorname{sen}(e^x)$ para $0 \le x \le 1{,}5$. La siguiente figura muestra el gráfico de f.



(a) Halle la intersección del gráfico de f con el eje x. [2]

(b) La región delimitada por el gráfico de f, el eje y y el eje x se rota 360° alrededor del eje x.

Halle el volumen del sólido generado. [3]

4. [Puntuación máxima: 7]

El primer término de una progresión geométrica infinita es 4. La suma de los infinitos términos de la progresión es igual a 200.

(a) Halle la razón común. [2]

(b) Halle la suma de los 8 primeros términos. [2]

(c) Halle el menor valor de n para el cual $S_n > 163$. [3]

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No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

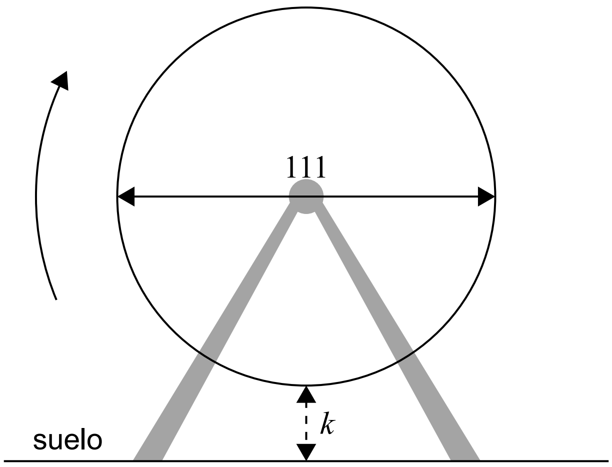
5. [Puntuación máxima: 6]

Considere el desarrollo de $\left(2x+\dfrac{k}{x}\right)^9$, donde $k>0$. El coeficiente del término en x^3 es igual al coeficiente del término en x^5. Halle k.

6. [Puntuación máxima: 8]

En un parque de atracciones, una noria de 111 metros de diámetro está girando a velocidad constante. La parte inferior de la noria está situada a k metros del suelo. Un asiento empieza el viaje situado en la parte inferior de la noria.

la figura no está dibujada a escala



La noria tarda 16 minutos en dar una vuelta completa.

(a) Al cabo de 8 minutos, el asiento está a $117\,\mathrm{m}$ del suelo. Halle k. [2]

Al cabo de t minutos, la altura del asiento respecto al suelo viene dada por

$h(t) = 61{,}5 + a\cos\left(\frac{\pi}{8}t\right)$, para $0 \le t \le 32$.

(b) Halle el valor de a. [3]

(c) Halle en qué instante se encontrará el asiento por tercera vez a $30\,\mathrm{m}$ del suelo. [3]

(Esta pregunta continúa en la página siguiente)

(Pregunta 6: continuación)

7. [Puntuación máxima: 7]

Sea $f(x)=\dfrac{8x-5}{cx+6}$ para $x \neq -\dfrac{6}{c}$, $c \neq 0$.

(a) La recta $x=3$ es una asíntota vertical del gráfico de f. Halle el valor de c. [2]

(b) Escriba la ecuación de la asíntota horizontal del gráfico de f. [2]

(c) La recta $y=k$, donde $k \in \mathbb{R}$, y el gráfico de $|f(x)|$ se cortan exactamente en un punto. Halle los posibles valores de k. [3]

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No escriba soluciones en esta página.

Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

8. [Puntuación máxima: 13]

Dos puntos P y Q tienen por coordenadas $(3, 2, 5)$ y $(7, 4, 9)$ respectivamente.

(a) (i) Halle $\overrightarrow{\mathrm{PQ}}$.

(ii) Halle $\left|\overrightarrow{\mathrm{PQ}}\right|$. [4]

Sea $\overrightarrow{\mathrm{PR}} = 6\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$.

(b) Halle el ángulo que forman PQ y PR. [4]

(c) Halle el área del triángulo PQR. [2]

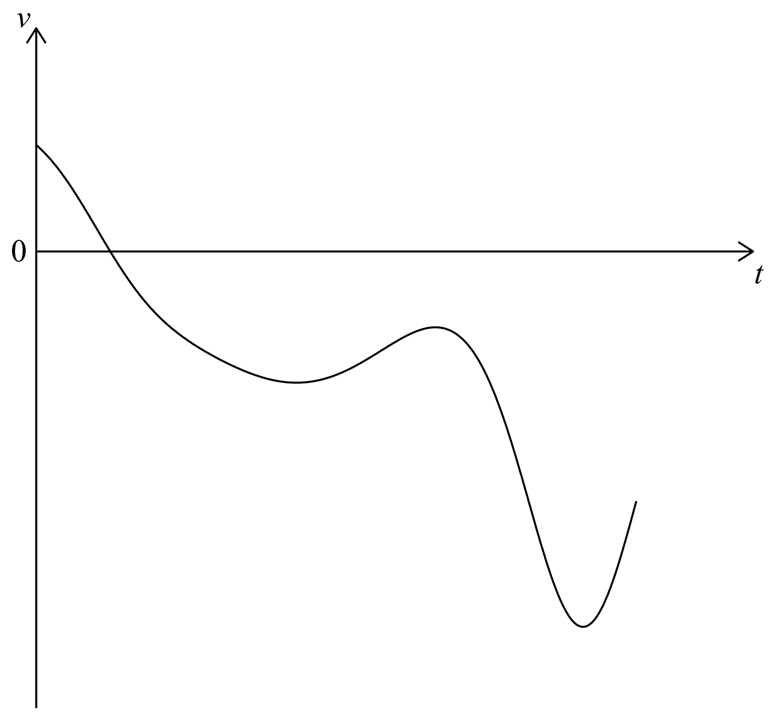
(d) A partir de lo anterior o de cualquier otro modo, halle la distancia más corta entre R y la recta que pasa por P y Q. [3]

No escriba soluciones en esta página.

9. [Puntuación máxima: 15]

Una partícula P se mueve a lo largo de una línea recta. La velocidad de P, $v\,\mathrm{m\,s^{-1}}$, en el instante t segundos viene dada por $v(t) = 7\cos t - 5t^{\cos t}$, para $0 \le t \le 7$.

La siguiente figura muestra el gráfico de v.



(a) Halle la velocidad inicial de P. [2]

(b) Halle la celeridad máxima de P. [3]

(c) Escriba el número de veces que la aceleración de P es igual a $0\,\mathrm{m\,s^{-2}}$. [3]

(d) Halle la aceleración de P cuando la partícula cambia de sentido. [4]

(e) Halle la distancia total que recorre P. [3]

No escriba soluciones en esta página.

10. [Puntuación máxima: 17]

La masa M, en gramos, de las manzanas sigue una distribución normal de media μ.
La siguiente tabla muestra la probabilidad de una serie de valores de M.

Valores de M	$M < 93$	$93 \le M \le 119$	$M > 119$
$\mathbf{P(X)}$	k	0,98	0,01

(a) (i) Escriba el valor de k.

(ii) Muestre que $\mu = 106$. [4]

(b) Halle $\mathrm{P}(M < 95)$. [5]

Las manzanas se venden en bolsas de diez unidades.

Una manzana se considera que es pequeña si tiene una masa inferior a $95\,\mathrm{g}$.

(c) Halle la probabilidad de que una bolsa de manzanas elegida al azar contenga a lo sumo una manzana pequeña. [3]

(d) Una caja de madera contiene 50 bolsas de manzanas. Se escoge una caja al azar.

(i) Halle el número esperado de bolsas de esa caja que contienen a lo sumo una manzana pequeña.

(ii) Halle la probabilidad de que en esta caja haya al menos 48 bolsas que contengan a lo sumo una manzana pequeña. [5]

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Las respuestas que se escriban en esta página no serán corregidas.

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Las respuestas que se escriban en esta página no serán corregidas.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathématiques
Niveau moyen
Épreuve 2

Jeudi 3 mai 2018 (matin)

Numéro de session du candidat

1 heure 30 minutes

Instructions destinées aux candidats

- Écrivez votre numéro de session dans les cases ci-dessus.
- N'ouvrez pas cette épreuve avant d'y être autorisé(e).
- Une calculatrice à écran graphique est nécessaire pour cette épreuve.
- Section A : répondez à toutes les questions. Rédigez vos réponses dans les cases prévues à cet effet.
- Section B : répondez à toutes les questions sur le livret de réponses prévu à cet effet. Écrivez votre numéro de session sur la première page du livret de réponses, et attachez ce livret à cette épreuve d'examen et à votre page de couverture en utilisant l'attache fournie.
- Sauf indication contraire dans l'intitulé de la question, toutes les réponses numériques devront être exactes ou correctes à trois chiffres significatifs près.
- Un exemplaire non annoté du **livret de formules pour le cours de mathématiques NM** est nécessaire pour cette épreuve.
- Le nombre maximum de points pour cette épreuve d'examen est de **[90 points]**.

2218 – 7308

13 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Le total des points ne sera pas nécessairement attribué pour une réponse correcte si le raisonnement n'a pas été indiqué. Les réponses doivent être appuyées par un raisonnement et/ou des explications. En particulier, les solutions obtenues à l'aide d'une calculatrice à écran graphique doivent être accompagnées d'un raisonnement adéquat. Par exemple, si des représentations graphiques sont utilisées pour trouver la solution, veuillez inclure une esquisse de ces représentations graphiques dans votre réponse. Lorsque la réponse est fausse, certains points peuvent être attribués si la méthode utilisée est correcte, pour autant que le raisonnement soit indiqué par écrit. On vous recommande donc de montrer tout votre raisonnement.

Section A

Répondez à **toutes** les questions. Rédigez vos réponses dans les cases prévues à cet effet. Si cela est nécessaire, vous pouvez poursuivre votre raisonnement en dessous des lignes.

1. [Note maximale : 6]

Le tableau suivant montre le poids moyen, y kg, d'enfants âgés de x ans.

Âge (x ans)	1,25	2,25	3,5	4,4	5,85
Poids (y kg)	10	13	14	17	19

La relation entre les variables est modélisée par la droite de régression d'équation $y = ax + b$.

(a) (i) Trouvez la valeur de a et celle de b.

(ii) Écrivez le coefficient de corrélation. [4]

(b) Utilisez votre équation pour estimer le poids moyen d'un enfant âgé de $1{,}95$ an. [2]

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2. [Note maximale : 6]

Le diagramme suivant montre le quadrilatère ABCD.

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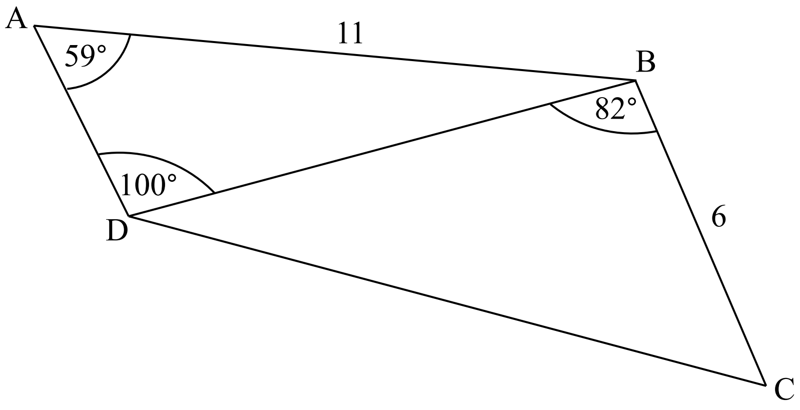


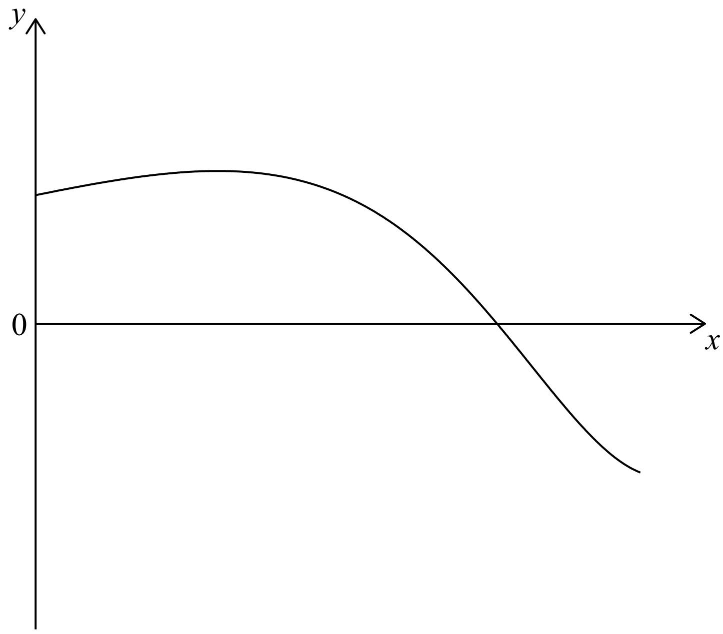
$$\mathrm{AB} = 11\,\mathrm{cm},\ \mathrm{BC} = 6\,\mathrm{cm},\ \mathrm{B\hat{A}D} = 59^\circ,\ \mathrm{A\hat{D}B} = 100^\circ \text{ et } \mathrm{C\hat{B}D} = 82^\circ$$

(a) Trouvez DB. [3]

(b) Trouvez DC. [3]

3. [Note maximale : 5]

Soit $f(x) = \sin(e^x)$ pour $0 \le x \le 1,5$. Le diagramme suivant montre la représentation graphique de f.



(a) Trouvez l'abscisse à l'origine de la représentation graphique de f. [2]

(b) La région délimitée par la représentation graphique de f, l'axe des ordonnées et l'axe des abscisses subit une rotation de 360° autour de l'axe des abscisses.

Trouvez le volume du solide formé. [3]

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4. [Note maximale : 7]

Le premier terme d'une suite géométrique infinie est 4. La somme de la suite infinie est 200.

(a) Trouvez la raison. [2]

(b) Trouvez la somme des 8 premiers termes. [2]

(c) Trouvez la plus petite valeur de n pour laquelle $S_n > 163$. [3]

Veuillez ne **pas** écrire sur cette page.

Les réponses rédigées sur cette page ne seront pas corrigées.

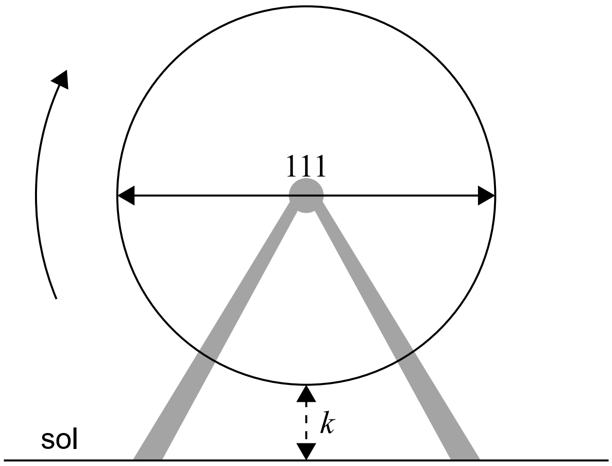
5. [Note maximale : 6]

Considérez le développement de $\left(2x+\frac{k}{x}\right)^9$, où $k>0$. Le coefficient du terme en x^3 est égal au coefficient du terme en x^5. Trouvez k.

6. [Note maximale : 8]

Dans un parc d'attractions, une grande roue dont le diamètre est de 111 mètres tourne à une vitesse constante. Le bas de la roue est k mètres au-dessus du sol. Un siège part du bas de la roue.

la figure n'est pas à l'échelle



La roue complète un tour en 16 minutes.

(a) Après 8 minutes, le siège est $117\,\mathrm{m}$ au-dessus du sol. Trouvez k. [2]

Après t minutes, la hauteur du siège au-dessus du sol est donnée par $h(t) = 61{,}5 + a\cos\left(\frac{\pi}{8}t\right)$, pour $0 \le t \le 32$.

(b) Trouvez la valeur de a. [3]

(c) Trouvez quand le siège est $30\,\mathrm{m}$ au-dessus du sol pour la troisième fois. [3]

(Suite de la question à la page suivante)

(Suite de la question 6)

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7. [Note maximale : 7]

Soit $f(x)=\dfrac{8x-5}{cx+6}$ pour $x\neq-\dfrac{6}{c}$, $c\neq 0$.

(a) La droite $x=3$ est une asymptote verticale de la représentation graphique de f. Trouvez la valeur de c. [2]

(b) Écrivez l'équation de l'asymptote horizontale de la représentation graphique de f. [2]

(c) La droite $y=k$, où $k\in\mathbb{R}$, coupe la représentation graphique de $\left|f(x)\right|$ en exactement un point. Trouvez les valeurs possibles de k. [3]

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N'écrivez **pas** vos solutions sur cette page.

Section B

Répondez à **toutes** les questions sur le livret de réponses fourni. Veuillez répondre à chaque question sur une nouvelle page.

8. [Note maximale : 13]

Les points P et Q ont respectivement pour coordonnées $(3\,;2\,;5)$ et $(7\,;4\,;9)$.

(a) (i) Trouvez $\overrightarrow{\mathrm{PQ}}$.

(ii) Trouvez $\left|\overrightarrow{\mathrm{PQ}}\right|$. [4]

Soit $\overrightarrow{\mathrm{PR}} = 6\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$.

(b) Trouvez l'angle entre PQ et PR. [4]

(c) Trouvez l'aire du triangle PQR. [2]

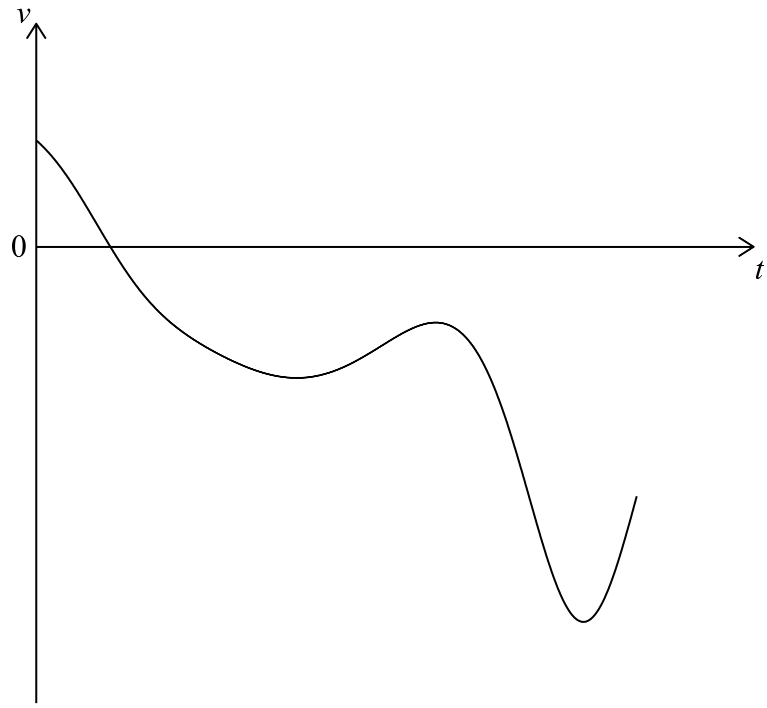
(d) À partir de là ou par toute autre méthode, trouvez la distance la plus courte entre R et la droite passant par P et Q. [3]

N’écrivez **pas** vos solutions sur cette page.

9. [Note maximale : 15]

Une particule P se déplace le long d’une droite. Le vecteur vitesse $v\,\mathrm{m\,s}^{-1}$ de P après t secondes est donnée par $v(t) = 7\cos t - 5t^{\cos t}$, pour $0 \le t \le 7$.

Le diagramme suivant montre la représentation graphique de v.



(a) Trouvez le vecteur vitesse initiale de P. [2]

(b) Trouvez la vitesse maximale de P. [3]

(c) Écrivez le nombre de fois où l’accélération de P est $0\,\mathrm{m\,s}^{-2}$. [3]

(d) Trouvez l’accélération de P lorsque la particule change de direction. [4]

(e) Trouvez la distance totale parcourue par P. [3]

N'écrivez **pas** vos solutions sur cette page.

10. [Note maximale : 17]

La masse M de pommes, en grammes, est normalement distribuée avec une moyenne μ.
Le tableau suivant montre les probabilités pour des valeurs de M.

Valeurs de M	$M < 93$	$93 \leq M \leq 119$	$M > 119$
$\mathbf{P}(X)$	k	0,98	0,01

(a) (i) Écrivez la valeur de k.

(ii) Montrez que $\mu = 106$. [4]

(b) Trouvez $\mathrm{P}(M < 95)$. [5]

Les pommes sont emballées dans des sacs de dix.

Toute pomme dont la masse est inférieure à $95\,\mathrm{g}$ est classée comme étant petite.

(c) Trouvez la probabilité qu'un sac de pommes choisi au hasard contienne au plus une petite pomme. [3]

(d) Une caisse contient 50 sacs de pommes. Une caisse est choisie au hasard.

(i) Trouvez le nombre espéré de sacs de cette caisse qui contiennent au plus une petite pomme.

(ii) Trouvez la probabilité qu'au moins 48 sacs de cette caisse contiennent au plus une petite pomme. [5]

Veuillez ne **pas** écrire sur cette page.

Les réponses rédigées sur cette page ne seront pas corrigées.

Veuillez ne **pas** écrire sur cette page.

Les réponses rédigées sur cette page ne seront pas corrigées.

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Les réponses rédigées sur cette page ne seront pas corrigées.

M18/5/MATSD/SP1/ENG/TZ1/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

25 pages

2218 – 7403

Please **do not** write on this page.

Answers written on this page
will not be marked.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. Each year the soccer team, Peterson United, plays 25 games at their home stadium. The owner of Peterson United claimed that last year the mean attendance per game at their home stadium was $24\,500$.

(a) Based on the owner's claim, calculate the total attendance for the games at Peterson United's home stadium last year. [2]

The actual total attendance last year was $617\,700$.

(b) Calculate the percentage error in the owner's claim. [2]

(c) Write down your answer to **part (b)** in the form $a \times 10^k$ where $1 \le a < 10$, $k \in \mathbb{Z}$. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

2. Each month the number of days of rain in Cardiff is recorded.
The following data was collected over a period of 10 months.

$$11 \quad 13 \quad 8 \quad 11 \quad 8 \quad 7 \quad 8 \quad 14 \quad x \quad 15$$

For these data the **median** number of days of rain per month is 10.

(a) Find the value of x. [2]

(b) Find

(i) the standard deviation;

(ii) the interquartile range. [4]

Working:

Answers:

(a) . . . .

(b) (i) . . . .

(ii) . . . .

3. Consider the following propositions.

p: my Mathematical Studies homework is due tomorrow
q: today is Wednesday

(a) Write down in words the compound proposition $\neg p \Rightarrow q$. [2]

(b) Complete the truth table. [3]

p	q	$\neg p$	$\neg p \Rightarrow q$	$\neg p \wedge q$	$(\neg p \Rightarrow q) \vee (\neg p \wedge q)$
T	T	F			
T	F	F			
F	T	T			
F	F	T			

(c) State whether the compound proposition $(\neg p \Rightarrow q) \vee (\neg p \wedge q)$ is a tautology, contradiction or neither. [1]

Working:

Answers:

(a) . . . .

. . . .

. . . .

(c) . . . .

4. A scientist measures the concentration of dissolved oxygen, in milligrams per litre (y), in a river. She takes 10 readings at different temperatures, measured in degrees Celsius (x).

The results are shown in the table.

Temperature, °C (x)	20	24	25	28	29	32	27	25	23	21
Dissolved Oxygen, mg l^{-1} (y)	10.9	9.7	9.2	7.6	7.3	6.4	7.9	8.4	9.4	9.9

It is believed that the concentration of dissolved oxygen in the river varies linearly with the temperature.

(a) For these data, find

(i) Pearson's product-moment correlation coefficient, r;

(ii) the equation of the regression line y on x. [4]

(b) Using the equation of the regression line, estimate the concentration of dissolved oxygen in the river when the temperature is $18\,^\circ\mathrm{C}$. [2]

Working:

Answers:

(a) (i) . . . .

(ii) . . . .

(b) . . . .

5. The point A has coordinates $(4, -8)$ and the point B has coordinates $(-2, 4)$.

(a) Write down the coordinates of C, the midpoint of line segment AB. [2]

The point D has coordinates $(-3, 1)$.

(b) Find the gradient of the line DC. [2]

(c) Find the equation of the line DC. Write your answer in the form $ax + by + d = 0$ where a, b and d are integers. [2]

Working:

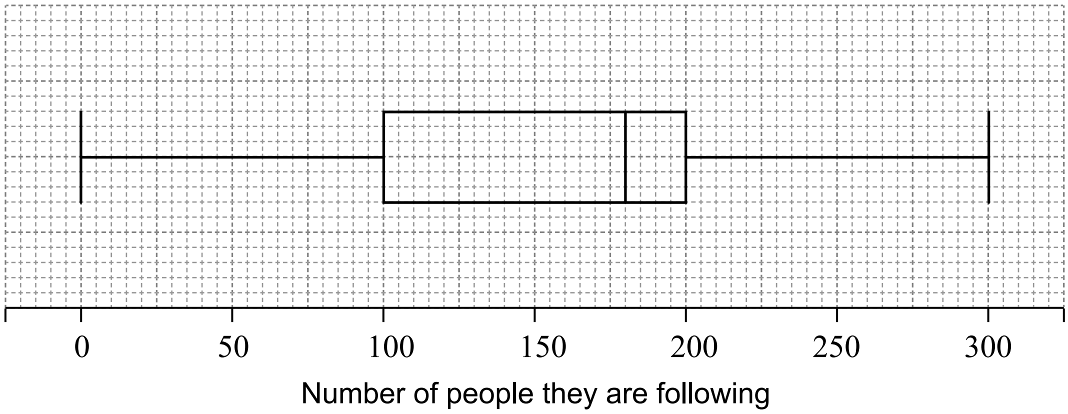
Answers:

(a) . . . .

(b) . . . .

(c) . . . .

6. In a high school, 160 students completed a questionnaire which asked for the number of people they are following on a social media website. The results were recorded in the following box-and-whisker diagram.



(a) Write down the median. [1]

The following incomplete table shows the distribution of the responses from these 160 students.

Number of people they are following (x)	**Number of high school students**
$0 \le x \le 50$	4
$50 < x \le 100$	
$100 < x \le 150$	34
$150 < x \le 200$	46
$200 < x \le 250$	
$250 < x \le 300$	16

(b) Complete the table. [2]

(c) (i) Write down the mid-interval value for the $100 < x \le 150$ group.

(ii) Using the table, calculate an estimate for the mean number of people being followed on the social media website by these 160 students. [3]

(This question continues on the following page)

(Question 6 continued)

Working:

Answers:

(a) . . . .

(c) (i) . . . .

(ii) . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.

7. Sergei is training to be a weightlifter. Each day he trains at the local gym by lifting a metal bar that has heavy weights attached. He carries out successive lifts. After each lift, the same amount of weight is **added** to the bar to increase the weight to be lifted.

The weights of each of Sergei's lifts form an arithmetic sequence.

Sergei's friend, Yuri, records the weight of each lift. Unfortunately, last Monday, Yuri misplaced all but two of the recordings of Sergei's lifts.

On that day, Sergei lifted $21\,\text{kg}$ on the third lift and $46\,\text{kg}$ on the eighth lift.

(a) For that day

(i) find how much weight was added after each lift;

(ii) find the weight of Sergei's first lift. [4]

On that day, Sergei made 12 successive lifts.

(b) Find the **total** combined weight of these lifts. [2]

Working:

Answers:

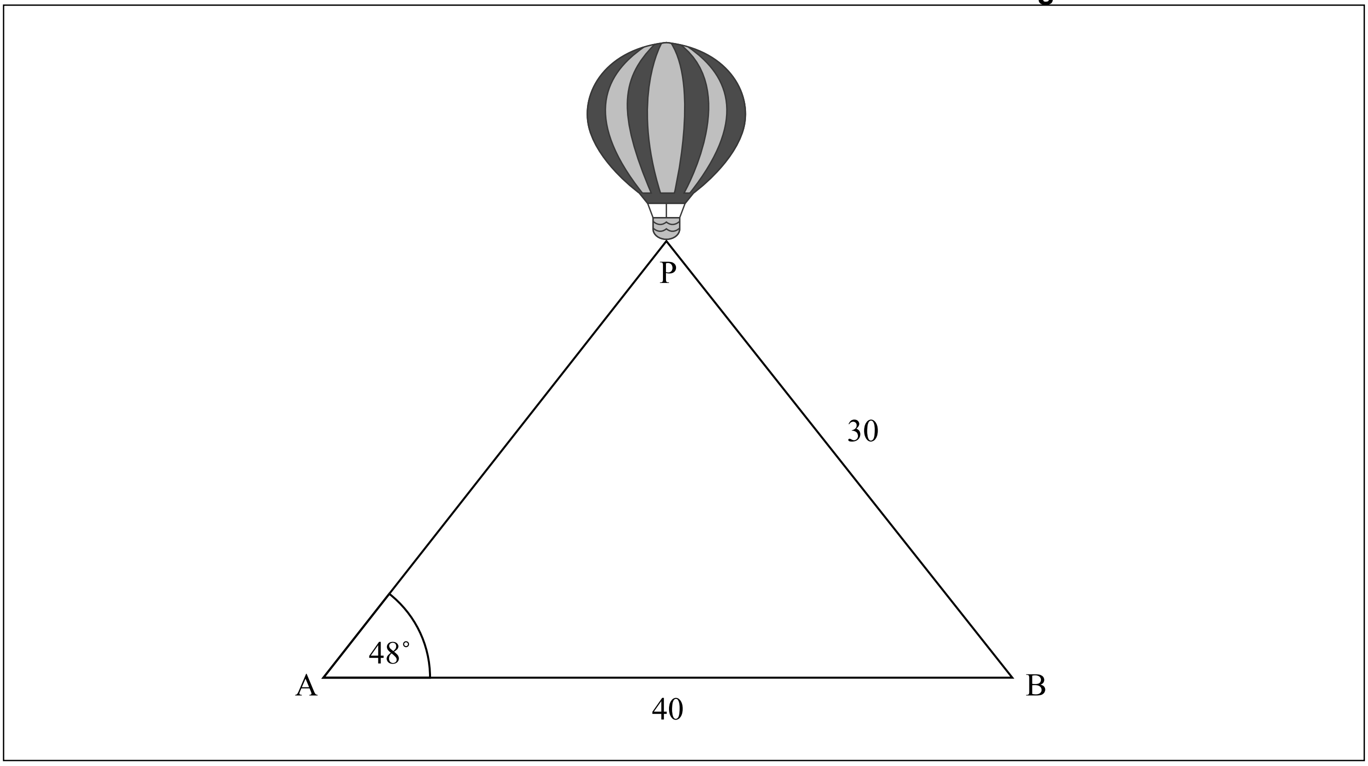
(a) (i) . . . .

(ii) . . . .

(b) . . . .

8. Two fixed points, A and B, are 40 m apart on horizontal ground. Two straight ropes, AP and BP, are attached to the same point, P, on the base of a hot air balloon which is vertically above the line AB. The length of BP is 30 m and angle BAP is 48°.

diagram not to scale



(a) On the diagram, draw and label with an x the angle of depression of B from P. [1]

Angle APB is acute.

(b) Find the size of angle APB. [3]

(c) Find the size of the angle of depression of B from P. [2]

(This question continues on the following page)

(Question 8 continued)

Working:

Answers:

(b) . . . .

(c) . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.

9. In an experiment, a number of fruit flies are placed in a container. The population of fruit flies, P, increases and can be modelled by the function

$$P(t) = 12 \times 3^{0.498t},\ t \geq 0,$$

where t is the number of days since the fruit flies were placed in the container.

(a) Find the number of fruit flies

(i) which were placed in the container;

(ii) that are in the container after 6 days. [4]

The maximum capacity of the container is 8000 fruit flies.

(b) Find the number of days until the container reaches its maximum capacity. [2]

Working:

Answers:

(a) (i) . . . .

(ii) . . . .

(b) . . . .

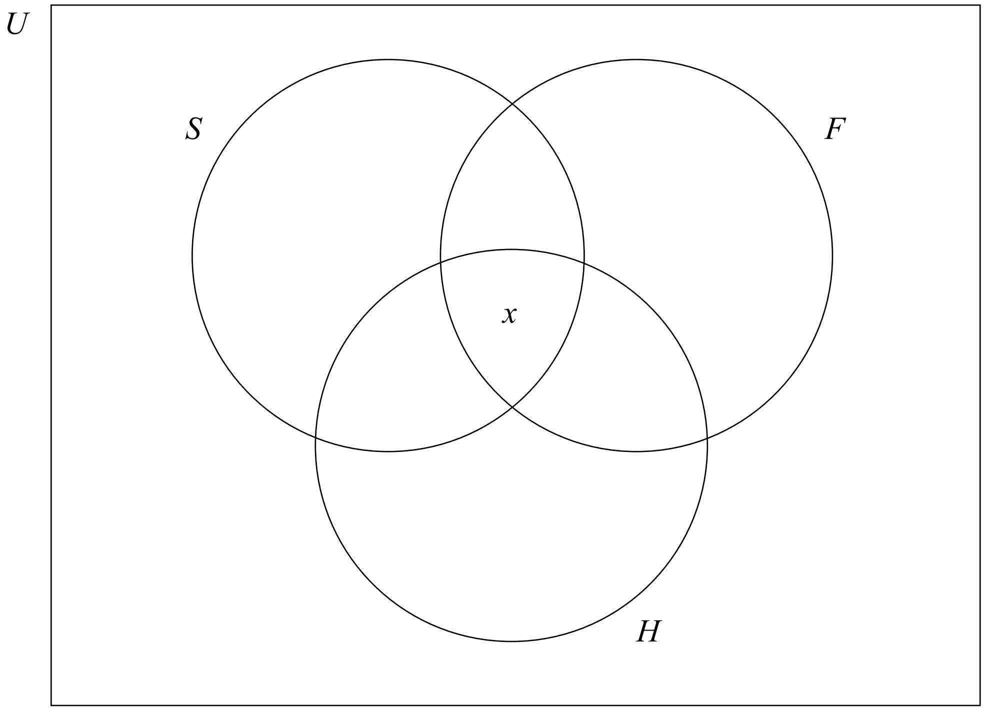
10. A group of 60 sports enthusiasts visited the PyeongChang 2018 Winter Olympic games to watch a variety of sporting events.

The most popular sports were snowboarding (S), figure skating (F) and ice hockey (H).

For this group of 60 people:

4 did not watch any of the most popular sports,
x watched all three of the most popular sports,
9 watched snowboarding only,
11 watched figure skating only,
15 watched ice hockey only,
7 watched snowboarding and figure skating,
13 watched figure skating and ice hockey,
11 watched snowboarding and ice hockey.

(a) Complete the Venn diagram using the given information. [3]



(b) Find the value of x. [2]

(c) Write down the value of $n\left((F \cup H) \cap S'\right)$. [1]

(This question continues on the following page)

(Question 10 continued)

Working:

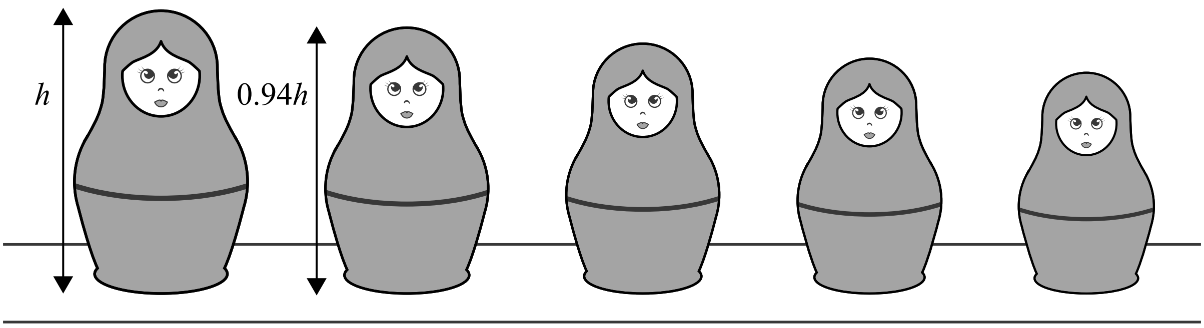
Answers:

(b) . . . .

(c) . . . .

11. Matryoshka dolls, or Russian dolls, are similarly designed dolls which open up and fit inside each other.

The largest set of these type of dolls is a 51 piece set which was completed in 2003. The height of the largest piece in this set is $54\,\text{cm}$. The heights of successive smaller dolls are $94\,\%$ of the preceding larger doll, as shown.



(a) Find the height of the smallest doll in this set. [3]

(b) Find the **total** height if all 51 dolls were placed one on top of another. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

12. Consider the quadratic function $f(x) = ax^2 + bx + 22$.
The equation of the line of symmetry of the graph $y = f(x)$ is $x = 1.75$.

(a) Using only this information, write down an equation in terms of a and b. [1]

The graph intersects the x-axis at the point $(-2, 0)$.

(b) Using this information, write down a second equation in terms of a and b. [1]

(c) Hence find the value of a and of b. [2]

The graph intersects the x-axis at a second point, P.

(d) Find the x-coordinate of P. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

(d) . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.

13. Malthouse school opens at 08:00 every morning.

The daily arrival times of the 500 students at Malthouse school follow a normal distribution. The mean arrival time is 52 minutes after the school opens and the standard deviation is 5 minutes.

(a) Find the probability that a student, chosen at random

(i) arrives at least 60 minutes after the school opens;

(ii) arrives between 45 minutes and 55 minutes after the school opens. [4]

A second school, Mulberry Park, also opens at 08:00 every morning. The arrival times of the students at this school follows exactly the same distribution as Malthouse school.

(b) Given that, on one morning, 15 students arrive at least 60 minutes after the school opens, estimate the number of students at Mulberry Park school. [2]

Working:

Answers:

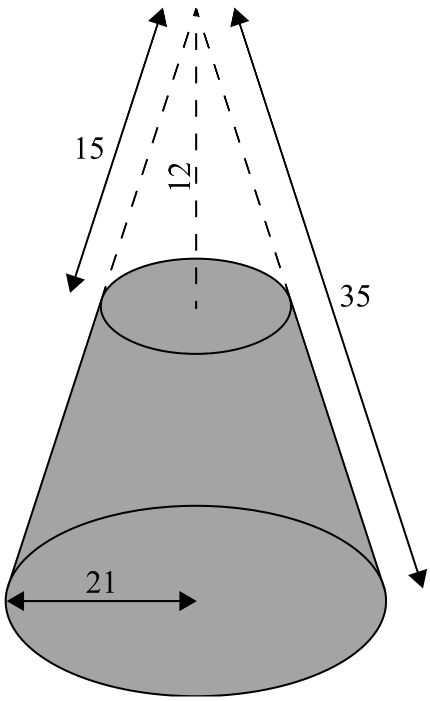
(a) (i) . . . .

(ii) . . . .

(b) . . . .

14. A solid right circular cone has a base radius of 21 cm and a slant height of 35 cm. A smaller right circular cone has a height of 12 cm and a slant height of 15 cm, and is removed from the top of the larger cone, as shown in the diagram.

diagram not to scale



(a) Calculate the radius of the base of the cone which has been removed. [2]

(b) Calculate the curved surface area of the cone which has been removed. [2]

(c) Calculate the curved surface area of the remaining solid. [2]

(This question continues on the following page)

(Question 14 continued)

Working:

Answers:

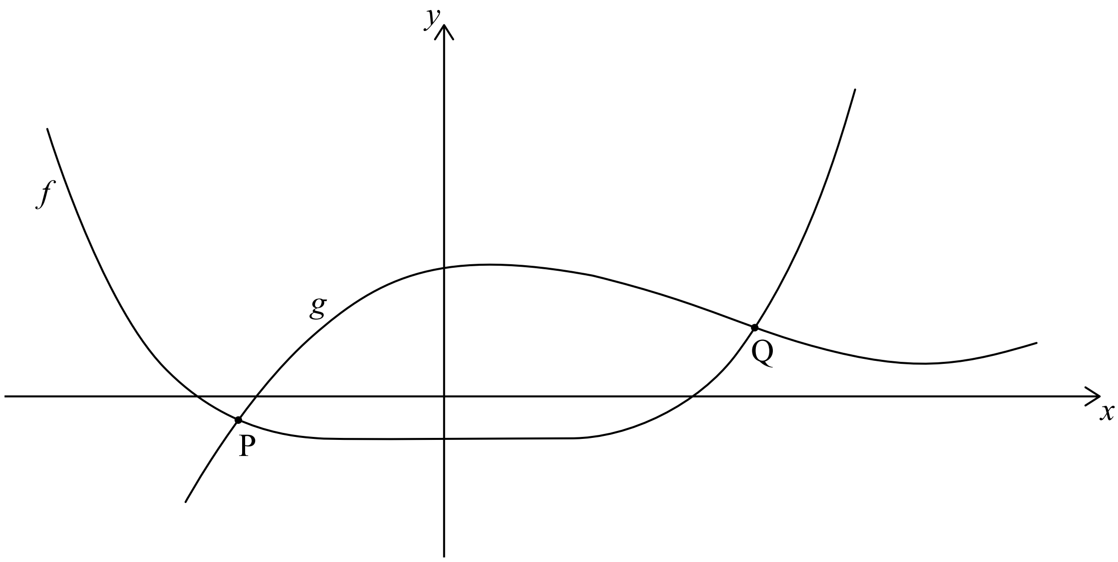
(a) . . . .

(b) . . . .

(c) . . . .

15. Consider the functions $f(x) = x^4 - 2$ and $g(x) = x^3 - 4x^2 + 2x + 6$.

The functions intersect at points P and Q. Part of the graph of $y = f(x)$ and part of the graph of $y = g(x)$ are shown on the diagram.



(a) Find the range of f. [2]

(b) Write down the x-coordinate of P and the x-coordinate of Q. [2]

(c) Write down the values of x for which $f(x) > g(x)$. [2]

(This question continues on the following page)

(Question 15 continued)

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

. . . .

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will not be marked.

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Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Mathematical studies

Standard level

Paper 1

23 pages

Paper 1 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

The number of marks for each question is 6.

1 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

C Marks awarded for **Correct** answers (irrespective of working shown)

R Marks awarded for clear **Reasoning**

ft Marks that can be awarded as **follow through** from previous results in the question

2 Method of Marking

(a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b) If the candidate has full marks on a question use the ***C6*** annotation, if the candidate has made an attempt but scores zero marks use ***C0***. If there is no attempt use the No response button. If a candidate does not score full or zero marks then full annotations **MUST** be shown.

(c) In this paper, if the **correct answer is seen on the answer line the** maximum mark is awarded. **There is no need to check the working!** Award ***C*** marks and move on.

(d) If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.

(e) If the **answer is wrong**, marks should be awarded for the working according to the markscheme.

(f) Working crossed out by the candidate should not be awarded any marks. Where candidates have written two solutions to a question, only the first solution should be marked.

(g) A correct answer in the working box transcribed inaccurately to the answer line can receive full marks.

(h) If correct working results in a correct answer **in the working box** but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer **as long as it is unambiguous**. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***(A1)*** *(ignore the further working)*
2.	$(x-6)(x+1)$	$x=6$ and -1	Do **not** award the final ***(A1)*** *(see next example)*

Example: Factorise $x^2 - 5x - 6$

Markscheme	Candidates' Scripts	Marking
$(x-6)(x+1)$ ***(A1)(A1)***	(i) Answer line: $(x+6)(x+1)$	***(A0)(A1)***
	(ii) Working box: $(x-6)(x+1)$	***(A1)***
	followed by $x = 6$ and -1, or just $6, -1$ in either working box or on answer line.	***(A0)***

3 Follow through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final ***A*** mark should not be awarded.

(c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ ***(M1)(A1)***	(a) $\frac{\sin A}{4} = \frac{\sin 30}{3}$	***(M1)(A0)*** *(use of sine rule but with wrong values)*
$A = 22.0^\circ\ (22.0243\ldots)$ ***(A1)***	$A = 41.8^\circ$	***(A0)***
	*(**Note:** the 2nd **(A1)** here was not marked* **(ft)** *and cannot be awarded because there was an earlier error in the **same** question part.)*	
(b) $x = 7\tan(22.0243\ldots^\circ)$ ***(M1)***	(b) case (i) $x = 7\tan 41.8^\circ$	***(M1)***
$= 2.83\,(2.83163\ldots)$ ***(A1)*****(ft)**	$= 6.26$	***(A1)*****(ft)**
	but case (ii) 6.26	***(C0)*** *since no working shown*

4 Using the Markscheme

(a) ***A*** marks are **dependent** on the preceding ***M*** mark being awarded, it is **not** possible to award ***(M0)(A1)***. Once an ***(M0)*** has been awarded, all subsequent ***A*** marks are lost in that part of the question, even if calculations are performed correctly, until the next ***M*** mark.
The only exception will be for an answer where the accuracy is specified in the question – see section 5.

(b) ***A*** marks are **dependent** on the ***R*** mark being awarded, it is **not** possible to award ***(A1)(R0)***. Hence the ***(A1)*** is not awarded for a correct answer if no reason or the wrong reason is given.

(c) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(d) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value $\left(\text{for example } \frac{2}{3} \text{ if applicable}\right)$;
the full calculator display in the form $2.83163\ldots$ as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(e) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; $1\cdot7$; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval: $3 < x < 5$; $(3, 5)$; $]\,3, 5\,[$.

Different forms of notation for set properties (*e.g.* complement): A'; $\bar{A}$; A^c; $U - A$;$(A$;$U \setminus A$.
Different forms of logic notation: $\neg p$; p'; $\tilde{p}$; $\overline{p}$; $\sim p$.
$p \Rightarrow q$; $p \to q$; $q \Leftarrow p$.

Significance level may be written as α .

(f) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award ***(A1)*** and ignore subsequent rounding.

Note: The unrounded answer may appear in either the working box or on the final answer line.

2. If the candidate's unrounded answer is **not** seen then award ***(A1)*** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise ***(A0)***.

Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples which follow.

	If candidates final answer is given …					
	Exact or to 4 or more sf (and would **round to the correct 3 sf**)	**Correct to 3 sf**	**Incorrect to 3 sf**	Correct to 2 sf [3]	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen[1]	Award the final ***(A1)*** irrespective of correct or incorrect rounding					
Unrounded answer not seen[2]	***(A1)***	***(A1)***	***(A0)***	***(A1)***	***(A0)***	***(A0)***
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen.[3]			

Examples:

Markscheme	Candidates' Scripts	Marking
$9.43\ (9.43398\ldots)$ ***(A1)***	(i) $9.43398\ldots$ is seen in the working box followed by 9; 9.4; 9.43; 9.434 *etc*. (correctly rounded)	***(A1)***
	(ii) $9.43398\ldots$ is seen in the working box followed by 9.433; 9.44 *etc*. (incorrectly rounded)	***(A1)***
	(iii) 9.4	***(A1)***
	(iv) 9	***(A0)*** *(correct to 1 sf)*
	(v) 9.3	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 9.44	***(A0)*** *(incorrectly rounded to 3 sf)*

Markscheme	Candidates' Scripts	Marking
$7.44\ (7.43798\ldots)$ ***(A1)***	(i) $7.43798\ldots$ is seen in the working box followed by 7; 7.4; 7.44; 7.438 *etc*. (correctly rounded)	***(A1)***
	(ii) $7.43798\ldots$ is seen in the working box followed by 7.437; 7.43 *etc.* (incorrectly rounded)	***(A1)***
	(iii) 7.4	***(A1)***
	(iv) 7	***(A0)*** *(correct to 1 sf)*
	(v) 7.5	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 7.43	***(A0)*** *(incorrectly rounded to 3 sf)*

Example: ABC is a right angled triangle with angle $ABC = 90^\circ$, $AC = 32\text{ cm}$ and $AB = 30\text{ cm}$. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $BC = \sqrt{32^2 - 30^2}$ ***(M1)*** *Award **(M1)** for correct substitution in Pythagoras' formula* $= 11.1\ \left(\sqrt{124}, 11.1355...\right)(\text{cm})$ ***(A1)***	(a) $BC = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$	***(M1)*** ***(A1)*** *(2 sf answer only seen, but correct)*
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11.1355...$ ***(M1)*** *Award **(M1)** for correct substitution in area of triangle formula* $= 167(167.032...)(\text{cm}^2)$ ***(A1)*(ft)**	(b) case (i) $\text{Area} = \frac{1}{2} \times 30 \times 11$ $= 165\,(\text{cm}^2)$ case (ii) $= 165\,(\text{cm}^2)$	***(M1)*** *(working shown)* ***(A1)*(ft)** ***(M0)(A0)*(ft)** *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)*

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*e.g.* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg,* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding ***(M0)***.

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared $7.68\ (7.67543\ldots)$ ***(A2)***	(a) 7.7	***(A2)***
	(b) 7.67	***(A1)***
	(c) 7.6	***(A1)***
	(d) 8	***(A0)***
	(e) 7	***(A0)***
	(e) 7.66	***(A0)***

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ ***(A2)*** $(y = 0.887686\ldots x + 13.4895\ldots)$ If an answer is not in the form of an equation award at most ***(A1)(A0)***.	(a) $y = 0.89x + 13$	***(A2)*** *(both accepted)*
	(b) $y = 0.88x + 13$	***(A1)*** *(one rounding error)*
	(c) $y = 0.88x + 14$	***(A1)*** *(rounding error repeated)*
	(d) (i) $y = 0.9x + 13$	
	(ii) $y = 0.8x + 13$	***(A1)*** *(1 sf not accepted)*
	(e) $0.88x + 14$	***(A0)*** *(two rounding errors and not an equation)*

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ ***(A1)(A1)*** $(2.06020\ldots, 4.49253\ldots)$	(a) $(2.1, 4.5)$	***(A1)(A1)*** *(both accepted)*
	(b) $(2.0, 4.4)$	***(A1)*** *(same rounding error twice)*
	(c) $(2.06, 4.4)$	***(A1)*** *(one rounding error)*
	(d) $(2, 4.4)$	***(A0)*** *(1sf not accepted, one rounding error)*

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) ***(A1)***	(i) 231.6	***(A0)***
	(ii) 232	***(A0)*** *(Correct rounding to incorrect level)*
	(iii) 231.61	***(A0)***
	(iv) 232.00	***(A0)*** *(Parts (iii) and (iv) are both incorrect rounding to correct level)*

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one or two marks per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded ***(A1)*** under the accuracy rules given in Section 5.

Markscheme	Candidates' Scripts	Marking
(a) $37000\ \text{m}^2$ ***(A1)***	(a) $36000\ \text{m}^2$	***(A0)*** *(Incorrect answer so units not considered)*
(b) $3200\ \text{m}^3$ ***(A1)***	(b) $3200\ \text{m}^2$	***(A0)*** *(Incorrect units)*

If no method is shown and the answer is correct but with incorrect or missing units award the C marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often obtain solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment "I used my GDC" cannot receive a method mark.

1. (a) $24\,500 \times 25$ ***(M1)***

Note: Award ***(M1)*** for multiplying $24\,500$ by 25.

$= 613\,000 \quad (612\,500)$ ***(A1)*** ***(C2)***

[2 marks]

(b) $\left|\dfrac{612\,500 - 617\,700}{617\,700}\right| \times 100$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the percentage error formula.

$= 0.842 \quad (0.841832)$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

(c) 8.42×10^{-1} ***(A1)*****(ft)*****(A1)*****(ft)** ***(C2)***

Note: Award ***(A0)(A0)*** for answers of the type 84.2×10^{-2}. Follow through from part (b). Ignore '%' sign.

[2 marks]

Total [6 marks]

2. (a) $\dfrac{x+11}{2} = 10$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into median formula or for arranging all 9 values into ascending/descending order.

$(x =)\ 9$ ***(A1)*** ***(C2)***

[2 marks]

(b) (i) $2.69 \quad (2.69072\ldots)$ ***(A2)*****(ft)**

Note: Follow through from part (a).

(ii) $13 - 8$ ***(M1)***

Note: Award ***(M1)*** for 13 and 8 seen.

$= 5$ ***(A1)*****(ft)** ***(C4)***

Note: Follow through from part (a).

[4 marks]

Total [6 marks]

3. (a) If my Mathematical Studies homework is not due in tomorrow then today is Wednesday. ***(A1)(A1)*** ***(C2)***

> **Note:** Award ***(A1)*** for If… then…
> Award ***(A1)*** for correct propositions, *my Mathematical Studies homework is **not** due in tomorrow* and *today is Wednesday*, in the correct order.
> Award ***(A1)(A0)*** for "*If* $\neg p$ *then q*".

[2 marks]

(b)

p	q	$\neg p$	$\neg p \Rightarrow q$	$\neg p \wedge q$	$(\neg p \Rightarrow q) \vee (\neg p \wedge q)$
T	T	F	**T**	**F**	**T**
T	F	F	**T**	**F**	**T**
F	T	T	**T**	**T**	**T**
F	F	T	**F**	**F**	**F**

(A1)(A1)(A1)*(ft)** ***(C3)

[3 marks]

(c) neither ***(A1)*(ft)** ***(C1)***

> **Note:** Follow through from the final column of their truth table.

[1 mark]

Total [6 marks]

4. (a) (i) -0.974 $(-0.973745\ldots)$ ***(A2)***

> **Note:** Award ***(A1)*** for an answer of 0.974 (minus sign omitted).
> Award ***(A1)*** for an answer of -0.973 (incorrect rounding).

(ii) $y = -0.365x + 17.9$ $(y = -0.365032\ldots x + 17.9418\ldots)$ ***(A1)(A1)*** ***(C4)***

> **Note:** Award ***(A1)*** for $-0.365x$, ***(A1)*** for 17.9. Award at most ***(A1)(A0)*** if not an equation or if the values are reversed (*eg* $y = 17.9x - 0.365$).

[4 marks]

(b) $y = -0.365032\ldots \times 18 + 17.9418\ldots$ ***(M1)***

> **Note:** Award ***(M1)*** for correctly substituting 18 into their part (a)(ii).

$= 11.4$ $(11.3712\ldots)$ ***(A1)*(ft)** ***(C2)***

> **Note:** Follow through from part (a)(ii).

[2 marks]

Total [6 marks]

5. (a) $(1, -2)$ ***(A1)(A1)*** ***(C2)***

> **Note:** Award ***(A1)*** for 1 and ***(A1)*** for -2, seen as a coordinate pair.
> Accept $x=1,\ y=-2$. Award ***(A1)(A0)*** if x and y coordinates are reversed.

[2 marks]

(b) $\dfrac{1-(-2)}{-3-1}$ ***(M1)***

> **Note:** Award ***(M1)*** for correct substitution, of their part (a), into gradient formula.

$= -\dfrac{3}{4}\ \ (-0.75)$ ***(A1)*(ft)** ***(C2)***

> **Note:** Follow through from part (a).

[2 marks]

(c) $y-1=-\dfrac{3}{4}(x+3)$ **OR** $y+2=-\dfrac{3}{4}(x-1)$ **OR** $y=-\dfrac{3}{4}x-\dfrac{5}{4}$ ***(M1)***

> **Note:** Award ***(M1)*** for correct substitution of their part (b) and a given point.

OR

$1=-\dfrac{3}{4}\times -3+c$ **OR** $-2=-\dfrac{3}{4}\times 1+c$ ***(M1)***

> **Note:** Award ***(M1)*** for correct substitution of their part (b) and a given point.

$3x+4y+5=0$ (accept any integer multiple, including negative multiples) ***(A1)*(ft)** ***(C2)***

> **Note:** Follow through from parts (a) and (b). Where the gradient in part (b) is found
> to be $\dfrac{5}{0}$, award at most ***(M1)(A0)*** for either $x=-3$ or $x+3=0$.

[2 marks]

Total [6 marks]

6. (a) 180 ***(A1)*** ***(C1)***

[1 mark]

(b) $36, 24$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A0)(A1)*** for two incorrect values that add up to 60.

[2 marks]

(c) (i) 125 (accept 125.5) ***(A1)***

(ii) $$\frac{4\times 25+36\times 75+34\times 125+46\times 175+24\times 225+16\times 275}{160}$$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their mid-interval values, multiplied by their frequencies, into mean formula.

$=156$ (155.625) ***(A1)*(ft)** ***(C3)***

Note: Follow through from parts (b) and (c)(i).

[3 marks]

Total [6 marks]

7. (a) (i) $5d = 46 - 21$ **OR** $u_1 + 2d = 21$ and $u_1 + 7d = 46$ ***(M1)***

Note: Award ***(M1)*** for a correct equation in d or for two correct equations in u_1 and d.

$(d =)\ 5\ \text{(kg)}$ ***(A1)*** ***(C2)***

(ii) $u_1 + 2 \times 5 = 21$ ***(M1)***

OR

$u_1 + 7 \times 5 = 46$ ***(M1)***

Note: Award ***(M1)*** for substitution of their d into either of the two equations.

$(u_1 =)\ 11\ \text{(kg)}$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a)(i).

[4 marks]

(b) $\frac{12}{2}\left(2 \times 11 + (12-1) \times 5\right)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into arithmetic series formula.

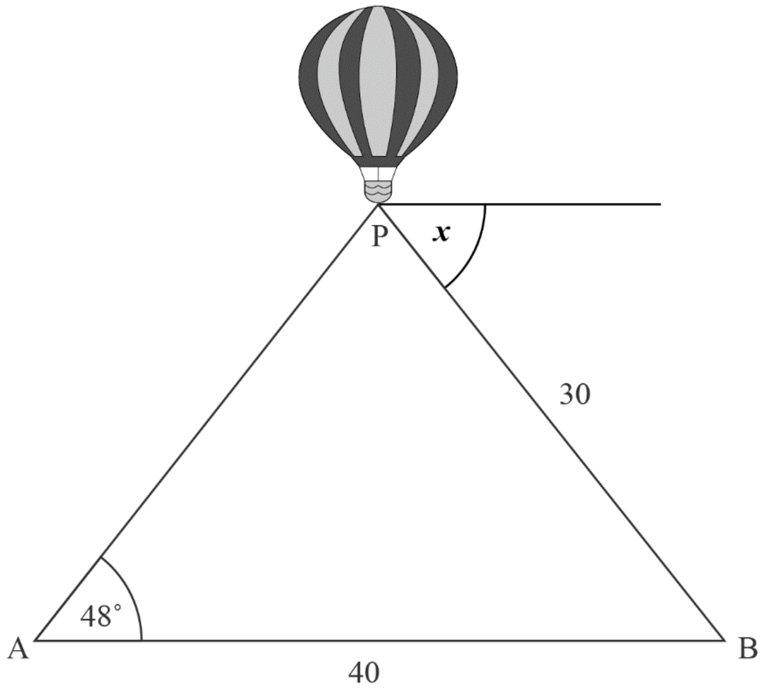
$= 462\ \text{(kg)}$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from parts (a) and (b).

[2 marks]

Total [6 marks]

8. (a)



(A1) ***(C1)***
[1 mark]

(b) $\dfrac{40}{\sin \text{APB}} = \dfrac{30}{\sin 48^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into sine rule, ***(A1)*** for correct substitution.

(angle $\text{APB} =$) 82.2° $(82.2473\ldots^\circ)$ ***(A1)*** ***(C3)***

[3 marks]

(c) $180 - 48 - 82.2473\ldots$ ***(M1)***
49.8° $(49.7526\ldots^\circ)$ ***(A1)*****(ft)*** ***(C2)***

Note: Follow through from parts (a) and (b).

[2 marks]

Total [6 marks]

9. (a) (i) $12 \times 3^{0.498 \times 0}$ ***(M1)***

Note: Award ***(M1)*** for substituting zero into the equation.

$= 12$ ***(A1)*** ***(C2)***

(ii) $12 \times 3^{0.498 \times 6}$ ***(M1)***

Note: Award ***(M1)*** for substituting 6 into the equation.

320 ***(A1)*** ***(C2)***

Note: Accept an answer of $319.756\ldots$ or 319.

[4 marks]

(b) $8000 = 12 \times 3^{0.498t}$ ***(M1)***

Note: Award ***(M1)*** for equating equation to 8000. Award ***(M1)*** for a sketch of $P(t)$ intersecting with the straight line $y = 8000$.

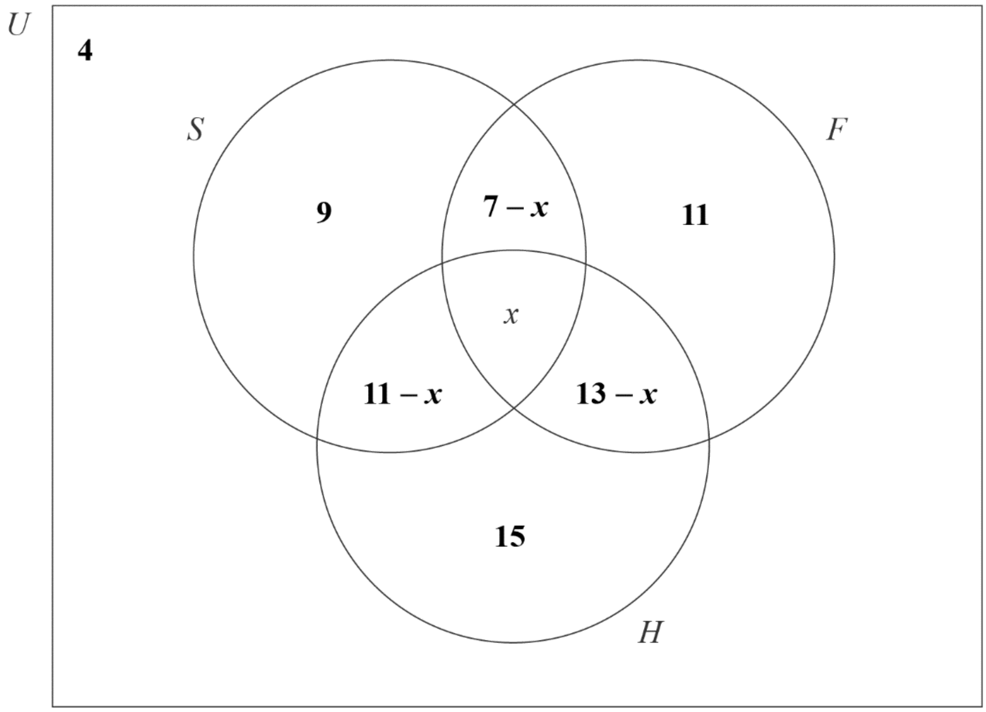
$= 11.9 \quad (11.8848\ldots)$ ***(A1)*** ***(C2)***

Note: Accept an answer of 11 or 12.

[2 marks]

Total [6 marks]

10. (a)



(A1)(A1)(A1) ***(C3)***

Note: Award ***(A1)*** for 4 in correct place. Award ***(A1)*** for 9, 11, 15 in correct place. Award ***(A1)*** for $7-x$, $13-x$, $11-x$ in correct place. Accept 2, 8 and 6 in place of $7-x$, $13-x$, $11-x$.

[3 marks]

(b) $4+9+11+15+x+(7-x)+(11-x)+(13-x)=60$ ***(M1)***

Note: Award ***(M1)*** for equating the sum of at least seven of the entries in their Venn diagram to 60.

$(x=)$ 5 ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a), but only if answer is positive.

[2 marks]

(c) 34 ***(A1)*(ft)** ***(C1)***

Note: Follow through from their Venn diagram.

[1 mark]

Total [6 marks]

11. (a) $54 \times (0.94)^{50}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into geometric sequence formula, ***(A1)*** for correct substitution.

2.45 (cm) $(2.44785\ldots$ cm$)$ ***(A1)*** ***(C3)***

[3 marks]

(b) $\dfrac{54 \times \left(1 - (0.94)^{51}\right)}{1 - 0.94}$ (or equivalent) ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substitution into geometric series formula, ***(A1)*****(ft)** for correct substitution using their common ratio from part (a).

$= 862$ (cm) $\left(861.650\ldots(\text{cm})\right)$ ***(A1)*****(ft)** ***(C3)***

[3 marks]

Total [6 marks]

12. (a) $1.75 = \frac{-b}{2a}$ (or equivalent) ***(A1)*** ***(C1)***

> **Note:** Award ***(A1)*** for $f(x) = (1.75)^2 a + 1.75b + 22$ or for $y = (1.75)^2 a + 1.75b + 22$ or for $f(1.75) = (1.75)^2 a + 1.75b + 22$.

[1 mark]

(b) $(-2)^2 \times a + (-2) \times b + 22 = 0$ (or equivalent) ***(A1)*** ***(C1)***

> **Note:** Award ***(A1)*** for $(-2)^2 \times a + (-2) \times b + 22 = 0$ seen.
> Award ***(A0)*** for $y = (-2)^2 \times a + (-2) \times b + 22$.

[1 mark]

(c) $a = -2, \; b = 7$ ***(A1)*****(ft)*****(A1)*****(ft)** ***(C2)***

> **Note:** Follow through from parts (a) and (b).
> Accept answers(s) embedded as a coordinate pair.

[2 marks]

(d) $-2x^2 + 7x + 22 = 0$ ***(M1)***

> **Note:** Award ***(M1)*** for correct substitution of a and b into equation and setting to zero. Follow through from part (c).

$(x =)$ 5.5 ***(A1)*****(ft)** ***(C2)***

> **Note:** Follow through from parts (a) and (b).

OR

x-coordinate $= 1.75 + \left(1.75 - (-2)\right)$ ***(M1)***

> **Note:** Award ***(M1)*** for correct use of axis of symmetry and given intercept.

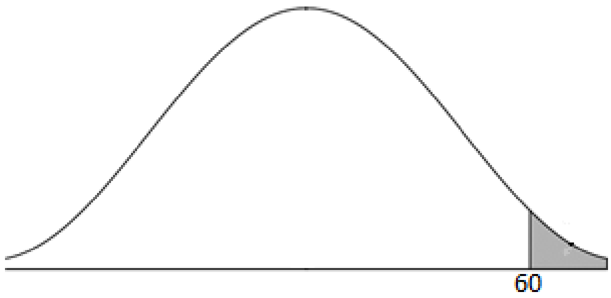
$(x =)$ 5.5 ***(A1)*** ***(C2)***

[2 marks]

Total [6 marks]

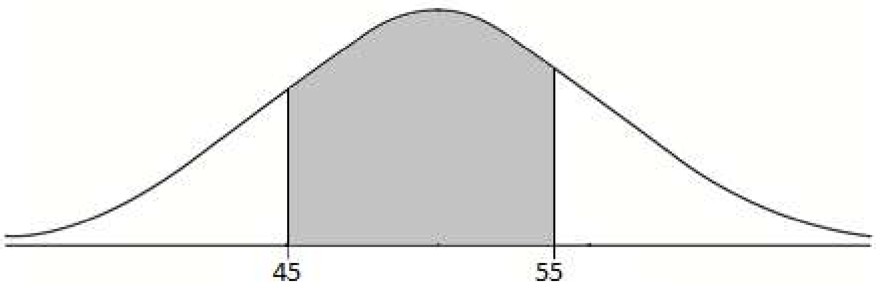
13. (a) (i) 0.0548 $(0.054799\ldots, 5.48\%)$ ***(A2)*** ***(C2)***

Note: Award ***(M1)(A0)*** for a correct probability statement, $\mathrm{P}(X > 60)$, or normal distribution graph with correctly shaded region, leading to incorrect or no answer.



(ii) 0.645 $(0.644990\ldots, 64.5\%)$ ***(A2)*** ***(C2)***

Note: Award ***(M1)(A0)*** for a correct probability statement, $\mathrm{P}(45 < X < 55)$, or normal distribution graph with correctly shaded region, leading to incorrect or no answer.



[4 marks]

(b) $\dfrac{15}{0.0548}$ ***(M1)***

Note: Award ***(M1)*** for dividing 15 by their part (a)(i). Accept an equation of the form $15 = x \times 0.0548$ for ***(M1)***.

274 $(273.722\ldots)$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a)(i). Accept 273.

[2 marks]

Total [6 marks]

14. (a) $\sqrt{15^2 - 12^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras theorem.

OR

$\frac{\text{radius}}{21} = \frac{15}{35}$ ***(M1)***

Note: Award ***(M1)*** for a correct equation.

$= 9 \text{ (cm)}$ ***(A1)*** ***(C2)***

[2 marks]

(b) $\pi \times 9 \times 15$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into curved surface area of a cone formula.

$= 424 \text{ cm}^2 \quad \left(135\pi,\ 424.115\ldots \text{cm}^2\right)$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

(c) $\pi \times 21 \times 35 - 424.115\ldots$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into curved surface area of a cone formula and for subtracting their part (b).

$= 1880 \text{ cm}^2 \quad \left(600\pi,\ 1884.95\ldots \text{ cm}^2\right)$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (b).

[2 marks]

Total [6 marks]

15. (a) $[-2, \infty[$ or $[-2, \infty)$ **OR** $f(x) \geq -2$ or $y \geq -2$ **OR** $-2 \leq f(x) < \infty$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for -2 and ***(A1)*** for completely correct mathematical notation, including weak inequalities. Accept $f \geq -2$.

[2 marks]

(b) -1 and 1.52 $(1.51839\ldots)$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for -1 and ***(A1)*** for 1.52 (1.51839).

[2 marks]

(c) $x < -1,\ x > 1.52$ **OR** $(-\infty, -1) \cup (1.52, \infty)$. ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: Award ***(A1)*(ft)** for **both** critical values in inequality or range statements such as $x < -1$, $(-\infty, -1)$, $x > 1.52$ or $(1.52, \infty)$.
Award the second ***(A1)*(ft)** for correct strict inequality statements used with their critical values. If an incorrect use of strict and weak inequalities has already been penalized in (a), condone weak inequalities for this second mark and award ***(A1)*(ft)**.

[2 marks]

Total [6 marks]

M18/5/MATSD/SP1/ENG/TZ2/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

21 pages

2218 – 7405

International Baccalaureate
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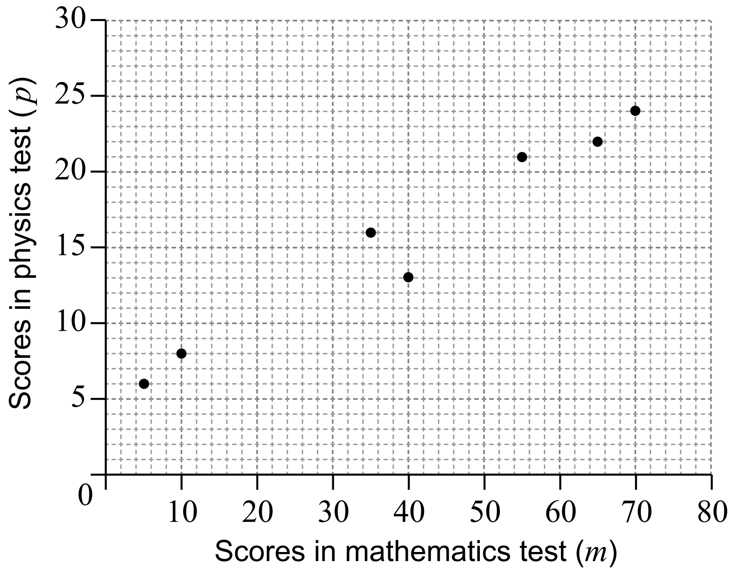
Please **do not** write on this page.

Answers written on this page
will not be marked.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. The following scatter diagram shows the scores obtained by seven students in their mathematics test, m, and their physics test, p.



The mean point, M, for these data is $(40, 16)$.

(a) Plot and label the point $\mathrm{M}(\bar{m}, \bar{p})$ on the scatter diagram. [2]

(b) Draw the line of best fit, by eye, on the scatter diagram. [2]

(c) Using your line of best fit, estimate the physics test score for a student with a score of 20 in their mathematics test. [2]

Working:

Answer:

(c) . . . .

2. Consider the following propositions.

p: the baby cries
q: the baby is happy
r: the baby wants to play

(a) Write down, in words, $(q \wedge r) \Rightarrow \neg p$. [3]

(b) Complete the following truth table. [2]

p	q	r	$\neg p$	$(q \wedge r)$	$(q \wedge r) \Rightarrow \neg p$
T	T	T	F		
T	T	F	F		
T	F	T	F		
T	F	F	F		
F	T	T	T		
F	T	F	T		
F	F	T	T		
F	F	F	T		

(c) State whether $(q \wedge r) \Rightarrow \neg p$ is a tautology, contradiction or neither. [1]

Working:

Answers:

(a) . . . .

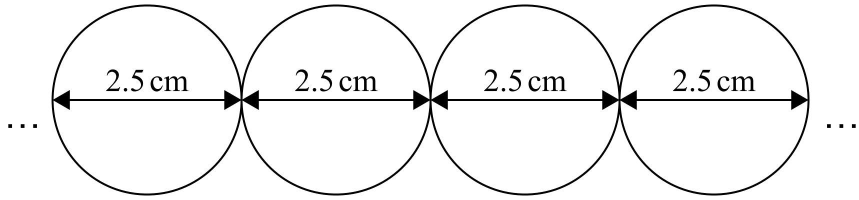
. . . .

. . . .

(c) . . . .

3. Last year a South American candy factory sold 4.8×10^8 spherical sweets. Each sweet has a diameter of $2.5\,\text{cm}$.

The factory is producing an advertisement showing all of these sweets placed in a straight line.



(a) Find the length, in cm, of this line. Give your answer in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$. [3]

The advertisement claims that the length of this line is x times the length of the Amazon River. The length of the Amazon River is $6400\,\text{km}$.

(b) (i) Write down the length of the Amazon River in cm.

(ii) Find the value of x. [3]

Working:

Answers:

(a) . . . .

(b) (i) . . . .

(ii) . . . .

4. The following table shows four different sets of numbers: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$.

Set	Example of a number in the set
$\mathbb{N}$	
$\mathbb{Z}$	
$\mathbb{Q}$	
$\mathbb{R}$	

(a) Complete the second column of the table by giving **one** example of a number from each set. [4]

Josh states: “Every integer is a natural number”.

(b) Write down whether Josh’s statement is correct. Justify your answer. [2]

Working:

Answer:

(b) . . . .

. . . .

. . . .

5. **In this question, give all answers to two decimal places.**

Karl invests 1000 US dollars (USD) in an account that pays a nominal annual interest of 3.5%, **compounded quarterly**. He leaves the money in the account for 5 years.

(a) (i) Calculate the amount of money he has in the account after 5 years;

(ii) Write down the amount of **interest** he earned after 5 years. [4]

Karl decides to donate this **interest** to a charity in France. The charity receives 170 euros (EUR). The exchange rate is $1\,\text{USD} = t\,\text{EUR}$.

(b) Calculate the value of t. [2]

Working:

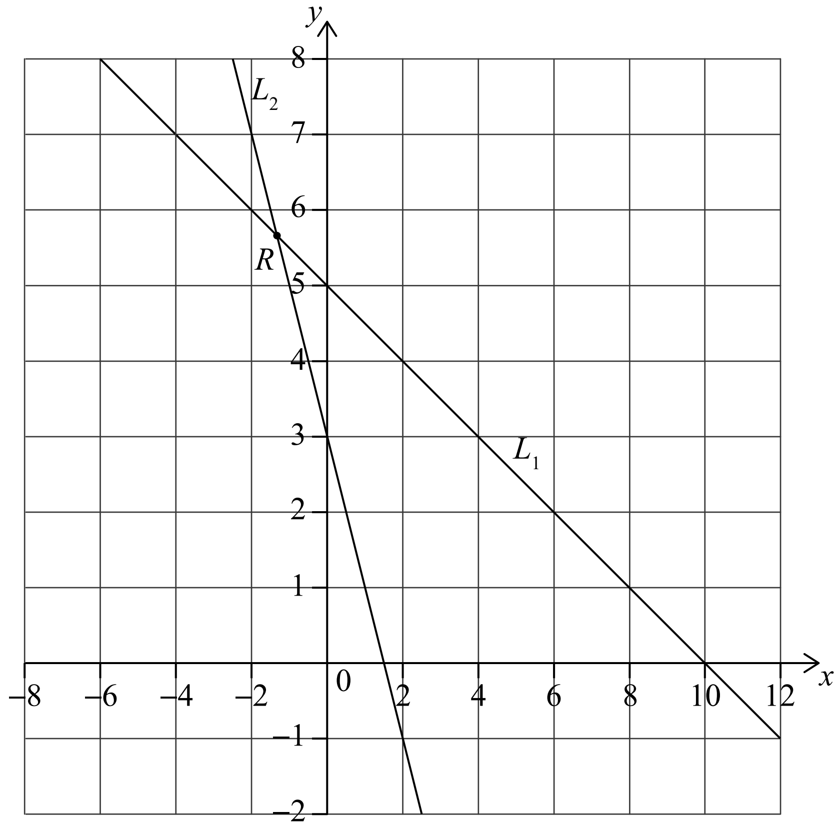
Answers:

(a) (i) . . . .

(ii) . . . .

(b) . . . .

6. Consider the straight lines L_1 and L_2. R is the point of intersection of these lines.



The equation of line L_1 is $y = ax + 5$.

(a) Find the value of a. [2]

The equation of line L_2 is $y = -2x + 3$.

(b) Find the coordinates of R. [2]

Line L_3 is parallel to line L_2 and passes through the point $(2, 3)$.

(c) Find the equation of line L_3. Give your answer in the form $y = mx + c$. [2]

(This question continues on the following page)

(Question 6 continued)

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

7. In an international competition, participants can answer questions in **only one** of the three following languages: Portuguese, Mandarin or Hindi. 80 participants took part in the competition. The number of participants answering in Portuguese, Mandarin or Hindi is shown in the table.

		Languages			
		Portuguese	Mandarin	Hindi	**Total**
Participants	Boys	20	18	5	**43**
	Girls	18	7	12	**37**
	Total	**38**	**25**	**17**	**80**

(a) State the number of boys who answered questions in Portuguese. [1]

A boy is chosen at random.

(b) Find the probability that the boy answered questions in Hindi. [2]

Two girls are selected at random.

(c) Calculate the probability that one girl answered questions in Mandarin and the other answered questions in Hindi. [3]

Working:

Answers:

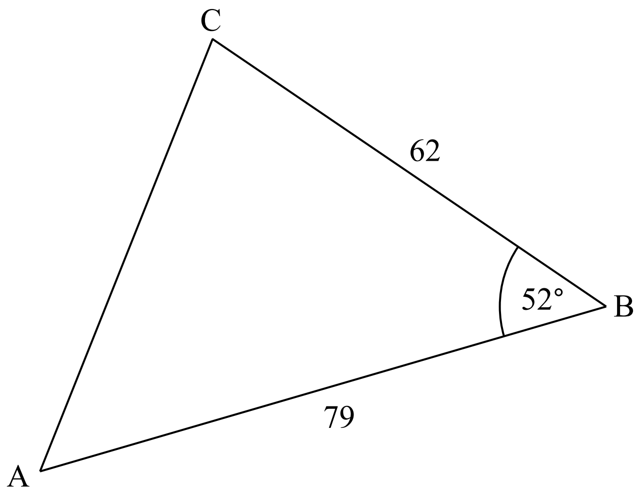
(a) . . . .

(b) . . . .

(c) . . . .

8. A park in the form of a triangle, ABC, is shown in the following diagram. AB is $79\,\mathrm{km}$ and BC is $62\,\mathrm{km}$. Angle $\mathrm{A\hat{B}C}$ is 52°.

diagram not to scale



(a) Calculate the length of side AC in km. [3]

(b) Calculate the area of the park. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

9. Consider the following Venn diagrams.

Diagram 1

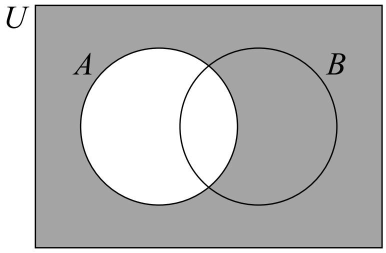


Diagram 2

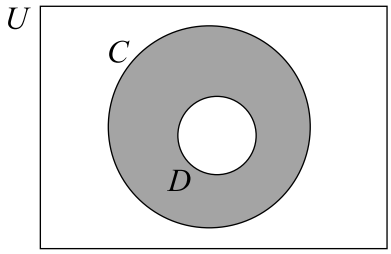
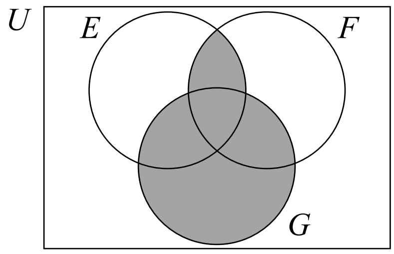


Diagram 3



(a) Write down an expression, in set notation, for the **shaded** region represented by

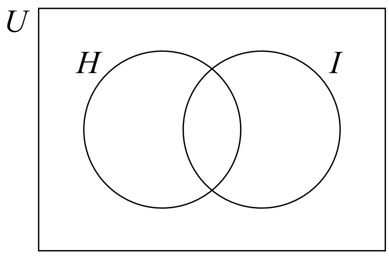
(i) Diagram 1;

(ii) Diagram 2;

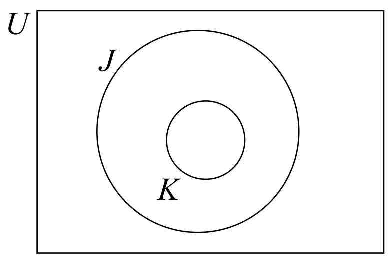
(iii) Diagram 3. [4]

(b) Shade, on the Venn diagram, the region represented by the set

(i) $(H \cup I)'$;



(ii) $J \cap K$. [2]



(This question continues on the following page)

(Question 9 continued)

Working:

Answers:

(a) (i) . . . .

(ii) . . . .

(iii) . . . .

10. The following function models the growth of a bacteria population in an experiment,

$$P(t) = A \times 2^t,\ t \geq 0$$

where A is a constant and t is the time, in hours, since the experiment began.

Four hours after the experiment began, the bacteria population is 6400.

(a) Find the value of A. [2]

(b) Interpret what A represents in this context. [1]

(c) Find the time since the experiment began for the bacteria population to be equal to $40A$. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

. . . .

(c) . . . .

11. Consider the graph of the function $f(x) = \frac{3}{x} - 2$, $x \neq 0$.

(a) Write down the equation of the vertical asymptote. [2]

(b) Write down the equation of the horizontal asymptote. [2]

(c) Calculate the value of x for which $f(x) = 0$. [2]

Working:

Answers:

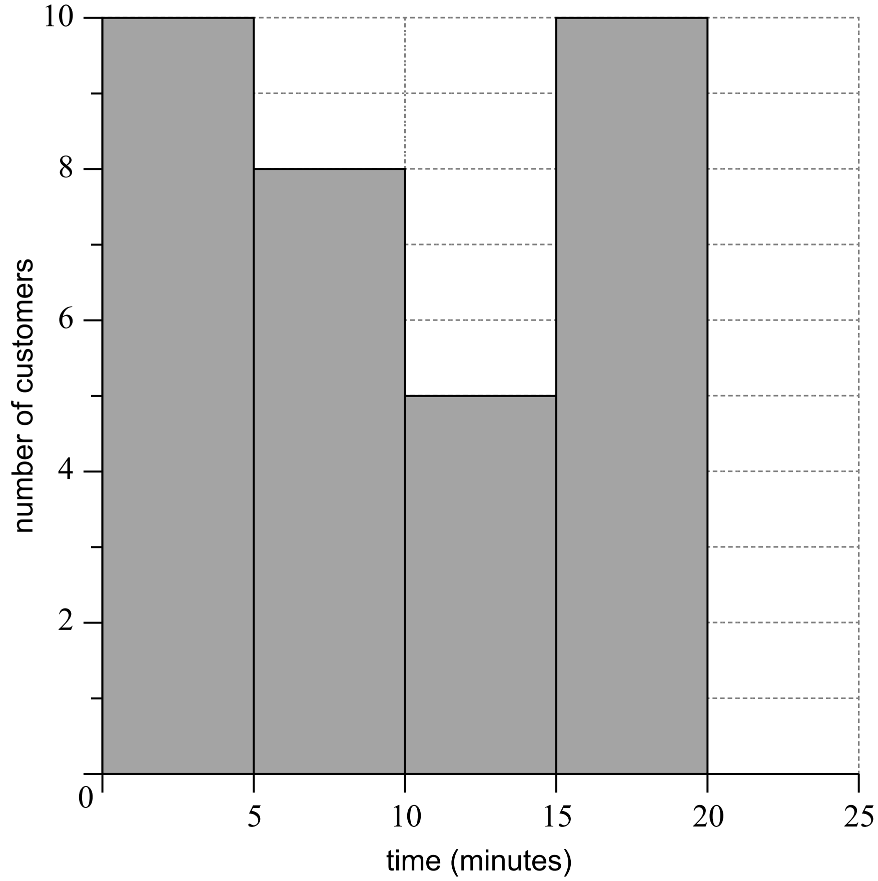
(a) . . . .

(b) . . . .

(c) . . . .

12. The histogram shows the time, t, in minutes, that it takes the customers of a restaurant to eat their lunch on one particular day. Each customer took less than 25 minutes**.**

The histogram is incomplete, and only shows data for $0 \le t < 20$.



(a) Write down the mid-interval value for $10 \le t < 15$. [1]

The mean time it took **all** customers to eat their lunch was estimated to be 12 minutes.

It was found that k customers took between 20 and 25 minutes to eat their lunch.

(b) (i) Write down the total number of customers in terms of k.

(ii) Calculate the value of k. [4]

(c) Hence, complete the histogram. [1]

(This question continues on the following page)

(Question 12 continued)

Working:

Answers:

(a) ....

(b) (i) ....

(ii) ....

13. A factory produces shirts. The cost, C, in Fijian dollars (FJD), of producing x shirts can be modelled by

$$C(x) = (x - 75)^2 + 100.$$

(a) Find the cost of producing 70 shirts. [2]

The cost of production should not exceed 500 FJD. To do this the factory needs to produce at least 55 shirts and at most s shirts.

(b) Find the value of s. [2]

(c) Find the number of shirts produced when the cost of production is lowest. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

14. Consider the function $f(x) = \frac{x^4}{4}$.

(a) Find $f'(x)$. [1]

(b) Find the gradient of the graph of f at $x = -\frac{1}{2}$. [2]

(c) Find the x-coordinate of the point at which the **normal** to the graph of f has gradient $-\frac{1}{8}$. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

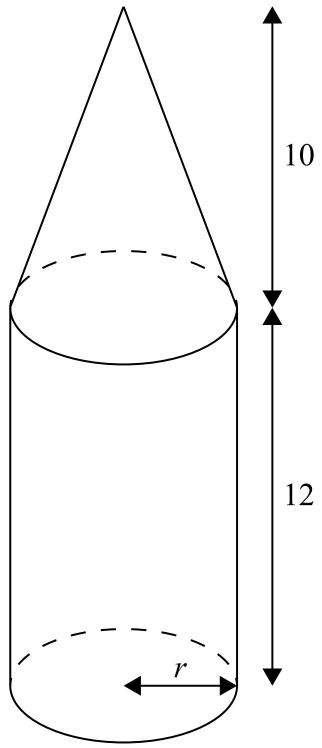
(c) . . . .

15. Julio is making a wooden pencil case in the shape of a large pencil. The pencil case consists of a cylinder attached to a cone, as shown.

The cylinder has a radius of r cm and a height of 12 cm.

The cone has a base radius of r cm and a height of 10 cm.

diagram not to scale



(a) Find an expression for the slant height of the cone **in terms of r**. [2]

The total external surface area of the pencil case rounded to 3 significant figures is $570\,\text{cm}^2$.

(b) Using your graphic display calculator, calculate the value of r. [4]

(This question continues on the following page)

(Question 15 continued)

Working:

Answers:

(a) . . . .

(b) . . . .

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will not be marked.

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will not be marked.

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M18/5/MATSD/SP1/ENG/TZ2/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Mathematical studies

Standard level

Paper 1

23 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Paper 1 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

The number of marks for each question is 6.

1 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

C Marks awarded for **Correct** answers (irrespective of working shown)

R Marks awarded for clear **Reasoning**

ft Marks that can be awarded as **follow through** from previous results in the question

2 Method of Marking

(a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b) If the candidate has full marks on a question use the ***C6*** annotation, if the candidate has made an attempt but scores zero marks use ***C0***. If there is no attempt use the No response button. If a candidate does not score full or zero marks then full annotations **MUST** be shown.

(c) In this paper, if the **correct answer is seen on the answer line the** maximum mark is awarded. **There is no need to check the working!** Award ***C*** marks and move on.

(d) If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.

(e) If the **answer is wrong**, marks should be awarded for the working according to the markscheme.

(f) Working crossed out by the candidate should not be awarded any marks. Where candidates have written two solutions to a question, only the first solution should be marked.

(g) A correct answer in the working box transcribed inaccurately to the answer line can receive full marks.

(h) If correct working results in a correct answer **in the working box** but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer **as long as it is unambiguous**. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***(A1)*** *(ignore the further working)*
2.	$(x-6)(x+1)$	$x=6$ and -1	Do **not** award the final ***(A1)*** *(see next example)*

Example: Factorise $x^2 - 5x - 6$

Markscheme	Candidates' Scripts	Marking
$(x-6)(x+1)$ ***(A1)(A1)***	(i) Answer line: $(x+6)(x+1)$	***(A0)(A1)***
	(ii) Working box: $(x-6)(x+1)$	***(A1)***
	followed by $x=6$ and -1, or just $6, -1$ in either working box or on answer line.	***(A0)***

3 Follow through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final ***A*** mark should not be awarded.

(c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ ***(M1)(A1)***	(a) $\frac{\sin A}{4} = \frac{\sin 30}{3}$	***(M1)(A0)*** *(use of sine rule but with wrong values)*
$A = 22.0^\circ$ $(22.0243\ldots)$ ***(A1)***	$A = 41.8^\circ$	***(A0)***
	*(**Note:** the 2nd **(A1)** here was not marked **(ft)** and cannot be awarded because there was an earlier error in the **same** question part.)*	
(b) $x = 7\tan(22.0243\ldots^\circ)$ ***(M1)***	(b) case (i) $x = 7\tan 41.8^\circ$	***(M1)***
$= 2.83\,(2.83163\ldots)$ ***(A1)*****(ft)**	$= 6.26$	***(A1)*****(ft)**
	but case (ii) 6.26	***(C0)*** *since no working shown*

4 Using the Markscheme

(a) ***A*** marks are **dependent** on the preceding ***M*** mark being awarded, it is **not** possible to award ***(M0)(A1)****.* Once an ***(M0)*** has been awarded, all subsequent ***A*** marks are lost in that part of the question, even if calculations are performed correctly, until the next ***M*** mark.
The only exception will be for an answer where the accuracy is specified in the question – see section 5.

(b) ***A*** marks are **dependent** on the ***R*** mark being awarded, it is **not** possible to award ***(A1)(R0)***. Hence the ***(A1)*** is not awarded for a correct answer if no reason or the wrong reason is given.

(c) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(d) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:

the 3 significant figure answer worked through from full calculator display;

the exact value $\left(\text{for example } \frac{2}{3} \text{ if applicable}\right)$;

the full calculator display in the form $2.83163\ldots$ as in the example above.

Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(e) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; $1\cdot7$; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval: $3 < x < 5$; (3, 5);] 3, 5 [.

Different forms of notation for set properties (*e.g.* complement): A'; $\bar{A}$; A^c; $U - A$;$(A$;$U \setminus A$.

Different forms of logic notation: $\neg p$; p'; $\tilde{p}$; $\bar{p}$; $\sim p$.

$p \Rightarrow q$; $p \to q$; $q \Leftarrow p$.

Significance level may be written as α.

(f) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award ***(A1)*** and ignore subsequent rounding.
Note: The unrounded answer may appear in either the working box or on the final answer line.

2. If the candidate's unrounded answer is **not** seen then award ***(A1)*** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise ***(A0)***.
Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples which follow.

<table>
<tr><td></td><td colspan="6">If candidates final answer is given …</td></tr>
<tr><td></td><td>Exact or to 4 or more sf (and would round to the correct 3 sf)</td><td>Correct to 3 sf</td><td>Incorrect to 3 sf</td><td>Correct to 2 sf [3]</td><td>Incorrect to 2 sf</td><td>Correct or incorrect to 1 sf</td></tr>
<tr><td>Unrounded answer seen[1]</td><td colspan="6">Award the final (A1) irrespective of correct or incorrect rounding</td></tr>
<tr><td>Unrounded answer not seen[2]</td><td>(A1)</td><td>(A1)</td><td>(A0)</td><td>(A1)</td><td>(A0)</td><td>(A0)</td></tr>
<tr><td>Treatment of subsequent parts</td><td colspan="2">As per MS</td><td colspan="4">Treat as follow through, only if working is seen.[3]</td></tr>
</table>

Examples:

Markscheme		Candidates' Scripts		Marking
9.43 (9.43398…)	***(A1)***	(i)	9.43398… is seen in the working box followed by 9; 9.4; 9.43; 9.434 *etc*. (correctly rounded)	***(A1)***
		(ii)	9.43398… is seen in the working box followed by 9.433; 9.44 *etc*. (incorrectly rounded)	***(A1)***
		(iii)	9.4	***(A1)***
		(iv)	9	***(A0)*** *(correct to 1 sf)*
		(v)	9.3	***(A0)*** *(incorrectly rounded to 2 sf)*
		(vi)	9.44	***(A0)*** *(incorrectly rounded to 3 sf)*

Markscheme		Candidates' Scripts		Marking
7.44 (7.43798…)	***(A1)***	(i)	7.43798… is seen in the working box followed by 7; 7.4; 7.44; 7.438 *etc*. (correctly rounded)	***(A1)***
		(ii)	7.43798… is seen in the working box followed by 7.437; 7.43 *etc.* (incorrectly rounded)	***(A1)***
		(iii)	7.4	***(A1)***
		(iv)	7	***(A0)*** *(correct to 1 sf)*
		(v)	7.5	***(A0)*** *(incorrectly rounded to 2 sf)*
		(vi)	7.43	***(A0)*** *(incorrectly rounded to 3 sf)*

Example: ABC is a right angled triangle with angle $\text{ABC} = 90^\circ$, $\text{AC} = 32\text{ cm}$ and $\text{AB} = 30\text{ cm}$. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $\text{BC} = \sqrt{32^2 - 30^2}$ ***(M1)*** *Award* ***(M1)*** *for correct substitution in Pythagoras' formula* $= 11.1\ \left(\sqrt{124}, 11.1355...\right)(\text{cm})$ ***(A1)***	(a) $\text{BC} = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$	***(M1)*** ***(A1)*** *(2 sf answer only seen, but correct)*
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11.1355...$ ***(M1)*** *Award* ***(M1)*** *for correct substitution in area of triangle formula* $= 167 (167.032...)\,(\text{cm}^2)$ ***(A1)*(ft)**	(b) case (i) $\text{Area} = \frac{1}{2} \times 30 \times 11$ $= 165\,(\text{cm}^2)$ case (ii) $= 165\,(\text{cm}^2)$	***(M1)*** *(working shown)* ***(A1)*(ft)** ***(M0)(A0)*(ft)** *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)*

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*e.g.* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg,* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding ***(M0)***.

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared 7.68 (7.67543…) ***(A2)***	(a) 7.7	***(A2)***
	(b) 7.67	***(A1)***
	(c) 7.6	***(A1)***
	(d) 8	***(A0)***
	(e) 7	***(A0)***
	(e) 7.66	***(A0)***

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ ***(A2)*** $(y = 0.887686\ldots x + 13.4895\ldots)$ If an answer is not in the form of an equation award at most ***(A1)(A0)***.	(a) $y = 0.89x + 13$	***(A2)*** *(both accepted)*
	(b) $y = 0.88x + 13$	***(A1)*** *(one rounding error)*
	(c) $y = 0.88x + 14$	***(A1)*** *(rounding error repeated)*
	(d) (i) $y = 0.9x + 13$	
	(ii) $y = 0.8x + 13$	***(A1)*** *(1 sf not accepted)*
	(e) $0.88x + 14$	***(A0)*** *(two rounding errors and not an equation)*

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ ***(A1)(A1)*** $(2.06020\ldots, 4.49253\ldots)$	(a) $(2.1, 4.5)$	***(A1)(A1)*** *(both accepted)*
	(b) $(2.0, 4.4)$	***(A1)*** *(same rounding error twice)*
	(c) $(2.06, 4.4)$	***(A1)*** *(one rounding error)*
	(d) $(2, 4.4)$	***(A0)*** *(1sf not accepted, one rounding error)*

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) ***(A1)***	(i) 231.6	***(A0)***
	(ii) 232	***(A0)*** *(Correct rounding to incorrect level)*
	(iii) 231.61	***(A0)***
	(iv) 232.00	***(A0)*** *(Parts (iii) and (iv) are both incorrect rounding to correct level)*

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one or two marks per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded ***(A1)*** under the accuracy rules given in Section 5.

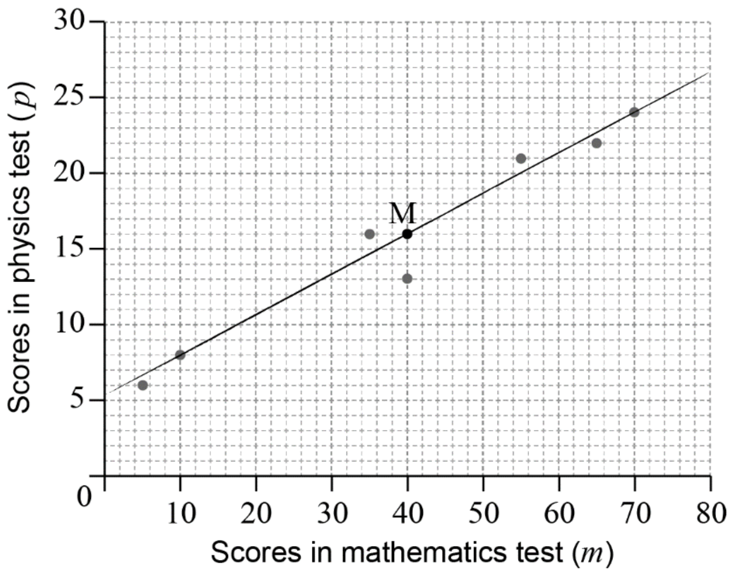
Markscheme	Candidates' Scripts	Marking
(a) $37\,000\text{ m}^2$ ***(A1)***	(a) $36\,000\text{ m}^2$	***(A0)*** *(Incorrect answer so units not considered)*
(b) 3200 m^3 ***(A1)***	(b) 3200 m^2	***(A0)*** *(Incorrect units)*

If no method is shown and the answer is correct but with incorrect or missing units award the C marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often obtain solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment “I used my GDC” cannot receive a method mark.

1. (a)



(A1)(A1) ***(C2)***

Note: Award ***(A1)*** for mean point plotted and ***(A1)*** for labelled M.

[2 marks]

(b) straight line through their mean point crossing the p-axis at 5 ± 2

(A1)*(ft)*(A1)*(ft)** ***(C2)

Note: Award ***(A1)*(ft)** for a **straight** line through their **mean** point. Award ***(A1)*(ft)** for a correct p-intercept if line is extended.

[2 marks]

(c) point on line where $m = 20$ identified and an attempt to identify y-coordinate ***(M1)***

10.5 ***(A1)*(ft)** ***(C2)***

Note: Follow through from their line in part (b).

[2 marks]

Total [6 marks]

2. (a) if the baby is happy and wants to play then the baby does not cry

(A1)(A1)(A1) ***(C3)***

Note: Award ***(A1)*** for "If… then…"; ***(A1)*** for "the baby is happy **and** wants to play", ***(A1)*** for "the baby does not cry". Crying must be negated.

[3 marks]

(b)

p	q	r	$\neg p$	$q \wedge r$	$(q \wedge r) \Rightarrow \neg p$
T	T	T	F	**T**	**F**
T	T	F	F	**F**	**T**
T	F	T	F	**F**	**T**
T	F	F	F	**F**	**T**
F	T	T	T	**T**	**T**
F	T	F	T	**F**	**T**
F	F	T	T	**F**	**T**
F	F	F	T	**F**	**T**

(A1)(A1) ***(C2)***

Note: Award ***(A1)*** for each correct column.

[2 marks]

(c) Neither ***(A1)*****(ft)** ***(C1)***

Note: Follow through from the last column in their part (b).

[1 mark]

Total [6 marks]

3. (a) $4.8\times10^{8}\times2.5$ ***(M1)***

Note: Award ***(M1)*** for multiplying by 2.5.

1.2×10^{9} (cm) ***(A1)*(ft)*(A1)*(ft)** ***(C3)***

Note: Award ***(A0)(A0)*** for answers of the type 12×10^{8}.

[3 marks]

(b) (i) $640\,000\,000$ (cm) $\left(6.4\times10^{8}\text{ (cm)}\right)$ ***(A1)***

(ii) $\dfrac{1.2\times10^{9}}{6.4\times10^{8}}$ ***(M1)***

Note: Award ***(M1)*** for division by $640\,000\,000$.

$=1.88$ (1.875) ***(A1)*(ft)** ***(C3)***

Note: Follow through from part (a) and part (b)(i).

[3 marks]

Total [6 marks]

4. (a)

Set	Example of a number in the set
$\mathbb{N}$	***Any natural number***
$\mathbb{Z}$	***Any integer***
$\mathbb{Q}$	***Any rational number***
$\mathbb{R}$	***Any real number***

(A1)(A1)(A1)(A1)(C4)
[4 marks]

(b) Incorrect ***(A1)***
Natural numbers are positive integers. Integers can also be negative. (or equivalent) ***(R1)*** ***(C2)***

Note: Accept a correct justification. Do not award ***(R0)(A1)***.
Accept: a statement with an example of an integer which is not natural.

[2 marks]

Total [6 marks]

5. (a) (i) $1000\left(1+\frac{3.5}{4\times 100}\right)^{4\times 5}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in compound interest formula, ***(A1)*** for correct substitution.

OR

$N = 5$
$I = 3.5$
$PV = 1000$
$P/Y = 1$
$C/Y = 4$

Note: Award ***(A1)*** for $C/Y = 4$ seen, ***(M1)*** for other correct entries.

OR

$N = 5\times 4$
$I = 3.5$
$PV = 1000$
$P/Y = 4$
$C/Y = 4$

Note: Award ***(A1)*** for $C/Y = 4$ seen, ***(M1)*** for other correct entries.

$= 1190.34$ (USD) ***(A1)***

Note: Award ***(M1)*** for substitution in compound interest formula, ***(A1)*** for correct substitution.

(ii) 190.34 (USD) ***(A1)*(ft)** ***(C4)***

Note: Award ***(A1)*(ft)** for subtraction of 1000 from their part (a)(i). Follow through from (a)(i).

[4 marks]

(b) $\frac{170}{190.34}$ ***(M1)***

Note: Award ***(M1)*** for division of 170 by their part (a)(ii).

$= 0.89$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their part (a)(ii).

[2 marks]

Total [6 marks]

6. (a) $0 = 10a + 5$ ***(M1)***

Note: Award ***(M1)*** for correctly substituting any point from L_1 into the equation.

OR

$\dfrac{0-5}{10-0}$ ***(M1)***

Note: Award ***(M1)*** for correctly substituting any two points on L_1 into the gradient formula.

$-\dfrac{5}{10}$ $\left(-\dfrac{1}{2}, -0.5\right)$ ***(A1)*** ***(C2)***

[2 marks]

(b) $(-1.33,\ 5.67)$ $\left(\left(-\dfrac{4}{3}, \dfrac{17}{3}\right), \left(-1\dfrac{1}{3}, 5\dfrac{2}{3}\right), (-1.33333\ldots, 5.66666\ldots)\right)$

(A1)**(ft)*****(A1)*****(ft)*** ***(C2)***

Note: Award ***(A1)*** for x-coordinate and ***(A1)*** for y-coordinate. Follow through from their part (a). Award ***(A1)(A0)*** if brackets are missing. Accept $x = -1.33,\ y = 5.67$.

[2 marks]

(c) $3 = -2(2) + c$ ***(M1)***

Note: Award ***(M1)*** for correctly substituting –2 and the given point into the equation of a line.

$y = -2x + 7$ ***(A1)*** ***(C2)***

Note: Award ***(A0)*** if the equation is not written in the form $y = mx + c$.

[2 marks]

Total [6 marks]

7. (a) 20 ***(A1)*** ***(C1)***
[1 mark]

(b) $\frac{5}{43}$ $(0.11627\ldots, 11.6279\ldots\%)$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for correct numerator, ***(A1)*** for correct denominator.

[2 marks]

(c) $\frac{7}{37}\times\frac{12}{36}+\frac{12}{37}\times\frac{7}{36}$ ***(A1)(M1)***

Note: Award ***(A1)*** for first or second correct product seen, ***(M1)*** for adding their two products or for multiplying their product by two.

$=\frac{14}{111}$ $(0.12612\ldots, 12.6126\%)$ ***(A1)*** ***(C3)***

[3 marks]

Total [6 marks]

8. (a) $(\text{AC}^2 =)\ 62^2+79^2-2\times62\times79\times\cos(52^\circ)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituting in the cosine rule formula, ***(A1)*** for correct substitution.

63.7 $(63.6708\ldots)$ (km) ***(A1)*** ***(C3)***

[3 marks]

(b) $\frac{1}{2}\times62\times79\times\sin(52^\circ)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituting in the area of triangle formula, ***(A1)*** for correct substitution.

$1930\,\text{km}^2$ $(1929.83\ldots\text{km}^2)$ ***(A1)*** ***(C3)***

[3 marks]

Total [6 marks]

9. (a) (i) A' ***(A1)***

Note: Accept alternative set notation for complement such as $U - A$.

(ii) $C \cap D'$ **OR** $D' \cap C$ ***(A1)***

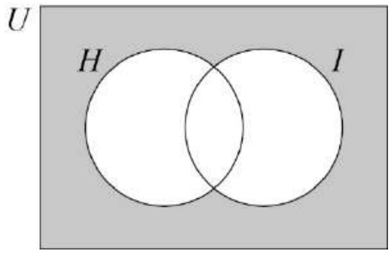
Note: Accept alternative set notation for complement.

(iii) $(E \cap F) \cup G$ **OR** $G \cup (E \cap F)$ ***(A2)*** ***(C4)***

Note: Accept equivalent answers, for example $(E \cup G) \cap (F \cup G)$.

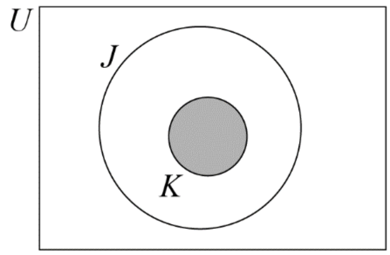
[4 marks]

(b) (i)



(A1)

(ii)



(A1) ***(C2)***
[2 marks]

Total [6 marks]

10. (a) $6400 = A \times 2^4$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 4 and 6400 in equation.

$(A =)\ 400$ ***(A1)*** ***(C2)***

[2 marks]

(b) the initial population **OR** the population at the start of experiment ***(A1)*** ***(C1)***

[1 mark]

(c) $40A = A \times 2^t$ **OR** $40 \times 400 = 400 \times 2^t$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into equation. Follow through with their A from part (a).

$40 = 2^t$ ***(M1)***

Note: Award ***(M1)*** for simplifying.

5.32 $(5.32192\ldots)$ (hours) **OR** 5 hours 19.3 $(19.3156....)$ minutes ***(A1)*** ***(C3)***

[3 marks]

Total [6 marks]

11. (a) $x = 0$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $x =$"a constant" ***(A1)*** for $= 0$. Award ***(A0)(A0)*** for an answer of "0".

[2 marks]

(b) $f(x) = -2$ $(y = -2)$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $y =$"a constant" ***(A1)*** for $= -2$. Award ***(A0)(A0)*** for an answer of "-2".

[2 marks]

(c) $\frac{3}{x} - 2 = 0$ ***(M1)***

Note: Award ***(M1)*** for equating $f(x)$ to 0.

$(x =)\frac{3}{2}$ (1.5) ***(A1)*** ***(C2)***

[2 marks]

Total [6 marks]

12. (a) 12.5 ***(A1)*** ***(C1)***

[1 mark]

(b) (i) $33+k$ **OR** $10+8+5+10+k$ ***(A1)***

Note: Award ***(A1)*** for "number of customers $=33+k$".

(ii) $$\frac{2.5\times 10+7.5\times 8+...+22.5\times k}{33+k}=12$$ ***(M1)(A1)*(ft)**

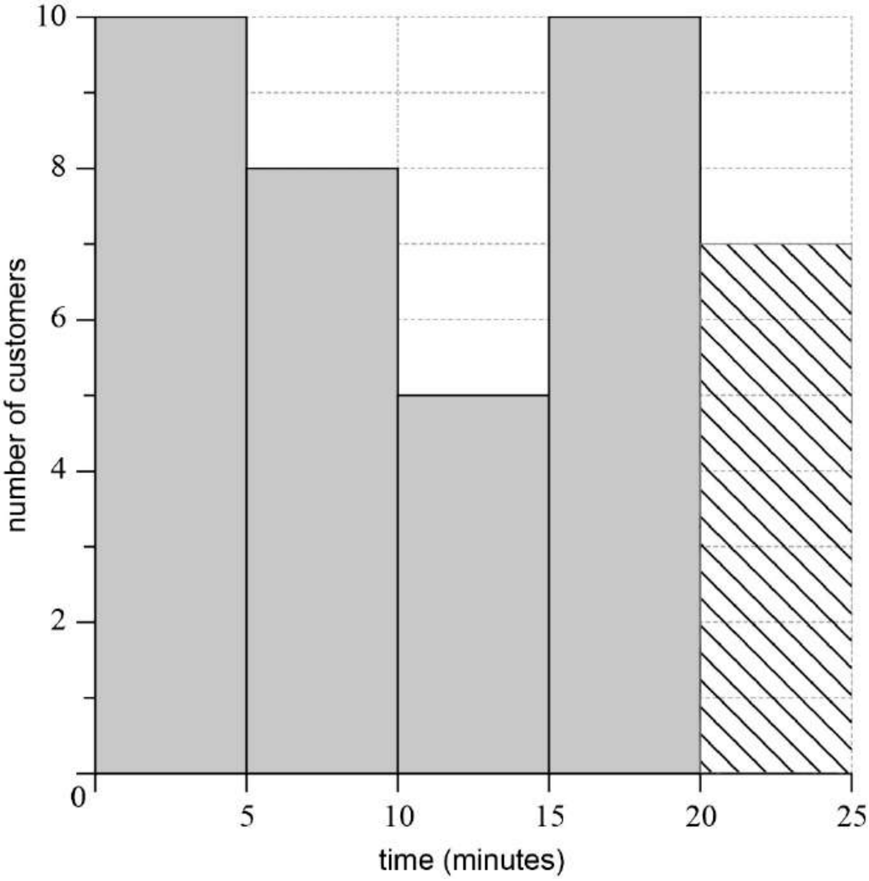
Note: Award ***(M1)*** for substitution into the mean formula and equating to 12, ***(A1)*(ft)** for their correct substitutions.

$(k=)\ 7$ ***(A1)*(ft)** ***(C4)***

Note: Follow through from part (b)(i) and their mid-interval values, consistent with part (a). Do not award final ***(A1)*** if answer is not an integer.

[4 marks]

(c)



(A1)*(ft)** ***(C1)

Note: Follow through from their part (b)(ii) but only if the value is between 1 and 10, inclusive.

[1 mark]

Total [6 marks]

13. (a) $(70-75)^2+100$ ***(M1)***

Note: Award ***(M1)*** for substituting in $x=70$.

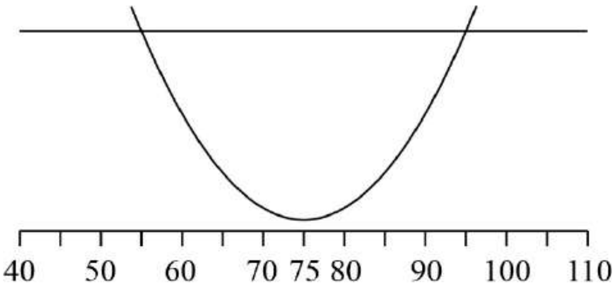
125 ***(A1)*** ***(C2)***

[2 marks]

(b) $(s-75)^2+100=500$ ***(M1)***

Note: Award ***(M1)*** for equating $C(x)$ to 500. Accept an inequality instead of =.

OR



(M1)

Note: Award ***(M1)*** for sketching correct graph(s).

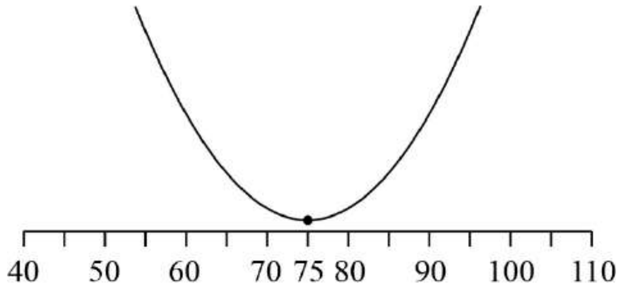
$(s=)\ 95$ ***(A1)*** ***(C2)***

[2 marks]

continued…

Question 13 continued

(c)



(M1)

Note: Award ***(M1)*** for an attempt at finding the minimum point using graph.

OR

$$\frac{95+55}{2}$$ ***(M1)***

Note: Award ***(M1)*** for attempting to find the mid-point between their part (b) and 55.

OR

$\left(C'(x)=\right)\ 2x-150=0$ ***(M1)***

Note: Award ***(M1)*** for an attempt at differentiation that is correctly equated to zero.

75 ***(A1) (C2)***

[2 marks]

Total [6 marks]

14. (a) x^3 ***(A1)*** ***(C1)***

[1 mark]

Note: Award ***(A0)*** for $\frac{4x^3}{4}$ and not simplified to x^3.

(b) $\left(-\frac{1}{2}\right)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of $-\frac{1}{2}$ into their derivative.

$-\frac{1}{8}$ (-0.125) ***(A1)*(ft)** ***(C2)***

Note: Follow through from their part (a).

[2 marks]

(c) $x^3 = 8$ ***(A1)(M1)***

Note: Award ***(A1)*** for 8 seen maybe seen as part of an equation $y = 8x + c$, ***(M1)*** for equating their derivative to 8.

$(x =)\ 2$ ***(A1)*** ***(C3)***

Note: Do not accept $(2, 4)$.

[3 marks]

Total [6 marks]

15. (a) (slant height^2 =) $10^2 + r^2$ ***(M1)***

Note: For correct substitution of 10 and r into Pythagoras' Theorem.

$\sqrt{10^2 + r^2}$ ***(A1)*** ***(C2)***

[2 marks]

(b) $\pi r^2 + 2\pi r \times 12 + \pi r\sqrt{100 + r^2} = 570$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correct substitution in curved surface area of cylinder and area of the base, ***(M1)*** for their correct substitution in curved surface area of cone, ***(M1)*** for adding their 3 surface areas and equating to 570. Follow through their part (a).

$= 4.58$ $(4.58358...)$ ***(A1)*****(ft)** ***(C4)***

Note: Last line must be seen to award final ***(A1)***. Follow through from part (a).

[4 marks]

Total [6 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Estudios matemáticos
Nivel medio
Prueba 1

Miércoles 2 de mayo de 2018 (tarde)

Número de convocatoria del alumno

1 hora 30 minutos

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Para esta prueba, se necesita una copia sin anotaciones del **cuadernillo de fórmulas de estudios matemáticos NM**.
- Conteste todas las preguntas.
- Escriba sus respuestas en las casillas provistas a tal efecto.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán darse como valores exactos o con una aproximación de tres cifras significativas.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

21 páginas

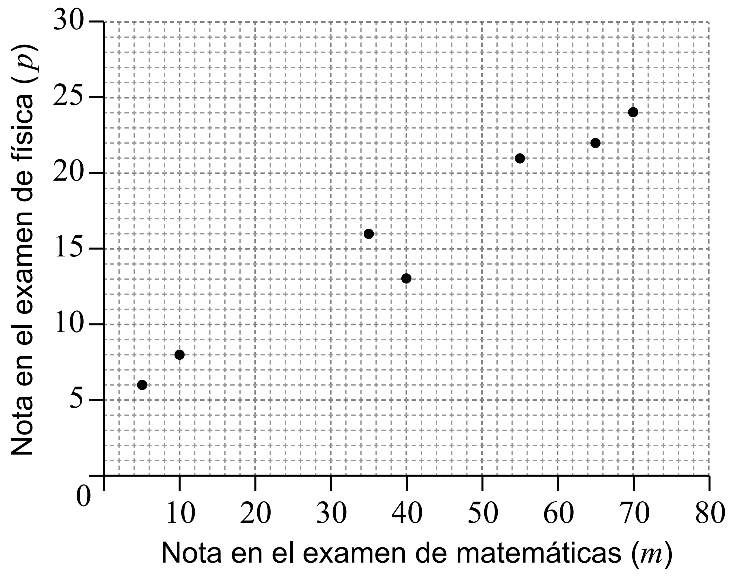
2218 – 7409

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

Se otorgará la máxima puntuación a las respuestas correctas. Aun cuando una respuesta sea incorrecta, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Escriba sus respuestas en las casillas provistas a tal efecto. Para los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido hasta su obtención. Por ejemplo, cuando deba utilizar un gráfico de una calculadora de pantalla gráfica para hallar soluciones, deberá dibujar aproximadamente esas gráficas en su respuesta.

1. El siguiente diagrama de dispersión muestra las notas que obtuvieron siete alumnos en un examen de matemáticas, m, y en otro de física, p.



La media, M, correspondiente a estos datos es $(40, 16)$.

(a) En el diagrama de dispersión, sitúe y rotule el punto $\mathrm{M}(\bar{m}, \bar{p})$. [2]

(b) En el mismo diagrama de dispersión, dibuje por aproximación (es decir, “a ojo”) la recta de ajuste óptimo. [2]

(c) Utilizando esa recta de ajuste óptimo, estime qué nota sacará en el examen de física un alumno que tenga una nota de 20 en el examen de matemáticas. [2]

Operaciones:

Respuesta:

(c) . . . .

2. Considere las siguientes proposiciones.

p: el bebé llora
q: el bebé está contento
r: el bebé quiere jugar

(a) Escriba con palabras, $(q \wedge r) \Rightarrow \neg p$. [3]

(b) Complete la siguiente tabla de verdad. [2]

p	q	r	$\neg p$	$(q \wedge r)$	$(q \wedge r) \Rightarrow \neg p$
V	V	V	F		
V	V	F	F		
V	F	V	F		
V	F	F	F		
F	V	V	V		
F	V	F	V		
F	F	V	V		
F	F	F	V		

(c) Indique si $(q \wedge r) \Rightarrow \neg p$ es una tautología, una contradicción o ninguna de las dos cosas. [1]

Operaciones:

Respuestas:

(a) . . . .

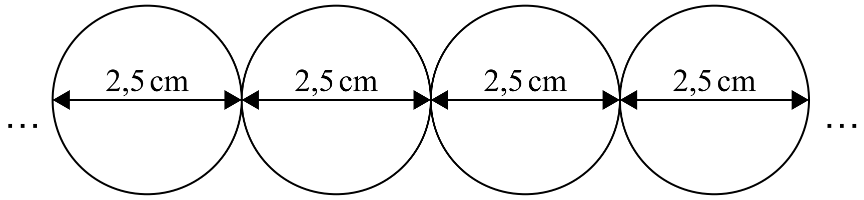
. . . .

. . . .

(c) . . . .

3. El año pasado, una fábrica de golosinas sudamericana vendió $4{,}8 \times 10^{8}$ caramelos esféricos. Cada caramelo tiene un diámetro de $2{,}5\,\text{cm}$.

La fábrica está elaborando un anuncio donde muestra todos estos caramelos colocados en fila, formando una línea recta.



(a) Halle la longitud, en cm, que tiene esta recta. Dé la respuesta en la forma $a \times 10^{k}$, donde $1 \le a < 10$ y $k \in \mathbb{Z}$. [3]

En el anuncio se afirma que la longitud de esta recta es igual a x veces la longitud del río Amazonas. El río Amazonas mide $6400\,\text{km}$ de longitud.

(b) (i) Escriba la longitud del río Amazonas en cm.

(ii) Halle el valor de x. [3]

Operaciones:

Respuestas:

(a) . . . .

(b) (i) . . . .

(ii) . . . .

4. La siguiente tabla muestra cuatro conjuntos distintos de números: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ y $\mathbb{R}$.

Conjunto	**Ejemplo de un número perteneciente al conjunto**
$\mathbb{N}$	
$\mathbb{Z}$	
$\mathbb{Q}$	
$\mathbb{R}$	

(a) Complete la segunda columna de la tabla dando **un** ejemplo de un número que pertenezca a cada conjunto. [4]

Josh afirma: "Todos los números enteros son números naturales".

(b) Escriba si la afirmación de Josh es correcta. Justifique su respuesta. [2]

Operaciones:

Respuesta:

(b) . . . .
. . . .
. . . .

5. En esta pregunta, dé todas las respuestas redondeando a dos lugares decimales.

Karl invierte 1000 dólares estadounidenses (USD) en una cuenta que paga un interés nominal anual del $3,5\%$, **compuesto trimestralmente**. Karl deja el dinero en esa cuenta durante 5 años.

(a) (i) Calcule la cantidad de dinero que Karl tendrá en la cuenta al cabo de 5 años;

(ii) Escriba la cantidad de **intereses** que habrá recibido al cabo de esos 5 años. [4]

Karl decide donar estos **intereses** a una ONG de Francia. La ONG recibe 170 euros (EUR). El tipo de cambio es $1\,\mathrm{USD} = t\,\mathrm{EUR}$.

(b) Calcule el valor de t. [2]

Operaciones:

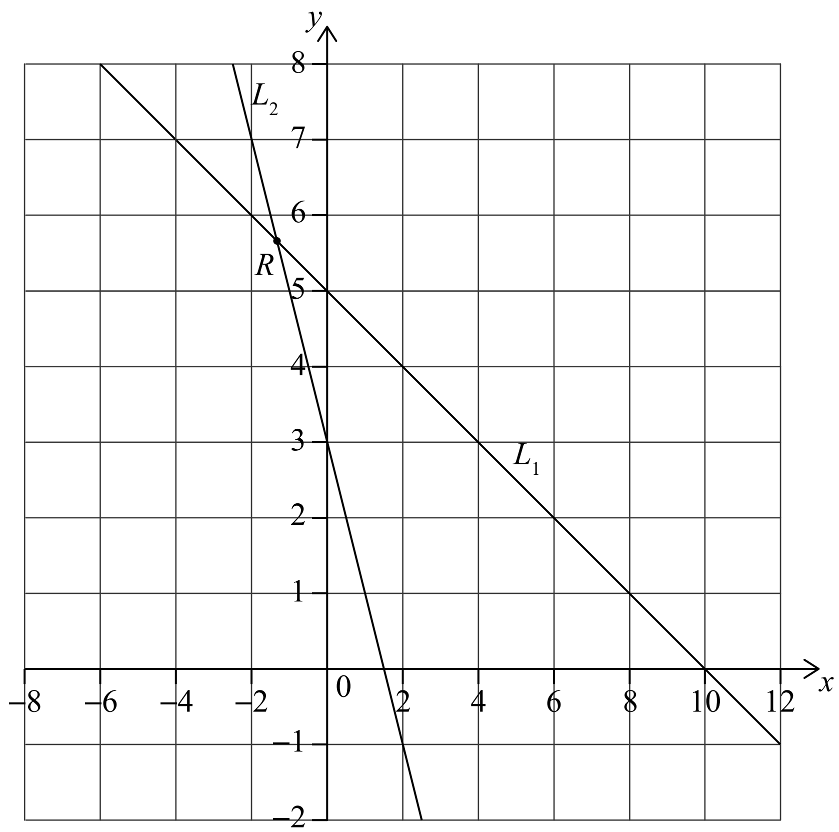
Respuestas:

(a) (i) . . . .

(ii) . . . .

(b) . . . .

6. Considere las rectas L_1 y L_2. R es el punto de intersección de estas dos rectas.



La ecuación de la recta L_1 es $y = ax + 5$.

(a) Halle el valor de a. [2]

La ecuación de la recta L_2 es $y = -2x + 3$.

(b) Halle las coordenadas de R. [2]

La recta L_3 es paralela a la recta L_2 y pasa por el punto $(2, 3)$.

(c) Halle la ecuación de la recta L_3. Dé la respuesta en la forma $y = mx + c$. [2]

(Esta pregunta continúa en la página siguiente)

(Pregunta 6: continuación)

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

7. En una competición internacional, los participantes pueden responder a las preguntas **solamente en una** de estas tres lenguas: portugués, chino mandarín o hindi. En la competición hubo 80 participantes. En la siguiente tabla se indica cuántos de estos participantes contestaron en portugués, cuántos en mandarín y cuántos en hindi.

		Lenguas			
		Portugués	Mandarín	Hindi	**Total**
Participantes	Varones	20	18	5	**43**
	Mujeres	18	7	12	**37**
	Total	**38**	**25**	**17**	**80**

(a) Indique cuántos varones contestaron a las preguntas en portugués. [1]

Se escoge un varón al azar.

(b) Halle la probabilidad de que el varón haya contestado a las preguntas en hindi. [2]

Ahora se escogen dos mujeres al azar.

(c) Calcule la probabilidad de que una mujer haya contestado a las preguntas en mandarín y la otra haya contestado a las preguntas en hindi. [3]

Operaciones:

Respuestas:

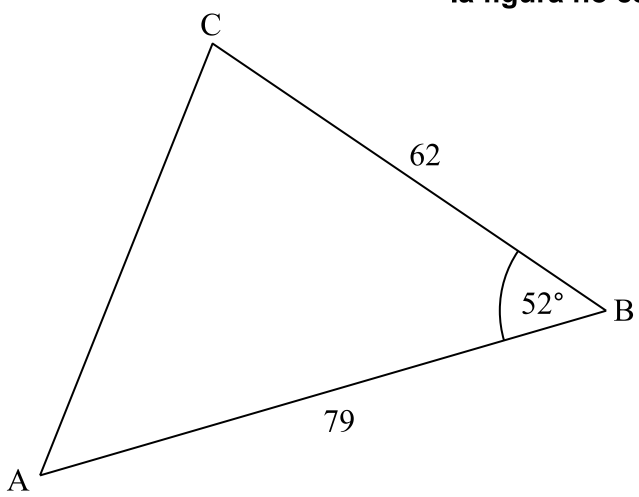
(a) . . . .

(b) . . . .

(c) . . . .

8. En la siguiente figura se muestra un parque nacional que tiene forma de triángulo ABC.
AB mide $79\,\mathrm{km}$ y BC mide $62\,\mathrm{km}$. El ángulo $\mathrm{A\hat{B}C}$ mide 52°.

la figura no está dibujada a escala



(a) Calcule la longitud del lado AC en km. [3]

(b) Calcule el área de este parque nacional. [3]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

9. Considere los siguientes diagramas de Venn.

Diagrama 1

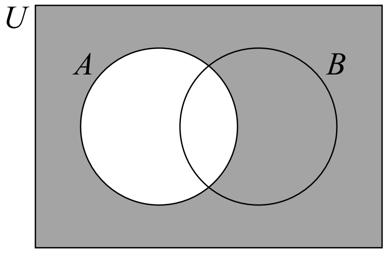


Diagrama 2

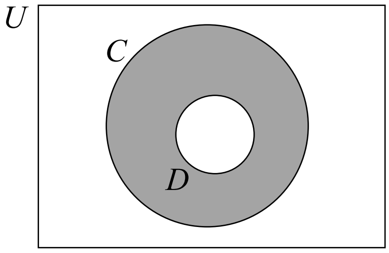
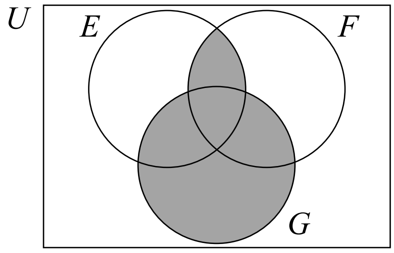


Diagrama 3



(a) Escriba una expresión, utilizando notación de conjuntos, que represente la región **sombreada** del

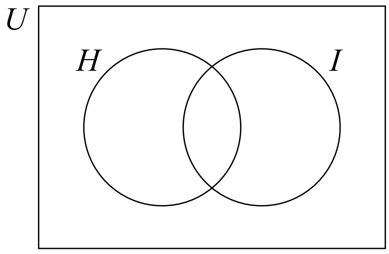
(i) Diagrama 1;

(ii) Diagrama 2;

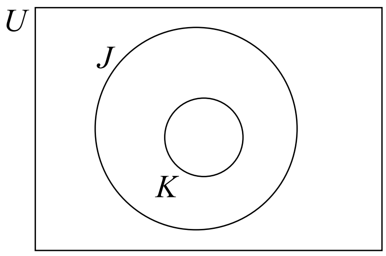
(iii) Diagrama 3. [4]

(b) En cada uno de los siguientes diagramas de Venn, sombree la región que representa al conjunto

(i) $(H \cup I)'$;



(ii) $J \cap K$. [2]



(Esta pregunta continúa en la página siguiente)

(Pregunta 9: continuación)

Operaciones:

Respuestas:

(a) (i) . . . .

(ii) . . . .

(iii) . . . .

10. La siguiente función modeliza el crecimiento de una población de bacterias en un experimento,

$$P(t) = A \times 2^t,\ t \geq 0$$

donde A es una constante y t es el tiempo, en horas, que ha transcurrido desde el inicio del experimento.

Cuatro horas después de que empezara el experimento, la población de bacterias era de 6400.

(a) Halle el valor de A. [2]

(b) Interprete qué representa A en este contexto. [1]

(c) Halle cuánto tiempo tuvo que transcurrir desde el inicio del experimento para que la población de bacterias fuera igual a $40A$. [3]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

. . . .

(c) . . . .

11. Considere el gráfico de la función $f(x)=\frac{3}{x}-2$, $x \neq 0$.

(a) Escriba la ecuación de la asíntota vertical. [2]

(b) Escriba la ecuación de la asíntota horizontal. [2]

(c) Calcule el valor de x para el cual $f(x)=0$. [2]

Operaciones:

Respuestas:

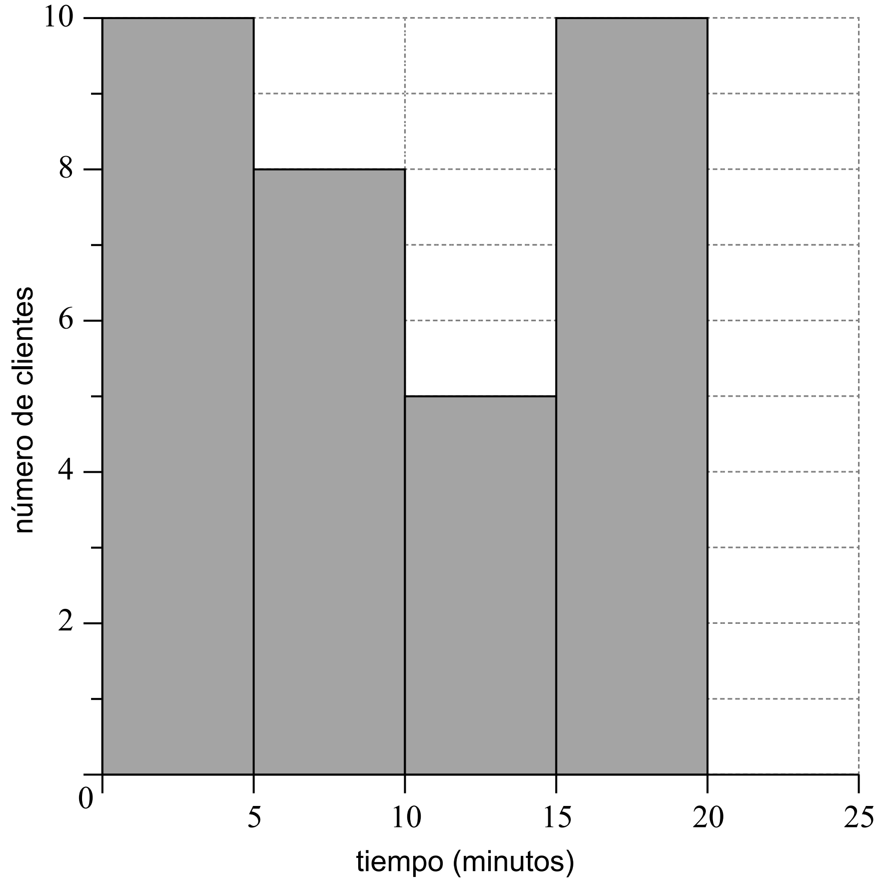
(a) . . . .

(b) . . . .

(c) . . . .

12. El siguiente histograma muestra el tiempo, t, en minutos, que tardan en comer los clientes de un restaurante, en un día determinado. Todos los clientes tardaron en comer menos de 25 minutos**.**

El histograma está incompleto, pues solo muestra los datos correspondientes a $0 \le t < 20$.



(a) Escriba el valor central del intervalo correspondiente a $10 \le t < 15$. [1]

La media del tiempo que tardaron en comer **todos** los clientes se estimó que fue de 12 minutos.

Se sabe que hubo k clientes que tardaron en comer entre 20 y 25 minutos.

(b) (i) Escriba, en función de k, el número total de clientes.

(ii) Calcule el valor de k. [4]

(c) A partir de lo anterior, complete el histograma. [1]

(Esta pregunta continúa en la página siguiente)

(Pregunta 12: continuación)

Operaciones:

Respuestas:

(a) . . . .

(b) (i) . . . .

(ii) . . . .

13. Una fábrica elabora camisas. El costo, C, en dólares de Fiji (FJD), de producir x camisas está modelado por

$$C(x) = (x - 75)^2 + 100\,.$$

(a) Halle el costo de producir 70 camisas. [2]

El costo de producción no debe sobrepasar los $500\ \mathrm{FJD}$. Para lograrlo, la fábrica tiene que producir al menos 55 camisas y como mucho s camisas.

(b) Halle el valor de s. [2]

(c) Halle el número de camisas que se producen cuando el costo de producción es lo más bajo posible. [2]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

14. Considere la función $f(x)=\dfrac{x^4}{4}$.

(a) Halle $f'(x)$. [1]

(b) Halle la pendiente del gráfico de f en $x=-\dfrac{1}{2}$. [2]

(c) Halle la coordenada x del punto en el que la **normal** al gráfico de f tiene pendiente $-\dfrac{1}{8}$. [3]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

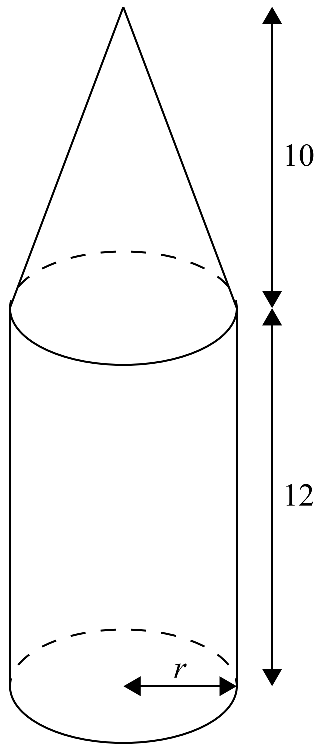
(c) . . . .

15. Julio está haciendo un estuche de madera para lápices que tiene forma de lápiz de gran tamaño. El estuche para lápices consta de un cilindro que está unido a un cono, como se muestra en la figura.

El cilindro mide r cm de radio y $12\,\mathrm{cm}$ de altura.

En el cono, la altura son $10\,\mathrm{cm}$ y el radio de la base mide r cm.

la figura no está dibujada a escala



(a) Halle una expresión que permita calcular la generatriz del cono **en función de r**. [2]

El área total de la superficie externa del estuche de lápices es igual a $570\,\mathrm{cm}^2$, si redondeamos a 3 cifras significativas.

(b) Utilizando la calculadora de pantalla gráfica, calcule el valor de r. [4]

(Esta pregunta continúa en la página siguiente)

(Pregunta 15: continuación)

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

M18/5/MATSD/SP1/SPA/TZ0/XX/M

Esquema de calificación

Mayo de 2018

Estudios matemáticos

Nivel medio

Prueba 1

23 páginas

Esquema de calificación de la Prueba 1
Instrucciones para los examinadores

Observaciones: **En caso de dudas sobre estas instrucciones o temas relacionados con la asignación de puntuaciones, póngase en contacto con su jefe de equipo.**

La puntuación máxima para cada pregunta es 6.

1 Siglas

En el esquema de calificación pueden aparecer las siguientes siglas:

M Puntos otorgados por el **método**

A Puntos otorgados por una **respuesta** o por la **precisión**

C Puntos otorgados por respuestas **correctas** (independientemente del trabajo mostrado)

R Puntos otorgados por un **razonamiento** claro

ft Se pueden otorgar puntos por la **coherencia** con resultados previos en la pregunta

2 Método para corregir

(a) Todas las correcciones deben realizarse en RM Assessor, utilizando las anotaciones de Estudios Matemáticos y de acuerdo con el documento vigente de orientaciones para la corrección electrónica de Estudios Matemáticos NM. Es fundamental leer el documento antes de empezar a corregir.

(b) Si el alumno obtiene la puntuación máxima en una pregunta utilice la anotación ***C6***, si lo ha intentado pero obtiene cero puntos utilice ***C0***. Si no lo ha intentado utilice la tecla de No respuesta. Si un alumno no ha obtenido ni la puntuación máxima ni cero puntos, entonces se **DEBEN** mostrar todas las anotaciones.

(c) En esta prueba, **si aparece la respuesta correcta en la línea de respuesta** se otorgará la puntuación máxima. **¡No es necesario comprobar el trabajo! Otorgue puntos C y siga adelante.**

(d) Si la respuesta no aparece en la línea de respuesta, pero la respuesta correcta se encuentra en el cuadro de operaciones sin trabajo posterior, otorgue la puntuación máxima.

(e) Si la **respuesta es incorrecta**, se deben otorgar puntos por el trabajo realizado, de acuerdo con el esquema de calificación.

(f) No se debe otorgar ningún punto al trabajo tachado por el alumno. Si el alumno ha dado dos respuestas a una pregunta, solo se debe corregir la primera respuesta.

(g) Una respuesta correcta en el cuadro de trabajo transcrita erróneamente a la línea de respuesta puede recibir la puntuación máxima.

(e) Si el procedimiento adecuado lleva a la respuesta correcta, pero esta va seguida de más desarrollos que indican una falta de comprensión matemática, entonces **no** se puede otorgar la puntuación máxima. En la mayoría de estos casos se perderá solo el último punto por respuesta. Una excepción podría ser las respuestas numéricas, donde una respuesta exacta que es correcta está seguida de una expresión decimal incorrecta.

Ejemplo:

	Se ve la respuesta correcta	**Hay más desarrollo**	**Acción**
1.	$8\sqrt{2}$	$5{,}65685...$ *(valor decimal incorrecto)*	Otorgue el ultimo ***(A1)*** *(ignore el desarrollo posterior)*
2.	$(x-6)\,(x+1)$	$x=6$ and -1	**No** otorgue el último ***(A1)***

Ejemplo: Factorice $x^2 - 5x - 6$

Esquema de calificación	Examen del alumno	Corrección
$(x-6)(x+1)$ ***(A1)(A1)***	(i) Línea de respuesta: $(x+6)(x+1)$	***(A0)(A1)***
	(ii) Cuadro de operaciones: $(x-6)(x+1)$	***(A1)***
	seguido de $x=6$ y -1, o simplemente $6, -1$ bien en el cuadro de trabajo o en la línea de respuesta.	***(A0)***

3 Puntos por la coherencia (ft)

En cualquier paso de una resolución, un error puede afectar a todo el trabajo posterior. Para limitar la gravedad de la penalización, se pueden otorgar **puntos por la coherencia**. Los esquemas de calificación indicarán, con **"(ft)"**, dónde es adecuado aplicar el criterio de coherencia en una pregunta.

(a) La coherencia se aplica solo de un apartado de una pregunta a otro apartado posterior de la pregunta. No se aplica dentro del mismo apartado.

(b) Si una respuesta que es consecuencia de un procedimiento de coherencia es muy poco realista (por ejemplo, distancias negativas o de un orden de magnitud excesivo) entonces el punto ***A*** final no se debe otorgar.

(c) Si, debido a un error, una pregunta queda transformada en otra **distinta**, **mucho más sencilla**, entonces el criterio de coherencia podría no ser aplicado.

(d) Para otorgar puntos por coherencia en un apartado de una pregunta, **es necesario que haya un trabajo explícito relacionado con ese apartado**. Una respuesta aislada, obtenida por coherencia, pero sin un desarrollo explícito, se considera incorrecta, y no recibirá ningún punto **aunque se aproxime a la respuesta correcta**.

(e) Una excepción a lo anterior sería una pregunta donde lo que se evalúa es el manejo del alumno en el uso de la calculadora de pantalla gráfica, y donde no se pretende que se muestre ningún desarrollo escrito. **En el esquema de calificación vendrá indicado claramente cuando sea el caso.**

(f) El uso inadvertido de radianes será penalizado la primera vez que aparezca. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez un punto por el uso de radianes.

Ejemplo: Cálculo de ángulos y longitudes en trigonometría

Esquema de calificación		Examen del alumno	Corrección
(a) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$	***(M1)(A1)***	(a) $\frac{\text{sen } A}{4} = \frac{\text{sen } 30}{3}$	***(M1)(A0)*** *(uso del teorema del seno, pero con valores incorrectos)*
$A = 22,0^\circ\ (22,0243\ldots)$	***(A1)***	$A = 41,8^\circ$	***(A0)*** ***(Nota:*** *Aquí, el segundo* ***(A1)*** *no ha sido corregido como* **(ft)** *y no se puede otorgar porque existe un error previo en el* ***mismo*** *apartado de la pregunta.)*
(b) $x = 7\text{tg}\,(22,0243\ldots^\circ)$ $= 2,83\,(2,831639\ldots)$	***(M1)*** ***(A1)*(ft)**	(b) caso (i) $x = 7\text{tg}\ 41,8^\circ$ $= 6,26$ ***pero*** caso (ii) $6,26$	***(M1)*** ***(A1)*(ft)** ***(C0)*** *pues no aparece un desarrollo explícito*

4 Uso del Esquema de calificación

(a) Los puntos ***A*** **dependen** de los puntos ***M*** obtenidos previamente, **no** es posible otorgar ***(M0)(A1)***. Una vez que se otorga un ***(M0)***, se pierden todos los puntos ***A*** que le siguen en ese apartado de la pregunta, incluso si los cálculos son correctos, hasta el siguiente punto ***M***.
La única excepción a lo anterior se hará en el caso de una respuesta donde la precisión se especifique en la pregunta (ver apartado 5).

(b) Los puntos ***A*** **dependen** de los puntos ***R*** obtenidos, **no** es posible otorgar ***(A1)(R0)***. Así pues, el ***(A1)*** no se otorga a una respuesta correcta si no aparece el razonamiento, o este es incorrecto.

(c) Los **métodos alternativos** no siempre se incluyen. Así, si una respuesta es errónea, entonces se debe analizar cuidadosamente el procedimiento, de modo que se asignen puntos por un método distinto de forma coherente con el esquema de calificación.
Cuando en el esquema de calificación se incluyen métodos alternativos para una pregunta, ello viene indicado mediante "**O**" *etc*.

(d) A menos que en la pregunta se especifique lo contrario, se aceptan **expresiones equivalentes**. Por ejemplo: $\frac{\text{sen}\,\theta}{\cos\theta}$ por $\text{tg}\,\theta$. En el esquema de calificación, estas expresiones algebraica o numéricamente equivalentes aparecerán a veces escritas entre paréntesis junto a la respuesta requerida.
Cuando se requieran respuestas numéricas como respuesta final de un apartado de una pregunta, en el esquema de calificación se mostrará en este orden;
la respuesta con 3 cifras significativas a partir de la solución completa de la calculadora;
el valor exacto (por ejemplo $\sqrt{3}$ si corresponde);
la solución completa de la calculadora en la forma $2{,}83163\ldots$ como en el ejemplo anterior.
Cuando se dan las respuestas con 3 cifras significativas y se utilizan después en apartados posteriores de la pregunta llevando a una solución con 3 cifras significativas **distinta**, también se darán estas soluciones.

(e) Dado que este es un examen internacional, se aceptarán todas las **formas de notación alternativas**. Algunos ejemplos:

Comas decimales: $1{,}7$**;** $1’7$; $1 \cdot 7$; $1;7$.
Los números decimales menores que 1 podrían aparecer escritos sin un cero en el frente: 0.49 o $.49$.
Descripciones de un intervalo: $3 < x < 5$; $(3;\ 5)$; $]\ 3;\ 5\ [$.
Distintas formas de notación de las propiedades de los conjuntos (por ejemplo complementario): A'; $\bar{A}$; A^c; $U - A$;$(A$;$U \setminus A$.
Distintas formas de notación lógica: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p$.
$p \Rightarrow q$; $p \to q$; .
El nivel de significación podría escribirse como α .

(f) Puntos discrecionales: habrá ocasiones excepcionales en las que el esquema de calificación no cubra el trabajo que aparece. En estos casos se utilizará la nota DM para indicar que el examinador ha utilizado su criterio. La discrecionalidad debe utilizarse con moderación, y si existe duda se debe plantear una excepción a través de RM Assessor al jefe del equipo.

A partir de noviembre de 2011 ya no se aplicarán las penalizaciones AP, FP y UP. La precisión y las unidades serán evaluados en preguntas específicas y los puntos se otorgarán de acuerdo a las reglas dadas en los apartados 5, 6 y 7.

5 Precisión de las respuestas

Instrucciones: Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas correctamente a 3 cifras significativas.

1. Si el alumno muestra la solución a 4 o más cifras significativas, la cual redondeada a 3 cifras significativas daría la respuesta requerida entonces otorgue ***(A1)*** e ignore cualquier redondeo posterior. **Nota:** La solución sin redondear puede estar tanto en la línea de respuesta como en el cuadro de operaciones.

2. Si el alumno **no** muestra la solución sin redondear, entonces otorgue ***(A1)*** si la solución dada está aproximada correctamente a 2 o más cifras significativas, y ***(A0)*** en cualquier otro caso.

3. Si una solución aproximada a 2 cifras significativas se utiliza en apartados posteriores, entonces para otorgar más puntos debe aparecer el desarrollo. (Este criterio es el mismo que en el que se aplica coherencia a partir de una respuesta errónea.)

Estos 3 casos (vea los superíndices) se han resumido en la siguiente tabla e ilustrado después con ejemplos.

	Si la solución final de alumno se expresa…				
	Exacta o a 4 o más cifras significativas (y que redondeada **a 3 cifras significativas daría la respuesta correcta**)	**Aproximada incorrectamente a 3 cifras significativas**	Aproximada correctamente a 2 cifras significativas[3]	Aproximada incorrectamente a 2 cifras significativas	Aproximada correcta o incorrectamente a 1 cifra significativa
Aparece la solución sin redondear[1]	Otorgue ***(A1)*** independientemente del redondeo correcto o incorrecto				
No aparece la solución sin redondear[2]	***(A1)***	***(A0)***	***(A1)***	***(A0)***	***(A0)***
Criterio para los apartados posteriores	Como en MS	Como en el criterio de coherencia, solo si se muestra el desarrollo[3]			

Ejemplos:

Esquema de calificación	Examen del alumno	Corrección
9,43 (9,43398…) ***(A1)***	(i) 9,43398… se muestra en el cuadro de trabajo seguido de 9; 9,4; 9,43; 9,434 (redondeados correctamente)	***(A1)***
	(ii) 9,43398… se muestra en el cuadro de trabajo seguido de 9,433; 9,44, *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 9,4	***(A1)***
	(iv) 9	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 9,3	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 9,44	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Esquema de calificación	Examen del alumno	Corrección
7,44 (7,43798…) ***(A1)***	(i) 7,43798… se muestra en el cuadro de trabajo seguido de 7; 7,4; 7,44; 7,438 *etc.* (redondeados correctamente)	***(A1)***
	(ii) 7,43798… se muestra en el cuadro de trabajo seguido de 7,437 ó 7,43 *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 7,4	***(A1)***
	(iv) 7	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 7,5	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 7,43	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Ejemplo: ABC es un triángulo rectángulo con el ángulo $\mathrm{ABC} = 90^{\circ}$, $\mathrm{AC} = 32\ \mathrm{cm}$ y $\mathrm{AB} = 30\ \mathrm{cm}$. Halle (a) la longitud de BC, (b) el área del triángulo ABC.

Esquema de calificación	Examen del alumno	Corrección
(a) $\mathrm{BC} = \sqrt{32^2 - 30^2}$ ***(M1)***	(a) $\mathrm{BC} = \sqrt{32^2 - 30^2}$	***(M1)***
*Otorgue **(M1)** por la sustitución correcta en el teorema de Pitágoras*	$11\,(\mathrm{cm})$	***(A1)***
$= 11{,}1 \left(\sqrt{124}\,11{,}1355...\right)(\mathrm{cm})$ ***(A1)***	*(solo se muestra la aproximación a 2 cifras significativas, pero correcta)*	
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11{,}1355...$ ***(M1)***	(b) caso (i) $\text{Área} = \frac{1}{2} \times 30 \times 11$	***(M1)***
*Otorgue **(M1)** por la sustitución correcta en la fórmula del área de un triángulo*	*(se muestra el desarrollo)*	
$= 167\ \ (167{,}032...)\,(\mathrm{cm}^2)$ ***(A1)*(ft)**	$= 165\,(\mathrm{cm}^2)$	***(A1)*(ft)**
	caso (ii) $= 165\,(\mathrm{cm}^2)$	***(M0)(A0)*(ft)**
	(No se muestra el desarrollo, la solución 11 se trata como procedimiento de coherencia, por lo que no se deben otorgar puntos en este caso)	

La aproximación a 3 cifras significativas de una solución exacta **se debe aceptar si se realiza correctamente**.

Las soluciones exactas del tipo $\frac{1}{4}$ se pueden escribir como decimales con menos de 3 cifras significativas si el resultado sigue siendo exacto. La simplificación de una fracción a su expresión irreducible **no** es imprescindible. Fracciones que incluyen un decimal en el numerador y/o en el denominador son aceptadas para demostrar una substitución correcta, sin embargo, no como respuesta final.

Razones de π y respuestas con expresiones de raíces cuadradas de enteros o cualquier potencia racional de un entero (por ejemplo, $\sqrt{13}, 2^{2/3}, \sqrt[4]{5}$), se pueden aceptar como respuestas exactas. Todas las otras potencias (por ejemplo de no enteros) y valores de funciones transcendentes tales como seno y coseno se deben evaluar.

Si el nivel de precisión viene especificado en la pregunta, se asignarán puntos por dar la respuesta con la precisión requerida. En **todos** estos casos no se obtiene el punto final si el redondeo no sigue las instrucciones dadas en la pregunta. El punto por la precisión especificada se puede considerar como punto por coherencia **(ft)** con independencia de un ***(M0)*** inmediatamente anterior.

6 Nivel de precisión en las preguntas sobre cuestiones financieras

El nivel de precisión de las respuestas se especificará en todas las preguntas que se relacionen con valores en dinero. Generalmente será a la unidad más cercana o de dos cifras decimales. La primera respuesta que no venga dada con el nivel de precisión especificado no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez la un punto por la precisión incorrecta en una pregunta sobre cuestiones financieras.

Ejemplo: En una pregunta sobre cuestiones financieras se pide una precisión de dos cifras decimales.

Esquema de calificación	Examen del alumno	Corrección
$\$231,62\ (231,6189)$ ***(A1)***	(i) $231,6$	***(A0)***
	(ii) 232 *(Aproximación correcta con un nivel de precisión distinto del requerido)*	***(A0)***
	(iii) $231,61$	***(A0)***
	(iv) $232,00$ *(Aproximación incorrecta al nivel de precisión requerido)*	***(A0)***

7 Unidades de medida en las respuestas

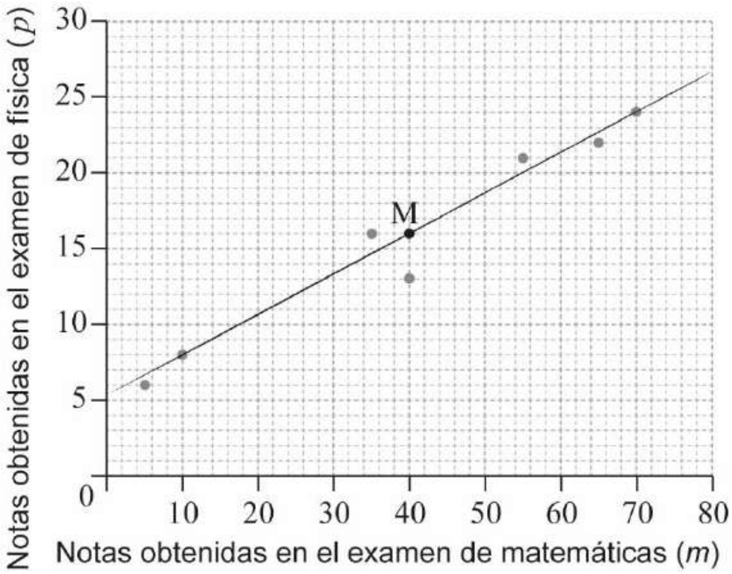
En determinadas preguntas será necesario expresar las unidades de medida, y esto vendrá claramente indicado en el esquema de calificación. La primera respuesta correcta pero sin unidades o con unidades incorrectas no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la falta de unidades o unidades incorrectas. Las unidades de medida se tendrán en cuenta solamente cuando se ha otorgado ***(A1)*** a la respuesta numérica siguiendo las indicaciones sobre la precisión dadas en el apartado 5.

Esquema de calificación	Examen del alumno	Corrección
(a) $37000\ \mathrm{m}^2$ ***(A1)***	(a) $36000\ \mathrm{m}^2$ *(respuesta incorrecta, por lo que no se consideran las unidades)*	***(A0)***
(b) $3200\ \mathrm{m}^3$ ***(A1)***	(b) $3200\ \mathrm{m}^2$ *(unidades incorrectas)*	***(A0)***

8 Calculadoras de pantalla gráfica

Con frecuencia los alumnos obtienen las soluciones directamente de la calculadora. Deben utilizar la notación matemática y no la notación de la calculadora. No se puede otorgar puntos por método a respuestas incorrectas basadas únicamente en notación de calculadora. No se puede otorgar puntos por método al comentario "usé la calculadora de pantalla gráfica".

1. (a)



(A1)(A1) ***(C2)***

Nota: Conceda ***(A1)*** por graficar la media y ***(A1)*** si está rotulado con una M.

[2 puntos]

(b) línea recta que pasa por la media y que corta al eje p en 5 ± 2

(A1)*(ft)*(A1)*(ft)** ***(C2)

Nota: Conceda ***(A1)*(ft)** por una línea **recta** que pase por la **media**. Conceda ***(A1)*(ft)** por un corte con el eje p correcto al extender la recta.

[2 puntos]

(c) ha identificado el punto de la recta para el cual $m = 20$ y ha intentado identificar la coordenada y de ese punto. ***(M1)***

$10{,}5$ ***(A1)*(ft)** ***(C2)***

Nota: Arrastre de error de su línea en parte (b).

[2 puntos]
Total [6 puntos]

2. (a) si el bebé está contento y quiere jugar, entonces el bebé no llora.

(A1)(A1)(A1) ***(C3)***

> **Nota:** Conceda ***(A1)*** por el "Si… entonces …".***(A1)*** por "el bebé está contento **y** quiere jugar", ***(A1)*** por "el bebé **no** llora". El llorar debe aparecer negado.

[3 puntos]

(b)

p	q	r	$\neg p$	$q \wedge r$	$(q \wedge r) \Rightarrow \neg p$
V	V	V	F	**V**	**F**
V	V	F	F	**F**	**V**
V	F	V	F	**F**	**V**
V	F	F	F	**F**	**V**
F	V	V	V	**V**	**V**
F	V	F	V	**F**	**V**
F	F	V	V	**F**	**V**
F	F	F	V	**F**	**V**

(A1)(A1) ***(C2)***

> **Nota:** Conceda ***(A1)*** por cada columna correcta.

[2 puntos]

(c) Ninguna de las dos cosas ***(A1)*(ft)** ***(C1)***

> **Nota:** Arrastre de error (FT) desde la última columna del apartado (b).

[1 punto]

Total [6 puntos]

3. (a) $4{,}8 \times 10^8 \times 2{,}5$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por multiplicar por 2,5.

$1{,}2 \times 10^9$ (cm) ***(A1)*****(ft)*****(A1)*****(ft)** ***(C3)***

> **Nota:** Conceda ***(A0)(A0)*** por respuestas del tipo 12×10^8.

[3 puntos]

(b) (i) $640\ 000\ 000$ (cm) $\left(6{,}4 \times 10^8 \text{(cm)}\right)$ ***(A1)***

(ii) $\dfrac{1{,}2 \times 10^9}{6{,}4 \times 10^8}$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por la división entre (por) 640 000 000.

$= 1{,}88\ (1{,}875)$ ***(A1)*****(ft)** ***(C3)***

> **Nota:** Arrastre de error (FT) desde el apartado (a) y el apartado (b)(i).

[3 puntos]

Total [6 puntos]

4. (a)

Conjunto	Ejemplo de un número perteneciente al conjunto
$\mathbb{N}$	***Cualquier número natural***
$\mathbb{Z}$	***Cualquier número entero***
$\mathbb{Q}$	***Cualquier número racional***
$\mathbb{R}$	***Cualquier número real***

(A1)(A1)(A1)(A1)(C4)
[4 puntos]

(b) Incorrecta ***(A1)***
Los números naturales son números enteros positivos. Los números enteros también pueden ser negativos. (o equivalente) ***(R1)*** ***(C2)***

> **Nota:** Acepte solo una justificación que sea correcta. No conceda ***(R0)(A1)***.
> Acepte: Una justificación con un ejemplo de un entero que no es natural.

[2 puntos]

Total [6 puntos]

5. (a) (i) $1000\left(1+\frac{3.5}{4\times 100}\right)^{4\times 5}$ ***(M1)(A1)***

> **Nota:** Conceda ***(M1)*** por sustituir los datos en la fórmula del interés compuesto, ***(A1)*** por una sustitución correcta.

O BIEN

$N = 5$
$I = 3{,}5$
$PV = 1000$
$P/Y = 1$
$C/Y = 4$

> **Nota:** Conceda ***(A1)*** si ha escrito $C/Y = 4$, ***(M1)*** por otras entradas correctas.

O BIEN

$N = 5\times 4$
$I = 3{,}5$
$PV = 1000$
$P/Y = 4$
$C/Y = 4$

> **Nota:** Conceda ***(A1)*** si ha escrito $C/Y = 4$, ***(M1)*** por otras entradas correctas.

$= 1190{,}34$ (USD) ***(A1)***

(ii) $190{,}34$ (USD) ***(A1)*(ft)** ***(C4)***

> **Nota:** Conceda ***(A1)*(ft)** por restar 1000 de su resultado en el apartado (a)(i). Arrastre de error (FT) desde (a)(i).

[4 puntos]

(b) $\frac{170}{190{,}34}$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por dividir 170 entre (por) el resultado del apartado (a)(ii).

$= 0{,}89$ ***(A1)*(ft)** ***(C2)***

> **Nota:** Arrastre de error (FT) desde el resultado del apartado (a)(ii).

[2 puntos]
Total [6 puntos]

6. (a) $0 = 10a + 5$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente en la ecuación cualquier punto perteneciente a L_1.

O BIEN

$\frac{0-5}{10-0}$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente dos puntos cualesquiera de L_1 en la fórmula de la pendiente.

$-\frac{5}{10}\ \left(-\frac{1}{2};\ -0,5\right)$ ***(A1)*** ***(C2)***

[2 puntos]

(b) $(-1,33;\ 5,67)\ \left(\left(-\frac{4}{3};\ \frac{17}{3}\right),\ \left(-1\frac{1}{3};\ 5\frac{2}{3}\right),\ (-1,33333\ldots;\ 5,66666\ldots)\right)$

(A1)**(ft)*****(A1)*****(ft)*** ***(C2)***

Nota: Conceda ***(A1)*** por la coordenada x y ***(A1)*** por la coordenada y. Arrastre de error del resultado del apartado (a). Conceda ***(A1)(A0)*** si faltan los paréntesis. Acepte $x = -1,33,\ y = 5,67$.

[2 puntos]

(c) $3 = -2(2) + c$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente –2 y el punto dado en la ecuación de la recta.

$y = -2x + 7$ ***(A1)*** ***(C2)***

Nota: Conceda ***(A0)*** si la ecuación no está escrita en la forma $y = mx + c$.

[2 puntos]

Total [6 puntos]

7. (a) 20 ***(A1)*** ***(C1)***

[1 punto]

(b) $\frac{5}{43}$ $(0{,}11627\ldots; 11{,}6279\ldots\%)$ ***(A1)(A1)*** ***(C2)***

> **Nota:** Conceda ***(A1)*** por un numerador correcto, ***(A1)*** por un denominador correcto.

[2 puntos]

(c) $\frac{7}{37}\times\frac{12}{36}+\frac{12}{37}\times\frac{7}{36}$ ***(A1)(M1)***

> **Nota:** Conceda ***(A1)*** por el primer o segundo producto correcto, ***(M1)*** por sumar esos dos productos o por multiplicar ese producto por dos.

$=\frac{14}{111}$ $(0{,}12612\ldots;\ 12{,}6126\%)$ ***(A1)*** ***(C3)***

[3 puntos]

Total [6 puntos]

8. (a) $(\mathrm{AC}^2 =)\ 62^2+79^2-2\times 62\times 79\times\cos(52^\circ)$ ***(M1)(A1)***

> **Nota:** Conceda ***(M1)*** por sustituir en la fórmula del coseno, ***(A1)*** por una sustitución correcta.

$63{,}7\quad (63{,}6708\ldots)\ (\mathrm{km})$ ***(A1)*** ***(C3)***

[3 puntos]

(b) $\frac{1}{2}\times 62\times 79\times \operatorname{sen}(52^\circ)$ ***(M1)(A1)***

> **Nota:** Conceda ***(M1)*** por sustituir en la fórmula del área de un triángulo, ***(A1)*** por una sustitución correcta.

$1930\,\mathrm{km}^2\left(1929{,}83\ldots\mathrm{km}^2\right)$ ***(A1)*** ***(C3)***

[3 puntos]

Total [6 puntos]

9. (a) (i) A' ***(A1)***

Nota: Acepte una notación de conjuntos alternativa para indicar el complemento como *U* – *A.*

(ii) $C \cap D'$ **O** $D' \cap C$ ***(A1)***

Nota: Acepte una notación de conjuntos alternativa para indicar el complemento.

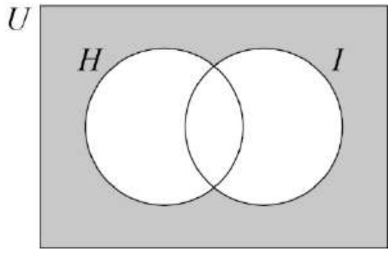
(iii) $(E \cap F) \cup G$ **O** $G \cup (E \cap F)$ ***(A2)*** ***(C4)***

Nota: Acepte respuestas equivalentes; por ejemplo, $(E \cup G) \cap (F \cup G)$.

[4 puntos]

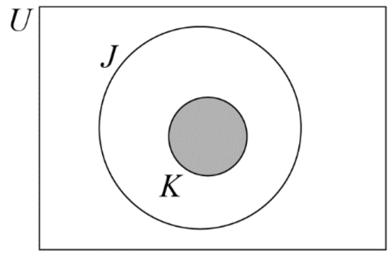
(b)

(i)



(A1)

(ii)



(A1) ***(C2)***
[2 puntos]

Total [6 puntos]

10. (a) $6400 = A \times 2^4$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido correctamente 4 y 6400 en la ecuación.

$(A =)\ 400$ ***(A1)*** ***(C2)***

[2 puntos]

(b) La población inicial **O BIEN** la población al comenzar el experimento ***(A1)*** ***(C1)***

[1 punto]

(c) $40A = A \times 2^t$ **O BIEN** $40 \times 400 = 400 \times 2^t$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por una sustitución correcta en la ecuación. Arrastre de error (FT) con su valor de A del apartado (a).

$40 = 2^t$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber simplificado.

$5{,}32$ $(5{,}32192\ldots)$ (horas) **O BIEN** 5 horas $19{,}3$ $(19{,}3156....)$ minutos ***(A1)*** ***(C3)***

[3 puntos]

Total [6 puntos]

11. (a) $x = 0$ ***(A1)(A1)*** ***(C2)***

> **Nota:** Conceda ***(A1)*** por $x =$ "una constante" ***(A1)*** por $= 0$. Conceda ***(A0)(A0)*** si la respuesta es solo "0".

[2 puntos]

(b) $f(x) = -2$ ($y = -2$) ***(A1)(A1)*** ***(C2)***

> **Nota:** Conceda ***(A1)*** por $y =$"una constante" o por $f(x) =$ "una constante" ***(A1)*** por $= -2$.

[2 puntos]

(c) $\frac{3}{x} - 2 = 0$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por igualar $f(x)$ a 0.

$(x =)\frac{3}{2}$ $(1{,}5)$ ***(A1)*** ***(C2)***

[2 puntos]

Total [6 puntos]

12. (a) $12{,}5$ ***(A1)*** ***(C1)***

[1 punto]

(b) (i) $33+k$ **O BIEN** $10+8+5+10+k$ ***(A1)***

Nota: Conceda ***(A1)*** por "número de clientes = 33 + k"

(ii) $$\frac{2{,}5\times 10+7{,}5\times 8+\ldots+22{,}5\times k}{33+k}=12$$ ***(M1)(A1)*(ft)**

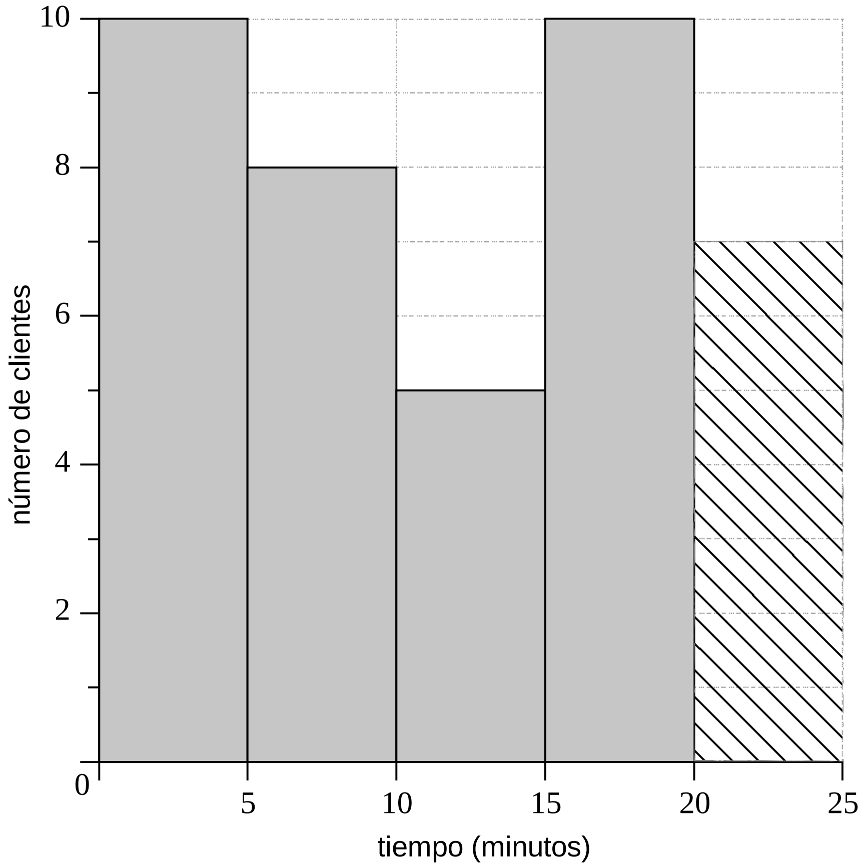
Nota: Conceda ***(M1)*** por haber sustituido en la fórmula de la media y haberla igualado a 12, ***(A1)*(ft)** si las sustituciones fueron correctas.

$(k=)\ 7$ ***(A1)*(ft)** ***(C4)***

Nota: Arrastre de error (FT) del apartado (b)(i) y los valores centrales del intervalo, que han de concordar con el apartado (a). No conceda el ***(A1)*** final si la respuesta dada no es un número entero.

[4 puntos]

(c)



(A1)*(ft)** ***(C1)

Nota: Arrastre de error (FT) desde la respuesta dada en el apartado (b)(ii), pero solo si el valor estaba comprendido entre 1 y 10, ambos inclusive.

[1 punto]

Total [6 puntos]

13. (a) $(70-75)^2+100$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir en $x=70$.

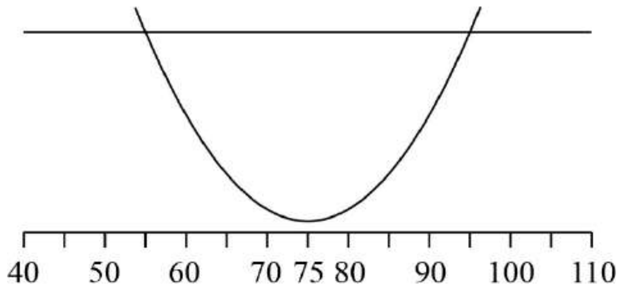
125 ***(A1)*** ***(C2)***

[2 puntos]

(b) $(s-75)^2+100=500$ ***(M1)***

Nota: Conceda ***(M1)*** por igualar $C(x)$ a 500. Acepte una desigualdad en vez de =.

O BIEN



(M1)

Nota: Conceda ***(M1)*** por dibujar aproximadamente gráfico(s) correcto(s).

$(s=)$ 95 ***(A1)*** ***(C2)***

[2 puntos]

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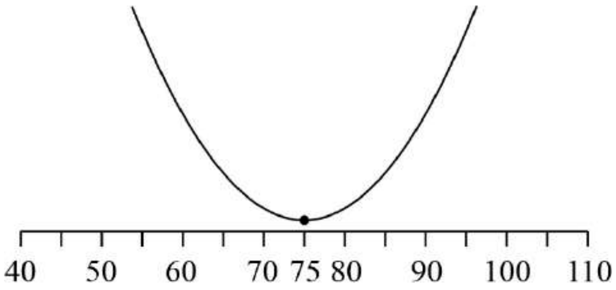
Continuación de la Pregunta 13

(c)



(M1)

Nota: Conceda ***(M1)*** por un intento de hallar el mínimo de la función utilizando un gráfico.

O BIEN

$$\frac{95+55}{2}$$ ***(M1)***

Nota: Conceda ***(M1)*** por tratar de hallar el punto medio entre la respuesta dada en el apartado (b) y 55.

O BIEN

$\left(C'(x)=\right)2x-150=0$ ***(M1)***

Nota: Conceda ***(M1)*** por un intento de hallar la derivada que se haya igualado correctamente a cero.

75 ***(A1)*** ***(C2)***

[2 puntos]

Total [6 puntos]

14. (a) x^3 ***(A1)*** ***(C1)***

Nota: Conceda ***(A0)*** por $\frac{4x^3}{4}$ y sin simplificar a x^3.

[1 punto]

(b) $\left(-\frac{1}{2}\right)^3$ ***(M1)***

Nota: Conceda ***(M1)*** por una sustitución correcta de $-\frac{1}{2}$ en su derivada.

$-\frac{1}{8}$ $(-0{,}125)$ ***(A1)*(ft)** ***(C2)***

Nota: Arrastre de error (FT) desde su apartado (a).

[2 puntos]

(c) $x^3 = 8$ ***(A1)(M1)***

Nota: Conceda ***(A1)*(ft)** por 8 visto quizás como parte de una ecuación $y = 8x + c$, ***(M1)*** por igualar su derivada a 8.

$(x =)\ 2$ ***(A1)*** ***(C3)***

Nota: No acepte $(2, 4)$.

[3 puntos]

Total [6 puntos]

15. (a) (generatriz$^2 =$)$10^2 + r^2$ ***(M1)***

Nota: Por una sustitución correcta de 10 y r en el teorema de Pitágoras.

$\sqrt{10^2 + r^2}$ ***(A1)*** ***(C2)***

[2 puntos]

(b) $\pi r^2 + 2\pi r \times 12 + \pi r\sqrt{100 + r^2} = 570$ ***(M1)(M1)(M1)***

Nota: Conceda ***(M1)*** por una sustitución correcta en las fórmulas del área de la superficie curva de un cilindro y del área de la base, ***(M1)*** por una sustitución correcta en la fórmula del área de la superficie curva de un cono, ***(M1)*** por sumar esas 3 áreas y igualarlas a 570. Arrastre de error (FT) desde el apartado (a).

$= 4{,}58 \ \ (4{,}58358...)$ ***(A1)*****(ft)** ***(C4)***

Nota: La última línea debe verse para otorgar el ***(A1)*** final. Error de arrastre desde la parte (a).

[4 puntos]

Total [6 puntos]

Diploma Programme
Programme du diplôme
Programa del Diploma

Études mathématiques
Niveau moyen
Épreuve 1

Mercredi 2 mai 2018 (après-midi)

Numéro de session du candidat

1 heure 30 minutes

Instructions destinées aux candidats

- Écrivez votre numéro de session dans les cases ci-dessus.
- N'ouvrez pas cette épreuve avant d'y être autorisé(e).
- Une calculatrice à écran graphique est nécessaire pour cette épreuve.
- Un exemplaire non annoté du **livret de formules pour le cours d'études mathématiques NM** est nécessaire pour cette épreuve.
- Répondez à toutes les questions.
- Rédigez vos réponses dans les espaces prévus à cet effet.
- Sauf indication contraire dans l'intitulé de la question, toutes les réponses numériques devront être exactes ou correctes à trois chiffres significatifs près.
- Le nombre maximum de points pour cette épreuve d'examen est de **[90 points]**.

2218 – 7407

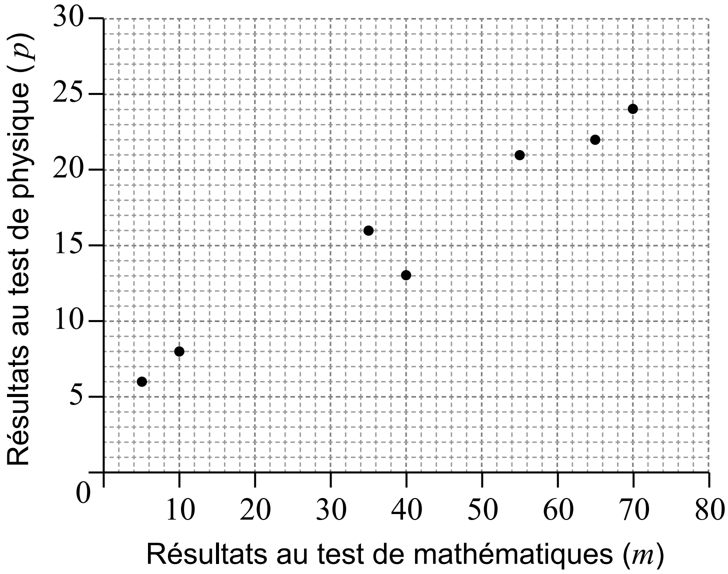
21 pages

Veuillez ne **pas** écrire sur cette page.

Les réponses rédigées sur cette page ne seront pas corrigées.

Le total des points sera attribué pour une réponse correcte. Lorsque la réponse est fausse, certains points peuvent être attribués si la méthode utilisée est correcte, pour autant que le raisonnement soit indiqué par écrit. Rédigez vos réponses dans les cases prévues à cet effet. Les solutions obtenues à l'aide d'une calculatrice à écran graphique doivent être accompagnées d'un raisonnement adéquat. Par exemple, si des représentations graphiques sont utilisées pour trouver la solution, veuillez inclure une esquisse de ces représentations graphiques dans votre réponse.

1. Le diagramme de dispersion suivant montre les résultats obtenus par sept élèves à leur test de mathématiques, m, et à leur test de physique, p.



Le point moyen, M, pour ces données est $(40\,;\,16)$.

(a) Placez et légendez le point $\mathrm{M}(\bar{m},\ \bar{p})$ sur le diagramme de dispersion. [2]

(b) Dessinez la droite de régression trouvée visuellement sur le diagramme de dispersion. [2]

(c) En utilisant votre droite de régression, estimez le résultat au test de physique d'un élève ayant obtenu un résultat de 20 à son test de mathématiques. [2]

Résolution :

Réponse :

(c) . . . .

2. Considérez les énoncés suivants.

p : le bébé pleure
q : le bébé est heureux
r : le bébé veut jouer

(a) Écrivez, en mots, $(q \wedge r) \Rightarrow \neg p$. [3]

(b) Complétez la table de vérité suivante. [2]

p	q	r	$\neg p$	$(q \wedge r)$	$(q \wedge r) \Rightarrow \neg p$
V	V	V	F		
V	V	F	F		
V	F	V	F		
V	F	F	F		
F	V	V	V		
F	V	F	V		
F	F	V	V		
F	F	F	V		

(c) Indiquez si $(q \wedge r) \Rightarrow \neg p$ est une tautologie, une contradiction logique ou ni l'une ni l'autre. [1]

Résolution :

Réponses :

(a) . . . .

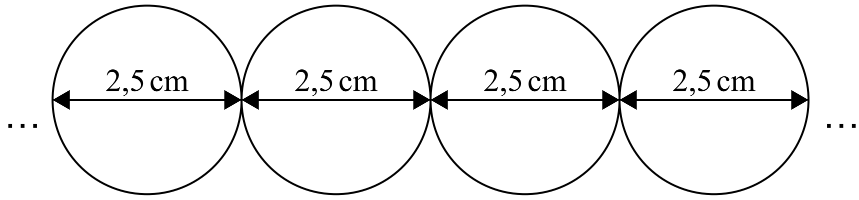
. . . .

. . . .

(c) . . . .

3. L'année dernière, une entreprise sud-américaine fabriquant des friandises a vendu $4,8 \times 10^8$ bonbons sphériques. Chaque bonbon a un diamètre de $2,5\,\text{cm}$.

L'entreprise produit une publicité qui présente tous ces bonbons placés sur une ligne droite.



(a) Trouvez la longueur, en cm, de cette ligne droite. Donnez votre réponse sous la forme $a \times 10^k$, où $1 \le a < 10$ et $k \in \mathbb{Z}$. [3]

La publicité prétend que la longueur de cette ligne droite équivaut à x fois la longueur du fleuve Amazone. La longueur du fleuve Amazone est de $6400\,\text{km}$.

(b) (i) Écrivez la longueur du fleuve Amazone en cm.

(ii) Trouvez la valeur de x. [3]

Résolution :

Réponses :

(a) . . . .

(b) (i) . . . .

(ii) . . . .

4. Le tableau suivant montre quatre ensembles de nombres différents : $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ et $\mathbb{R}$.

Ensemble	**Exemple d'un nombre faisant partie de l'ensemble**
$\mathbb{N}$	
$\mathbb{Z}$	
$\mathbb{Q}$	
$\mathbb{R}$	

(a) Complétez la deuxième colonne du tableau en donnant **un** exemple d'un nombre pour chaque ensemble. [4]

Josh affirme : « Tout entier relatif est un entier naturel ».

(b) Écrivez si l'affirmation de Josh est correcte. Justifiez votre réponse. [2]

Résolution :

Réponse :

(b) . . . .

. . . .

. . . .

5. Dans cette question, donnez toutes vos réponses avec une précision de deux décimales.

Karl investit 1000 dollars américains (USD) dans un compte lui versant un intérêt annuel nominal de $3,5\%$, **composé trimestriellement**. Il laisse l'argent dans le compte pour une période de 5 ans.

(a) (i) Calculez le montant d'argent qu'il aura dans son compte après 5 ans ;

(ii) Écrivez le montant d'**intérêt** qui lui a été versé après 5 ans. [4]

Karl décide de faire don de cet **intérêt** à un organisme de charité en France. L'organisme de charité reçoit 170 euros (EUR). Le taux de change est $1\,\text{USD} = t\,\text{EUR}$.

(b) Calculez la valeur de t. [2]

Résolution :

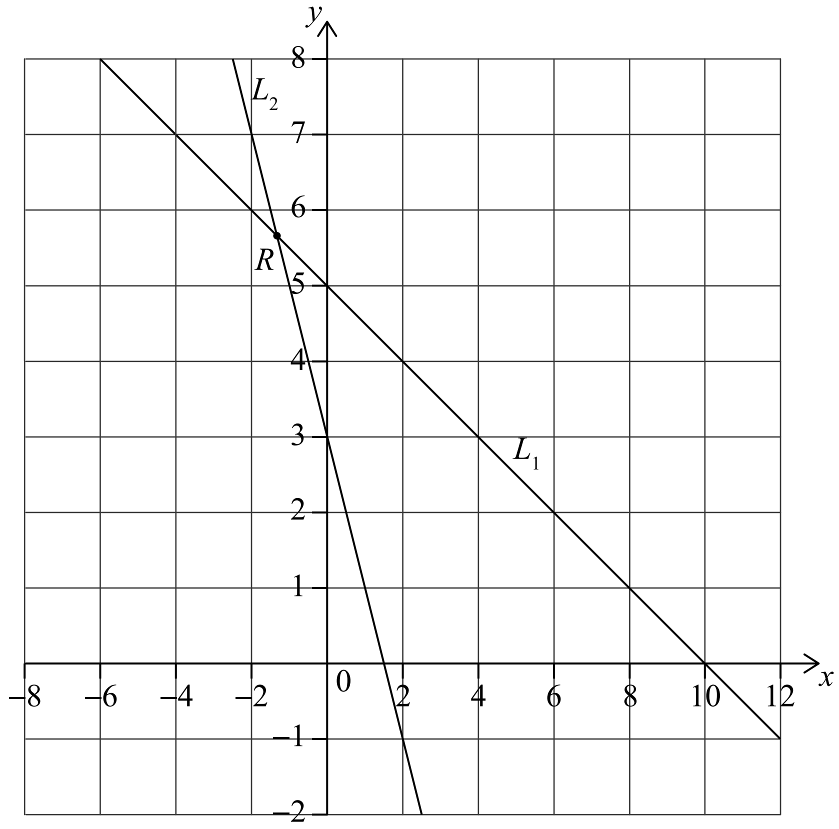
Réponses :

(a) (i) . . . .

(ii) . . . .

(b) . . . .

6. Considérez les droites L_1 et L_2. R est le point d'intersection de ces droites.



L'équation de la droite L_1 est $y = ax + 5$.

(a) Trouvez la valeur de a. [2]

L'équation de la droite L_2 est $y = -2x + 3$.

(b) Trouvez les coordonnées de R. [2]

La droite L_3 est parallèle à la droite L_2 et passe par le point $(2\,;\,3)$.

(c) Trouvez l'équation de la droite L_3. Donnez votre réponse sous la forme $y = mx + c$. [2]

(Suite de la question à la page suivante)

(Suite de la question 6)

Résolution :

Réponses :

(a) . . . .

(b) . . . .

(c) . . . .

7. Dans le cadre d'une compétition internationale, les participants ne peuvent répondre aux questions que dans **une seule** des trois langues suivantes : portugais, mandarin ou hindi. Il y a 80 participants qui ont pris part à la compétition. Le nombre de participants ayant répondu en portugais, mandarin ou hindi est montré dans le tableau.

		Langues			
		Portugais	Mandarin	Hindi	**Total**
Participants	Garçons	20	18	5	$\mathbf{43}$
	Filles	18	7	12	$\mathbf{37}$
	Total	$\mathbf{38}$	$\mathbf{25}$	$\mathbf{17}$	$\mathbf{80}$

(a) Indiquez le nombre de garçons ayant répondu aux questions en portugais. [1]

Un garçon est choisi au hasard.

(b) Trouvez la probabilité que ce garçon ait répondu aux questions en hindi. [2]

Deux filles sont choisies au hasard.

(c) Calculez la probabilité qu'une fille ait répondu aux questions en mandarin et l'autre ait répondu aux questions en hindi. [3]

Résolution :

Réponses :

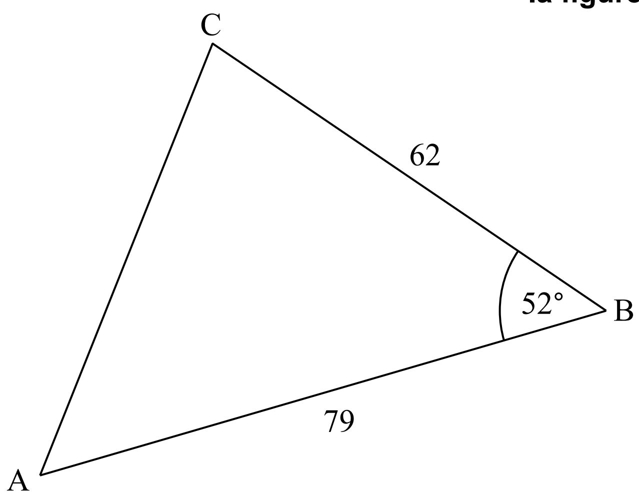
(a) . . . .

(b) . . . .

(c) . . . .

8. Un parc de forme triangulaire, ABC, est montré dans le diagramme suivant. AB mesure $79\,\mathrm{km}$ et BC mesure $62\,\mathrm{km}$. L'angle $\mathrm{A\hat{B}C}$ mesure 52°.

la figure n'est pas à l'échelle



(a) Calculez la longueur du côté AC en km. [3]

(b) Calculez l'aire du parc. [3]

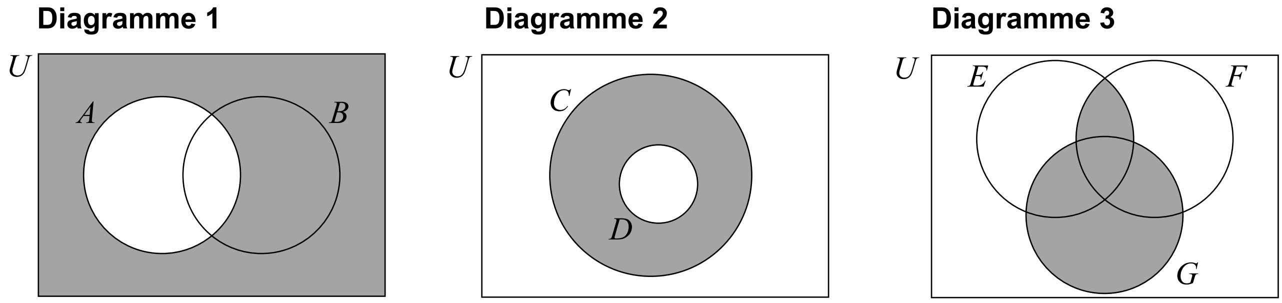
Résolution :

Réponses :

(a) . . . .

(b) . . . .

9. Considérez les diagrammes de Venn suivants.



(a) Écrivez une expression, en notation ensembliste, pour la région **grisée** représentée par

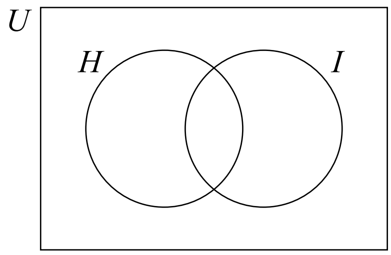
(i) le diagramme 1 ;

(ii) le diagramme 2 ;

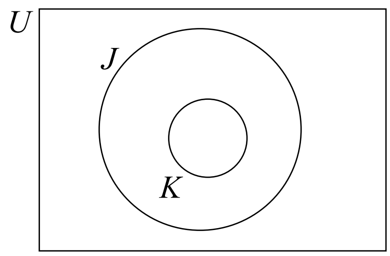
(iii) le diagramme 3. [4]

(b) Hachurez, sur le diagramme de Venn, la région représentée par l'ensemble

(i) $(H \cup I)'$;



(ii) $J \cap K$. [2]



(Suite de la question à la page suivante)

(Suite de la question 9)

Résolution :

Réponses :

(a) (i) . . . .

(ii) . . . .

(iii) . . . .

10. La fonction suivante modélise la croissance d'une population de bactéries lors d'une expérience,

$$P(t) = A \times 2^t,\ t \geq 0$$

où A est une constante et t est le temps, en heures, depuis le début de l'expérience.

Quatre heures après le début de l'expérience, la population de bactéries est de 6400.

(a) Trouvez la valeur de A. [2]

(b) Interprétez ce que A représente dans ce contexte. [1]

(c) Trouvez le temps nécessaire, depuis début de l'expérience, pour que la population de bactéries soit égale à $40A$. [3]

Résolution :

Réponses :

(a) . . . .

(b) . . . .

. . . .

(c) . . . .

11. Considérez la représentation graphique de la fonction $f(x)=\dfrac{3}{x}-2$, $x \neq 0$.

(a) Écrivez l'équation de l'asymptote verticale. [2]

(b) Écrivez l'équation de l'asymptote horizontale. [2]

(c) Calculez la valeur de x pour laquelle $f(x)=0$. [2]

Résolution :

Réponses :

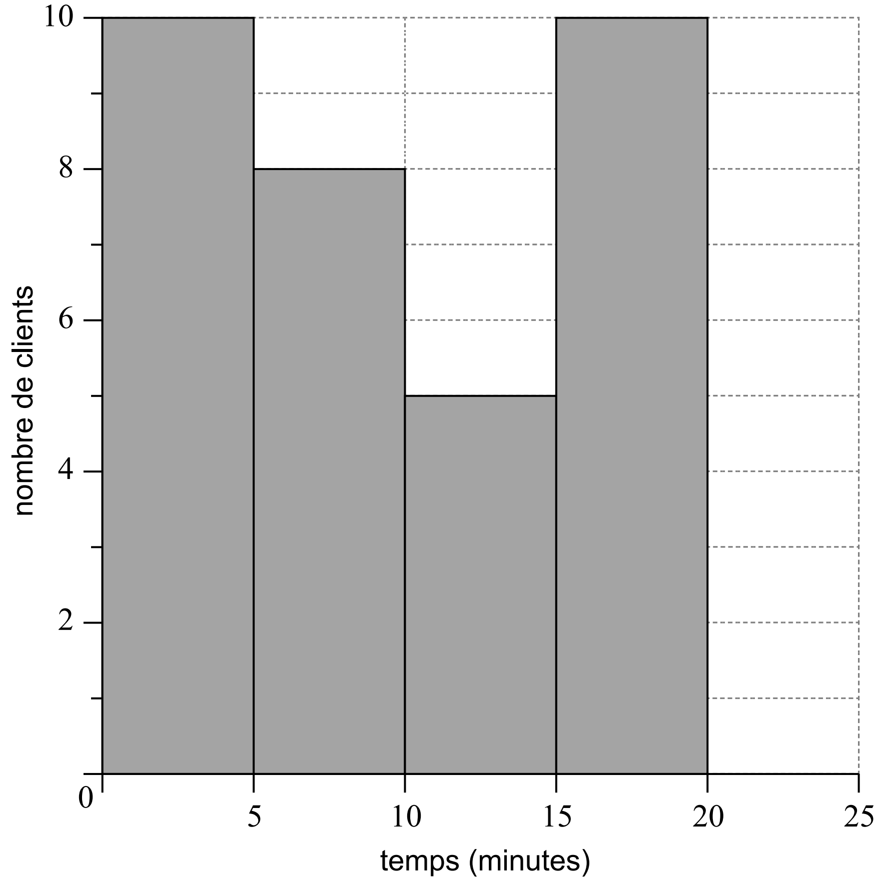
(a) . . . .

(b) . . . .

(c) . . . .

12. L'histogramme montre le temps, t, en minutes, qu'il faut aux clients d'un restaurant pour manger leur déjeuner lors d'une journée donnée. Chaque client a pris moins de 25 minutes**.**

L'histogramme est incomplet et ne montre que les données pour $0 \le t < 20$.



(a) Écrivez la valeur centrale de l'intervalle pour $10 \le t < 15$. [1]

Le temps moyen que **tous** les clients ont pris pour manger leur déjeuner a été estimé à 12 minutes.

On a trouvé que k clients ont pris entre 20 et 25 minutes pour manger leur déjeuner.

(b) (i) Écrivez le nombre total de clients en fonction de k.

(ii) Calculez la valeur de k. [4]

(c) À partir de là, complétez l'histogramme. [1]

(Suite de la question à la page suivante)

(Suite de la question 12)

Résolution :

Réponses :

(a) . . . .

(b) (i) . . . .

(ii) . . . .

13. Une usine produit des chemises. Le coût de production, C, en dollars fidjiens (FJD), de x chemises peut être modélisé par

$$C(x) = (x - 75)^2 + 100\,.$$

(a) Trouvez le coût de production de 70 chemises. [2]

Le coût de production ne doit pas dépasser 500 FJD. Pour ce faire, l'usine doit produire au moins 55 chemises et au plus s chemises.

(b) Trouvez la valeur de s. [2]

(c) Trouvez le nombre de chemises produites lorsque le coût de production est minimal. [2]

Résolution :

Réponses :

(a) . . . .

(b) . . . .

(c) . . . .

14. Considérez la fonction $f(x) = \frac{x^4}{4}$.

(a) Trouvez $f'(x)$. [1]

(b) Trouvez la pente de la représentation graphique de f en $x = -\frac{1}{2}$. [2]

(c) Trouvez l'abscisse du point où la **normale** à la représentation graphique de f a une pente de $-\frac{1}{8}$. [3]

Résolution :

Réponses :

(a) . . . .

(b) . . . .

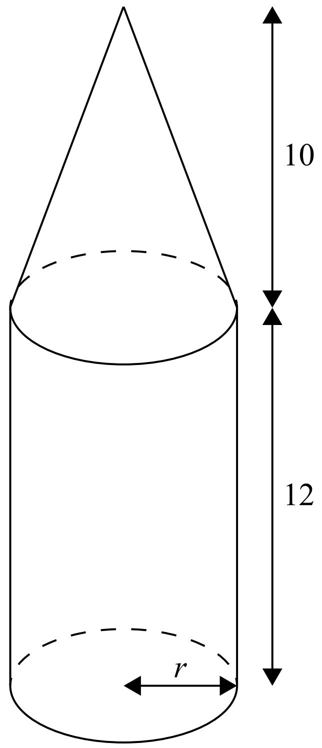
(c) . . . .

15. Julio fabrique un étui à crayons en bois ayant la forme d'un gros crayon. L'étui est constitué d'un cylindre attaché à un cône, tel que montré sur la figure suivante.

Le cylindre a un rayon de r cm et une hauteur de 12 cm.

Le cône a une base dont le rayon est de r cm et une hauteur de 10 cm.

la figure n'est pas à l'échelle



(a) Trouvez une expression pour la hauteur oblique (l'apothème) du cône **en fonction de r**. [2]

L'aire extérieure totale de l'étui à crayons, arrondie à 3 chiffres significatifs, est de $570\,\text{cm}^2$.

(b) En utilisant votre calculatrice à écran graphique, calculez la valeur de r. [4]

(Suite de la question à la page suivante)

(Suite de la question 15)

Résolution :

Réponses :

(a) . . . .

(b) . . . .

Veuillez ne **pas** écrire sur cette page.

Les réponses rédigées sur cette page ne seront pas corrigées.

Veuillez ne **pas** écrire sur cette page.

Les réponses rédigées sur cette page ne seront pas corrigées.

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Les réponses rédigées sur cette page ne seront pas corrigées.

Mathematical studies
Standard level
Paper 2

Thursday 3 May 2018 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

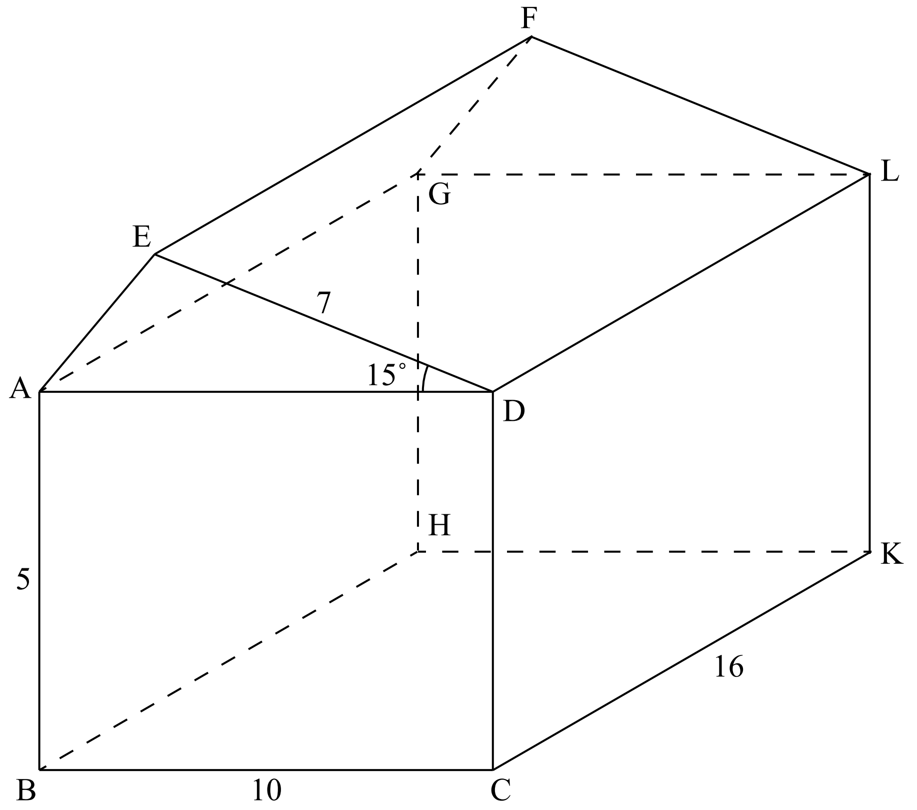
2218 – 7404

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 18]

Farmer Brown has built a new barn, on horizontal ground, on his farm. The barn has a cuboid base and a triangular prism roof, as shown in the diagram.

diagram not to scale



The cuboid has a width of $10\,\text{m}$, a length of $16\,\text{m}$ and a height of $5\,\text{m}$.
The roof has two sloping faces and two vertical and identical sides, ADE and GLF.
The face DEFL slopes at an angle of 15° to the horizontal and $\text{ED} = 7\,\text{m}$.

(a) Calculate the area of triangle EAD. [3]

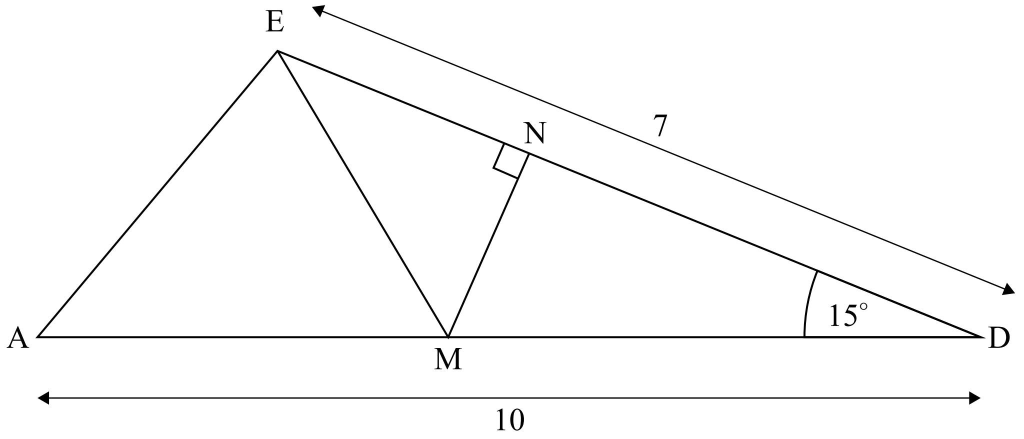
(b) Calculate the **total** volume of the barn. [3]

(This question continues on the following page)

(Question 1 continued)

The roof was built using metal supports. Each support is made from **five** lengths of metal AE, ED, AD, EM and MN, and the design is shown in the following diagram.

diagram not to scale



$\mathrm{ED} = 7\,\mathrm{m}$, $\mathrm{AD} = 10\,\mathrm{m}$ and angle $\mathrm{ADE} = 15^\circ$.
M is the midpoint of AD.
N is the point on ED such that MN is at right angles to ED.

(c) Calculate the length of MN. [2]

(d) Calculate the length of AE. [3]

Farmer Brown believes that N is the midpoint of ED.

(e) Show that Farmer Brown is incorrect. [3]

(f) Calculate the **total** length of metal required for one support. [4]

2. [Maximum mark: 16]

On one day 180 flights arrived at a particular airport. The distance travelled and the arrival status for each incoming flight was recorded. The flight was then classified as on time, slightly delayed, or heavily delayed.

The results are shown in the following table.

		Distance travelled			
		At most 500 km	**Between 500 km and 5000 km**	**At least 5000 km**	**TOTAL**
Arrival Status	**On time**	19	17	16	52
	Slightly delayed	13	18	14	45
	Heavily delayed	28	15	40	83
	TOTAL	60	50	70	180

A χ^2 test is carried out at the 10% significance level to determine whether the arrival status of incoming flights is independent of the distance travelled.

(a) State the alternative hypothesis. [1]

(b) Calculate the expected frequency of flights travelling at most $500\,\text{km}$ and arriving slightly delayed. [2]

(c) Write down the number of degrees of freedom. [1]

(d) Write down

(i) the χ^2 statistic;

(ii) the associated p-value. [3]

(This question continues on the following page)

(Question 2 continued)

The critical value for this test is 7.779.

(e) State, with a reason, whether you would reject the null hypothesis. [2]

A flight is chosen at random from the 180 recorded flights.

(f) Write down the probability that this flight arrived on time. [2]

(g) Given that this flight was not heavily delayed, find the probability that it travelled between $500\,\text{km}$ and $5000\,\text{km}$. [2]

Two flights are chosen at random from those which were slightly delayed.

(h) Find the probability that each of these flights travelled at least $5000\,\text{km}$. [3]

3. [Maximum mark: 11]

Give your answers to parts (b), (c) and (d) to the nearest whole number.

Harinder has $14\,000$ US Dollars (USD) to invest for a period of five years. He has two options of how to invest the money.

Option A: Invest the full amount, in USD, in a fixed deposit account in an American bank.

The account pays a nominal annual interest rate of $r\%$, **compounded yearly**, for the five years. The bank manager says that this will give Harinder a return of $17\,500\,\text{USD}$.

(a) Calculate the value of r. [3]

Option B: Invest the full amount, in Indian Rupees (INR), in a fixed deposit account in an Indian bank. The money must be converted from USD to INR before it is invested.

The exchange rate is $1\ \text{USD} = 66.91\,\text{INR}$.

(b) Calculate $14\,000\,\text{USD}$ in INR. [2]

The account in the Indian bank pays a nominal annual interest rate of 5.2% **compounded monthly**.

(c) Calculate the amount of this investment, in INR, in this account after five years. [3]

Harinder chose option B. At the end of five years, Harinder converted this investment back to USD. The exchange rate, at that time, was $1\,\text{USD} = 67.16\,\text{INR}$.

(d) Calculate how much **more** money, in USD, Harinder earned by choosing option B instead of option A. [3]

4. [Maximum mark: 14]

Consider the function $f(x)=\frac{48}{x}+kx^2-58$, where $x>0$ and k is a constant.

The graph of the function passes through the point with coordinates $(4, 2)$.

(a) Find the value of k. [2]

(b) Using your value of k, find $f'(x)$. [3]

P is the minimum point of the graph of $f(x)$.

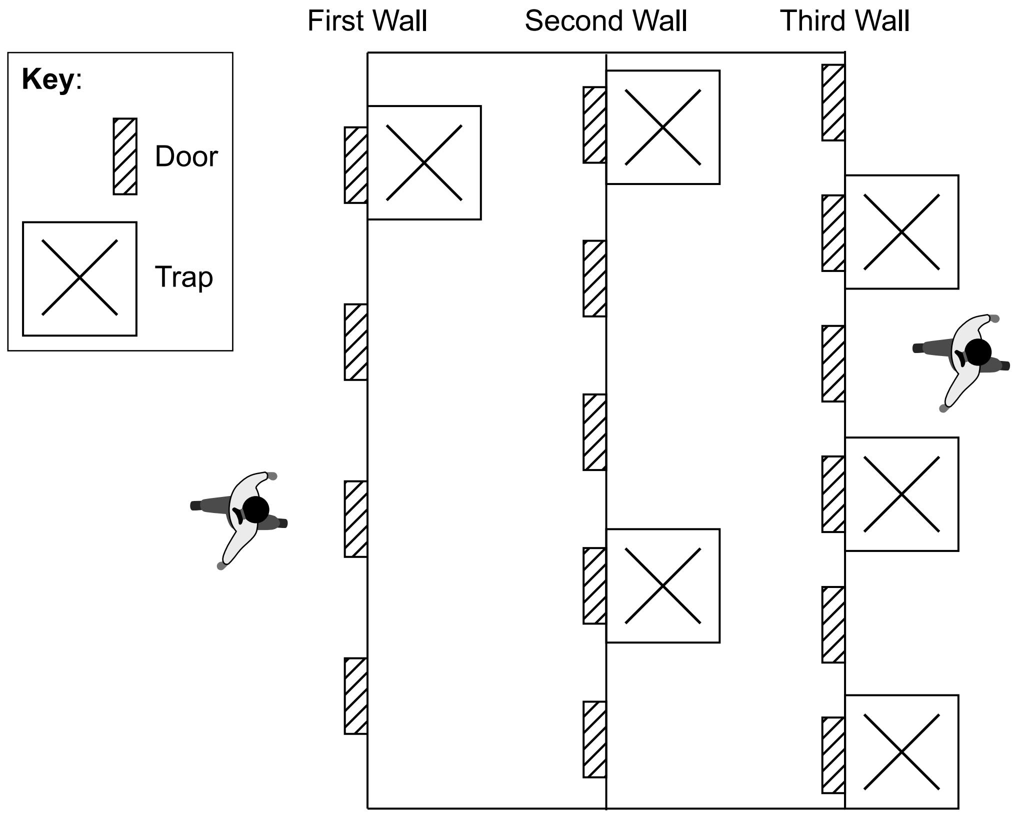
(c) **Use your answer** to part (b) to show that the minimum value of $f(x)$ is -22. [3]

(d) Write down the **two** values of x which satisfy $f(x)=0$. [2]

(e) Sketch the graph of $y=f(x)$ for $0<x\le 6$ and $-30\le y\le 60$.
Clearly indicate the minimum point P and the x-intercepts on your graph. [4]

5. [Maximum mark: 15]

Contestants in a TV gameshow try to get through three walls by passing through doors without falling into a trap. Contestants choose doors at random.
If they avoid a trap they progress to the next wall.
If a contestant falls into a trap they exit the game before the next contestant plays.
Contestants are not allowed to watch each other attempt the game.



The first wall has four doors with a trap behind one door.

Ayako is a contestant.

(a) Write down the probability that Ayako avoids the trap in this wall. [1]

Natsuko is the second contestant.

(b) Find the probability that only one of Ayako and Natsuko falls into a trap while attempting to pass through a door **in the first wall**. [3]

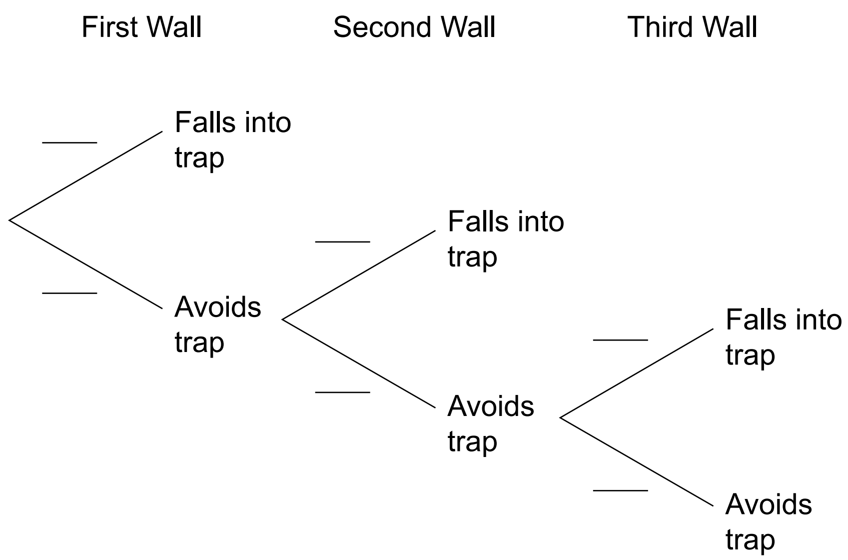
The second wall has five doors with a trap behind two of the doors.

The third wall has six doors with a trap behind three of the doors.

(This question continues on the following page)

(Question 5 continued)

The following diagram shows the branches of a probability tree diagram for a contestant in the game.



(c) **Copy** the probability tree diagram and write down the relevant probabilities along the branches. [3]

(d) A contestant is chosen at random. Find the probability that this contestant

(i) fell into a trap while attempting to pass through a door in the second wall;

(ii) fell into a trap. [5]

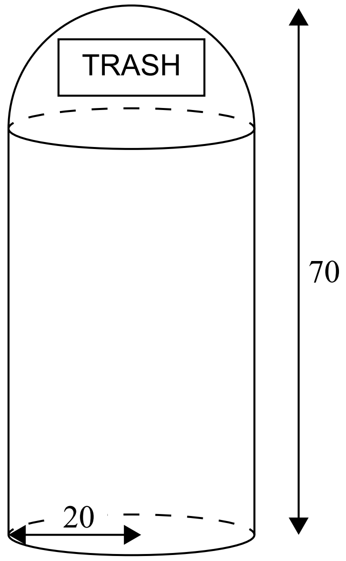
120 contestants attempted this game.

(e) Find the expected number of contestants who fell into a trap while attempting to pass through a door in the third wall. [3]

6. [Maximum mark: 16]

A manufacturer makes trash cans in the form of a cylinder with a hemispherical top. The trash can has a height of $70\,\text{cm}$. The base radius of both the cylinder and the hemispherical top is $20\,\text{cm}$.

diagram not to scale

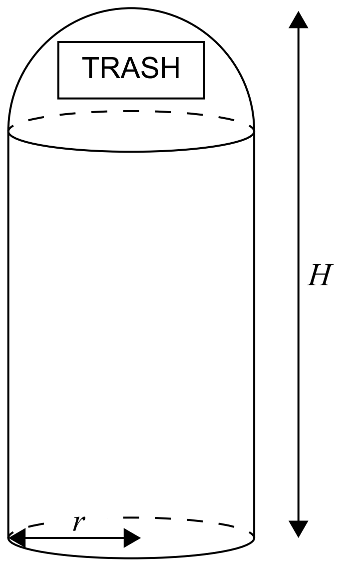


(a) Write down the height of the cylinder. [1]

(b) Find the total volume of the trash can. [4]

A designer is asked to produce a new trash can.
The new trash can will also be in the form of a cylinder with a hemispherical top.
This trash can will have a height of $H\,\text{cm}$ and a base radius of $r\,\text{cm}$.

diagram not to scale



(This question continues on the following page)

(Question 6 continued)

There is a design constraint such that $H + 2r = 110\,\text{cm}$.
The designer has to maximize the volume of the trash can.

(c) Find the height of the **cylinder**, h, of the new trash can, in terms of r. [2]

(d) Show that the volume, $V\,\text{cm}^3$, of the new trash can is given by

$$V = 110\pi r^2 - \frac{7}{3}\pi r^3.$$ [3]

(e) Using your graphic display calculator, find the value of r which maximizes the value of V. [2]

The designer claims that the new trash can has a capacity that is at least $40\,\%$ greater than the capacity of the original trash can.

(f) State whether the designer's claim is correct. Justify your answer. [4]

M18/5/MATSD/SP2/ENG/TZ1/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Mathematical studies

Standard level

Paper 2

24 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Paper 2 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

1 Abbreviations

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

R Marks awarded for clear **Reasoning**

G Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

AG **Answer Given** in the question and consequently, marks not awarded.

ft Marks that can be awarded as **follow through** from previous results in the question.

2 Method of Marking

(a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the ***A0*** annotation, otherwise full annotations must be shown.

(c) Working crossed out by the candidate should not be awarded any marks.

(d) Where candidates have written two solutions to a question, only the first solution should be marked.

(e) If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***(A1)*** *(ignore the further working)*
2.	$(x-6)(x+1)$	$x=6$ and -1	Do **not** award the final ***(A1)***

Example: Calculate the gradient of the line passing through the points $(5, 3)$ and $(0, 9)$.

Markscheme	**Candidates' Scripts**	**Marking**
$\frac{9-3}{0-5}$ ***(M1)*** *Award **(M1)** for correct substitution in gradient formula*	(i) $\frac{9-3}{0-5}=-\frac{6}{5}$	***(M1)***
	Gradient is $=-\frac{6}{5}$	***(A1)***
$=-\frac{6}{5}$ ***(A1)***	*(There is clear understanding of the gradient.)* $y=-\frac{6}{5}x+9$	
	(ii) $\frac{9-3}{0-5}=-\frac{6}{5}$	***(M1)***
	$y=-\frac{6}{5}x+9$	***(A0)***
	(There is confusion about what is required.)	

3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final ***A*** mark should not be awarded.

(c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f) Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3}=\frac{\sin 30}{4}$ ***(M1)(A1)*** *Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.*	(a) $\frac{\sin A}{4}=\frac{\sin 30}{3}$	***(M1)(A0)*** *(use of sine rule but with wrong values)*
$A=22.0^\circ\ (22.0243\ldots)$ ***(A1)(G2)***	$A=41.8^\circ$	***(A0)*** *(**Note:** the 2nd **(A1)** here was not marked* **(ft)** *and cannot be awarded because there was an earlier error in the **same** question part.)*
(b) $x=7\tan(22.0243\ldots^\circ)$ ***(M1)*** $=2.83\,(2.83163\ldots)$ ***(A1)*****(ft)**	(b) case (i) $x=7\tan 41.8^\circ$ $=6.26$	***(M1)*** ***(A1)*****(ft)**
	but case (ii) 6.26	***(G0)*** *since no working shown*

4 Using the Markscheme

(a) ***A*** marks are **dependent** on the preceding ***M*** mark being awarded, it is **not** possible to award ***(M0)(A1)***. Once an ***(M0)*** has been awarded, all subsequent ***A*** marks are lost in that part of the question, even if calculations are performed correctly, until the next ***M*** mark.
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

(b) ***A*** marks are **dependent** on the ***R*** mark being awarded, it is **not** possible to award ***(A1)(R0)***. Hence the ***(A1)*** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.

(c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated ***G*** in the mark scheme as an alternative to the full marks. Example ***(M1)(A1)(A1)(G2)***.

Example: Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ ***(M1)(A1)***	(i) $\frac{\sin A}{3} = \frac{\sin 30}{4}$	***(M1)(A1)***
*Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.*	$A = 22.0^\circ$	***(A1)***
	(ii) $A = 22.0^\circ$	***(G2)***
$A = 22.0^\circ$ $(22.0243\ldots)$ ***(A1)(G2)***	***Note: G*** *marks are used only if no working has been shown and the answer is correct.*	

(d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(e) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value $\left(\text{for example } \frac{2}{3} \text{ if applicable}\right)$;
the full calculator display in the form $2.83163\ldots$ as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1’7; $1\cdot 7$; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval: $3 < x < 5$; (3, 5);] 3, 5 [.

Different forms of notation for set properties (*eg,* complement): A'; $\bar{A}$; A^c; $U - A$;$(A$;$U \setminus A$.

Different forms of logic notation: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p$.
$p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

Significance level may be written as α .

(g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award ***(A1)*** and ignore subsequent rounding.

2. If the candidate's unrounded answer is **not** seen then award ***(A1)*** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise ***(A0)***.

Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given …					
	Exact or to 4 or more sf (and would **round to the correct 3 sf**)	**Correct to 3 sf**	**Incorrect to 3 sf**	Correct to 2 sf [3]	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen[1]	Award the final ***(A1)*** irrespective of correct or incorrect rounding					
Unrounded answer not seen[2]	***(A1)***	***(A1)***	***(A0)***	***(A1)***	***(A0)***	***(A0)***
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen.[3]			

Examples:

Markscheme	Candidates' Scripts	Marking
$9.43\,(9.43398\ldots)$ ***(A1)***	(i) $9.43398\ldots$ is seen followed by 9; 9.4; 9.43; 9.434 *etc.* (correctly rounded)	***(A1)***
	(ii) $9.43398\ldots$ is seen followed by 9.433; 9.44 *etc*. (incorrectly rounded)	***(A1)***
	(iii) 9.4	***(A1)***
	(iv) 9	***(A0)*** *(correct to 1 sf)*
	(v) 9.3	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 9.44	***(A0)*** *(incorrectly rounded to 3 sf)*

Markscheme	Candidates' Scripts	Marking
$7.44\,(7.43798\ldots)$ ***(A1)***	(i) $7.43798\ldots$ is seen followed by 7; 7.4; 7.44; 7.438 *etc.* (correctly rounded)	***(A1)***
	(ii) $7.43798\ldots$ is seen followed by 7.437; 7.43 *etc*. (incorrectly rounded)	***(A1)***
	(iii) 7.4	***(A1)***
	(iv) 7	***(A0)*** *(correct to 1 sf)*
	(v) 7.5	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 7.43	***(A0)*** *(incorrectly rounded to 3 sf)*

Example: ABC is a right angled triangle with angle $\text{ABC} = 90^\circ$, $\text{AC} = 32\text{ cm}$ and $\text{AB} = 30\text{ cm}$. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $\text{BC} = \sqrt{32^2 - 30^2}$ ***(M1)*** *Award **(M1)** for correct substitution in Pythagoras' formula* $= 11.1\ \left(\sqrt{124}, 11.1355...\right)(\text{cm})$ ***(A1)*** (b) $\text{Area} = \frac{1}{2} \times 30 \times 11.1355...$ ***(M1)*** *Award **(M1)** for correct substitution in area of triangle formula* $= 167 (167.032...) (\text{cm}^2)$ ***(A1)*(ft)**	(a) $\text{BC} = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$ (b) case (i) $\text{Area} = \frac{1}{2} \times 30 \times 11$ $= 165\,(\text{cm}^2)$ case (ii) $= 165\,(\text{cm}^2)$	***(M1)*** ***(A1)*** *(2 sf answer only seen, but correct)* ***(M1)*** *(working shown)* ***(A1)*(ft)** ***(M0)(A0)*(ft)** *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)*

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared $7.68\,(7.67543\ldots)$ ***(A2)***	(a) 7.7	***(G2)***
	(b) 7.67	***(G1)***
	(c) 7.6	***(G1)***
	(d) 8	***(G0)***
	(e) 7	***(G0)***
	(e) 7.66	***(G0)***

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ ***(A2)*** $(y = 0.887686\ldots x + 13.4895\ldots)$ If an answer is not in the form of an equation award at most ***(A1)(A0)***.	(a) $y = 0.89x + 13$	***(G2)*** *(both accepted)*
	(b) $y = 0.88x + 13$	***(G1)*** *(one rounding error)*
	(c) $y = 0.88x + 14$	***(G1)*** *(rounding error repeated)*
	(d) (i) $y = 0.9x + 13$ (ii) $y = 0.8x + 13$	***(G1)*** *(1 sf not accepted)*
	(e) $0.88x + 13$	***(G0)*** *(one rounding error and not an equation)*

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ ***(A1)(A1)*** $(2.06020\ldots, 4.49253\ldots)$	(a) $(2.1, 4.5)$	***(A1)(A1)*** *(both accepted)*
	(b) $(2.0, 4.4)$	***(A1)*** *(same rounding error twice)*
	(c) $(2.06, 4.4)$	***(A1)*** *(one rounding error)*
	(d) $(2, 4.4)$	***(A0)*** *(1sf not accepted, one rounding error)*

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*eg,* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg,* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding ***(M0)***.

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) ***(A1)***	(i) 231.6	***(A0)***
	(ii) 232	***(A0)*** *(Correct rounding to incorrect level)*
	(iii) 231.61	***(A0)***
	(iv) 232.00	***(A0)*** *(Parts (iii) and (iv) are both incorrect rounding to correct level)*

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded ***(A1)*** under the accuracy rules given in Section 5.

Example:

Markscheme	Candidates' Scripts	Marking
(a) 37000 m^2 ***(A1)***	(a) 36000 m^2	***(A0)*** *(Incorrect answer so units not considered)*
(b) 3200 m^3 ***(A1)***	(b) 3200 m^2	***(A0)*** *(Incorrect units)*

If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a) (Area of $\text{EAD} =$) $\frac{1}{2} \times 10 \times 7 \times \sin 15$ ***(M1)(A1)***

> **Note:** Award ***(M1)*** for substitution into area of a triangle formula, ***(A1)*** for correct substitution. Award ***(M0)(A0)(A0)*** if EAD or AED is considered to be a right-angled triangle.

$= 9.06 \text{ m}^2 \quad (9.05866\ldots \text{ m}^2)$ ***(A1)*** ***(G3)***

[3 marks]

(b) $(10 \times 5 \times 16) + (9.05866\ldots \times 16)$ ***(M1)(M1)***

> **Note:** Award ***(M1)*** for correct substitution into volume of a cuboid, ***(M1)*** for adding the correctly substituted volume of their triangular prism.

$= 945 \text{ m}^3 \quad (944.938\ldots \text{ m}^3)$ ***(A1)*(ft)** ***(G3)***

> **Note:** Follow through from part (a).

[3 marks]

(c) $\frac{\text{MN}}{5} = \sin 15$ ***(M1)***

> **Note:** Award ***(M1)*** for correct substitution into trigonometric equation.

$(\text{MN} =) \; 1.29 \text{(m)} \quad (1.29409\ldots \text{(m)})$ ***(A1)*** ***(G2)***

[2 marks]

(d) $(\text{AE}^2 =) \; 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 15$ ***(M1)(A1)***

> **Note:** Award ***(M1)*** for substitution into cosine rule formula, and ***(A1)*** for correct substitution.

$(\text{AE} =) \; 3.71 \text{(m)} \quad (3.71084\ldots \text{(m)})$ ***(A1)*** ***(G2)***

[3 marks]

continued…

Question 1 continued

(e) $\text{ND}^2 = 5^2 - (1.29409\ldots)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras theorem.

$(\text{ND} =)\ \ 4.83\ \ (4.82962\ldots)$ ***(A1)*(ft)**

Note: Follow through from part (c).

OR

$\dfrac{1.29409\ldots}{\text{ND}} = \tan 15^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into tangent.

$(\text{ND} =)\ \ 4.83\ \ (4.82962\ldots)$ ***(A1)*(ft)**

Note: Follow through from part (c).

OR

$\dfrac{\text{ND}}{5} = \cos 15^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine.

$(\text{ND} =)\ \ 4.83\ \ (4.82962\ldots)$ ***(A1)*(ft)**

Note: Follow through from part (c).

OR

$\text{ND}^2 = 1.29409...^2 + 5^2 - 2 \times 1.29409... \times 5 \times \cos 75^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine rule.

$(\text{ND} =)\ \ 4.83\ \ (4.82962\ldots)$ ***(A1)*(ft)**

Note: Follow through from part (c).

continued…

Question 1(e) continued

$4.82962\ldots \neq 3.5 \quad (\text{ND} \neq 3.5)$ ***(R1)*(ft)**

OR

$4.82962\ldots \neq 2.17038... \quad (\text{ND} \neq \text{NE})$ ***(R1)*(ft)**
(hence Farmer Brown is incorrect)

Note:	Do not award ***(M0)(A0)(R1)*(ft)**. Award ***(M0)(A0)(R0)*** for a correct conclusion without any working seen.

[3 marks]

(f) $(\text{EM}^2 =) \ 1.29409\ldots^2 + (7 - 4.82962\ldots)^2$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into Pythagoras theorem.

OR

$(\text{EM}^2 =) \ 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos 15$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine rule formula.

$(\text{EM} =) \ 2.53\,(\text{m}) \quad (2.52689...\,(\text{m}))$ ***(A1)*(ft)** ***(G2)*(ft)**

Note: Follow through from parts (c), (d) and (e).

(Total length $=$) $2.52689\ldots + 3.71084\ldots + 1.29409\ldots + 10 + 7$ ***(M1)***

Note: Award ***(M1)*** for adding their EM, their parts (c) and (d), and 10 and 7.

$= 24.5\,(\text{m}) \quad (24.5318\ldots\,(\text{m}))$ ***(A1)*(ft)** ***(G4)***

Note: Follow through from parts (c) and (d).

[4 marks]

Total [18 marks]

2. (a) The arrival status is dependent on the distance travelled by the incoming flight ***(A1)***

Note: Accept "associated" or "not independent".

[1 mark]

(b) $\frac{60\times 45}{180}$ **OR** $\frac{60}{180}\times\frac{45}{180}\times 180$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into expected value formula.

$=15$ ***(A1)*** ***(G2)***

[2 marks]

(c) 4 ***(A1)***

Note: Award ***(A0)*** if "$2+2=4$" is seen.

[1 mark]

(d) (i) $9.55\ \ (9.54671\ldots)$ ***(G2)***

Note: Award ***(G1)*** for an answer of 9.54.

(ii) $0.0488\ \ (0.0487961\ldots)$ ***(G1)***

[3 marks]

(e) Reject the Null Hypothesis ***(A1)*****(ft)**

Note: Follow through from their hypothesis in part (a).

$9.55\ \ (9.54671\ldots) > 7.779$ ***(R1)*****(ft)**

OR

$0.0488\ \ (0.0487961\ldots) < 0.1$ ***(R1)*****(ft)**

Note: Do not award ***(A1)*****(ft)*****(R0)*****(ft)**. Follow through from part (d). Award ***(R1)*****(ft)** for a correct comparison, ***(A1)*****(ft)** for a consistent conclusion with the answers to parts (a) and (d). Award ***(R1)*****(ft)** for $\chi^2_{calc} > \chi^2_{crit}$, provided the calculated value is explicitly seen in part (d)(i).

[2 marks]

continued…

Question 2 continued

(f) $\frac{52}{180}$ $\left(0.289, \frac{13}{45}, 28.9\%\right)$ ***(A1)(A1)*** ***(G2)***

Note: Award ***(A1)*** for correct numerator, ***(A1)*** for correct denominator.

[2 marks]

(g) $\frac{35}{97}$ $(0.361, 36.1\%)$ ***(A1)(A1)*** ***(G2)***

Note: Award ***(A1)*** for correct numerator, ***(A1)*** for correct denominator.

[2 marks]

(h) $\frac{14}{45} \times \frac{13}{44}$ ***(A1)(M1)***

Note: Award ***(A1)*** for two correct fractions and ***(M1)*** for multiplying their two fractions***.***

$= \frac{182}{1980}$ $\left(0.0919, \frac{91}{990}, 0.091919..., 9.19\%\right)$ ***(A1)*** ***(G2)***

[3 marks]

Total [16 marks]

3. (a) $17\,500 = 14\,000\left(1+\frac{r}{100}\right)^5$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into the compound interest formula, ***(A1)*** for correct substitution. Award at most ***(M1)(A0)*** if not equated to $17\,500$.

OR

$N = 5$
$PV = \pm 14\,000$
$FV = \mp 17\,500$
$P/Y = 1$
$C/Y = 1$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 1$ seen, ***(M1)*** for **all** other correct entries. FV and PV must have opposite signs.

$= 4.56\ (\%)\ \ (4.56395\ldots\ (\%))$ ***(A1)*** ***(G3)***

[3 marks]

(b) $14\,000 \times 66.91$ ***(M1)***

Note: Award ***(M1)*** for multiplying $14\,000$ by 66.91.

$936\,740\ \ (\text{INR})$ ***(A1)*** ***(G2)***

Note: Answer must be given to the nearest whole number.

[2 marks]

continued…

Question 3 continued

(c) $936\,740 \times \left(1 + \frac{5.2}{12 \times 100}\right)^{12 \times 5}$ ***(M1)(A1)*(ft)**

Note:	Award ***(M1)*** for substitution into the compound interest formula, ***(A1)*(ft)** for their correct substitution.

OR

$N = 60$
$I\,\% = 5.2$
$PV = \pm 936\,740$
$P\,/\,Y = 12$
$C\,/\,Y = 12$ ***(A1)(M1)***

Note:	Award ***(A1)*** for $C\,/\,Y = 12$ seen, ***(M1)*** for **all** other correct entries.

OR

$N = 5$
$I\,\% = 5.2$
$PV = \pm 936\,740$
$P\,/\,Y = 1$
$C\,/\,Y = 12$ ***(A1)(M1)***

Note:	Award ***(A1)*** for $C\,/\,Y = 12$ seen, ***(M1)*** for **all** other correct entries.

$= 1\,214\,204 \;\; (\text{INR})$ ***(A1)*(ft)** ***(G3)***

Note:	Follow through from part (b). Answer must be given to the nearest whole number.

[3 marks]

(d) $\frac{1\,214\,204}{67.16}$ ***(M1)***

Note:	Award ***(M1)*** for dividing their (c) by 67.16.

$\left(\frac{1\,214\,204}{67.16}\right) - 17\,500 = 579 \text{ (USD)}$ ***(M1)(A1)*(ft)** ***(G3)***

Note:	Award ***(M1)*** for finding the difference between their conversion and 17500. Answer must be given to the nearest whole number. Follow through from part (c).

[3 marks]
Total [11 marks]

4. (a) $\frac{48}{4}+k\times 4^2-58=2$ ***(M1)***

> **Note:** Award ***(M1)*** for correct substitution of $x=4$ and $y=2$ into the function.

$k=3$ ***(A1)*** ***(G2)***

[2 marks]

(b) $\frac{-48}{x^2}+6x$ ***(A1)(A1)(A1)*(ft)** ***(G3)***

> **Note:** Award ***(A1)*** for -48, ***(A1)*** for x^{-2}, ***(A1)*(ft)** for their $6x$. Follow through from part (a). Award at most ***(A1)(A1)(A0)*** if additional terms are seen.

[3 marks]

(c) $\frac{-48}{x^2}+6x=0$

(M1)

> **Note:** Award ***(M1)*** for equating their part (b) to zero.

$x=2$ ***(A1)*(ft)**

> **Note:** Follow through from part (b). Award ***(M1)(A1)*** for $\frac{-48}{(2)^2}+6(2)=0$ seen.
>
> Award ***(M0)(A0)*** for $x=2$ seen either from a graphical method or without working.

$\frac{48}{2}+3\times 2^2-58 \quad (=-22)$ ***(M1)***

> **Note:** Award ***(M1)*** for substituting their 2 into their function, but only if the final answer is -22. Substitution of the known result invalidates the process; award ***(M0)(A0)(M0)***.

-22 ***(AG)***

[3 marks]

(d) $0.861 \quad (0.860548\ldots)$, $3.90 \quad (3.90307\ldots)$ ***(A1)*(ft)*(A1)*(ft)** ***(G2)***

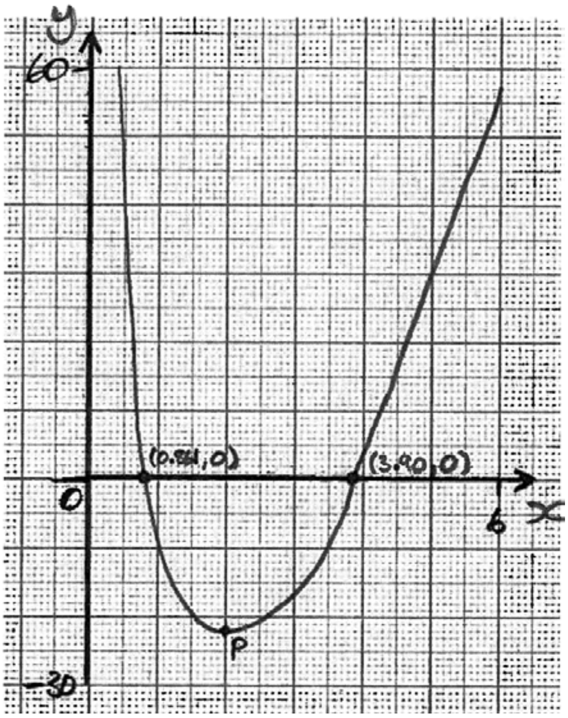
> **Note:** Follow through from part (a) but only if the answer is positive. Award at most ***(A1)*(ft)*(A0)*** if answers are given as coordinate pairs or if extra values are seen. The function $f(x)$ only has two x-intercepts within the domain. Do not accept a negative x-intercept.

[2 marks]

continued…

Question 4 continued

(e)



***(A1)(A1)*(ft)*(A1)*(ft)*(A1)*(ft)**

Note:	Award ***(A1)*** for correct window. Axes must be labelled. ***(A1)*(ft)** for a smooth curve with correct shape and zeros in approximately correct positions relative to each other. ***(A1)*(ft)** for point P indicated in approximately the correct position. Follow through from their x-coordinate in part (c). ***(A1)*(ft)** for two x-intercepts identified on the graph and curve reflecting asymptotic properties.

[4 marks]

Total [14 marks]

5. (a) $\frac{3}{4}$ $(0.75,\ 75\%)$ ***(A1)***

[1 mark]

(b) $\frac{3}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}$ **OR** $2\times\frac{3}{4}\times\frac{1}{4}$ ***(M1)(M1)***

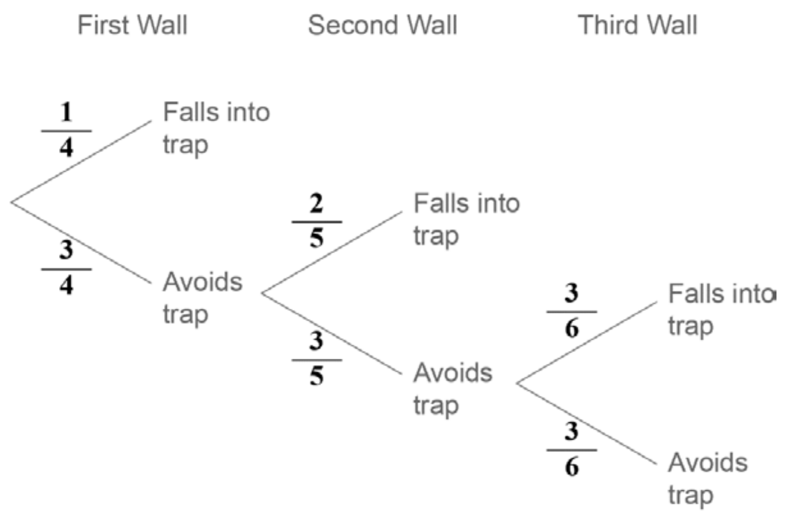
Note:	Award ***(M1)*** for their product $\frac{1}{4}\times\frac{3}{4}$ seen, and ***(M1)*** for adding their two products or multiplying their product by 2.

$=\frac{3}{8}$ $\left(\frac{6}{16},\ 0.375,\ 37.5\%\right)$ ***(A1)*(ft)** ***(G3)***

Note:	Follow through from part (a), but only if the sum of their two fractions is 1.

[3 marks]

(c)



(A1)*(ft)*(A1)(A1)

Note:	Award ***(A1)*** for each correct pair of branches. Follow through from part (a).

[3 marks]

continued…

Question 5 continued

(d) (i) $\frac{3}{4}\times\frac{2}{5}$ ***(M1)***

Note: Award ***(M1)*** for correct probabilities multiplied together.

$=\frac{3}{10}$ $(0.3,\ 30\%)$ ***(A1)*(ft)** ***(G2)***

Note: Follow through from their tree diagram or part (a).

(ii) $1-\frac{3}{4}\times\frac{3}{5}\times\frac{3}{6}$ **OR** $\frac{1}{4}+\frac{3}{4}\times\frac{2}{5}+\frac{3}{4}\times\frac{3}{5}\times\frac{3}{6}$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\frac{3}{4}\times\frac{3}{5}\times\frac{3}{6}$ and ***(M1)*** for subtracting their correct probability from 1, or adding to their $\frac{1}{4}+\frac{3}{4}\times\frac{2}{5}$.

$=\frac{93}{120}$ $\left(\frac{31}{40},\ 0.775,\ 77.5\%\right)$ ***(A1)*(ft)** ***(G2)***

Note: Follow through from their tree diagram.

[5 marks]

(e) $\frac{3}{4}\times\frac{3}{5}\times\frac{3}{6}\times 120$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\frac{3}{4}\times\frac{3}{5}\times\frac{3}{6}$ $\left(\frac{3}{4}\times\frac{3}{5}\times\frac{1}{2}\right.$ **OR** $\frac{27}{120}$ **OR** $\left.\frac{9}{40}\right)$ and ***(M1)*** for multiplying by 120.

$=27$ ***(A1)*(ft)** ***(G3)***

Note: Follow through from their tree diagram or their $\frac{3}{4}\times\frac{3}{5}\times\frac{3}{6}$ from their calculation in part (d)(ii).

[3 marks]

Total [15 marks]

6. (a) 50 (cm) ***(A1)***

[1 mark]

(b) $\pi \times 50 \times 20^2 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 20^3$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for their correctly substituted volume of cylinder, ***(M1)*** for correctly substituted volume of sphere formula, ***(M1)*** for halving the substituted volume of sphere formula. Award at most ***(M1)(M1)(M0)*** if there is no addition of the volumes.

$= 79\,600 \text{ (cm}^3) \quad \left(79\,587.0\ldots \text{ (cm}^3), \frac{76\,000}{3}\pi\right)$ ***(A1)*(ft)** ***(G3)***

Note: Follow through from part (a).

[4 marks]

(c) $h = H - r$ (or equivalent) **OR** $H = 110 - 2r$ ***(M1)***

Note: Award ***(M1)*** for writing h in terms of H and r or for writing H in terms of r.

$(h =)\ 110 - 3r$ ***(A1)*** ***(G2)***

[2 marks]

(d) $(V =)\ \frac{2}{3}\pi r^3 + \pi r^2 \times (110 - 3r)$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for volume of hemisphere, ***(M1)*** for correct substitution of their *h* into the volume of a cylinder, ***(M1)*** for addition of two correctly substituted volumes leading to the given answer. Award at most ***(M1)(M1)(M0)*** for subsequent working that does not lead to the given answer. Award at most ***(M1)(M0)(M0)*** for substituting $H = 110 - 2r$ as their *h*.

$V = 110\pi r^2 - \frac{7}{3}\pi r^3$ ***(AG)***

[3 marks]

(e) $(r =)\ 31.4$ (cm) $(31.4285\ldots \text{ (cm)})$ ***(G2)***

OR

$(\pi)(220r - 7r^2) = 0$ ***(M1)***

Note: Award ***(M1)*** for setting the correct derivative equal to zero.

$(r =)\ 31.4$ (cm) $(31.4285\ldots \text{ (cm)})$ ***(A1)***

[2 marks]

continued…

Question 6 continued

(f) $(V =)\ 110\pi(31.4285\ldots)^2 - \frac{7}{3}\pi(31.4285\ldots)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their $31.4285\ldots$ into the given equation.

$= 114\,000 \quad (113\,781\ldots)$ ***(A1)*****(ft)**

Note: Follow through from part (e).

$\left(\text{increase in capacity} =\right)\ \frac{113\,781\ldots - 79\,587.0\ldots}{79\,587.0\ldots} \times 100 = 43.0\,(\%)$ ***(R1)*****(ft)**

Note: Award ***(R1)*****(ft)** for finding the correct percentage increase from their two volumes.

OR

$1.4 \times 79\,587.0\ldots = 111421.81...$ ***(R1)*****(ft)**

Note: Award ***(R1)*****(ft)** for finding the capacity of a trash can 40% larger than the original.

Claim is correct ***(A1)*****(ft)**

Note: Follow through from parts (b), (e) and within part (f). The final ***(R1)(A1)*****(ft)** can be awarded for their correct reason and conclusion. Do not award ***(R0)(A1)*****(ft)**.

[4 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Thursday 3 May 2018 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

2218 – 7406

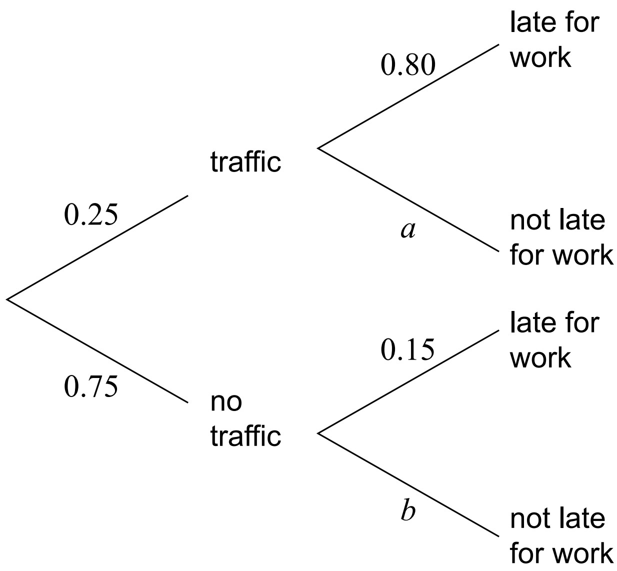
Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 16]

In a company it is found that 25% of the employees encountered traffic on their way to work. From those who encountered traffic the probability of being late for work is 80%.

From those who did not encounter traffic, the probability of being late for work is 15%.

The tree diagram illustrates the information.



(a) Write down the value of

(i) a;

(ii) b. [2]

(b) Use the tree diagram to find the probability that an employee

(i) encountered traffic and was late for work;

(ii) was late for work;

(iii) encountered traffic given that they were late for work. [8]

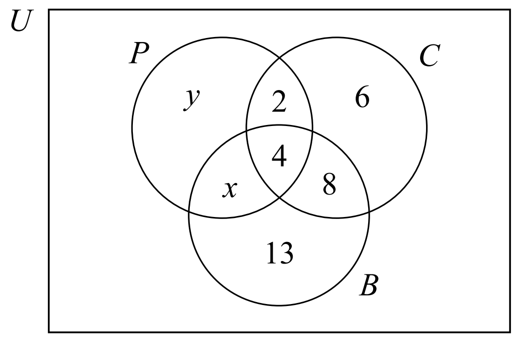
(This question continues on the following page)

(Question 1 continued)

The company investigates the different means of transport used by their employees in the past year to travel to work. It was found that the three most common means of transport used to travel to work were public transportation (P), car (C) and bicycle (B).

The company finds that 20 employees travelled by car, 28 travelled by bicycle and 19 travelled by public transportation in the last year.

Some of the information is shown in the Venn diagram.



(c) Find the value of

(i) x;

(ii) y. [2]

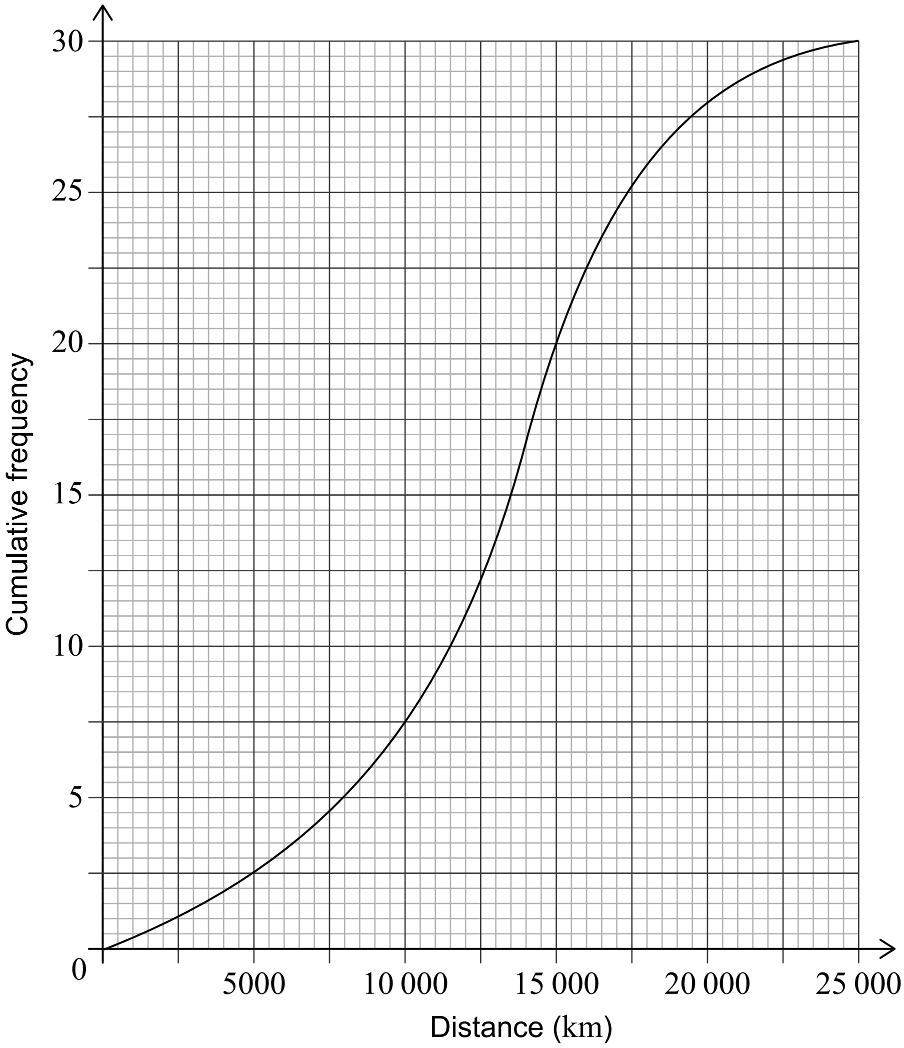
There are 54 employees in the company.

(d) Find the number of employees who, in the last year, did not travel to work by car, bicycle or public transportation. [2]

(e) Find $n\left((C \cup B) \cap P'\right)$. [2]

2. [Maximum mark: 15]

A transportation company owns 30 buses. The distance that each bus has travelled since being purchased by the company is recorded. The cumulative frequency curve for these data is shown.



(a) Find the number of buses that travelled a distance between $15\,000$ and $20\,000$ kilometres. [2]

(b) Use the cumulative frequency curve to find the

(i) median distance;

(ii) lower quartile;

(iii) upper quartile. [4]

(This question continues on the following page)

(Question 2 continued)

(c) Hence write down the interquartile range. [1]

(d) Write down the percentage of buses that travelled a distance greater than the upper quartile. [1]

(e) Find the number of buses that travelled a distance less than or equal to $12\,000\,\text{km}$. [1]

It is known that 8 buses travelled more than m kilometres.

(f) Find the value of m. [2]

The smallest distance travelled by one of the buses was $2500\,\text{km}$.
The longest distance travelled by one of the buses was $23\,000\,\text{km}$.

(g) **On graph paper**, draw a box-and-whisker diagram for these data. Use a scale of $2\,\text{cm}$ to represent $5000\,\text{km}$. [4]

3. [Maximum mark: 14]

The weight, W, of basketball players in a tournament is found to be normally distributed with a mean of $65\,\text{kg}$ and a standard deviation of $5\,\text{kg}$.

(a) (i) Find the probability that a basketball player has a weight that is less than $61\,\text{kg}$.

In a training session there are 40 basketball players.

(ii) Find the expected number of players with a weight less than $61\,\text{kg}$ in this training session. [4]

(b) The probability that a basketball player has a weight that is within 1.5 standard deviations of the mean is q.

(i) Sketch a normal curve to represent this probability.

(ii) Find the value of q. [3]

(c) Given that $\text{P}(W > k) = 0.225$, find the value of k. [2]

A basketball coach observed 60 of her players to determine whether their performance and their weight were independent of each other. Her observations were recorded as shown in the table.

		Performance	
		Satisfactory	**Excellent**
Weight	**Below average**	6	10
	Average	7	15
	Above average	12	10

She decided to conduct a χ^2 test for independence at the 5% significance level.

(d) For this test,

(i) state the null hypothesis; [1]

(ii) find the p-value. [2]

(e) State a conclusion for this test. Justify your answer. [2]

4. [Maximum mark: 16]

A new café opened and during the first week their profit was $\$60$.

The café's profit increases by $\$10$ every week.

(a) Find the café's profit during the 11th week. [3]

(b) Calculate the café's **total** profit for the first 12 weeks. [3]

A new tea-shop opened at the same time as the café. During the first week their profit was also $\$60$.

The tea-shop's profit increases by 10% every week.

(c) Find the tea-shop's profit during the 11th week. [3]

(d) Calculate the tea-shop's **total** profit for the first 12 weeks. [3]

In the mth week the tea-shop's **total** profit exceeds the café's **total** profit, for the first time since they both opened.

(e) Find the value of m. [4]

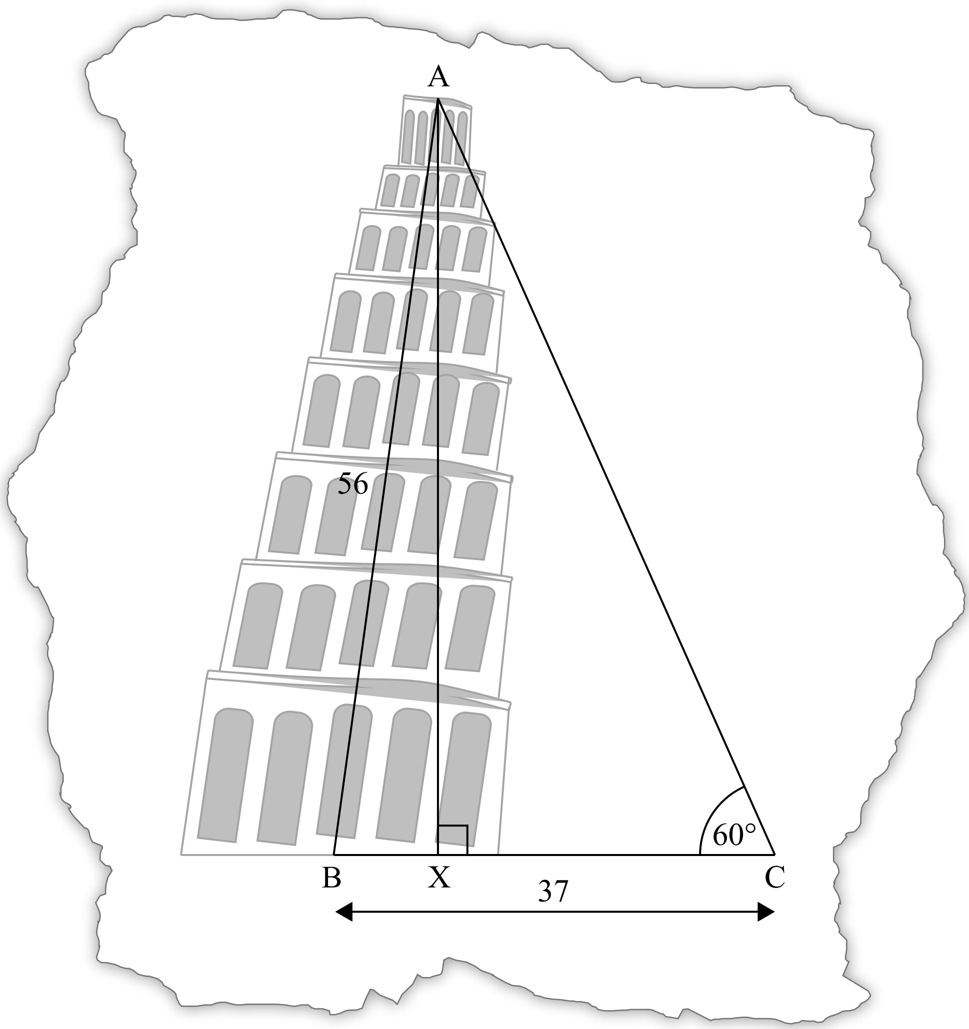
5. [Maximum mark: 14]

The Tower of Pisa is well known worldwide for how it leans.

Giovanni visits the Tower and wants to investigate how much it is leaning. He draws a diagram showing a non-right triangle, ABC.

On Giovanni's diagram the length of AB is $56\,\mathrm{m}$, the length of BC is $37\,\mathrm{m}$, and angle ACB is 60°. AX is the perpendicular height from A to BC.

diagram not to scale



(a) Use Giovanni's diagram to

(i) show that angle ABC, the angle at which the Tower is leaning relative to the horizontal, is 85° to the nearest degree.

(ii) calculate the length of AX.

(iii) find the length of BX, the horizontal displacement of the Tower. [9]

(This question continues on the following page)

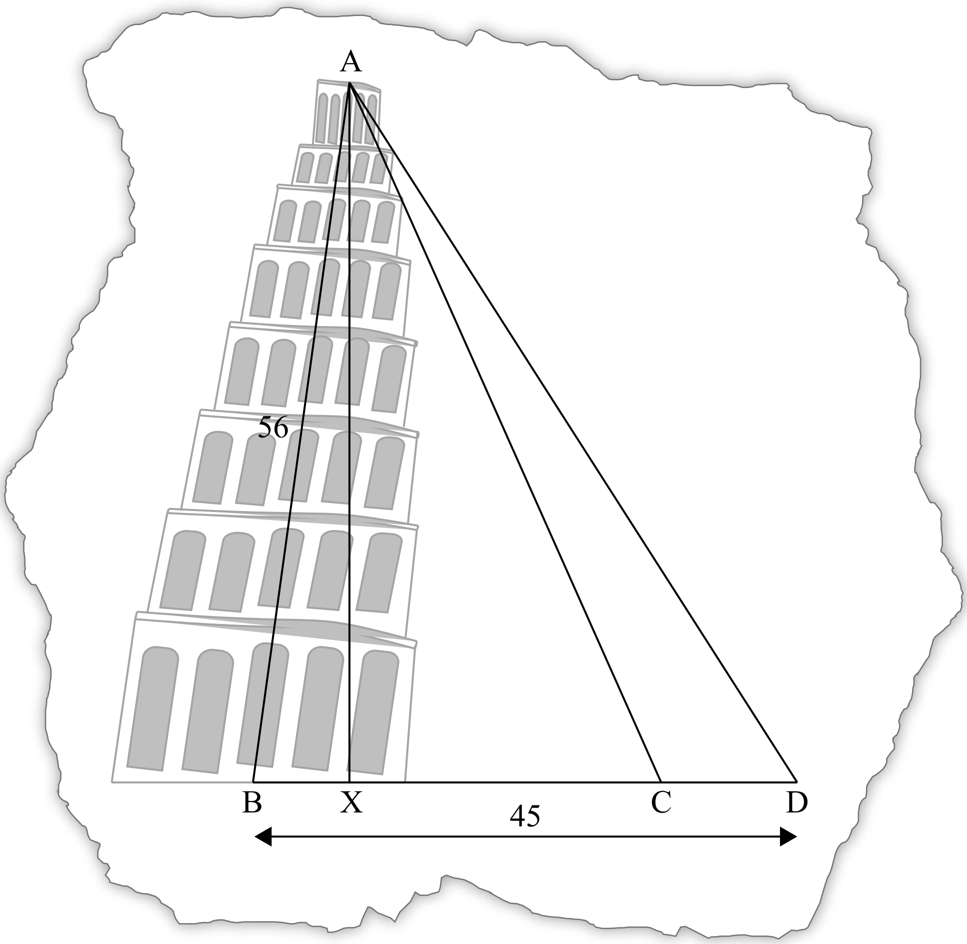
(Question 5 continued)

Giovanni's tourist guidebook says that the actual horizontal displacement of the Tower, BX, is 3.9 metres.

(b) Find the percentage error on Giovanni's diagram. [2]

Giovanni adds a point D to his diagram, such that $\mathrm{BD} = 45\,\mathrm{m}$, and another triangle is formed.

diagram not to scale



(c) Find the angle of elevation of A from D. [3]

6. [Maximum mark: 15]

Consider the curve $y = 2x^3 - 9x^2 + 12x + 2$, for $-1 < x < 3$.

(a) Sketch the curve for $-1 < x < 3$ and $-2 < y < 12$. [4]

(b) A teacher asks her students to make some observations about the curve.

Three students responded.
Nadia said "*The x-intercept of the curve is between -1 and zero*".
Rick said "*The curve is decreasing when $x < 1$*".
Paula said "*The gradient of the curve is less than zero between $x = 1$ and $x = 2$*".

State the name of the student who made an **incorrect** observation. [1]

(c) Find the value of y when $x = 1$. [2]

(d) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(e) Show that the stationary points of the curve are at $x = 1$ and $x = 2$. [2]

(f) Given that $2x^3 - 9x^2 + 12x + 2 = k$ has **three** solutions, find the possible values of k. [3]

Markscheme

May 2018

Mathematical studies

Standard level

Paper 2

Paper 2 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

1 Abbreviations

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

R Marks awarded for clear **Reasoning**

G Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

AG **Answer Given** in the question and consequently, marks not awarded.

ft Marks that can be awarded as **follow through** from previous results in the question.

2 Method of Marking

(a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the ***A0*** annotation, otherwise full annotations must be shown.

(c) Working crossed out by the candidate should not be awarded any marks.

(d) Where candidates have written two solutions to a question, only the first solution should be marked.

(e) If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***(A1)*** *(ignore the further working)*
2.	$(x-6)(x+1)$	$x=6$ and -1	Do **not** award the final ***(A1)***

Example: Calculate the gradient of the line passing through the points $(5,3)$ and $(0,9)$.

Markscheme	**Candidates' Scripts**	**Marking**
$\frac{9-3}{0-5}$ ***(M1)*** *Award **(M1)** for correct substitution in gradient formula* $=-\frac{6}{5}$ ***(A1)***	(i) $\frac{9-3}{0-5}=-\frac{6}{5}$ Gradient is $=-\frac{6}{5}$ *(There is clear understanding of the gradient.)* $y=-\frac{6}{5}x+9$ (ii) $\frac{9-3}{0-5}=-\frac{6}{5}$ $y=-\frac{6}{5}x+9$ *(There is confusion about what is required.)*	***(M1)*** ***(A1)*** ***(M1)*** ***(A0)***

3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final ***A*** mark should not be awarded.

(c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f) Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3}=\frac{\sin 30}{4}$ ***(M1)(A1)*** *Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.*	(a) $\frac{\sin A}{4}=\frac{\sin 30}{3}$	***(M1)(A0)*** *(use of sine rule but with wrong values)*
$A=22.0^{\circ}$ $(22.0243\ldots)$ ***(A1)(G2)***	$A=41.8^{\circ}$	***(A0)***
	(***Note:*** *the 2nd **(A1)** here was not marked* **(ft)** *and cannot be awarded because there was an earlier error in the **same** question part.)*	
(b) $x=7\tan(22.0243\ldots^{\circ})$ ***(M1)*** $=2.83\,(2.83163\ldots)$ ***(A1)*****(ft)**	(b) case (i) $x=7\tan 41.8^{\circ}$ $=6.26$	***(M1)*** ***(A1)*****(ft)**
	but case (ii) 6.26	***(G0)*** *since no working shown*

4 Using the Markscheme

(a) ***A*** marks are **dependent** on the preceding ***M*** mark being awarded, it is **not** possible to award ***(M0)(A1)***. Once an ***(M0)*** has been awarded, all subsequent ***A*** marks are lost in that part of the question, even if calculations are performed correctly, until the next ***M*** mark.
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

(b) ***A*** marks are **dependent** on the ***R*** mark being awarded, it is **not** possible to award ***(A1)(R0)***. Hence the ***(A1)*** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.

(c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated ***G*** in the mark scheme as an alternative to the full marks. Example ***(M1)(A1)(A1)(G2)***.

Example: Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ ***(M1)(A1)*** *Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.* $A = 22.0^\circ$ $(22.0243\ldots)$ ***(A1)(G2)***	(i) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ $A = 22.0^\circ$ (ii) $A = 22.0^\circ$ ***Note: G** marks are used only if no working has been shown and the answer is correct.*	***(M1)(A1)*** ***(A1)*** ***(G2)***

(d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(e) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value $\left(\text{for example } \frac{2}{3} \text{ if applicable}\right)$;
the full calculator display in the form $2.83163\ldots$ as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; $1 \cdot 7$; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval: $3 < x < 5$; (3, 5);] 3, 5 [.

Different forms of notation for set properties (*eg,* complement): A'; $\bar{A}$; A^c; $U - A$;$(A$;$U \setminus A$.

Different forms of logic notation: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p$.

$p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

Significance level may be written as α .

(g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award ***(A1)*** and ignore subsequent rounding.

2. If the candidate's unrounded answer is **not** seen then award ***(A1)*** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise ***(A0)***.

Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

<table>
<tr><td rowspan="2"></td><td colspan="6">If candidates final answer is given …</td></tr>
<tr><td>Exact or to 4 or more sf (and would round to the correct 3 sf)</td><td>Correct to 3 sf</td><td>Incorrect to 3 sf</td><td>Correct to 2 sf [3]</td><td>Incorrect to 2 sf</td><td>Correct or incorrect to 1 sf</td></tr>
<tr><td>Unrounded answer seen[1]</td><td colspan="6">Award the final (A1) irrespective of correct or incorrect rounding</td></tr>
<tr><td>Unrounded answer not seen[2]</td><td>(A1)</td><td>(A1)</td><td>(A0)</td><td>(A1)</td><td>(A0)</td><td>(A0)</td></tr>
<tr><td>Treatment of subsequent parts</td><td colspan="2">As per MS</td><td colspan="4">Treat as follow through, only if working is seen.[3]</td></tr>
</table>

Examples:

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398…) ***(A1)***	(i) 9.43398… is seen followed by 9; 9.4; 9.43; 9.434 *etc.* (correctly rounded)	***(A1)***
	(ii) 9.43398… is seen followed by 9.433; 9.44 *etc*. (incorrectly rounded)	***(A1)***
	(iii) 9.4	***(A1)***
	(iv) 9	***(A0)*** *(correct to 1 sf)*
	(v) 9.3	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 9.44	***(A0)*** *(incorrectly rounded to 3 sf)*

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798…) ***(A1)***	(i) 7.43798… is seen followed by 7; 7.4; 7.44; 7.438 *etc.* (correctly rounded)	***(A1)***
	(ii) 7.43798… is seen followed by 7.437; 7.43 *etc*. (incorrectly rounded)	***(A1)***
	(iii) 7.4	***(A1)***
	(iv) 7	***(A0)*** *(correct to 1 sf)*
	(v) 7.5	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 7.43	***(A0)*** *(incorrectly rounded to 3 sf)*

Example: ABC is a right angled triangle with angle $\text{ABC} = 90^\circ$, $\text{AC} = 32\text{ cm}$ and $\text{AB} = 30\text{ cm}$. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $\text{BC} = \sqrt{32^2 - 30^2}$ ***(M1)*** *Award **(M1)** for correct substitution in Pythagoras' formula* $= 11.1 \left(\sqrt{124}, 11.1355...\right)(\text{cm})$ ***(A1)***	(a) $\text{BC} = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$ *(2 sf answer only seen, but correct)*	***(M1)*** ***(A1)***
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11.1355...$ ***(M1)*** *Award **(M1)** for correct substitution in area of triangle formula* $= 167 (167.032...) (\text{cm}^2)$ ***(A1)*(ft)**	(b) case (i) $\text{Area} = \frac{1}{2} \times 30 \times 11$ *(working shown)* $= 165\,(\text{cm}^2)$ case (ii) $= 165\,(\text{cm}^2)$ *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)*	***(M1)*** ***(A1)*(ft)** ***(M0)(A0)*(ft)**

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared $7.68\,(7.67543\ldots)$ ***(A2)***	(a) 7.7	***(G2)***
	(b) 7.67	***(G1)***
	(c) 7.6	***(G1)***
	(d) 8	***(G0)***
	(e) 7	***(G0)***
	(e) 7.66	***(G0)***

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ ***(A2)*** $(y = 0.887686\ldots x + 13.4895\ldots)$ If an answer is not in the form of an equation award at most ***(A1)(A0)***.	(a) $y = 0.89x + 13$	***(G2)*** *(both accepted)*
	(b) $y = 0.88x + 13$	***(G1)*** *(one rounding error)*
	(c) $y = 0.88x + 14$	***(G1)*** *(rounding error repeated)*
	(d) (i) $y = 0.9x + 13$	
	(ii) $y = 0.8x + 13$	***(G1)*** *(1 sf not accepted)*
	(e) $0.88x + 13$	***(G0)*** *(one rounding error and not an equation)*

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ ***(A1)(A1)*** $(2.06020\ldots, 4.49253\ldots)$	(a) $(2.1, 4.5)$	***(A1)(A1)*** *(both accepted)*
	(b) $(2.0, 4.4)$	***(A1)*** *(same rounding error twice)*
	(c) $(2.06, 4.4)$	***(A1)*** *(one rounding error)*
	(d) $(2, 4.4)$	***(A0)*** *(1sf not accepted, one rounding error)*

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*eg,* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg,* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding ***(M0)***.

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) ***(A1)***	(i) 231.6	***(A0)***
	(ii) 232	***(A0)*** *(Correct rounding to incorrect level)*
	(iii) 231.61	***(A0)***
	(iv) 232.00	***(A0)*** *(Parts (iii) and (iv) are both incorrect rounding to correct level)*

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded ***(A1)*** under the accuracy rules given in Section 5.

Example:

Markscheme	Candidates' Scripts	Marking
(a) $37000\ \text{m}^2$ ***(A1)***	(a) $36000\ \text{m}^2$	***(A0)*** *(Incorrect answer so units not considered)*
(b) $3200\ \text{m}^3$ ***(A1)***	(b) $3200\ \text{m}^2$	***(A0)*** *(Incorrect units)*

If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a) $a = 0.2$, $b = 0.85$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct value.

[2 marks]

(b) (i) 0.25×0.8 ***(M1)***

Note: Award ***(M1)*** for a correct product.

$= 0.2 \ \left(\frac{1}{5}, 20\%\right)$ ***(A1)(G2)***

(ii) $0.25 \times 0.8 + 0.75 \times 0.15$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their (0.25×0.8) and (0.75×0.15), ***(M1)*** for adding two products.

$= 0.313 \ \left(0.3125, \frac{5}{16}, 31.3\%\right)$ ***(A1)*(ft)*(G3)***

Note: Award the final ***(A1)*(ft)** only if answer does not exceed 1. Follow through from part (b)(i).

(iii) $\dfrac{0.25 \times 0.8}{0.25 \times 0.8 + 0.75 \times 0.15}$ ***(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*(ft)** for a correct numerator (their part (b)(i)), ***(A1)*(ft)** for a correct denominator (their part (b)(ii)). Follow through from parts (b)(i) and (b)(ii).

$= 0.64 \ \left(\frac{16}{25}, 64\%\right)$ ***(A1)*(ft)*(G3)***

Note: Award final ***(A1)*(ft)** only if answer does not exceed 1.

[8 marks]

(c) (i) $(x =)\ 3$ ***(A1)***

(ii) $(y =)\ 10$ ***(A1)*(ft)**

Note: Following through from part (c)(i) but only if their x is less than or equal to 13.

[2 marks]

continued…

Question 1 continued

(d) $54-(10+3+4+2+6+8+13)$ ***(M1)***

Note: Award ***(M1)*** for subtracting their correct sum from 54. Follow through from their part (c).

$=8$ ***(A1)(ft)(G2)***

Note: Award ***(A1)*(ft)** only if their sum does not exceed 54. Follow through from their part (c).

[2 marks]

(e) $6+8+13$ ***(M1)***

Note: Award ***(M1)*** for summing 6, 8 and 13.

27 ***(A1)(G2)***

[2 marks]

Total [16 marks]

2. (a) $28-20$ ***(A1)***

Note: Award ***(A1)*** for 28 and 20 seen.

8 ***(A1)(G2)***

[2 marks]

(b) (i) 13500 ***(G2)***

Note: Accept an answer in the range 13500 to 13750.

(ii) 10000 ***(G1)***

Note: Accept an answer in the range 10000 to 10250.

(iii) 16000 ***(G1)***

Note: Accept an answer in the range 16000 to 16250.

[4 marks]

(c) 6000 ***(A1)*****(ft)**

Note: Follow through from their part (b)(ii) and (iii).

[1 mark]

(d) 25% ***(A1)***

[1 mark]

(e) 11 ***(G1)***

[1 mark]

(f) $30-8$ **OR** 22 ***(M1)***

Note: Award ***(M1)*** for subtracting $30-8$ or 22 seen.

15750 ***(A1)(G2)***

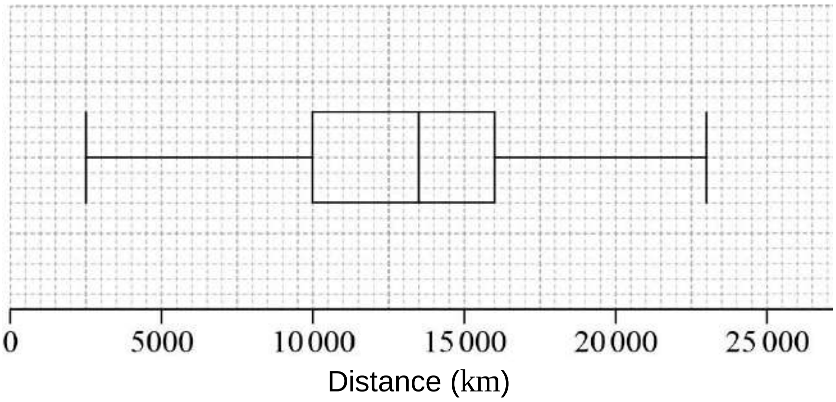
Note: Accept 15750 ± 250.

[2 marks]

continued…

Question 2 continued

(g)



(A1)(A1)(A1)(A1)

Note: Award ***(A1)*** for correct label and scale; accept "distance" or "km" for label.
***(A1)*(ft)** for correct median,
***(A1)*(ft)** for correct quartiles and box,
(A1) for endpoints at 2500 and $23\,000$ joined to box by straight lines.
Accept ± 250 for the median, quartiles and endpoints.
Follow through from their part (b).
The final ***(A1)*** is not awarded if the line goes through the box.

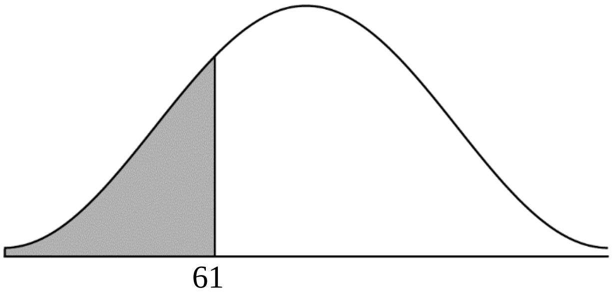
[4 marks]

Total [15 marks]

3. (a) (i) $\mathrm{P}(W < 61)$ ***(M1)***

Note: Award ***(M1)*** for correct probability statement.

OR



(M1)

Note: Award ***(M1)*** for correct region labelled and shaded on diagram.

$= 0.212\ (0.21185\ldots,\ 21.2\%)$ ***(A1)(G2)***

(ii) $40 \times 0.21185\ldots$ ***(M1)***

Note: Award ***(M1)*** for product of 40 and their 0.212.

$= 8.47\ \ (8.47421...)$ ***(A1)*****(ft)*****(G2)***

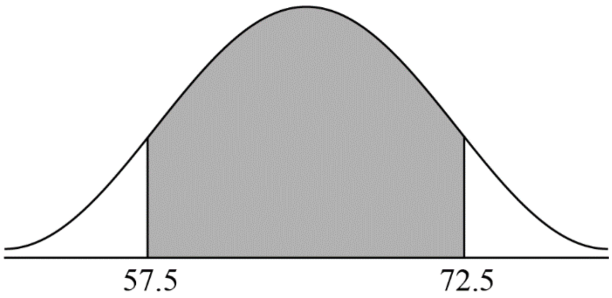
Note: Follow through from their part (a)(i) provided their answer to part (a)(i) is less than 1.

[4 marks]

continued…

Question 3 continued

(b) (i)



(A1)(M1)

Note:	Award ***(A1)*** for two correctly labelled vertical lines in approximately correct positions. The values 57.5 and 72.5, or $\mu - 1.5\sigma$ and $\mu + 1.5\sigma$ are acceptable labels. Award ***(M1)*** for correctly shaded region marked by their two vertical lines.

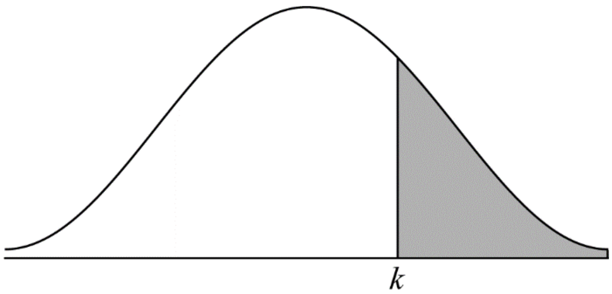
(ii) $0.866 \ \ (0.86638\ldots, 86.6\%)$ ***(A1)*(ft)**

Note:	Follow through from their part (b)(i) shaded region if their values are clear.

[3 marks]

(c) $\mathrm{P}(W < k) = 0.775$ ***(M1)***

OR



(M1)

Note:	Award ***(A1)*** for correct region labelled and shaded on diagram.

$(k =)$ $68.8 \ \ (68.7770\ldots)$ ***(A1)(G2)***

[2 marks]

continued…

Question 3 continued

(d) (i) (H_0:) performance (of players) and (their) weight are independent. ***(A1)***

Note: Accept "there is no association between performance (of players) and (their) weight". Do not accept "not related" or "not correlated" or " not influenced".

[1 mark]

(ii) 0.287 $(0.287436\ldots)$ ***(G2)***

[2 marks]

(e) accept/ do not reject null hypothesis/H_0 ***(A1)*(ft)**

OR

performance (of players) and (their) weight are independent. ***(A1)*(ft)**

$0.287 > 0.05$ ***(R1)*(ft)**

Note: Accept p-value>significance level provided their p-value is seen in b(ii). Accept $28.7\% > 5\%$. Do not award ***(A1)(R0)***. Follow through from part (d).

[2 marks]

Total [14 marks]

4. (a) $60+10\times10$ ***(M1)(A1)***

Note:	Award ***(M1)*** for substitution into the arithmetic sequence formula, ***(A1)*** for correct substitution.

$=(\$)\ 160$ ***(A1)(G3)***

[3 marks]

(b) $\frac{12}{2}\left(2\times60+11\times10\right)$ ***(M1)(A1)*****(ft)**

Note:	Award ***(M1)*** for substituting the arithmetic series formula, ***(A1)*****(ft)** for correct substitution. Follow through from their first term and common difference in part (a).

$=(\$)\ 1380$ ***(A1)*****(ft)*****(G2)***

[3 marks]

(c) 60×1.1^{10} ***(M1)(A1)***

Note:	Award ***(M1)*** for substituting the geometric progression *n*th term formula, ***(A1)*** for correct substitution.

$=(\$)\ 156\quad(155.624\ldots)$ ***(A1)(G3)***

Note:	Accept the answer if it rounds correctly to 3 sf, as per the accuracy instructions.

[3 marks]

(d) $\frac{60\,(1.1^{12}-1)}{1.1-1}$ ***(M1)(A1)(ft)***

Note:	Award ***(M1)*** for substituting the geometric series formula, ***(A1)*****(ft)** for correct substitution. Follow through from part (c) for their first term and common ratio.

$=(\$)\ 1280\quad(1283.05\ldots)$ ***(A1)(ft)(G2)***

[3 marks]

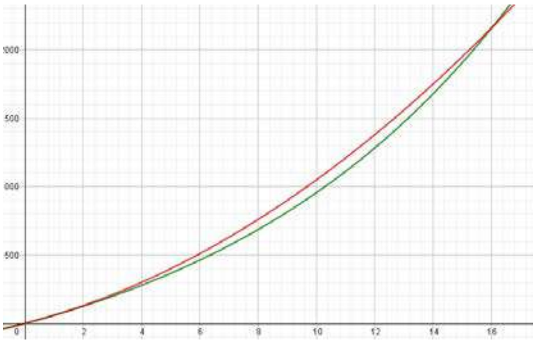
continued…

Question 4 continued

(e) $\frac{60(1.1^n - 1)}{1.1 - 1} > \frac{n}{2}\left(2 \times 60 + (n-1) \times 10\right)$ ***(M1)(M1)***

Note: Award ***(M1)*** for correctly substituted geometric and arithmetic series formula with n (accept other variable for "n"), ***(M1)*** for comparing their expressions consistent with their part (b) and part (d).

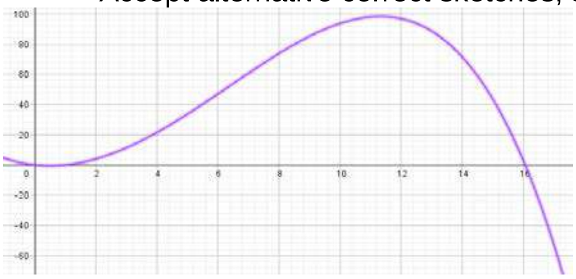
OR



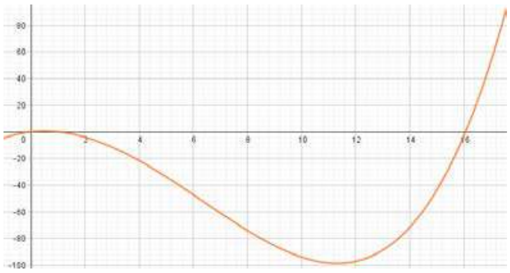
(M1)(M1)

Note: Award ***(M1)*** for two curves with approximately correct shape drawn in the first quadrant, ***(M1)*** for one point of intersection with approximate correct position.

Accept alternative correct sketches, such as



OR



Award ***(M1)*** for a curve with approximate correct shape drawn in the 1st (or 4th) quadrant and all above (or below) the x-axis, ***(M1)*** for one point of intersection with the x-axis with approximate correct position.

17 ***(A2)(ft)(G3)***

Note: Follow through from parts (b) and (d).
An answer of 16 is incorrect. Award at most ***(M1)(M1)(A0)(A0)*** with working seen.
Award ***(G0)*** if final answer is 16 without working seen.

[4 marks]

Total [16 marks]

5. (a) (i) $\frac{\sin \text{BAC}}{37} = \frac{\sin 60}{56}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituting the sine rule formula, ***(A1)*** for correct substitution.

angle $\text{B}\hat{\text{A}}\text{C} = 34.9034\ldots^\circ$ ***(A1)***

Note: Award ***(A0)*** if unrounded answer does not round to 35. Award ***(G2)*** if 34.9034… seen without working.

angle $\text{A}\hat{\text{B}}\text{C} = 180 - (34.9034\ldots + 60)$ ***(M1)***

Note: Award ***(M1)*** for subtracting their angle $\text{BAC} + 60$ from 180.

$85.0965\ldots^\circ$ ***(A1)***
85° ***(AG)***

Note: Both the unrounded and rounded value must be seen for the final ***(A1)*** to be awarded. If the candidate rounds $34.9034...^\circ$ to 35° while substituting to find angle $\text{A}\hat{\text{B}}\text{C}$, the final ***(A1)*** can be awarded but **only** if both $34.9034...^\circ$ and 35° are seen. If 85 is used as part of the workings, award at most (***M1)(A0)(A0)(M0)(A0)(AG)***. This is the reverse process and not accepted.

(ii) $\sin 85\ldots \times 56$ ***(M1)***
$= 55.8 \quad (55.7869\ldots) \quad \text{(m)}$ ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution in trigonometric ratio.

continued…

Question 5(a) continued

(iii) $\sqrt{56^2 - 55.7869\ldots^2}$ ***(M1)***

Note:	Award ***(M1)*** for correct substitution in the Pythagoras theorem formula. Follow through from part (a)(ii).

OR

$\cos(85) \times 56$ ***(M1)***

Note:	Award ***(M1)*** for correct substitution in trigonometric ratio.

$= 4.88\ (4.88072\ldots)\ \ (\text{m})$ ***(A1)*****(ft)*****(G2)***

Note:	Accept 4.73 $(4.72863\ldots)$ (m) from using their 3 s.f answer. Accept equivalent methods.

[9 marks]

continued…

Question 5 continued

(b) $\left|\frac{4.88-3.9}{3.9}\right|\times 100$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the percentage error formula.

$= 25.1 \quad (25.1282) \quad (\%)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (a)(iii).

[2 marks]

(c) $\tan^{-1}\left(\frac{55.7869\ldots}{40.11927\ldots}\right)$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their 40.11927… seen. Award ***(M1)*** for correct substitution into trigonometric ratio.

OR

$(37-4.88072...)^2+55.7869...^2$

$(\text{AC}=)\ 64.3725...$

$64.3726...^2+8^2-2\times 8\times 64.3726...\times\cos 120$

$(\text{AD}=)\ 68.7226...$

$\frac{\sin 120}{68.7226...}=\frac{\sin \text{A}\hat{\text{D}}\text{C}}{64.3725...}$ ***(A1)*(ft)*(M1)***

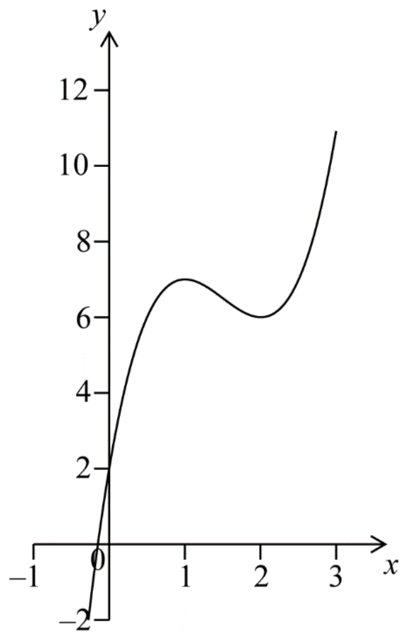
Nota: Award ***(A1)*(ft)** for their correct values seen, ***(M1)*** for correct substitution into the sine formula.

$= 54.3^\circ \quad (54.2781\ldots^\circ)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (a). Accept equivalent methods.

[3 marks]
Total [14 marks]

6. (a)



(A1)(A1)(A1)(A1)

Note:	Award ***(A1)*** for correct window (condone a window which is slightly off) and axes labels. An indication of window is necessary. -1 to 3 on the x-axis and -2 to 12 on the y-axis and a graph in that window. ***(A1)*** for correct shape (curve having cubic shape and must be smooth). ***(A1)*** for both stationary points in the 1st quadrant with approximate correct position, ***(A1)*** for intercepts (negative x-intercept and positive y intercept) with approximate correct position.

[4 marks]

(b) Rick ***(A1)***

Note: Award ***(A0)*** if extra names stated.

[1 mark]

(c) $2(1)^3 - 9(1)^2 + 12(1) + 2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into equation.

$= 7$ ***(A1)(G2)***

[2 marks]

continued…

Question 6 continued

(d) $6x^2 - 18x + 12$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term. Award at most ***(A1)(A1)(A0)*** if extra terms seen.

[3 marks]

(e) $6x^2 - 18x + 12 = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their derivative to 0. If the derivative is not explicitly equated to 0, but a subsequent solving of their correct equation is seen, award ***(M1)***.

$6(x-1)(x-2) = 0$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for correct factorization. The final ***(M1)*** is awarded only if answers are clearly stated.
Award ***(M0)(M0)*** for substitution of 1 and of 2 in their derivative.

$x = 1,\ x = 2$ ***(AG)***

[2 marks]

(f) $6 < k < 7$ ***(A1)(A1)(ft)(A1)***

Note: Award ***(A1)*** for an inequality with 6, award ***(A1)*****(ft)** for an inequality with 7 from their part (c) provided it is greater than 6, ***(A1)*** for their correct strict inequalities.
Accept $]\,6,\ 7[$ or $(6,\ 7)$.

[3 marks]

Total [15 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Estudios matemáticos
Nivel medio
Prueba 2

Jueves 3 de mayo de 2018 (mañana)

1 hora 30 minutos

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Para esta prueba, se necesita una copia sin anotaciones del **cuadernillo de fórmulas de estudios matemáticos NM**.
- Conteste todas las preguntas en el cuadernillo de respuestas provisto.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán darse como valores exactos o con una aproximación de tres cifras significativas.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

2218 – 7410

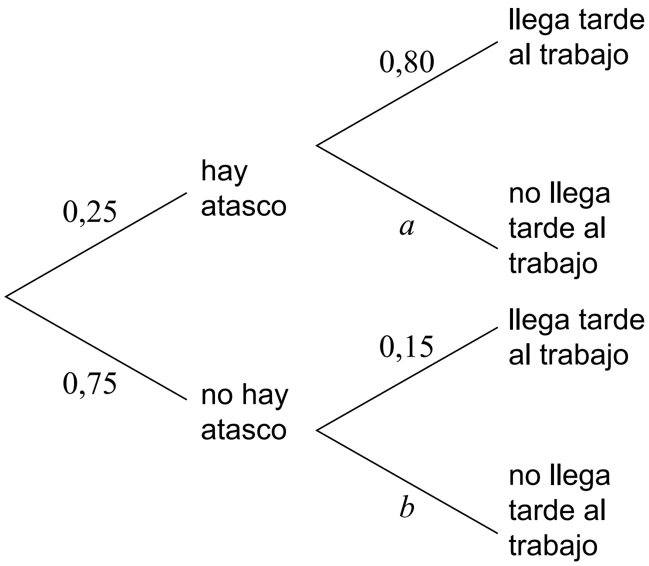
Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta. Se recomienda que muestre todos los cálculos, siempre que sea posible. Cuando la respuesta sea incorrecta se otorgarán algunos puntos siempre que aparezca el método empleado y éste sea correcto. Para los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el proceso seguido hasta su obtención. Por ejemplo, cuando deba utilizar un gráfico de una calculadora de pantalla gráfica para hallar soluciones, deberá dibujar esas gráficas en su respuesta.

1. [Puntuación máxima: 16]

En una empresa se sabe que el 25% de los empleados se encuentran con algún atasco de camino al trabajo. De entre aquellos que se encuentran con algún atasco, la probabilidad de que lleguen tarde al trabajo es del 80%.

De entre aquellos que no se encuentran ningún atasco, la probabilidad de que lleguen tarde al trabajo es del 15%.

El siguiente diagrama de árbol ilustra toda esta información.



(a) Escriba el valor de

(i) a;

(ii) b. [2]

(b) Utilice el diagrama de árbol para hallar la probabilidad de que un empleado

(i) se haya encontrado con un atasco y haya llegado tarde a trabajar;

(ii) haya llegado tarde a trabajar;

(iii) se haya encontrado con un atasco, sabiendo que llegó tarde a trabajar. [8]

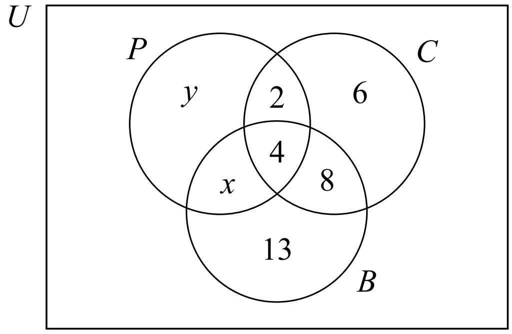
(Esta pregunta continúa en la página siguiente)

(Pregunta 1: continuación)

La empresa decide analizar los diferentes medios de transporte que han utilizado sus trabajadores en el último año para ir a trabajar. Vieron que los tres medios de transporte que más se utilizaron para ir a trabajar fueron el transporte público (P), el coche (C) y la bicicleta (B).

La empresa sabe que, en el último año, hubo 20 trabajadores que fueron en coche a trabajar, 28 que fueron en bicicleta y 19 que utilizaron el transporte público.

Algunos de estos datos se muestran en el siguiente diagrama de Venn.



(c) Halle el valor de

(i) x;

(ii) y. [2]

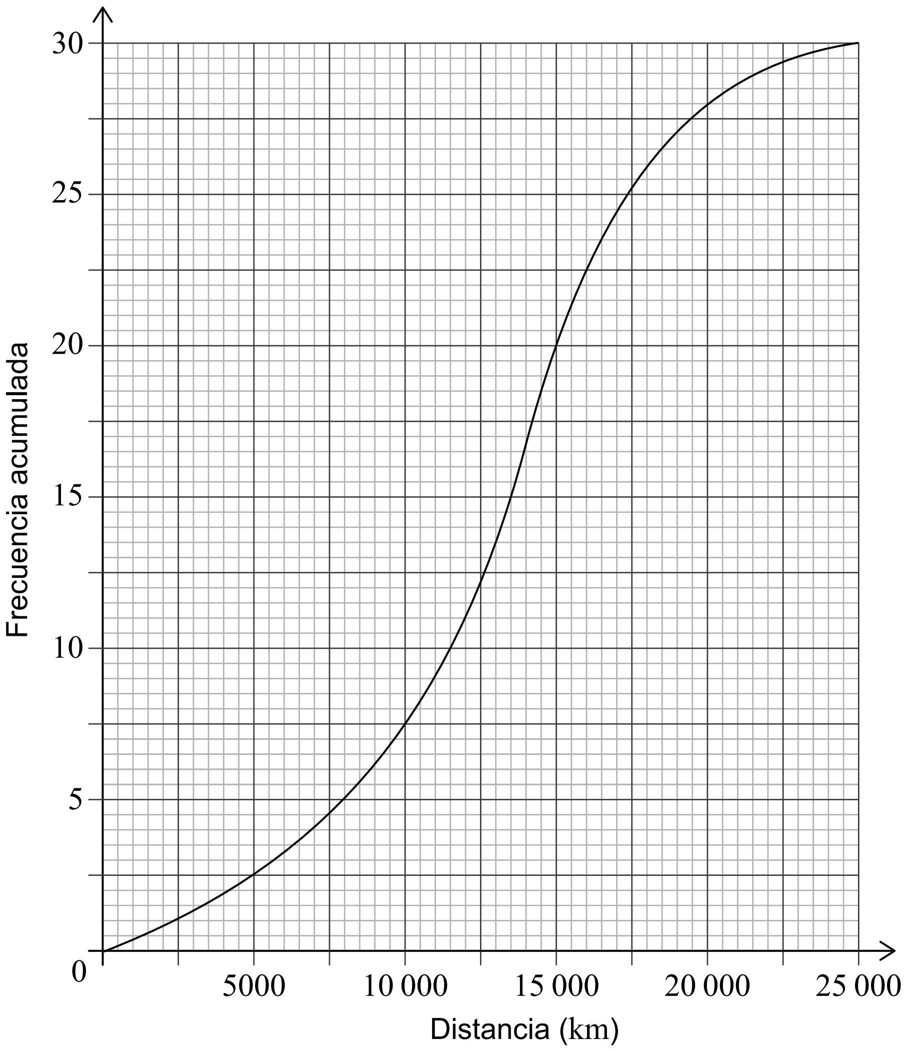
En la empresa hay 54 trabajadores.

(d) Halle cuántos trabajadores, en el último año, no fueron a trabajar ni en coche, ni en bicicleta ni en transporte público. [2]

(e) Halle $n\left((C \cup B) \cap P'\right)$. [2]

2. [Puntuación máxima: 15]

Una empresa de transportes posee 30 autobuses. Se va anotando la distancia que lleva recorrida cada autobús desde que lo compró la empresa. A continuación se muestra la curva de frecuencias acumuladas correspondiente a estos datos.



(a) Halle cuántos autobuses han recorrido una distancia comprendida entre $15\,000$ y $20\,000$ kilómetros. [2]

(b) Utilice la curva de frecuencias acumuladas para hallar

(i) la mediana de las distancias;

(ii) el primer cuartil;

(iii) el tercer cuartil. [4]

(Esta pregunta continúa en la página siguiente)

(Pregunta 2: continuación)

(c) A partir de lo anterior, escriba el rango intercuartil. [1]

(d) Escriba el porcentaje de autobuses que han recorrido una distancia mayor que el tercer cuartil. [1]

(e) Halle cuántos autobuses han recorrido una distancia menor o igual a $12\,000\,\text{km}$. [1]

Se sabe que hay 8 autobuses que han recorrido más de m kilómetros.

(f) Halle el valor de m. [2]

La menor distancia recorrida por uno de estos autobuses fue de $2500\,\text{km}$.
La mayor distancia recorrida por uno de estos autobuses fue de $23\,000\,\text{km}$.

(g) **En un papel milimetrado**, dibuje con precisión un diagrama de caja y bigotes que represente estos datos. Utilice la siguiente escala: cada $2\,\text{cm}$ han de representar $5000\,\text{km}$. [4]

3. [Puntuación máxima: 14]

El peso, W, de los jugadores de baloncesto que participan en un torneo sigue una distribución normal de media $65\,\text{kg}$ y una desviación típica de $5\,\text{kg}$.

(a) (i) Halle la probabilidad de que un jugador de baloncesto participante pese menos de $61\,\text{kg}$.

A una sesión de entrenamiento acuden 40 jugadores de baloncesto.

(ii) Halle el número esperado de jugadores que hay en esta sesión de entrenamiento con un peso inferior a $61\,\text{kg}$. [4]

(b) La probabilidad de que un jugador de baloncesto tenga un peso que esté a menos de $1{,}5$ desviaciones típicas de la media es igual a q.

(i) Dibuje aproximadamente la curva de una distribución normal para representar esta probabilidad.

(ii) Halle el valor de q. [3]

(c) Sabiendo que $\mathrm{P}(W > k) = 0{,}225$, halle el valor de k. [2]

Una entrenadora de baloncesto estuvo observando a 60 de sus jugadores para determinar si su rendimiento y su peso eran variables independientes la una de la otra. Fue anotando sus observaciones, tal y como se muestra en la siguiente tabla.

		Rendimiento	
		Satisfactorio	**Excelente**
Peso	**Por debajo de la media**	6	10
	Promedio	7	15
	Por encima de la media	12	10

Ella decide realizar una prueba de independencia de χ^2 a un nivel de significación del 5%.

(d) Para esta prueba,

(i) indique la hipótesis nula; [1]

(ii) halle el valor del parámetro p. [2]

(e) Indique la conclusión de esta prueba. Justifique su respuesta. [2]

4. [Puntuación máxima: 16]

Ha abierto una cafetería nueva y, durante la primera semana, obtuvo unos beneficios de $60\$$.

Los beneficios de la cafetería aumentan $10\$$ cada semana.

(a) Halle los beneficios que obtendrá la cafetería en su 11^{a} semana. [3]

(b) Calcule los beneficios **totales** que habrá obtenido la cafetería en las primeras 12 semanas. [3]

A la vez que abrió esta cafetería también abrió una casa de té nueva. En la primera semana, también obtuvo unos beneficios de $60\$$.

Los beneficios de la casa de té aumentan un 10% cada semana.

(c) Halle los beneficios que obtendrá la casa de té en su 11^{a} semana. [3]

(d) Calcule los beneficios **totales** que habrá obtenido la casa de té en las primeras 12 semanas. [3]

En la m-ésima semana, los beneficios **totales** de la casa de té superan a los beneficios **totales** de la cafetería por vez primera desde que ambas abrieron.

(e) Halle el valor de m. [4]

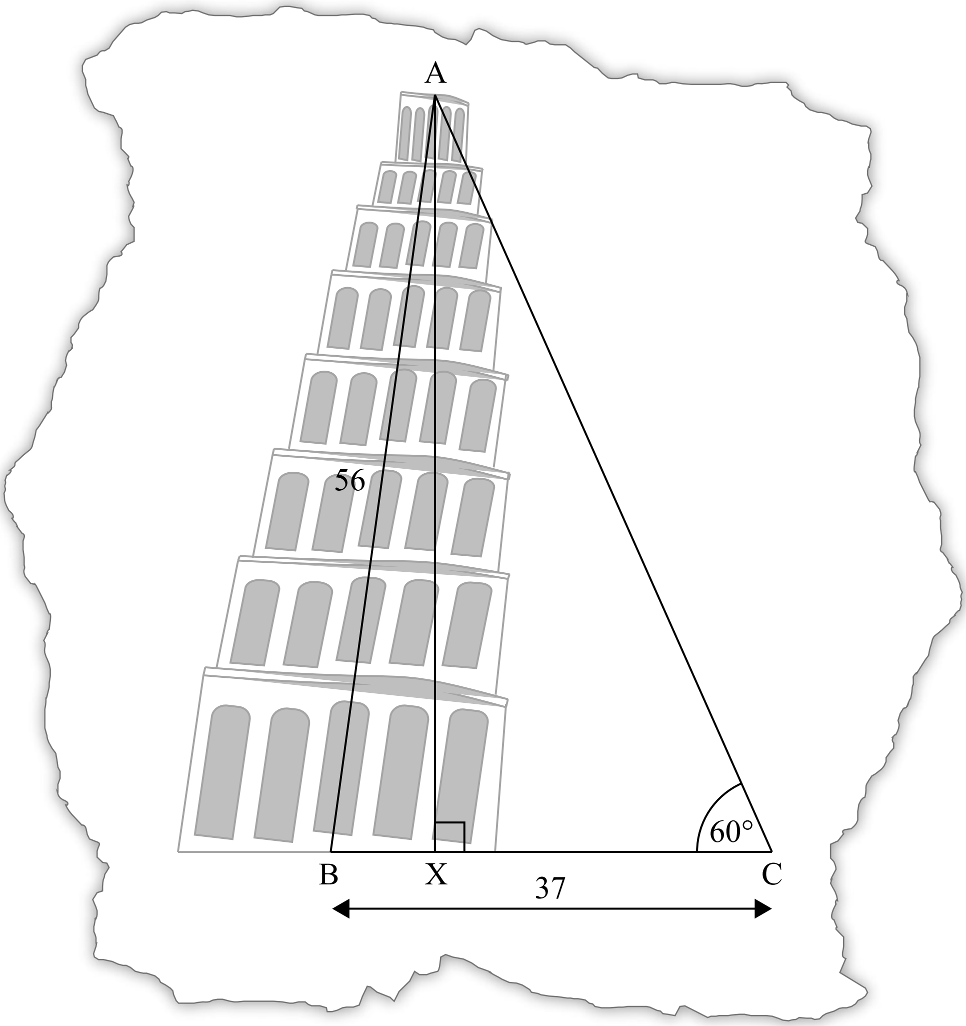
5. [Puntuación máxima: 14]

La torre de Pisa es muy famosa en todo el mundo porque está inclinada.

Giovanni va a ver la torre y quiere averiguar qué inclinación tiene. Para ello, dibuja una figura donde se incluye el triángulo no rectángulo ABC.

En la figura de Giovanni AB mide $56\,\text{m}$, BC mide $37\,\text{m}$, y el ángulo ACB son 60°. AX indica la altura del triángulo y es perpendicular a BC.

la figura no está dibujada a escala



(a) Utilice la figura de Giovanni para

(i) mostrar que el ángulo ABC —que es el ángulo que la torre, debido a su inclinación, forma con la horizontal— es igual a 85°, redondeado al grado más próximo;

(ii) calcule la longitud de AX;

(iii) halle la longitud de BX, que representa el desplazamiento horizontal de la torre. [9]

(Esta pregunta continúa en la página siguiente)

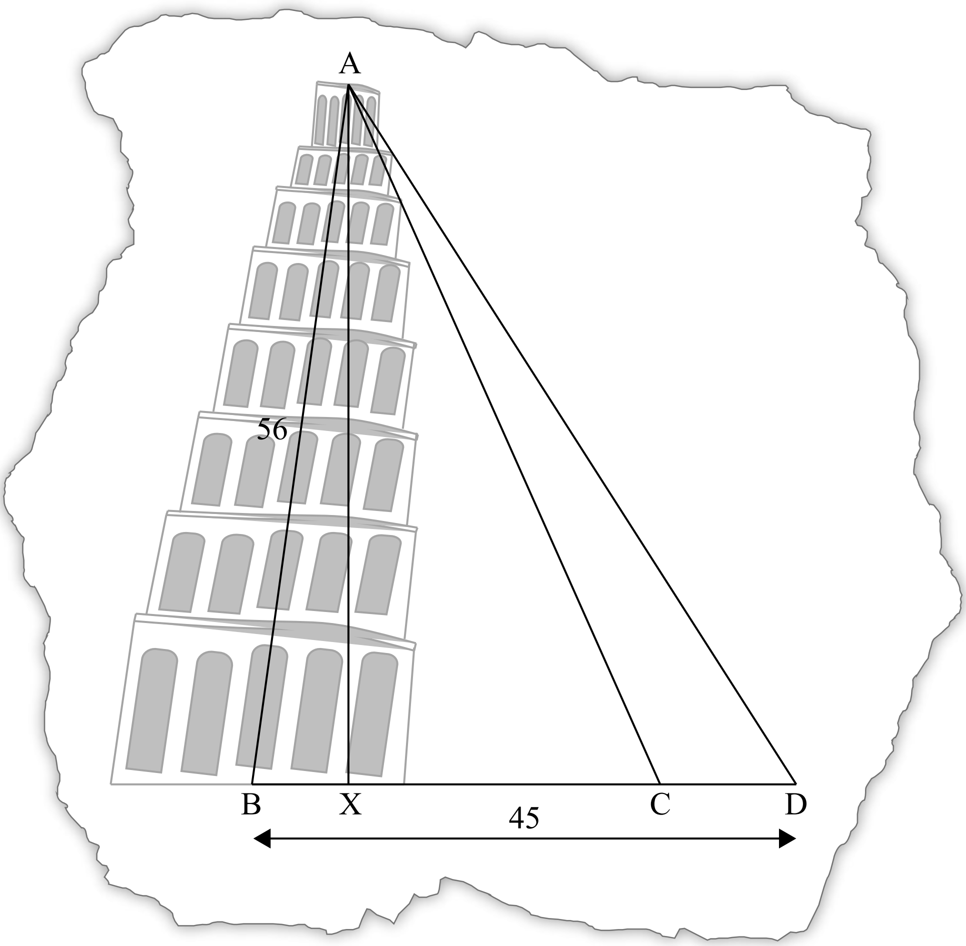
(Pregunta 5: continuación)

La guía turística que tiene Giovanni dice que el desplazamiento horizontal real de la torre, BX, son $3,9$ metros.

(b) Halle el porcentaje de error de la figura de Giovanni. [2]

Giovanni añade un punto D a su figura, de modo tal que $BD = 45\,m$. Con ello se forma otro triángulo.

la figura no está dibujada a escala



(c) Halle el ángulo de elevación de A visto desde D. [3]

6. [Puntuación máxima: 15]

Considere la curva $y = 2x^3 - 9x^2 + 12x + 2$, para $-1 < x < 3$.

(a) Dibuje aproximadamente la curva para $-1 < x < 3$, $-2 < y < 12$. [4]

(b) Un profesor les pide a sus alumnos que realicen algunas observaciones sobre la curva.

Tres alumnos respondieron lo siguiente.
Nadia dijo "*El punto de corte de la curva con el eje x está comprendido entre -1 y cero".*
Rick dijo *"La curva es decreciente para $x < 1$"*.
Paula dijo *"La pendiente de la curva es menor que cero entre $x = 1$ y $x = 2$"*.

Indique el nombre del alumno que ha realizado una observación **incorrecta**. [1]

(c) Halle el valor de y para $x = 1$. [2]

(d) Halle $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(e) Muestre que los puntos estacionarios de la curva están en $x = 1$ y $x = 2$. [2]

(f) Sabiendo que $2x^3 - 9x^2 + 12x + 2 = k$ tiene **tres** soluciones, halle los posibles valores de k. [3]

Diploma Programme
Programme du diplôme
Programa del Diploma

Esquema de calificación

Mayo de 2018

Estudios matemáticos

Nivel medio

Prueba 2

Esquema de calificación de la Prueba 2
Instrucciones para los examinadores

Observaciones: **En caso de dudas sobre estas instrucciones o temas relacionados con la asignación de puntuaciones, póngase en contacto con su jefe de equipo.**

1 Siglas

M Puntos otorgados por el **método**
A Puntos otorgados por una **respuesta** o por la **precisión**
R Puntos otorgados por un **razonamiento** claro
G Puntos otorgados por soluciones correctas obtenidas mediante la **calculadora de pantalla gráfica**, independientemente del trabajo mostrado.
AG **Respuesta incluida** en la pregunta y, en consecuencia; no se otorgan puntos.
ft Se pueden otorgar puntos por la **coherencia** con resultados previos en la pregunta.

2 Método para corregir

(a) Todas las correcciones deben realizarse en RM Assessor, utilizando las anotaciones de Estudios Matemáticos y de acuerdo con el documento vigente de orientaciones para la corrección electrónica de Estudios Matemáticos NM. Es fundamental leer el documento antes de empezar a corregir.

(b) Si un apartado de una pregunta es del todo correcto use las anotaciones tic con números para otorgar la puntuación máxima. Si un apartado es completamente erróneo use la nota ***A0***, de lo contrario se deben mostrar todas las anotaciones.

(c) No se debe otorgar ningún punto al trabajo tachado por el alumno.

(d) Si el alumno ha dado dos respuestas a una pregunta, solo se debe corregir la primera respuesta.

(e) Si el procedimiento adecuado lleva a la respuesta correcta, pero esta va seguida de más desarrollos que indican una falta de comprensión matemática, entonces **no** se puede otorgar la puntuación máxima. En la mayoría de estos casos se perderá solo el último punto por respuesta. Una excepción podría ser las respuestas numéricas, donde una respuesta exacta que es correcta está seguida de una expresión decimal incorrecta.

Ejemplo:

	Se ve la respuesta correcta	**Hay más desarrollo**	**Acción**
1.	$8\sqrt{2}$	$5,65685...$ *(valor decimal incorrecto)*	Otorgue el ultimo ***(A1)*** *(ignore el desarrollo posterior)*
2.	$(x-6)(x+1)$	$x=6$ and -1	**No** otorgue el último ***(A1)***

Ejemplo: Calcule la pendiente de la recta que pasa por los puntos $(5;3)$ y $(0;9)$.

Esquema de calificación	**Examen del alumno**	**Corrección**
$\frac{9-3}{0-5}$ ***(M1)*** *Otorgue (M1) por la sustitución correcta en la fórmula de la pendiente* $=-\frac{6}{5}$ ***(A1)***	(i) $\frac{9-3}{0-5}=-\frac{6}{5}$ La pendiente es $=-\frac{6}{5}$ *(Existe una comprensión clara de la pendiente.)* $y=-\frac{6}{5}x+9$ (ii) $\frac{9-3}{0-5}=-\frac{6}{5}$ $y=-\frac{6}{5}x+9$ *(Existe confusión sobre lo requerido.)*	***(M1)*** ***(A1)*** ***(M1)*** ***(A0)***

3 Puntos por la coherencia (ft)

En cualquier paso de una resolución, un error puede afectar a todo el trabajo posterior. Para limitar la gravedad de la penalización, se pueden otorgar puntos por la **coherencia (ft)**. Los esquemas de calificación indicarán, con **"(ft)"**, dónde es adecuado aplicar el criterio de coherencia en una pregunta.

(a) La coherencia se aplica solo de un apartado de una pregunta a otro apartado posterior de la pregunta. No se aplica dentro del mismo apartado.

(b) Si una respuesta que es consecuencia de un procedimiento de coherencia es muy poco realista (por ejemplo, distancias negativas o de un orden de magnitud excesivo) entonces el punto ***A*** final no se debe otorgar.

(c) Si, debido a un error, una pregunta queda transformada en otra **distinta**, **mucho más sencilla**, entonces el criterio de coherencia podría no ser aplicado.

(d) Para otorgar puntos por coherencia en un apartado de una pregunta, **es necesario que haya un trabajo explícito relacionado con ese apartado**. Una respuesta aislada, obtenida por coherencia, pero sin un desarrollo explícito, se considera incorrecta, y no recibirá ningún punto **aunque se aproxime a la respuesta correcta**.

(e) Una excepción a lo anterior sería una pregunta donde lo que se evalúa es el manejo del alumno en el uso de la calculadora de pantalla gráfica, y donde no se pretende que se muestre ningún desarrollo escrito. **En el esquema de calificación vendrá indicado claramente cuando sea el caso.**

(f) El uso inadvertido de radianes será penalizado la primera vez que aparezca. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo un punto por el uso de radianes.

Ejemplo: Cálculo de ángulos y longitudes en trigonometría

Esquema de calificación	Examen del alumno	Corrección
(a) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ ***(M1)(A1)*** *Otorgue **(M1)** por la sustitución en el teorema del seno, **(A1)** por las sustituciones correctas.*	(a) $\frac{\text{sen } A}{4} = \frac{\text{sen } 30}{3}$	***(M1)(A0)*** *(uso del teorema del seno, pero con valores incorrectos)*
$A = 22{,}0^\circ$ $(22{,}0243\ldots)$ ***(A1)(G2)***	$A = 41{,}8^\circ$	***(A0)*** *(**Observación**: Aquí, el segundo **(A1)** no ha sido corregido como* **(ft)** *y no se puede otorgar porque existe un error previo en el **mismo** apartado de la pregunta.)*
(b) $x = 7\text{tg}\,(22{,}0243\ldots^\circ)$ ***(M1)*** $= 2{,}83$ $(2{,}83163\ldots)$ ***(A1)*(ft)**	(b) caso (i) $x = 7\text{tg}\,41{,}8^\circ$ $= 6{,}26$ ***pero*** caso (ii) $6{,}26$	***(M1)*** ***(A1)*(ft)** ***(G0)*** *pues no aparece un desarrollo explícito*

4 Uso del Esquema de calificación

(a) Los puntos ***A* dependen** de los puntos ***M*** obtenidos previamente, **no** es posible otorgar ***(M0)(A1)***. Una vez que se otorga un ***(M0)***, se pierden todos los puntos ***A*** que le siguen en ese apartado de la pregunta, incluso si los cálculos son correctos, hasta el siguiente punto ***M***.

La única excepción a lo anterior se hará en el caso de una respuesta donde la precisión se especifique en la pregunta (ver apartado 5).

(b) Los puntos ***A* dependen** de los puntos ***R*** obtenidos, **no** es posible otorgar ***(A1)(R0)***. Así pues, el ***(A1)*** no se puede otorgar a una respuesta que sea correcta cuando no aparece el razonamiento, o este es incorrecto.

(c) En la prueba 2 se espera que los alumnos demuestren su destreza en la comunicación matemática mediante el uso de desarrollos adecuados. Las respuestas que sean correctas, pero no se basen en un desarrollo adecuado **no siempre recibirán la puntuación máxima**. Estas respuestas sin desarrollo que las sustente vienen designadas por ***G*** en el esquema de calificación, como una alternativa a la puntuación máxima. Ejemplo ***(M1)(A1)(A1)(G2).***

Ejemplo: Uso de la trigonometría para el cálculo de un ángulo de un triángulo.

Esquema de calificación	Examen del alumno	Corrección
(a) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ ***(M1)(A1)*** *Otorgue **(M1)** por la sustitución en el teorema del seno, **(A1)** por las sustituciones correctas.* $A = 22{,}0^\circ$ $(22{,}0243\ldots)$ ***(A1)(G2)***	(i) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ $A = 22{,}0^\circ$ (ii) $A = 22{,}0^\circ$ ***Observación:** Los puntos **G** se utilizan solo si no se muestra ningún desarrollo, pero la respuesta es correcta.*	***(M1)(A1)*** ***(A1)*** ***(G2)***

(d) Los **métodos alternativos** no siempre se incluyen. Así, si una respuesta es errónea, entonces se debe analizar cuidadosamente el procedimiento, de modo que se asignen puntos por un método distinto de forma coherente con el esquema de calificación.
Cuando en el esquema de calificación se incluyen métodos alternativos para una pregunta, ello viene indicado mediante "**O**" *etc*.

(e) A menos que en la pregunta se especifique lo contrario, se aceptan **expresiones equivalentes**.
Por ejemplo: $\frac{\text{sen } \theta}{\cos \theta}$ por $\text{tg } \theta$.

En el esquema de calificación, estas expresiones algebraica o numéricamente equivalentes aparecerán a veces escritas entre paréntesis junto a la respuesta requerida.
Cuando se requieran respuestas numéricas como respuesta final de un apartado de una pregunta, en el esquema de calificación se mostrará en este orden:
la respuesta con 3 cifras significativas a partir de la solución completa de la calculadora;
el valor exacto (por ejemplo $\frac{2}{3}$ si corresponde);
la solución completa de la calculadora en la forma $2{,}83163\ldots$ como en el ejemplo anterior.
Cuando se dan las respuestas con 3 cifras significativas y se utilizan después en apartados posteriores de la pregunta llevando a una solución con 3 cifras significativas **distinta**, también se darán estas soluciones.

(f) Dado que este es un examen internacional, se aceptarán todas las **formas de notación alternativas**. Algunos ejemplos:

Comas decimales: 1,7; 1'7; $1 \cdot 7$; 1,7.

Los números decimales menores que 1 podrían aparecer escritos sin un cero en el frente: 0.49 o .49.

Distintas descripciones de un intervalo: $3 < x < 5$; $(3; 5)$; $]\,3; 5\,[$.

Distintas formas de notación de las propiedades de los conjuntos (por ejemplo complementario): A'; $\overline{A}$; A^c; $U - A$; $(A$; $U \setminus A$.

Distintas formas de notación lógica: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p$.
$p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

El nivel de significación podría escribirse como α .

(g) Puntos discrecionales: habrá ocasiones excepcionales en las que el esquema de calificación no cubra el trabajo que aparece. En estos casos se utilizará la nota DM para indicar que el examinador ha utilizado su criterio. La discrecionalidad debe utilizarse con moderación, y si existe duda se debe plantear una excepción a través de RM Assessor al jefe del equipo.

A partir de noviembre de 2011 no habrá una única penalización por prueba por precisión AP, precisión financiera FP y unidades UP. En lugar de ello, estas destrezas serán evaluadas en preguntas específicas y los puntos se otorgarán de acuerdo a lo especificado en los apartados 5, 6 y 7.

5 Precisión de las respuestas

Instrucciones: Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas correctamente a 3 cifras significativas.

1. Si el alumno muestra la solución a 4 o más cifras significativas, la cual redondeada a 3 cifras significativas daría la respuesta requerida entonces otorgue ***(A1)*** e ignore cualquier redondeo posterior.
Observación: La solución sin redondear puede estar tanto en la línea de respuesta como en el cuadro de trabajo.

2. Si el alumno **no** muestra la solución sin redondear, entonces otorgue ***(A1)*** si la solución dada está aproximada **correctamente** a 2 o más cifras significativas, y ***(A0)*** en cualquier otro caso.

3. Si una solución aproximada a 2 cifras significativas se utiliza en apartados posteriores, entonces para otorgar más puntos debe aparecer el desarrollo. (Este criterio es el mismo que en el que se aplica coherencia a partir de una respuesta errónea.)

Estos 3 casos (vea los supra índices) se han resumido en la siguiente tabla e ilustrado después con ejemplos.

<table>
<tr><td></td><td colspan="5">Si la solución final de alumno se expresa…</td></tr>
<tr><td></td><td>Exacta o a 4 o más cifras significativas (y que redondeada a 3 cifras significativas daría la respuesta correcta)</td><td>Aproximada incorrectamente a 3 cifras significativas</td><td>Aproximada correctamente a 2 cifras significativas[3]</td><td>Aproximada incorrectamente a 2 cifras significativas</td><td>Aproximada correcta o incorrectamente a 1 cifra significativa</td></tr>
<tr><td>Aparece la solución sin redondear[1]</td><td colspan="5">Otorgue (A1) independientemente del redondeo correcto o incorrecto</td></tr>
<tr><td>No aparece la solución sin redondear[2]</td><td>(A1)</td><td>(A0)</td><td>(A1)</td><td>(A0)</td><td>(A0)</td></tr>
<tr><td>Criterio para los apartados posteriores</td><td>Como en MS</td><td colspan="4">Como en el criterio de coherencia, solo si se muestra el desarrollo[3]</td></tr>
</table>

Ejemplos:

Esquema de calificación	Examen del alumno	Corrección
$9,43\ (9,43398\ldots)$ ***(A1)***	(i) $9,43398\ldots$ se muestra en el cuadro de trabajo seguido de 9; 9,4; 9,43; 9,434 (redondeados correctamente)	***(A1)***
	(ii) $9,43398\ldots$ se muestra en el cuadro de trabajo seguido de 9,433; 9,44, *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 9,4	***(A1)***
	(iv) 9	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 9,3	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 9,44	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Esquema de calificación	Examen del alumno	Corrección
$7,44\ (7,43798\ldots)$ ***(A1)***	(i) $7,43798\ldots$ se muestra en el cuadro de trabajo seguido de 7; 7,4; 7,44; 7,438 *etc.* (redondeados correctamente)	***(A1)***
	(ii) $7,43798\ldots$ se muestra seguido de 7,437; 7,43 *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 7,4	***(A1)***
	(iv) 7	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 7,5	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 7,43	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Ejemplo: ABC es un triángulo rectángulo con el ángulo $\text{ABC} = 90^\circ$, $\text{AC} = 32\text{ cm}$ y $\text{AB} = 30\text{ cm}$. Halle (a) la longitud de BC, (b) el área del triángulo ABC.

Esquema de calificación	Examen del alumno	Corrección
(a) $\text{BC} = \sqrt{32^2 - 30^2}$ ***(M1)*** *Otorgue **(M1)** por la sustitución correcta en el teorema de Pitágoras* $= 11{,}1\left(\sqrt{124};\ 11{,}1355...\right)(\text{cm})$ ***(A1)***	(a) $\text{BC} = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$ *(solo se muestra la aproximación a 2 cifras significativas, pero correcta)*	***(M1)*** ***(A1)***
(b) $\text{Área} = \frac{1}{2} \times 30 \times 11{,}1355...$ ***(M1)*** *Otorgue **(M1)** por la sustitución correcta en la fórmula del área de un triángulo* $= 167(167{,}032...)(\text{cm}^2)$ ***(A1)*(ft)**	(b) caso (i) $\text{Área} = \frac{1}{2} \times 30 \times 11$ *(se muestra el desarrollo)* $= 165\,(\text{cm}^2)$ caso (ii) $= 165\,(\text{cm}^2)$ *(No se muestra el desarrollo, la solución 11 se trata como procedimiento de coherencia, por lo que no se deben otorgar puntos en este caso)*	***(M1)*** ***(A1)*(ft)** ***(M0)(A0)*(ft)**

La aproximación a 3 cifras significativas de una solución exacta **se debe aceptar si se realiza correctamente**.

Las soluciones exactas del tipo $\frac{1}{4}$ se pueden escribir como decimales con menos de 3 cifras significativas si el resultado sigue siendo exacto. La simplificación de una fracción a su expresión irreducible **no** es imprescindible. Las fracciones que contienen un numerador y/o un denominador decimal se aceptan para mostrar sustituciones pero no como respuesta final.

Razones de π y respuestas con expresiones de raíces cuadradas de enteros o cualquier potencia racional de un entero (por ejemplo, $\sqrt{13}, 2^{2/3}, \sqrt[4]{5}$), se pueden aceptar como respuestas exactas. Todas las otras potencias (por ejemplo de no enteros) y valores de funciones transcendentes tales como seno y coseno se deben evaluar.

Si el nivel de precisión viene especificado en la pregunta, se asignarán puntos por dar la respuesta con la precisión requerida. En **todos** estos casos no se obtiene el punto final si el redondeo no sigue las instrucciones dadas en la pregunta. El punto por la precisión especificada se puede considerar como punto por coherencia **(ft)** con independencia de un ***(M0)*** inmediatamente anterior.

6 Nivel de precisión en las preguntas sobre cuestiones financieras

El nivel de precisión de las respuestas se especificará en todas las preguntas que se relacionen con valores en dinero. Generalmente será a la unidad más cercana o a dos cifras decimales. La primera respuesta que no venga dada con el nivel de precisión especificado no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la precisión incorrecta en una pregunta sobre cuestiones financieras.

Ejemplo: En una pregunta sobre cuestiones financieras se pide una precisión de 2 cifras decimales.

Esquema de calificación	Examen del alumno	Corrección
$\$231,62\,(231,6189)$ ***(A1)***	(i) $231,6$	***(A0)***
	(ii) 232	***(A0)***
	(Aproximación correcta con un nivel de precisión distinto del requerido)	
	(iii) $231,61$	***(A0)***
	(iv) $232,00$	***(A0)***
	(Aproximación incorrecta al nivel de precisión requerido)	

7 Unidades de medida en las respuestas

En determinadas preguntas será necesario expresar las unidades de medida, y esto vendrá claramente indicado en el esquema de calificación. La primera respuesta correcta pero sin unidades o con unidades incorrectas no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la falta de unidades o unidades incorrectas.

Las unidades de medida se tendrán en cuenta solamente cuando se ha otorgado ***(A1)*** a la respuesta numérica siguiendo las indicaciones sobre la precisión dadas en el apartado 5.

Ejemplo:

Esquema de calificación	Examen del alumno	Corrección
(a) $37000\ \mathrm{m}^2$ ***(A1)***	(a) $36000\ \mathrm{m}^2$	***(A0)***
	(respuesta incorrecta, por lo que no se consideran las unidades)	
(b) $3200\ \mathrm{m}^3$ ***(A1)***	(b) $3200\ \mathrm{m}^2$	***(A0)***
	(unidades incorrectas)	

8 Calculadoras de pantalla gráfica

Con frecuencia los alumnos van a obtener las soluciones directamente de la calculadora. Deben utilizar la notación matemática y no la notación de la calculadora. No se puede otorgar puntos por método a respuestas incorrectas basadas únicamente en notación de calculadora. No se puede otorgar puntos por método al comentario “usé la calculadora de pantalla gráfica”.

1. (a) $a = 0,2\,; b = 0,85$ ***(A1)(A1)***

Nota: Conceda ***(A1)*** por cada valor correcto.

[2 puntos]

(b) (i) $0,25 \times 0,8$ ***(M1)***

Nota: Conceda ***(M1)*** por un producto correcto.

$= 0,2 \quad \left(\frac{1}{5},\ 20\,\%\right)$ ***(A1)(G2)***

(ii) $0,25 \times 0,8 + 0,75 \times 0,15$ ***(A1)*(ft)*(M1)***

Nota: Conceda ***(A1)*(ft)** por su $(0,25 \times 0,8)$ y $(0,75 \times 0,15)$, ***(M1)*** por sumar dos prodcutos.

$= 0,313 \left(0,3125;\ \frac{5}{16};\ 31,3\,\%\right)$ ***(A1)*(ft)*(G3)***

Nota: Conceda el ***(A1)*(ft)** final solo si la respuesta no excede 1. Arrastre de error (FT) desde el apartado (b)(i).

(iii) $\dfrac{0,25 \times 0,8}{0,25 \times 0,8 + 0,75 \times 0,15}$ ***(A1)*(ft)*(A1)*(ft)**

Nota: Conceda ***(A1)*(ft)** por un numerador correcto (el dado como respuesta en el apartado (b)(i)); ***(A1)*(ft)** por un denominador correcto (el dado como respuesta en el apartado (b)(ii)). Arrastre de error (FT) desde los apartados (b)(i) y (b)(ii).

$= 0,64 \left(\frac{16}{25};\ 64\,\%\right)$ ***(A1)*(ft)*(G3)***

Nota: Conceda el ***(A1)*(ft)** final solo si la respuesta no excede 1.

[8 puntos]

(c) (i) $(x =)\ 3$ ***(A1)***

(ii) $(y =)\ 10$ ***(A1)*(ft)**

Nota: Arrastre de error (FT) desde el apartado (c)(i), pero solo si el valor de x dado por el alumno es menor o igual que 13.

[2 puntos]

continúa en la pág. siguiente...

Continuación de la Pregunta 1

(d) $54-(10+3+4+2+6+8+13)$ ***(M1)***

Nota:	Conceda ***(M1)*** por restar de 54 el resultado de su suma (que ha de ser correcta). Arrastre de error (FT) desde el apartado (c).

$=8$ ***(A1)*(ft)*(G2)***

Nota:	Conceda ***(A1)*(ft)** solo si su suma no excede 54. Arrastre de error (FT) desde la respuesta que dio el alumno en el apartado (c).

[2 puntos]

(e) $6+8+13$ ***(M1)***

Nota:	Conceda ***(M1)*** por sumar 6, 8 y 13.

27 ***(A1)(G2)***

[2 puntos]

Total [16 puntos]

2. (a) $28-20$ ***(A1)***

Nota: Conceda ***(A1)*** por haber escrito 28 y 20.

8 ***(A1)(G2)***

[2 puntos]

(b) (i) $13\,500$ ***(G2)***

Nota: Acepte una respuesta en el rango 13500 a 13750.

(ii) $10\,000$ ***(G1)***

Nota: Acepte una respuesta en el rango 10000 a 10250.

(iii) $16\,000$ ***(G1)***

Nota: Acepte una respuesta en el rango 16000 a 16250.

[4 puntos]

(c) 6000 ***(A1)*(ft)**

Nota: Arrastre de error (FT) desde la respuestas dadas en los apartados (b)(ii) y (iii).

[1 punto]

(d) $25\ \%$ ***(A1)***

[1 punto]

(e) 11 ***(G1)***

[1 punto]

(f) $30-8$ o 22 ***(M1)***

Nota: Conceda ***(M1)*** por la resta $30-8$ o si ha escrito 22.

$15\,750$ ***(A1)(G2)***

Nota: Acepte $15\,750 \pm 250$.

[2 puntos]

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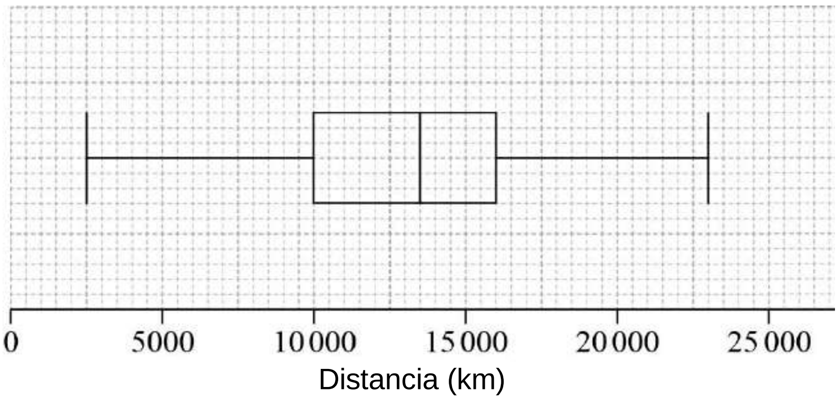
Continuación de la Pregunta 2

(g)



(A1)(A1)(A1)(A1)

Nota: Conceda ***(A1)*** por unos rótulos y una escala correctos. Acepte distancia o km como rótulo.
(A1)**(ft)** por una mediana correcta,
(A1)**(ft)** por unos cuartiles y una caja correctos,
(A1) por situar los extremos en 2500 y $23\,000$ y unirlos a la caja mediante líneas rectas.
Acepte ± 250 para la mediana, los cuartiles y los extremos.
Arrastre de error (FT) desde la respuesta dada en el apartado (b).

El ***(A1)*** final no se puede conceder si la recta atraviesa la caja.

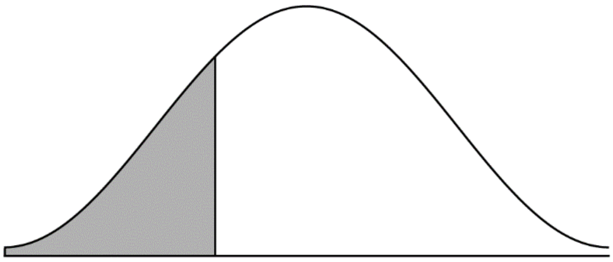
[4 puntos]

Total [15 puntos]

3. (a) (i) $\mathrm{P}(W < 61)$ ***(M1)***

Nota: Conceda ***(M1)*** por un enunciado correcto referido a dicha probabilidad.

O BIEN



(M1)

Nota: Conceda ***(M1)*** si ha rotulado y sombreado en la figura la región correcta.

$= 0{,}212\ (0{,}21185\ldots;\ 21{,}2\,\%)$ ***(A1)(G2)***

(ii) $40 \times 0{,}21185\ldots$ ***(M1)***

Nota: Conceda ***(M1)*** por el producto de 40 y su $0{,}212$.

$= 8{,}47\ (8{,}47421\ldots)$ ***(A1)*(ft)*(G2)***

Nota: Arrastre de error (FT) desde la respuesta dada en el apartado (a)(i), siempre y cuando su respuesta al apartado (a)(i) sea menor que 1.

[4 puntos]

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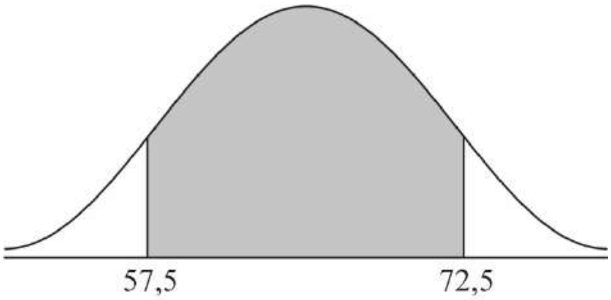
Continuación de la Pregunta 3

(b) (i)



(A1)(M1)

Nota: Conceda ***(A1)*** por dos rectas verticales rotuladas correctamente y que estén en posiciones aproximadamente correctas. Los valores $57{,}5$ y $72{,}5$, o $\mu - 1{,}5\sigma$ y $\mu + 1{,}5\sigma$ son rótulos aceptables.
Conceda ***(M1)*** por la region sombreada correcta delimitada por sus dos rectas verticales.

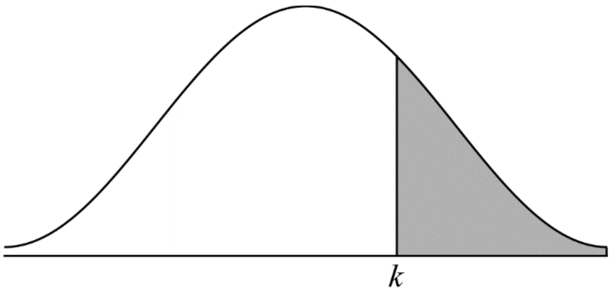
(ii) $0{,}866 \ (0{,}86638\ldots; 86{,}6\%)$ ***(A1)*(ft)**

Nota: Arrastre de error (FT) desde su región sombreada de su apartado (b)(i) solo si sus valores son claros.

[3 puntos]

(c) $\mathrm{P}(W < k) = 0{,}775$ ***(M1)***

O BIEN



(M1)

Nota: Conceda ***(A1)*** si ha rotulado y sombreado en la figura la región correcta.

$(k =) \ \ 68{,}8 \ \ (68{,}7770\ldots)$ ***(A1)(G2)***

[2 puntos]

continúa en la pág. siguiente...

Continuación de la Pregunta 3

(d) (i) (H_0:) *El rendimiento (de los jugadores) y (su) peso son independientes.* ***(A1)***

> **Nota:** Acepte "No hay asociación entre el rendimiento (de los jugadores) y (su) peso". No acepte "no se relacionan" o "no se correlacionan" or "no se influencian".

[1 punto]

(ii) 0,287 (0,287436…) ***(G2)***

[2 puntos]

(e) aceptar / no rechazar la hipótesis nula/H_0 ***(A1)*****(ft)**

O BIEN

el rendimiento (de los jugadores) y (su) peso son independientes. ***(A1)*****(ft)**

$0,287 > 0,05$ ***(R1)*****(ft)**

> **Nota:** Acepte (parámetro) p > nivel de significación solo si su valor de p está escrito en (d)(ii). Acepte 28,7% >5 %. No conceda ***(A1)(R0)***. Arrastre de error (FT) desde el apartado (d).

[2 puntos]

Total [14 puntos]

4. (a) $60+10\times 10$ ***(M1)(A1)***

Nota: Conceda ***(M1)*** por sustituir en la fórmula de la progresión aritmética, ***(A1)*** por una sustitución correcta.

$=(\$)\ 160$ ***(A1)(G3)***

[3 puntos]

(b) $\frac{12}{2}(2\times 60+11\times 10)$ ***(M1)(A1)*****(ft)**

Nota: Conceda ***(M1)*** por sustituir los datos en la fórmula de la suma de los términos de una progresión aritmética, ***(A1)*****(ft)** por una sustitución correcta. Arrastre de error (FT) desde el primer término y diferencia común usados en el apartado (a).

$=(\$)\ 1380$ ***(A1)*****(ft)*****(G2)***

[3 puntos]

(c) $60\times(1{,}1)^{10}$ ***(M1)(A1)***

Nota: Conceda ***(M1)*** por sustituir los datos en la fórmula del término enésimo de una progresión geométrica, ***(A1)*** por una sustitución correcta.

$=(\$)\ 156\quad (155{,}624\ldots)$ ***(A1)(G3)***

Nota: Acepte la respuesta si redondeada da la respuesta correcta a 3 cifras significativas tal y como lo especifica las instrucciones de precisión.

[3 puntos]

continúa en la pág. siguiente...

Continuación de la Pregunta 4

(d) $\dfrac{60(1{,}1^{12}-1)}{1{,}1-1}$ ***(M1)(A1)*****(ft)**

Nota:	Conceda ***(M1)*** por sustituir en la fórmula de la suma de los términos de una progresión geométrica, ***(A1)*****(ft)** por una sustitución correcta. Arrastre de error (FT) desde el apartado (c) desde el primer término y la razón común que dio el alumno.

$= (\$)\ 1280 \quad (1283{,}05\ldots)$ ***(A1)*****(ft)*****(G2)***

[3 puntos]

(e) $\dfrac{60(1{,}1^{n}-1)}{1{,}1-1} > \dfrac{n}{2}\left(2\times 60+(n-1)\times 10\right)$ ***(M1)(M1)***

Nota:	Conceda ***(M1)*** por haber sustituido correctamente los datos en las fórmulas de la suma de n términos de una progresión geométrica y aritmética (accepte el uso de una variable que no sea "n"), ***(M1)*** por haber comparado sus expresiones, conforme a las respuestas halladas en el apartado (b) y en el apartado (d).

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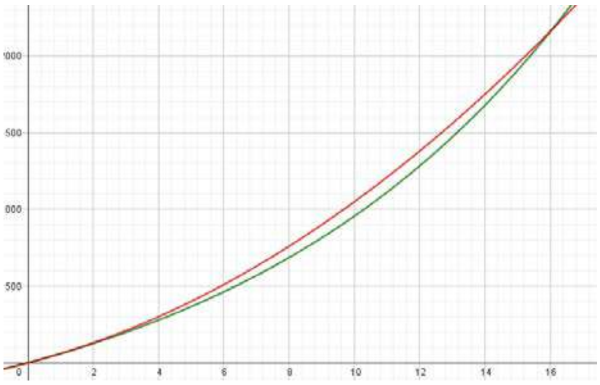
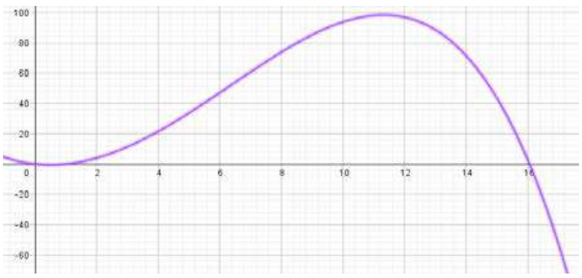
Continuación de la Pregunta 4(e)

O BIEN

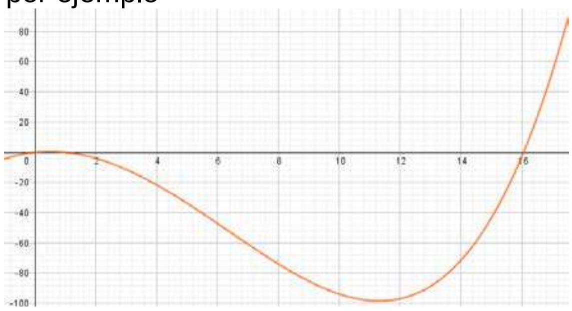


(M1)(M1)

Nota:	Conceda ***(M1)*** por dos curvas con formas aproximadamente correctas y que estén dibujadas en el primer cuadrante, ***(M1)*** por un punto de intersección en una posición aproximadamente correcta. Acepte dibujos aproximados alternativos como por ejemplo



O BIEN



Conceda ***(M1)*** por una curva con una forma aproximadamente correcta y que esté dibujada en el primer (o cuarto) cuadrante y por encima (o debajo) del eje x, ***(M1)*** por un punto de intersección con el eje x en una posición aproximadamente correcta.

17 ***(A2)*****(ft)*****(G3)***

Nota:	Arrastre de error (FT) desde los apartados (c) y (d). La respuesta 16 es incorrecta. Conceda a lo sumo ***(M1)(M1)(A0)(A0)*** si está acompañada por desarrollo. Conceda ***(G0)*** si la respuesta final es 16 y no está acompañada por desarrollo.

[4 puntos]

Total [16 puntos]

5. (a) (i) $\frac{\operatorname{sen} \mathrm{BAC}}{37} = \frac{\operatorname{sen} 60}{56}$ ***(M1)(A1)***

> **Nota:** Conceda ***(M1)*** por sustituir en la fórmula del teorema del seno, ***(A1)*** por una sustitución correcta.

ángulo $\mathrm{B\hat{A}C} = 34{,}9034\ldots^{\circ}$ ***(A1)***

> **Nota:** Conceda ***(A0)*** si su respuesta redondeada a 2 cifras significativas no es 35.
> Conceda ***(G2)*** si ha escrito 34,9034… sin acompañarlo del desarrollo del ejercicio.

ángulo $\mathrm{A\hat{B}C} = 180 - (34{,}9034\ldots + 60)$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por restar "ángulo $\mathrm{B\hat{A}C} + 60$" de 180.

$85{,}0965\ldots^{\circ}$ ***(A1)***

85° ***(AG)***

> **Nota:** El alumno, para que se le pueda conceder el ***(A1)*** final, tiene que haber escrito tanto el valor sin redondear como el valor redondeado. Si el alumno redondea $34{,}9034\ldots^{\circ}$ a 35° mientras está sustituyendo para hallar $\mathrm{A\hat{B}C}$, el ***(A1)*** final se puede conceder pero **solamente** si ha escrito ambos valores: $34{,}9034\ldots^{\circ}$ y 35°.
> Si han utilizado 85 como parte de su desarrollo, conceda a lo sumo (***M1)(A0)(A0)(M0)(A0)(AG)***.

continúa en la pág. siguiente...

Continuación de la Pregunta 5(a)

(ii) $\text{sen}\,85\ldots \times 56$ ***(M1)***

$= 55{,}8 \quad (55{,}7869\ldots) \quad (\text{m})$ ***(A1)(G2)***

Nota:	Conceda ***(M1)*** por una sustitución correcta en la razón trigonométrica.

(iii) $\sqrt{56^2 - 55{,}7869\ldots^2}$ ***(M1)***

Nota:	Conceda ***(M1)*** por una sustitución correcta en la fórmula del teorema de Pitágoras. Arrastre de error (FT) desde el apartado (a)(ii).

O BIEN

$\cos(85) \times 56$ ***(M1)***

Nota:	Conceda ***(M1)*** por sustitución correcta en razón trigonométrica.

$= 4{,}88 \ (4{,}88072\ldots) \quad (\text{m})$ ***(A1)*(ft)*(G2)***

Nota:	Acepte $4{,}73$ $(4{,}72863\ldots)$ (m) por usar su respuesta a 3 cifras significativas. Acepte otros métodos equivalentes.

[9 puntos]

(b) $\left|\dfrac{4{,}88 - 3{,}9}{3{,}9}\right| \times 100$ ***(M1)***

Nota:	Conceda ***(M1)*** por una sustitución correcta en la fórmula del porcentaje de error.

$= 25{,}1 \quad (25{,}1282) \quad (\%)$ ***(A1)*(ft)*(G2)***

Nota:	Arrastre de error (FT) desde el apartado (a)(iii).

[2 puntos]

continúa en la pág. siguiente...

Continuación de la Pregunta 5

(c) $\tan^{-1}\left(\dfrac{55,7869\ldots}{40,11927\ldots}\right)$ ***(A1)*(ft)*(M1)***

Nota:	Conceda ***(A1)*(ft)** por haber escrito $40,11927\ldots$. Conceda ***(M1)*** por una sustitución correcta en la razón trigonométrica.

O BIEN

$(37-4,88072...)^2+55,7869...^2$
$(\text{AC}=)\ 64,3725...$
$64,3726...^2+8^2-2\times 8\times 64,3726...\times\cos 120$
$(\text{AD}=)\ 68,7226...$
$\dfrac{\text{sen}120}{68,7226...}=\dfrac{\text{sen}\,\text{A}\hat{\text{D}}\text{C}}{64,3725...}$ ***(A1)*(ft)*(M1)***

Nota:	Conceda ***(A1)*(ft)** por ver escritos sus valores correctos, ***(M1)*** por una sustitución correcta en la fórmula del teorema del seno..

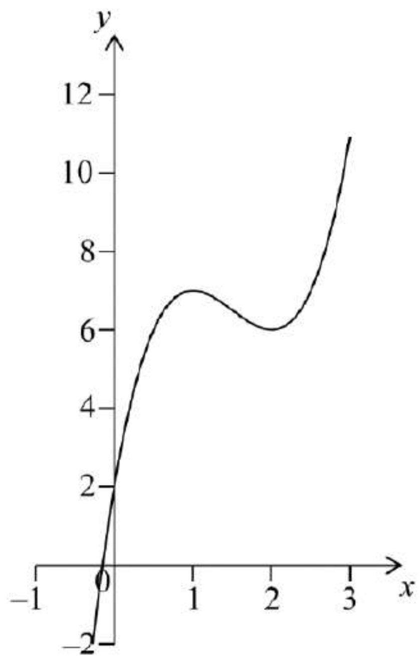
$=54.3^\circ\ \ (54.2781\ldots^\circ)$ ***(A1)*(ft)*(G2)***

Nota:	Arrastre de error (FT) desde el apartado (a). Acepte otros métodos equivalentes.

[3 puntos]

Total [14 puntos]

6. (a)



(A1)(A1)(A1)(A1)

Nota: Conceda ***(A1)*** por una ventana correcta (condone una ventana que esté ligeramente por fuera de la correcta) y unos ejes rotulados. Es necesario alguna indicación de ventana. -1 a 3 en el eje x y -2 a 12 en el eje y, y un gráfico en esa ventana.
(A1) por una forma correcta (la curva ha de tener una forma cúbica y debe ser suave),
(A1) por ambos puntos estacionarios situados en el primer cuadrante en una posición aproximadamente correcta,
(A1) por puntos de corte con los ejes (corte con el eje x a la izquierda del origen y corte con el eje y con una ordenada positiva) en una posición aproximadamente correcta.

[4 puntos]

(b) Rick ***(A1)***

Nota: Conceda ***(A0)*** si menciona además otros nombres.

[1 punto]

(c) $2(1)^3 - 9(1)^2 + 12(1) + 2$ ***(M1)***

Nota: Conceda ***(M1)*** por una sustitución correcta en la ecuación.

$= 7$ ***(A1)(G2)***

[2 puntos]

continúa en la pág. siguiente...

Continuación de la Pregunta 6

(d) $6x^2 - 18x + 12$ ***(A1)(A1)(A1)***

Nota:	Conceda ***(A1)*** por cada término correcto. Conceda como mucho ***(A1)(A1)(A0)*** si ha escrito otros términos adicionales.

[3 puntos]

(e) $6x^2 - 18x + 12 = 0$ ***(M1)***

Nota:	Conceda ***(M1)*** por igualar su derivada a 0. Si la derivada no se iguala a cero en forma explícita, pero luego se ve una resolución de su ecuación correcta, conceda ***(M1)***.

$6(x-1)(x-2) = 0$ (o equivalente) ***(M1)***

Nota:	Conceda ***(M1)*** por una factorización correcta. El ***(M1)*** final se concede únicamente si el alumno ha indicado claramente las respuestas. Conceda ***(M0)(M0)*** por la sustitución de 1 o de 2 en su derivada.

$x = 1,\ x = 2$ ***(AG)***

[2 puntos]

(f) $6 < k < 7$ ***(A1)(A1)*****(ft)*****(A1)***

Nota:	Conceda ***(A1)*** por una inecuación con el 6, conceda ***(A1)*****(ft)** por una inecuación con el 7 de su apartado (c) solo si es mayor que 6, y conceda ***(A1)*** por sus inecuaciones estrictas y correctas. Acepte $]6,\ 7[$ o bien $(6,\ 7)$.

[3 puntos]

Total [15 puntos]

Diploma Programme
Programme du diplôme
Programa del Diploma

Études mathématiques
Niveau moyen
Épreuve 2

Jeudi 3 mai 2018 (matin)

1 heure 30 minutes

Instructions destinées aux candidats

- N'ouvrez pas cette épreuve avant d'y être autorisé(e).
- Une calculatrice à écran graphique est nécessaire pour cette épreuve.
- Un exemplaire non annoté du **livret de formules pour le cours d'études mathématiques NM** est nécessaire pour cette épreuve.
- Répondez à toutes les questions dans le livret de réponses fourni.
- Sauf indication contraire dans l'intitulé de la question, toutes les réponses numériques devront être exactes ou correctes à trois chiffres significatifs près.
- Le nombre maximum de points pour cette épreuve d'examen est de **[90 points]**.

2218 – 7408

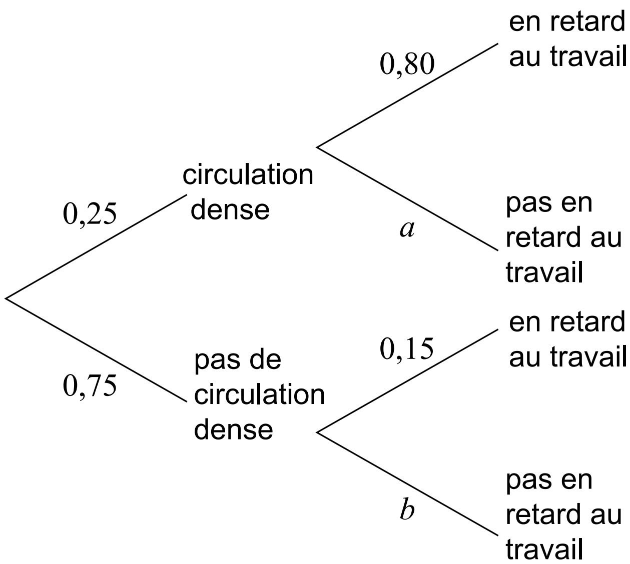
Répondez à **toutes** les questions dans le livret de réponses fourni. Veuillez répondre à chaque question sur une nouvelle page. On vous recommande d'indiquer votre raisonnement autant que possible. Lorsque la réponse est fausse, certains points seront accordés si la méthode utilisée est correcte, pour autant que le raisonnement soit indiqué par écrit. Les solutions obtenues à l'aide de calculatrices à écran graphique doivent être accompagnées d'un raisonnement adéquat. Par exemple, si des représentations graphiques sont utilisées pour trouver la solution, veuillez inclure une esquisse de ces représentations graphiques dans votre réponse.

1. [Note maximale : 16]

Dans une entreprise, on sait que 25% des employés ont eu à affronter une circulation dense en se rendant au travail. Pour ceux qui ont eu à affronter une circulation dense, la probabilité d'arriver en retard au travail était de 80%.

Pour ceux qui n'ont pas eu à affronter une circulation dense, la probabilité d'arriver en retard au travail était de 15%.

Le diagramme en arbre illustre ces informations.



(a) Écrivez la valeur de

(i) a ;

(ii) b. [2]

(b) Utilisez le diagramme en arbre pour trouver la probabilité qu'un employé

(i) ait eu à affronter une circulation dense et soit arrivé en retard au travail ;

(ii) soit arrivé en retard au travail ;

(iii) ait eu à affronter une circulation dense, étant donné qu'il est arrivé en retard au travail. [8]

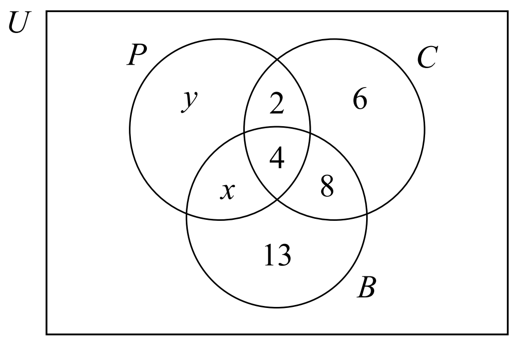
(Suite de la question à la page suivante)

(Suite de la question 1)

L'entreprise étudie les différents moyens de transport empruntés par ses employés pour se rendre au travail au cours de la dernière année. Il s'est avéré que les trois moyens de transport les plus utilisés pour se rendre au travail ont été les transports en commun (P), l'automobile (C) et le vélo (B).

L'entreprise trouve que 20 employés ont emprunté l'automobile, 28 ont emprunté le vélo et 19 ont emprunté les transports en commun au cours de la dernière année.

Une partie de ces informations est montrée dans le diagramme de Venn.



(c) Trouvez la valeur de

(i) x ;

(ii) y. [2]

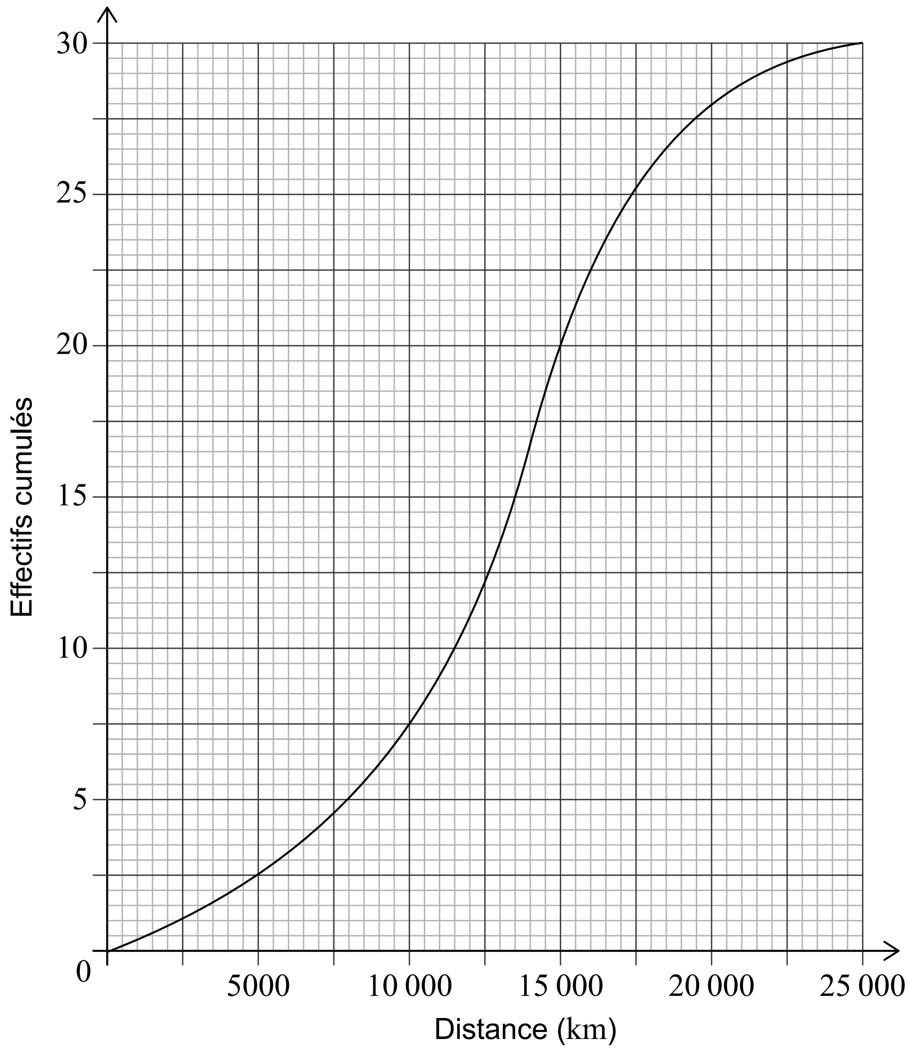
L'entreprise compte 54 employés.

(d) Trouvez le nombre d'employés qui, au cours de la dernière année, n'ont emprunté ni l'automobile ni le vélo ni les transports en commun pour se rendre au travail. [2]

(e) Trouvez $n\left((C \cup B) \cap P'\right)$. [2]

2. [Note maximale : 15]

Une entreprise de transport possède 30 autobus. La distance parcourue par chaque autobus depuis son achat par l'entreprise a été enregistrée. La courbe des effectifs cumulés pour ces données est présentée.



(a) Trouvez le nombre d'autobus ayant parcouru une distance entre 15 000 et 20 000 kilomètres. [2]

(b) Utilisez la courbe des effectifs cumulés pour trouver

(i) la distance médiane ;

(ii) le premier quartile ;

(iii) le troisième quartile. [4]

(Suite de la question à la page suivante)

(Suite de la question 2)

(c) À partir de là, écrivez l'écart interquartile. [1]

(d) Écrivez le pourcentage d'autobus ayant parcouru une distance supérieure au troisième quartile. [1]

(e) Trouvez le nombre d'autobus ayant parcouru une distance inférieure ou égale à $12\,000\,\mathrm{km}$. [1]

On sait que 8 autobus ont parcouru plus de m kilomètres.

(f) Trouvez la valeur de m. [2]

La plus courte distance parcourue par un des autobus a été de $2500\,\mathrm{km}$.
La plus longue distance parcourue par un des autobus a été de $23\,000\,\mathrm{km}$.

(g) **Sur du papier millimétré**, dessinez un diagramme en boîte à moustaches pour ces données. Utilisez une échelle de $2\,\mathrm{cm}$ pour représenter $5000\,\mathrm{km}$. [4]

3. [Note maximale : 14]

Le poids, W, des joueurs de basket-ball qui participent à un tournoi est normalement distribué avec une moyenne de $65\,\mathrm{kg}$ et un écart type de $5\,\mathrm{kg}$.

(a) (i) Trouvez la probabilité qu'un joueur de basket-ball ait un poids inférieur à $61\,\mathrm{kg}$.

Lors d'une séance d'entraînement, il y a 40 joueurs de basket-ball.

(ii) Trouvez le nombre espéré de joueurs ayant un poids inférieur à $61\,\mathrm{kg}$ dans cette séance d'entraînement. [4]

(b) La probabilité qu'un joueur de basket-ball ait un poids se situant à l'intérieur de $1{,}5$ écart type de la moyenne est q.

(i) Esquissez une courbe normale pour représenter cette probabilité.

(ii) Trouvez la valeur de q. [3]

(c) Étant donné que $\mathrm{P}(W > k) = 0{,}225$, trouvez la valeur de k. [2]

Une entraîneuse de basket-ball a observé 60 de ses joueurs afin de déterminer si leur performance et leur poids étaient indépendants. Ses observations ont été enregistrées dans le tableau suivant.

		Performance	
		Satisfaisante	**Excellente**
Poids	**En dessous de la moyenne**	6	10
	Dans la moyenne	7	15
	Au-dessus de la moyenne	12	10

Elle a décidé d'effectuer un test d'indépendance du χ^2 au seuil de signification de $5\,\%$.

(d) Pour ce test,

(i) indiquez l'hypothèse nulle ; [1]

(ii) trouvez la valeur p. [2]

(e) Indiquez une conclusion pour ce test. Justifiez votre réponse. [2]

4. [Note maximale : 16]

Un nouveau café a ouvert ses portes et au cours de la première semaine, son profit a été de 60 \$.

Le profit du café augmente de 10 \$ chaque semaine.

(a) Trouvez le profit du café au cours de la 11^{e} semaine. [3]

(b) Calculez le profit **total** du café pour les 12 premières semaines. [3]

Un nouveau salon de thé a ouvert ses portes en même temps que le café. Au cours de la première semaine, son profit a également été de 60 \$.

Le profit du salon de thé augmente de 10% chaque semaine.

(c) Trouvez le profit du salon de thé au cours de la 11^{e} semaine. [3]

(d) Calculez le profit **total** du salon de thé pour les 12 premières semaines. [3]

Au cours de la $m^{\text{ième}}$ semaine, le profit **total** du salon de thé dépasse le profit **total** du café pour la première fois depuis leur ouverture.

(e) Trouvez la valeur de m. [4]

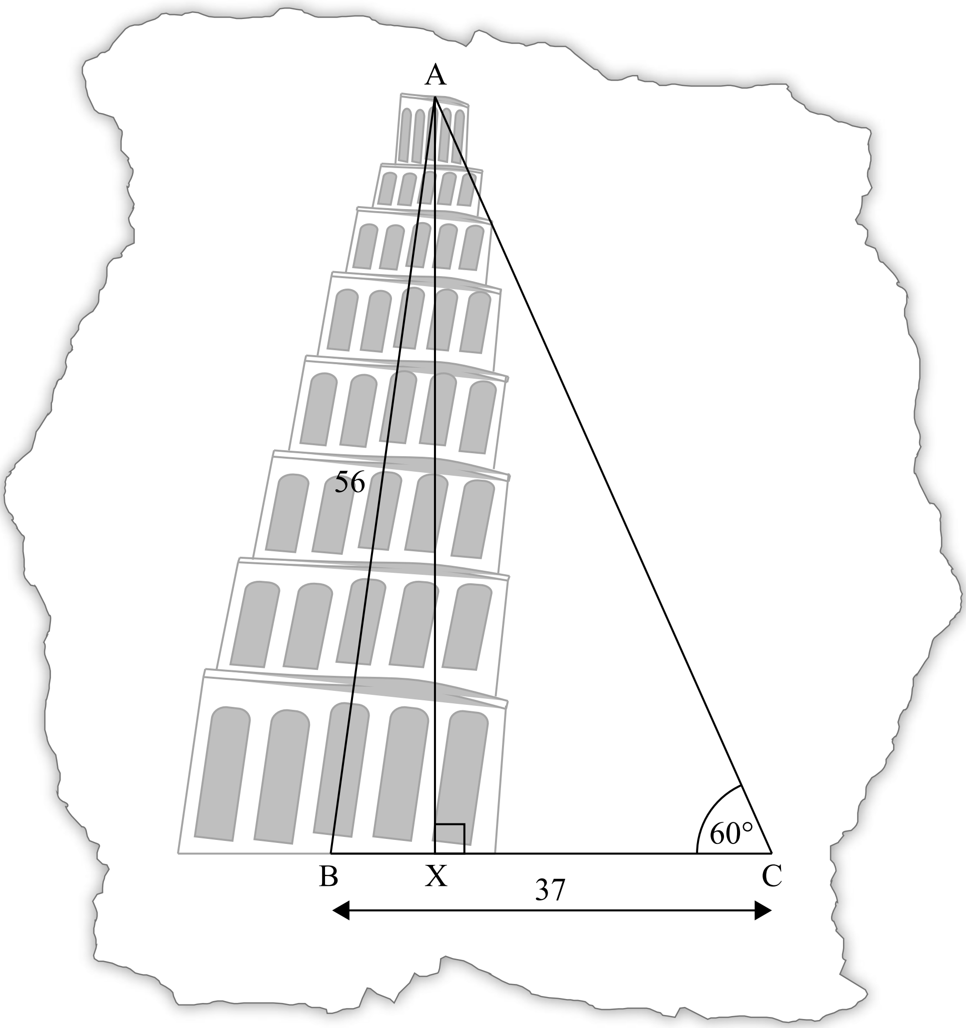
5. [Note maximale : 14]

La tour de Pise est bien connue à travers le monde pour son inclinaison.

Giovanni visite la tour et veut étudier quelle est son inclinaison. Il dessine un diagramme montrant un triangle non rectangle, ABC.

Sur le diagramme de Giovanni, la longueur de AB est de $56\,\text{m}$, la longueur de BC est de $37\,\text{m}$, et l'angle ACB mesure 60°. AX est la hauteur perpendiculaire de A sur BC.

la figure n'est pas à l'échelle



(a) Utilisez le diagramme de Giovanni pour

(i) montrer que l'angle ABC, l'angle d'inclinaison de la tour par rapport à l'horizontale, est de 85°, au degré près ;

(ii) calculer la longueur de AX ;

(iii) trouver la longueur de BX, le déplacement horizontal de la tour. [9]

(Suite de la question à la page suivante)

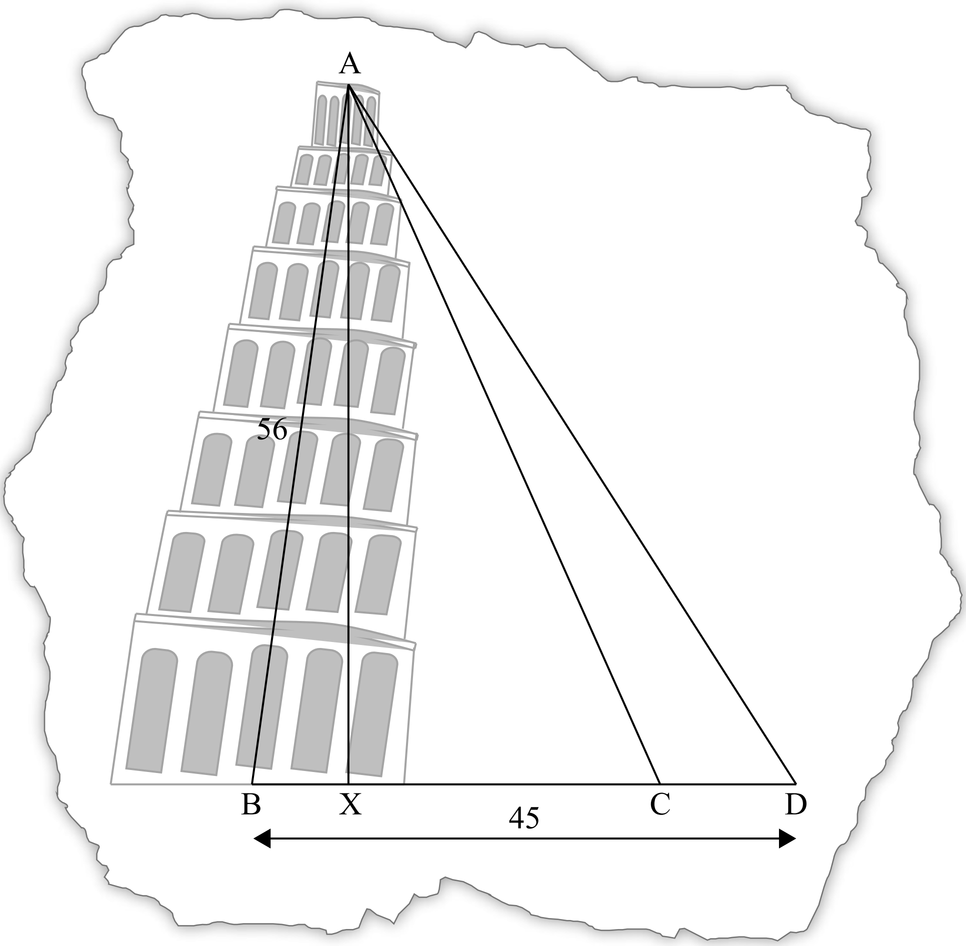
(Suite de la question 5)

Le guide touristique de Giovanni affirme que le véritable déplacement horizontal de la tour, BX, est de $3{,}9$ mètres.

(b) Trouvez le pourcentage d'erreur sur le diagramme de Giovanni. [2]

Giovanni ajoute un point D à son diagramme, tel que $\mathrm{BD} = 45\,\mathrm{m}$, et un autre triangle est formé.

la figure n'est pas à l'échelle



(c) Trouvez l'angle d'élévation de A à partir de D. [3]

6. [Note maximale : 15]

Considérez la courbe $y = 2x^3 - 9x^2 + 12x + 2$, pour $-1 < x < 3$.

(a) Esquissez la courbe pour $-1 < x < 3$ et $-2 < y < 12$. [4]

(b) Une enseignante demande à ses élèves de faire quelques observations à propos de la courbe.

Trois élèves répondent.
Nadia dit : « *L'abscisse à l'origine de la courbe se trouve entre* -1 *et zéro* ».
Rick dit : « *La courbe est décroissante lorsque* $x < 1$ ».
Paula dit : « *La pente de la courbe est inférieure à zéro entre* $x = 1$ *et* $x = 2$ ».

Indiquez le nom de l'élève qui a fait une observation **incorrecte**. [1]

(c) Trouvez la valeur de y lorsque $x = 1$. [2]

(d) Trouvez $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(e) Montrez que les points stationnaires de la courbe se trouvent en $x = 1$ et $x = 2$. [2]

(f) Étant donné que $2x^3 - 9x^2 + 12x + 2 = k$ admet **trois** solutions, trouvez les valeurs possibles de k. [3]

Diploma Programme
Programme du diplôme
Programa del Diploma

Further mathematics
Higher level
Paper 1

Thursday 17 May 2018 (afternoon)

2 hours 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[150 marks]**.

2218 – 7101

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 9]

(a) Use the Euclidean algorithm to find the greatest common divisor of 74 and 383. [4]

(b) Hence find integers s and t such that $74s + 383t = 1$. [5]

2. [Maximum mark: 6]

Let $\boldsymbol{A}^2 = 2\boldsymbol{A} + \boldsymbol{I}$ where $\boldsymbol{A}$ is a 2×2 matrix.

(a) Show that $\boldsymbol{A}^4 = 12\boldsymbol{A} + 5\boldsymbol{I}$. [3]

Let $\boldsymbol{B} = \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix}$.

(b) Given that $\boldsymbol{B}^2 - \boldsymbol{B} - 4\boldsymbol{I} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, find the value of k. [3]

3. [Maximum mark: 7]

(a) A number written in base 5 is 4303. Find this as a number written in base 10. [2]

(b) 1000 is a number written in base 10. Find this as a number written in base 7. [5]

4. [Maximum mark: 12]

The transformations T_1, T_2, T_3, T_4, in the plane are defined as follows:

T_1: A rotation of $360°$ about the origin
T_2: An anticlockwise rotation of $270°$ about the origin
T_3: A rotation of $180°$ about the origin
T_4: An anticlockwise rotation of $90°$ about the origin.

(a) Copy and complete the following Cayley table for the transformations of T_1, T_2, T_3, T_4, under the operation of composition of transformations.

	T_1	T_2	T_3	T_4
T_1	T_1	T_2	T_3	T_4
T_2	T_2			
T_3	T_3			
T_4	T_4			

[2]

(b) (i) Show that T_1, T_2, T_3, T_4 under the operation of composition of transformations form a group. Associativity may be assumed.

(ii) Show that this group is cyclic. [4]

The transformation T_5 is defined as a reflection in the x-axis.

(c) Write down the 2×2 matrices representing T_3, T_4 and T_5. [3]

(d) The transformation T is defined as the composition of T_3 followed by T_5 followed by T_4.

(i) Find the 2×2 matrix representing T.

(ii) Give a geometric description of the transformation T. [3]

5. [Maximum mark: 7]

Use the integral test to determine whether or not $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges. [7]

6. [Maximum mark: 9]

(a) Consider the integers between 1 and 20 inclusive.
Let $A = \{\text{multiples of } 2\}$, $B = \{\text{multiples of } 3\}$, $C = \{\text{multiples of } 4\}$.
Find the elements in each of the following sets,

(i) $A \cap (B \cup C)$;

(ii) $A \setminus (B \setminus C)$. [5]

(b) Let $M = \{x : x \text{ is an integer multiple of } 10\}$ and let $N = \{x : x \text{ is an integer multiple of } 5\}$
Prove that M is a proper subset of N. [4]

7. [Maximum mark: 9]

A sample of size 100 is taken from a normal population with unknown mean μ and known variance 36.

(a) An investigator wishes to test the hypotheses $H_0 : \mu = 65$, $H_1 : \mu > 65$.

He decides on the following acceptance criteria:
Accept H_0 if the sample mean $\bar{x} \le 66.5$
Accept H_1 if $\bar{x} > 66.5$

Find the probability of a Type I error. [3]

(b) Another investigator decides to use the same data to test the hypotheses
$H_0 : \mu = 65$, $H_1 : \mu = 67.9$.

(i) She decides to use the same acceptance criteria as the previous investigator.
Find the probability of a Type II error.

(ii) Find the critical value for $\bar{x}$ if she wants the probabilities of a Type I error and a Type II error to be equal. [6]

8. [Maximum mark: 13]

Consider the simultaneous linear equations

$$x + z = -1$$
$$3x + y + 2z = 1$$
$$2x + ay - z = b$$

where a and b are constants.

(a) Using row reduction, find the solutions in terms of a and b when $a \neq 3$. [8]

(b) Explain why the equations have no unique solution when $a = 3$. [1]

(c) Find all the solutions to the equations when $a = 3$, $b = 10$ in the form $\boldsymbol{r} = \boldsymbol{s} + \lambda \boldsymbol{t}$. [4]

9. [Maximum mark: 13]

(a) Given that A is the interval $\{x : 0 \leq x \leq 3\}$ and B is the interval $\{y : 0 \leq y \leq 4\}$ then describe $A \times B$ in geometric form. [3]

(b) Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ be defined by $f(x, y) = (x + 3y, 2x - y)$.

(i) Show that the function f is a bijection.

(ii) Hence find the inverse function f^{-1}. [10]

10. [Maximum mark: 12]

(a) By considering the images of the points (1, 0) and (0, 1),

(i) determine the 2×2 matrix $\boldsymbol{P}$ which represents a reflection in the line $y = (\tan\theta)x$;

(ii) determine the 2×2 matrix $\boldsymbol{Q}$ which represents an anticlockwise rotation of θ about the origin. [5]

(b) Describe the transformation represented by the matrix $\boldsymbol{PQ}$. [5]

(c) A matrix M is said to be orthogonal if $\boldsymbol{M}^T\boldsymbol{M} = \boldsymbol{I}$ where $\boldsymbol{I}$ is the identity. Show that $\boldsymbol{Q}$ is orthogonal. [2]

11. [Maximum mark: 12]

Given that y is a function of x, the function z is given by $z = \frac{y - x}{y + x}$, where $x \in \mathbb{R}, x \neq 3$, $y + x \neq 0$.

(a) Show that $\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{2}{(y+x)^2}\left(x\frac{\mathrm{d}y}{\mathrm{d}x} - y\right)$. [3]

(b) Show that the differential equation $f(x)\left(x\frac{\mathrm{d}y}{\mathrm{d}x} - y\right) = y^2 - x^2$ can be written as $f(x)\frac{\mathrm{d}z}{\mathrm{d}x} = 2z$. [2]

(c) Hence show that the solution to the differential equation $(x-3)\left(x\frac{\mathrm{d}y}{\mathrm{d}x} - y\right) = y^2 - x^2$ given that $x = 4$ when $y = 5$ is $\frac{y-x}{y+x} = \left(\frac{x-3}{3}\right)^2$. [7]

12. [Maximum mark: 15]

(a) Solve the recurrence relation $u_n = 4u_{n-1} - 4u_{n-2}$ given that $u_0 = u_1 = 1$. [6]

Consider v_n which satisfies the recurrence relation $2v_n = 7v_{n-1} - 3v_{n-2}$ subject to the initial conditions $v_0 = v_1 = 1$.

(b) Prove by using strong induction that $v_n = \frac{4}{5}\left(\frac{1}{2}\right)^n + \frac{1}{5}(3)^n$ for $n \in \mathbb{N}$. [9]

13. [Maximum mark: 9]

Consider the matrix $\boldsymbol{M} = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$.

(a) Show that the linear transformation represented by $\boldsymbol{M}$ transforms any point on the line $y = x$ to a point on the same line. [2]

(b) Explain what happens to points on the line $4y + x = 0$ when they are transformed by $\boldsymbol{M}$. [3]

(c) State the two eigenvalues of $\boldsymbol{M}$. [2]

(d) State two eigenvectors of $\boldsymbol{M}$ which correspond to the two eigenvalues. [2]

14. [Maximum mark: 8]

At an early stage in analysing the marks scored by candidates in an examination paper, the examining board takes a random sample of 250 candidates and finds that the marks, x, of these candidates give $\sum x = 10985$ and $\sum x^2 = 598736$.

(a) Calculate a 90% confidence interval for the population mean mark μ for this paper. [4]

(b) The null hypothesis $\mu = 46.5$ is tested against the alternative hypothesis $\mu < 46.5$ at the $\lambda\%$ significance level. Determine the set of values of λ for which the null hypothesis is rejected in favour of the alternative hypothesis. [4]

15. [Maximum mark: 9]

Given that the tangents at the points P and Q on the parabola $y^2 = 4ax$ are perpendicular, find the locus of the midpoint of PQ. [9]

M18/5/FURMA/HP1/ENG/TZ0/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Further mathematics

Higher level

Paper 1

21 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document "**Mathematics HL: Guidance for e-marking May 2018**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = \left(2\cos(5x-3)\right)5 \ \left(=10\cos(5x-3)\right)$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT***.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) $383 = 5\times 74 + 13$ ***M1***

$74 = 5\times 13 + 9$ ***A1***

$13 = 1\times 9 + 4$ ***(A1)***

$9 = 2\times 4 + 1$

$4 = 4\times 1 + 0$

$\Rightarrow$ gcd $(74, 383) = 1$ ***A1***

[4 marks]

(b) **EITHER**

$1 = 9 - 2\times 4$ ***(M1)***

$= 9 - 2(13 - 1\times 9) = 3\times 9 - 2\times 13$ ***(A1)***

$= 3(74 - 5\times 13) - 2\times 13 = 3\times 74 - 17\times 13$ ***(A1)***

$= 3\times 74 - 17(383 - 5\times 74) = 88\times 74 - 17\times 383$

OR

$13 = 383 - 5\times 74$ ***(M1)***

$9 = 74 - 5\times 13$

$= 74 - 5(383 - 5\times 74)$

$= 26\times 74 - 5\times 383$ ***(A1)***

$4 = 13 - 9$

$= (383 - 5\times 74) - (26\times 74 - 5\times 383)$

$= 6\times 383 - 31\times 74$ ***(A1)***

$1 = 9 - 2\times 4$

$= (26\times 74 - 5\times 383) - 2(6\times 383 - 31\times 74)$

$= 88\times 74 - 17\times 383$

THEN

$\Rightarrow s = 88$ and $t = -17$ ***A1A1***

[5 marks]

Total [9 marks]

2. (a) **METHOD 1**

$\boldsymbol{A}^4 = 4\boldsymbol{A}^2 + 4\boldsymbol{AI} + \boldsymbol{I}^2$ or equivalent ***M1A1***
$= 4(2\boldsymbol{A} + \boldsymbol{I}) + 4\boldsymbol{A} + \boldsymbol{I}$ ***A1***
$= 8\boldsymbol{A} + 4\boldsymbol{I} + 4\boldsymbol{A} + \boldsymbol{I}$
$= 12\boldsymbol{A} + 5\boldsymbol{I}$ ***AG***

[3 marks]

METHOD 2

$\boldsymbol{A}^3 = \boldsymbol{A}(2\boldsymbol{A} + \boldsymbol{I}) = 2\boldsymbol{A}^2 + \boldsymbol{AI} = 2(2\boldsymbol{A} + \boldsymbol{I}) + \boldsymbol{A} (= 5\boldsymbol{A} + 2\boldsymbol{I})$ ***M1A1***

$\boldsymbol{A}^4 = \boldsymbol{A}(5\boldsymbol{A} + 2\boldsymbol{I})$ ***A1***

$= 5\boldsymbol{A}^2 + 2\boldsymbol{A} = 5(2\boldsymbol{A} + \boldsymbol{I}) + 2\boldsymbol{A}$

$= 12\boldsymbol{A} + 5\boldsymbol{I}$ ***AG***

(b) $\boldsymbol{B}^2 = \begin{bmatrix} 18 & 2 \\ 1 & 11 \end{bmatrix}$ ***(A1)***

$\begin{bmatrix} 18 & 2 \\ 1 & 11 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ ***(A1)***

$\Rightarrow k = 10$ ***A1***

[3 marks]

Total [6 marks]

3. (a) $4303_5 = 4\times5^3+3\times5^2+0\times5^1+3\times5^0$ ***(M1)***
$=500+75+3$
$=578$ ***A1***

[2 marks]

(b) **METHOD 1**

$1000 = a_0 + 7a_1 + 49a_2 + 343a_3$ ***(M1)***

(Since $343a_3 < 1000$) $\Rightarrow a_3 = 2$ ***(A1)***

$1000 = a_0 + 7a_1 + 49a_2 + 686$

$a_0 + 7a_1 + 49a_2 = 314 \Rightarrow a_2 = 6$ ***(A1)***

$a_0 + 7a_1 = 20 \Rightarrow a_1 = 2,\ a_0 = 6$ ***(A1)***

$\Rightarrow 1000_{10} = 2626_7$ ***A1***

METHOD 2

$1000 = 7\times142+6$ ***(M1)***

$142 = 7\times20+2$ ***(A1)***

$20 = 7\times2+6$ ***(A1)***

$2 = 7\times0+2$ ***(A1)***

$\Rightarrow (1000)_{10} = 2626_7$ ***A1***

[5 marks]

Total [7 marks]

4. (a)

	T_1	T_2	T_3	T_4
T_1	T_1	T_2	T_3	T_4
T_2	T_2	$\boldsymbol{T_3}$	$\boldsymbol{T_4}$	$\boldsymbol{T_1}$
T_3	T_3	$\boldsymbol{T_4}$	$\boldsymbol{T_1}$	$\boldsymbol{T_2}$
T_4	T_4	$\boldsymbol{T_1}$	$\boldsymbol{T_2}$	$\boldsymbol{T_3}$

A2

[2 marks]

Note: Award ***A1*** for 6, 7 or 8 correct.

(b) (i) the table is closed – no new elements ***A1***
T_1 is the identity ***A1***
T_3 (and T_1) are self-inverse; T_2 and T_4 are an inverse pair. Hence every element has an inverse ***A1***
hence it is a group ***AG***

(ii) all elements in the group can be generated by T_2 (or T_4) ***R1***
hence the group is cyclic ***AG***

[4 marks]

continued…

Question 4 continued

(c) T_3 is represented by $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ ***A1***

T_4 is represented by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ***A1***

T_5 is represented by $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ***A1***

[3 marks]

(d) (i) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ***(M1)A1***

Note: Award ***M1A0*** for multiplying the matrices in the wrong order.

(ii) a reflection in the line $y = -x$ ***A1***

[3 marks]

Total [12 marks]

5. let $u = \ln x$ ***(M1)***

$\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$

$\int \frac{1}{x(\ln x)^2}\mathrm{d}x = \int \frac{1}{u^2}\mathrm{d}u$ ***(A1)***

$= -\frac{1}{u} = -\frac{1}{\ln x}$ ***(A1)***

$\int_2^m \frac{1}{x(\ln x)^2}\mathrm{d}x = \left[-\frac{1}{\ln x}\right]_2^m$ ***M1***

$= \left[-\frac{1}{\ln m} + \frac{1}{\ln 2}\right]$ ***A1***

as $m \to \infty$, $-\frac{1}{\ln m} \to 0$ ***(A1)***

$\int_2^\infty \frac{1}{x(\ln x)^2}\mathrm{d}x = \frac{1}{\ln 2}$ and hence the series converges ***R1***

[7 marks]

6. (a) (i) $B \cup C = \{3\ 4\ 6\ 8\ 9\ 12\ 15\ 16\ 18\ 20\}$ ***(M1)***

$\Rightarrow A \cap (B \cup C) = \{4\ 6\ 8\ 12\ 16\ 18\ 20\}$ ***A1***

(ii) $B \setminus C = B \cap C' = \{3\ 6\ 9\ 15\ 18\}$ ***(M1)(A1)***

$A \setminus (B \setminus C) = \left(A \cap (B \cap C')'\right) = \{2\ 4\ 8\ 10\ 12\ 14\ 16\ 20\}$ ***A1***

[5 marks]

(b) let x be any element of M

then $x = 10q$ for $q \in \mathbb{Z}$ ***M1***

hence $x = 5(2 \times q)$ ***A1***

since $2 \times q$ is an integer, x is an element of N ***R1***

since M is smaller than N, ***R1***

it is a proper subset ***AG***

[4 marks]

Total [9 marks]

7. (a) $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$

$\bar{X} \sim \mathrm{N}\left(65, \frac{36}{100}\right)$ ***(A1)***

$\mathrm{P}(\text{Type I Error}) = \mathrm{P}(\bar{X} > 66.5)$ ***(M1)***

$= 0.00621$ ***A1***

[3 marks]

(b) (i) $\mathrm{P}(\text{Type II Error}) = \mathrm{P}(\text{accept } H_0 \mid H_1 \text{ is true})$

$= \mathrm{P}(\bar{X} \le 66.5 \mid \mu = 67.9)$ ***(M1)***

$= \mathrm{P}(\bar{X} \le 66.5)$ when $\bar{X} \sim \mathrm{N}\left(67.9, \frac{36}{100}\right)$ ***(M1)***

$= 0.00982$ ***A1***

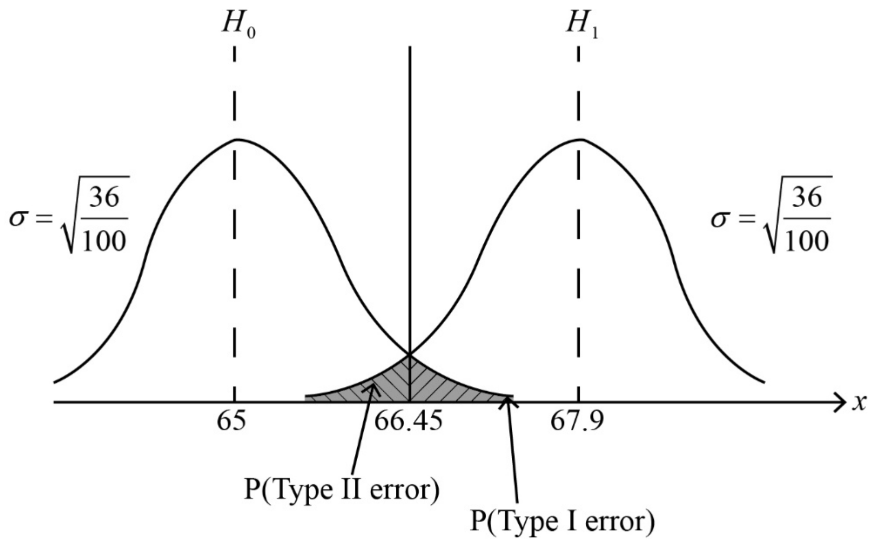
continued…

Question 7 continued

(ii) the variances of the distributions given by H_0 and H_1 are equal, ***(R1)***

by symmetry the value of $\overline{x}$ lies midway between 65 and 67.9 ***(M1)***

$\Rightarrow \overline{x} = \frac{1}{2}(65 + 67.9) = 66.45$ ***A1***



[6 marks]

Total [9 marks]

8. (a) $\begin{pmatrix} 1 & 0 & 1 & -1 \\ 3 & 1 & 2 & 1 \\ 2 & a & -1 & b \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 1 & -1 \\ 3a-2 & 0 & 2a+1 & a-b \\ 2 & a & -1 & b \end{pmatrix}$ or equivalent ***M1A1***

$$\begin{pmatrix} 0 & 0 & a-3 & -4a+2+b \\ 3a-2 & 0 & 2a+1 & a-b \\ 2 & a & -1 & b \end{pmatrix}$$

$z = \dfrac{-4a+b+2}{a-3}$ ***M1A1***

$x = -1 - z$ ***M1***

$$x = -1 - \left(\frac{-4a+b+2}{a-3}\right)$$

$$x = \frac{-a+3+4a-b-2}{a-3}$$

$x = \dfrac{3a-b+1}{a-3}$ ***A1***

$y = 1 - 3x - 2z$ ***M1***

$$y = 1 - 3\left(\frac{3a-b+1}{a-3}\right) - 2\left(\frac{-4a+b+2}{a-3}\right)$$

$$= \frac{a-3-9a+3b-3+8a-2b-4}{a-3}$$

$= \dfrac{b-10}{a-3}$ ***A1***

[8 marks]

(b) when $a = 3$ the denominator of x, y and $z = 0$ ***R1***

Note: Accept any valid reason.

hence no unique solutions ***AG***

[1 mark]

(c) For example let $z = \lambda$ ***(M1)***

$x = -1 - \lambda$ ***(A1)***

$y = 1 - 3(-1-\lambda) - 2\lambda$

$y = 4 + \lambda$ ***(A1)***

$r = \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ ***A1***

[4 marks]

Note: Accept answers which let $x = \lambda$ or $y = \lambda$.

Total [13 marks]

9. (a) $A \times B$ is a rectangle *A1*

vertices at $(0,0)$, $(3,0)$, $(0,4)$ and $(3,4)$ or equivalent description *A1*

and its interior *A1*

Note: Accept diagrammatic answers.

[3 marks]

(b) (i) need to prove it is injective and surjective *R1*

need to show if $f(x,y) = f(u,v)$ then $(x,y) = (u,v)$ *M1*

$\Rightarrow x + 3y = u + 3v$

$2x - y = 2u - v$ *A1*

Equation 2 – 2 Equation 1 $\Rightarrow y = v$

Equation 1 + 3 Equation 2 $\Rightarrow x = u$ *A1*

thus $(x,y) = (u,v) \Rightarrow f$ is injective

let (s,t) be any value in the co-domain $\mathbb{R} \times \mathbb{R}$

we must find (x,y) such that $f(x,y) = (s,t)$ *M1*

$s = x + 3y$ and $t = 2x - y$ *M1*

$\Rightarrow y = \frac{2s - t}{7}$ *A1*

and $x = \frac{s + 3t}{7}$ *A1*

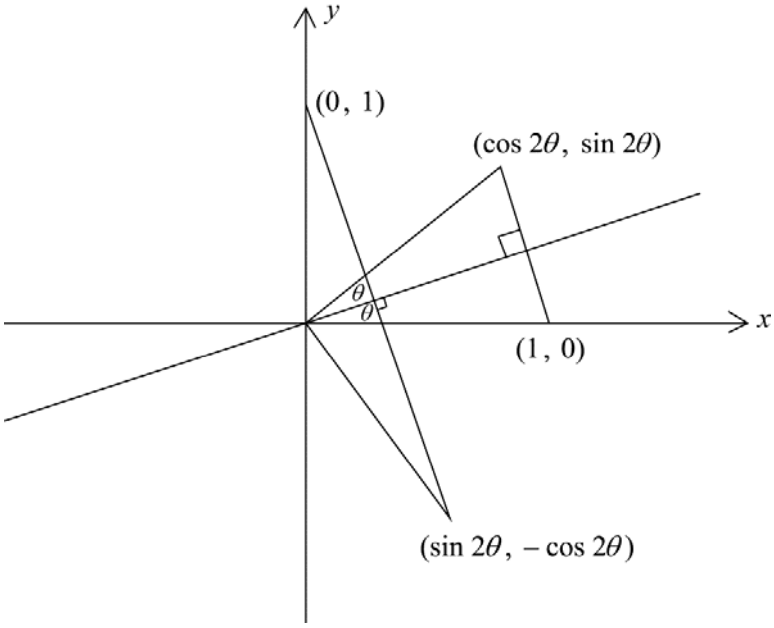
hence $f(x,y) = (s,t)$ and is therefore surjective

(ii) $f^{-1}(x,y) = \left(\frac{x+3y}{7}, \frac{2x-y}{7}\right)$ *A1A1*

[10 marks]

Total [13 marks]

10. (a) (i)



(M1)

using the transformation of the unit square:

$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}\cos 2\theta\\ \sin 2\theta\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}\sin 2\theta\\ -\cos 2\theta\end{pmatrix}$ ***(M1)***

hence the matrix $\boldsymbol{P}$ is $\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix}$ ***A1***

(ii) using the transformation of the unit square:

$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}\cos \theta\\ \sin \theta\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}-\sin \theta\\ \cos \theta\end{pmatrix}$ ***(M1)***

hence the matrix $\boldsymbol{Q}$ is $\begin{pmatrix}\cos \theta & -\sin \theta\\ \sin \theta & \cos \theta\end{pmatrix}$ ***A1***

[5 marks]

continued…

Question 10 continued

(b) $\boldsymbol{PQ} = \begin{pmatrix} \cos\theta\cos 2\theta + \sin\theta\sin 2\theta & \cos\theta\sin 2\theta - \sin\theta\cos 2\theta \\ -\cos 2\theta\sin\theta + \sin 2\theta\cos\theta & -\sin\theta\sin 2\theta - \cos\theta\cos 2\theta \end{pmatrix}$ ***M1A1***

$= \begin{pmatrix} \cos(2\theta-\theta) & \sin(2\theta-\theta) \\ \sin(2\theta-\theta) & -\cos(2\theta-\theta) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ ***M1A1***

this is a reflection in the line $y = \left(\tan\frac{1}{2}\theta\right)x$ ***A1***

[5 marks]

(c) $\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{Q} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$= \begin{pmatrix} \cos^2\theta + \sin^2\theta & -\cos\theta\sin\theta + \cos\theta\sin\theta \\ -\sin\theta\cos\theta + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$ ***M1A1***

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ***AG***

[2 marks]

Total [12 marks]

11. (a) $z = \dfrac{y-x}{y+x}$

$\Rightarrow \dfrac{\mathrm{d}z}{\mathrm{d}x} = \dfrac{(y+x)\left(\frac{\mathrm{d}y}{\mathrm{d}x} - 1\right) - (y-x)\left(\frac{\mathrm{d}y}{\mathrm{d}x} + 1\right)}{(y+x)^2}$ ***M1A1***

$\Rightarrow \dfrac{\mathrm{d}z}{\mathrm{d}x} = \dfrac{y\frac{\mathrm{d}y}{\mathrm{d}x} + x\frac{\mathrm{d}y}{\mathrm{d}x} - y - x - y\frac{\mathrm{d}y}{\mathrm{d}x} + x\frac{\mathrm{d}y}{\mathrm{d}x} - y + x}{(y+x)^2}$ ***A1***

$\Rightarrow \dfrac{\mathrm{d}z}{\mathrm{d}x} = \dfrac{2}{(y+x)^2}\left(x\dfrac{\mathrm{d}y}{\mathrm{d}x} - y\right)$ ***AG***

[3 marks]

(b) $f(x)\left(\dfrac{(y+x)^2}{2}\right)\dfrac{\mathrm{d}z}{\mathrm{d}x} = y^2 - x^2$ ***(M1)***

$f(x)\dfrac{\mathrm{d}z}{\mathrm{d}x} = 2\dfrac{(y-x)(y+x)}{(y+x)^2}$ ***A1***

$f(x)\dfrac{\mathrm{d}z}{\mathrm{d}x} = 2\dfrac{(y-x)}{(y+x)} = 2z$ ***AG***

[2 marks]

continued…

Question 11 continued

(c) **METHOD 1**

$$f(x)\frac{\mathrm{d}z}{\mathrm{d}x} = 2z$$

$$\frac{1}{z}\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{2}{f(x)}$$

$$\frac{1}{z}\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{2}{x-3}$$ ***M1A1***

EITHER

$\Rightarrow \ln z = 2\ln(x-3) + c$ ***A1***

when $y = 5, x = 4 \Rightarrow z = \frac{1}{9}$ ***M1***

$\Rightarrow c = \ln\frac{1}{9}$ ***A1***

$\Rightarrow \ln z = 2\ln(x-3) + \ln\frac{1}{9}$

$\Rightarrow \ln z = \ln(x-3)^2 - \ln 9$ ***A1***

$\Rightarrow \ln z = \ln\left(\frac{x-3}{3}\right)^2$ ***A1***

$\Rightarrow z = \left(\frac{x-3}{3}\right)^2$

OR

$\Rightarrow \ln z = 2\ln(x-3) + \ln c$ ***A1***

$z = c(x-3)^2$ ***M1A1***

when $y = 5, x = 4 \Rightarrow z = \frac{1}{9}$ ***M1***

$\Rightarrow c = \frac{1}{9}$ ***A1***

THEN

$\Rightarrow \frac{y-x}{y+x} = \left(\frac{x-3}{3}\right)^2$ ***AG***

continued…

Question 11 continued

METHOD 2

$f(x)\frac{\mathrm{d}z}{\mathrm{d}x} = 2z$

$f(x)\frac{\mathrm{d}z}{\mathrm{d}x} - 2z = 0$

$\frac{\mathrm{d}z}{\mathrm{d}x} - \frac{2z}{x-3} = 0$ ***M1***

integrating factor is $e^{\int \frac{-2}{x-3}\mathrm{d}x}$ ***A1***

$e^{\int \frac{-2}{x-3}\mathrm{d}x} = e^{-2\ln(x-3)}$

$= \frac{1}{(x-3)^2}$ ***A1***

hence $\frac{d}{\mathrm{d}x}\left[\frac{z}{(x-3)^2}\right] = 0$ ***M1***

$z = A(x-3)^2$ ***A1***

when $y = 5, x = 4 \Rightarrow z = \frac{1}{9}$ ***M1***

$\Rightarrow A = \frac{1}{9}$ ***A1***

$\Rightarrow \frac{y-x}{y+x} = \left(\frac{x-3}{3}\right)^2$ ***AG***

[7 marks]

Total [12 marks]

12. (a) auxiliary equation is $m^2 - 4m + 4 = 0$ ***M1A1***
hence m has a repeated root of 2 ***(A1)***
solution is of the form $u_n = a(2)^n + bn(2)^n$ ***M1***
using the initial conditions ***M1***
$\Rightarrow a = 1$ and $b = -\frac{1}{2}$
$\Rightarrow u_n = 2^n - \frac{n}{2}(2)^n$ ***A1***

[6 marks]

continued…

Question 12 continued

(b) $v_n = \frac{4}{5}\left(\frac{1}{2}\right)^n + \frac{1}{5}(3)^n$

let $n=0$ $v_0 = \frac{4}{5}\left(\frac{1}{2}\right)^0 + \frac{1}{5}(3)^0 = \frac{4}{5}+\frac{1}{5}=1$

let $n=1$ $v_1 = \frac{4}{5}\left(\frac{1}{2}\right)^1 + \frac{1}{5}(3)^1 = \frac{2}{5}+\frac{3}{5}=1$

hence true for $n=0$ and $n=1$ ***M1A1***

assume that $v_j = \frac{4}{5}\left(\frac{1}{2}\right)^j + \frac{1}{5}(3)^j$ is true for all $j<k+1$ ***M1***

hence $v_k = \frac{4}{5}\left(\frac{1}{2}\right)^k + \frac{1}{5}(3)^k$ and $v_{k-1} = \frac{4}{5}\left(\frac{1}{2}\right)^{k-1} + \frac{1}{5}(3)^{k-1}$

$$v_{k+1} = \frac{7v_k - 3v_{k-1}}{2}$$

$$v_{k+1} = \frac{7\left[\frac{4}{5}\left(\frac{1}{2}\right)^k + \frac{1}{5}(3)^k\right] - 3\left[\frac{4}{5}\left(\frac{1}{2}\right)^{k-1} + \frac{1}{5}(3)^{k-1}\right]}{2}$$ ***M1A1***

$$v_{k+1} = \frac{7\left[\frac{8}{5}\left(\frac{1}{2}\right)^{k+1} + \frac{1}{15}(3)^{k+1}\right] - 3\left[\frac{16}{5}\left(\frac{1}{2}\right)^{k+1} + \frac{1}{45}(3)^{k+1}\right]}{2}$$ ***(A1)***

$$v_{k+1} = \frac{\frac{56}{5}\left(\frac{1}{2}\right)^{k+1} + \frac{7}{15}(3)^{k+1} - \frac{48}{5}\left(\frac{1}{2}\right)^{k+1} - \frac{1}{15}(3)^{k+1}}{2}$$ ***(A1)***

$$v_{k+1} = \frac{\frac{8}{5}\left(\frac{1}{2}\right)^{k+1} + \frac{6}{15}(3)^{k+1}}{2}$$ ***(A1)***

Note: Only one of the above ***(A1)*** can be implied.

$$v_{k+1} = \frac{4}{5}\left(\frac{1}{2}\right)^{k+1} + \frac{1}{5}(3)^{k+1}$$

since the basis step and the inductive step have been verified, the Principle of Mathematical Induction tells us that $v_n = \frac{4}{5}\left(\frac{1}{2}\right)^n + \frac{1}{5}(3)^n$ is the general solution ***R1***

[9 marks]

Note: Only award final ***R1*** if at least 5 previous marks have been awarded.

Total [15 marks]

13. (a) $\begin{pmatrix} 2 & -4 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} -2k \\ -2k \end{pmatrix} \left(= -2\begin{pmatrix} k \\ k \end{pmatrix} \right)$ ***M1A1***

hence still on the line $y = x$ ***AG***

[2 marks]

(b) consider $\begin{pmatrix} 2 & -4 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 4k \\ -k \end{pmatrix}$ ***M1***

$= \begin{pmatrix} 12k \\ -3k \end{pmatrix} \left(= 3\begin{pmatrix} 4k \\ -k \end{pmatrix} \right)$ ***A1***

hence the line is invariant ***A1***

[3 marks]

(c) hence the eigenvalues are -2 and 3 ***A1A1***

[2 marks]

(d) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ or equivalent ***A1A1***

[2 marks]

Total [9 marks]

14. (a) $\bar{x} = 43.94$ ***(A1)***

unbiased variance estimate $= 466.0847$ ***(A1)***

Note: Accept sample variance $= 464.2204$.

$\Rightarrow$ 90% confidence interval is $(41.7, 46.2)$ ***A1A1***

[4 marks]

(b) Z-value is -1.87489 or -1.87866 ***(A1)***

probability is 0.0304 or 0.0301 ***(A1)***

$\Rightarrow \lambda \geq 3.01$ ***(M1)A1***

[4 marks]

Total [8 marks]

15. EITHER

attempt to differentiate ***(M1)***

let $y = 2at \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = 2a$ and $x = at^2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = 2at$ ***A1***

hence $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x} = \frac{2a}{2at} = \frac{1}{t}$ ***A1***

let P have coordinates $\left(at_1^2, 2at_1\right)$ and Q have coordinates $\left(at_2^2, 2at_2\right)$ ***(M1)***

therefore gradient of tangent at P is $\frac{1}{t_1}$ and gradient of tangent at Q is $\frac{1}{t_2}$ ***A1***

since these tangents are perpendicular $\frac{1}{t_1} \times \frac{1}{t_2} = -1 \Rightarrow t_1 t_2 = -1$ ***A1***

mid-point of PQ is $\left(\frac{a\left(t_1^2 + t_2^2\right)}{2}, a\left(t_1 + t_2\right)\right)$ ***A1***

$y^2 = a^2\left(t_1^2 + 2t_1t_2 + t_2^2\right)$ ***M1***

$y^2 = a^2\left(\frac{2x}{a} - 2\right)$ $\left(\Rightarrow y^2 = 2ax - 2a^2\right)$ ***A1***

OR

attempt to differentiate ***(M1)***

$2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a}{y}$

let coordinates of P be $\left(x_1, y_1\right)$ and the coordinates of Q be $\left(x_2, y_2\right)$ ***(M1)***

coordinates of midpoint of PQ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ ***M1***

if the tangents are perpendicular $\frac{2a}{y_1} \times \frac{2a}{y_2} = -1$ ***A1***

$\Rightarrow y_1 y_2 = -4a^2$

$y_1^2 + y_2^2 = 4a\left(x_1 + x_2\right)$ ***A1***

$\frac{y_1^2 + 2y_1y_2 + y_2^2}{4} = \frac{4a\left(x_1 + x_2\right) + 2y_1y_2}{4}$ ***M1***

$\left(\frac{y_1 + y_2}{2}\right)^2 = 2a\left(\frac{x_1 + x_2}{2}\right) - \frac{8a^2}{4}$ ***A1***

hence equation of locus is $y^2 = 2ax - 2a^2$ ***A1***

[9 marks]

Further mathematics
Higher level
Paper 2

Friday 18 May 2018 (morning)

2 hours 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[150 marks]**.

2218 – 7102

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 17]

The independent random variables X and Y are given by $X \sim \mathrm{N}\left(\mu_1, \sigma_1^2\right)$ and $Y \sim \mathrm{N}\left(\mu_2, \sigma_2^2\right)$.

(a) Write down the distribution of $aX + bY$ where $a, b \in \mathbb{R}$. [2]

(b) Two independent random variables X_1 and X_2 each have a normal distribution with a mean 3 and a variance 9. Four independent random variables Y_1, Y_2, Y_3, Y_4 each have a normal distribution with mean 2 and variance 25. Each of the variables Y_1, Y_2, Y_3, Y_4 is independent of each of the variables X_1, X_2. Find

(i) $\mathrm{P}\left(X_1 + Y_1 < 11\right)$;

(ii) $\mathrm{P}\left(3X_1 + 4Y_1 > 15\right)$;

(iii) $\mathrm{P}\left(X_1 + X_2 + Y_1 + Y_2 + Y_3 + Y_4 < 30\right)$. [10]

(c) Given that $\bar{X}$ and $\bar{Y}$ are the respective sample means, find $\mathrm{P}\left(\bar{X} > \bar{Y}\right)$. [5]

2. [Maximum mark: 17]

It is given that $(5x + y)\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x + 5y)$ and that when $x = 0$, $y = 2$.

(a) Use Euler's method with step length 0.1 to find an approximate value of y when $x = 0.4$. [5]

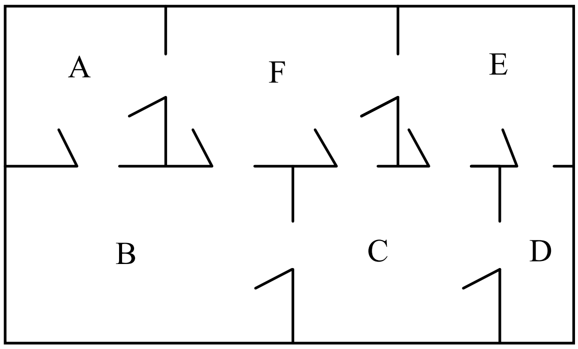
(b) (i) Show that $(5x + y)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 1 - \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$.

(ii) Show that $(5x + y)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3} = -5\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - 3\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$.

(iii) Find the Maclaurin expansion for y up to and including the term in x^3. [12]

3. [Maximum mark: 17]

While on holiday Pauline visits the local museum. On the ground floor of the museum there are six rooms, A, B, C, D, E and F. The doorways between the rooms are indicated on the following floorplan.



(a) Draw a graph G to represent this floorplan where the rooms are represented by the vertices and an edge represents a door between two rooms. [2]

(b) (i) Explain why the graph G has an Eulerian trail but not an Eulerian circuit.

(ii) Explain the consequences of having an Eulerian trail but not an Eulerian circuit, for Pauline's visit to the ground floor of the museum. [4]

(c) (i) Write down a Hamiltonian cycle for the graph G.

(ii) Explain the consequences of having a Hamiltonian cycle for Pauline's visit to the ground floor of the museum. [3]

(This question continues on the following page)

(Question 3 continued)

There are 6 museums in 6 towns in the area where Pauline is on holiday. The 6 towns and the roads connecting them can be represented by a graph. Each vertex represents a town, each edge represents a road and the weight of each edge is the distance between the towns using that road. The information is shown in the adjacency table.

Vertices	U	V	W	X	Y	Z
U	-	11	10	7	11	12
V	11	-	5	13	4	6
W	10	5	-	15	10	10
X	7	13	15	-	9	15
Y	11	4	10	9	-	7
Z	12	6	10	15	7	-

Pauline wants to visit each town and needs to start and finish in the same town.

(d) Use the nearest-neighbour algorithm to determine a possible route and an upper bound for the length of her route starting in town Z. [2]

(e) By removing Z, use the deleted vertex algorithm to determine a lower bound for the length of her route. [6]

4. [Maximum mark: 14]

(a) Draw slope fields for the following cases for $-2 \le x \le 2$, $-2 \le y \le 2$

(i) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$;

(ii) $\frac{\mathrm{d}y}{\mathrm{d}x} = x + 1$;

(iii) $\frac{\mathrm{d}y}{\mathrm{d}x} = x - 1$. [6]

(b) Explain what isoclines tell you about the slope field in each of the following cases,

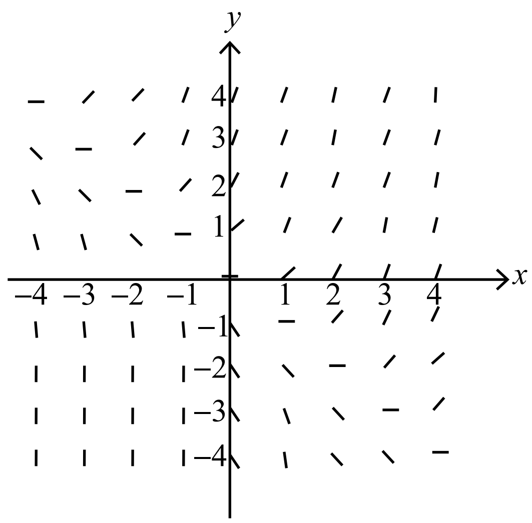
(i) $\frac{\mathrm{d}y}{\mathrm{d}x} =$ constant;

(ii) $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)$. [2]

(This question continues on the following page)

(Question 4 continued)

(c) The slope field for the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = x + y$ for $-4 \le x \le 4$, $-4 \le y \le 4$ is shown in the following diagram.



Explain why the slope field indicates that the only linear solution is $y = -x - 1$. [2]

(d) Given that all the isoclines from a slope field of a differential equation are straight lines through the origin, find two examples of the differential equation. [4]

5. [Maximum mark: 20]

Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(a) Show that the area enclosed by the ellipse is πab. [9]

(b) The area enclosed by the ellipse is 8π and $b = 2$.

(i) Determine which coordinate axis the major axis of the ellipse lies along.

(ii) Hence find the eccentricity.

(iii) Find the coordinates of the foci.

(iv) Find the equations of the directrices. [8]

(c) The centre of another ellipse is now given as the point $(2, 1)$. The minor and major axes are of lengths 3 and 5 and are parallel to the x and y axes respectively. Find the equation of the ellipse. [3]

6. [Maximum mark: 19]

The set of all integers from 0 to 99 inclusive is denoted by S. The binary operations $*$ and $\circ$ are defined on S by

$$a * b = [a + b + 20](\text{mod } 100)$$
$$a \circ b = [a + b - 20](\text{mod } 100).$$

(a) Find the identity element of S with respect to $*$. [3]

(b) Show that every element of S has an inverse with respect to $*$. [2]

(c) State which elements of S are self-inverse with respect to $*$. [2]

(d) Prove that the operation $\circ$ is not distributive over $*$. [5]

The equivalence relation R is defined by $aRb \Leftrightarrow \left(\sin\frac{\pi a}{5} = \sin\frac{\pi b}{5}\right)$.

(e) Determine the equivalence classes into which R partitions S, giving the first four elements of each class. [5]

(f) Find two elements in the same equivalence class which are inverses of each other with respect to $*$. [2]

7. [Maximum mark: 24]

(a) (i) In a triangle ABC, prove $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

(ii) Prove that the area of the triangle ABC is $\frac{1}{2}ab\sin C$.

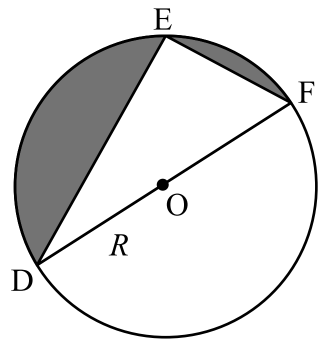
(iii) Given that R denotes the radius of the circumscribed circle prove that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

(iv) Hence show that the area of the triangle ABC is $\frac{abc}{4R}$. [10]

(This question continues on the following page)

(Question 7 continued)

(b) A new triangle DEF is positioned within a circle radius R such that DF is a diameter as shown in the following diagram.



(i) Find in terms of R, the two values of $(\mathrm{DE})^2$ such that the area of the shaded region is twice the area of the triangle DEF.

(ii) Using two diagrams, explain why there are two values of $(\mathrm{DE})^2$. [11]

(c) A parallelogram is positioned inside a circle such that all four vertices lie on the circle. Prove that it is a rectangle. [3]

8. [Maximum mark: 22]

The discrete random variable X follows a geometric distribution $\mathrm{Geo}(p)$ where

$$\mathrm{P}(X = x) = \begin{cases} pq^{x-1}, \text{ for } x = 1, 2... \\ 0, \text{ otherwise} \end{cases}.$$

(a) (i) Show that the probability generating function of X is given by

$$G(t) = \frac{pt}{1-qt}.$$

(ii) Deduce that $\mathrm{E}(X) = \dfrac{1}{p}$. [7]

(This question continues on the following page)

(Question 8 continued)

(b) Two friends A and B play a ball game with the following rules.

Each player starts with zero points. Player A serves first and then the players have alternate pairs of serves so that the service order is $\mathrm{A}, \mathrm{B}, \mathrm{B}, \mathrm{A}, \mathrm{A}, \ldots$ When player A serves, the probability of her scoring 1 point is p_A and the probability of B scoring 1 point is q_A, where $q_A = 1 - p_A$.

When player B serves, the probability of her scoring 1 point is p_B and the probability of A scoring 1 point is q_B, where $q_B = 1 - p_B$.

Show that, after the first 6 serves, the probability that each player has 3 points is

$$\sum_{x=0}^{x=3} \binom{3}{x}^2 (p_A)^x (p_B)^x (q_A)^{3-x} (q_B)^{3-x}.$$ [5]

(c) After 6 serves the score is 3 points each. Play continues and the game ends when one player has scored two more points than the other player. Let N be the number of further serves required before the game ends. Given that $p_A = 0.7$ and $p_B = 0.6$ find $\mathrm{P}(N = 2)$. [3]

(d) Let $M = \frac{1}{2}N$. Show that M has a geometric distribution and hence find the value of $\mathrm{E}(N)$. [7]

M18/5/FURMA/HP2/ENG/TZ0/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Further mathematics

Higher level

Paper 2

18 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document "**Mathematics HL: Guidance for e-marking May 2018**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = \left(2\cos(5x-3)\right)5 \ \left(=10\cos(5x-3)\right)$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

Note: In question 1, accept answers that round correctly to 2 significant figures.

1. (a) $aX + bY \sim \mathrm{N}\left(a\mu_1 + b\mu_2\ ,\ a^2\sigma_1^2 + b^2\sigma_2^2\right)$ ***A1A1***

Note: ***A1*** for N and the mean, ***A1*** for the variance.

[2 marks]

(b) (i) $X_1 + Y_1 \sim \mathrm{N}(5, 34)$ ***(A1)(A1)***

$\Rightarrow \mathrm{P}(X_1 + Y_1 < 11) = 0.848$ ***A1***

(ii) $3X_1 + 4Y_1 \sim \mathrm{N}(9 + 8\ ,\ 9 \times 9 + 16 \times 25)$ ***(A1)(M1)(A1)***

Note: Award ***(A1)*** for correct expectation, ***(M1)(A1)*** for correct variance.

$\sim \mathrm{N}(17, 481)$

$\Rightarrow \mathrm{P}(3X_1 + 4Y_1 > 15) = 0.536$ ***A1***

(iii) $X_1 + X_2 + Y_1 + Y_2 + Y_3 + Y_4 \sim \mathrm{N}(6 + 8,\ 2 \times 9 + 4 \times 25)$ ***(A1)(A1)***

$\sim \mathrm{N}(14, 118)$

$\Rightarrow \mathrm{P}(X_1 + X_2 + Y_1 + Y_2 + Y_3 + Y_4 < 30) = 0.930$ ***A1***

[10 marks]

(c) consider $\bar{X} - \bar{Y}$ ***(M1)***

$\mathrm{E}(\bar{X} - \bar{Y}) = 3 - 2 = 1$ ***A1***

$\mathrm{Var}(\bar{X} - \bar{Y}) = \frac{9}{2} + \frac{25}{4} (= 10.75)$ ***(M1)A1***

$\Rightarrow \mathrm{P}(\bar{X} - \bar{Y} > 0) = 0.620$ ***A1***

[5 marks]

Total [17 marks]

2. (a) Euler's method with step length $h=0.1$ to find y when $x=0.4$

x	y	$\frac{dy}{dx}$	$h\frac{dy}{dx}$	$y+h\frac{dy}{dx}$
0	2	5	0.5	2.5
0.1	2.5	4.2	0.42	2.92
0.2	2.92	3.7755	0.37755	3.29755
0.3	3.29755	3.49923	0.349923	3.64747
0.4	3.64747			

Note: Accept 3 significant figures in the table.

first line of table ***(M1)(A1)***
line 2 ***(A1)***
line 3 ***(A1)***
hence $y=3.65$ ***A1***

Note: Accept any answer that rounds to 3.65.

[5 marks]

(b) (i) $(5x+y)\frac{dy}{dx}=x+5y$

$$\left(5+\frac{dy}{dx}\right)\frac{dy}{dx}+(5x+y)\frac{d^2y}{dx^2}=1+5\frac{dy}{dx}$$ ***M1A1A1***

Note: Award ***M1*** for a valid attempt to differentiate, ***A1*** for LHS, ***A1*** for RHS.

$$(5x+y)\frac{d^2y}{dx^2}=1+5\frac{dy}{dx}-5\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^2$$

$$(5x+y)\frac{d^2y}{dx^2}=1-\left(\frac{dy}{dx}\right)^2$$ ***AG***

(ii) $(5x+y)\frac{d^2y}{dx^2}=1-\left(\frac{dy}{dx}\right)^2$

$$\left(5+\frac{dy}{dx}\right)\frac{d^2y}{dx^2}+(5x+y)\frac{d^3y}{dx^3}=-2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$$ ***M1A1A1A1***

$$(5x+y)\frac{d^3y}{dx^3}=-2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)-5\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$$

$$(5x+y)\frac{d^3y}{dx^3}=-5\frac{d^2y}{dx^2}-3\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$$ ***AG***

continued…

Question 2 continued

(iii) when $x = 0$ $y = 2$

when $x = 0$ $\frac{dy}{dx} = 5$ ***A1***

when $x = 0$ $\frac{d^2y}{dx^2} = -12$ ***A1***

when $x = 0$ $\frac{d^3y}{dx^3} = 120$ ***A1***

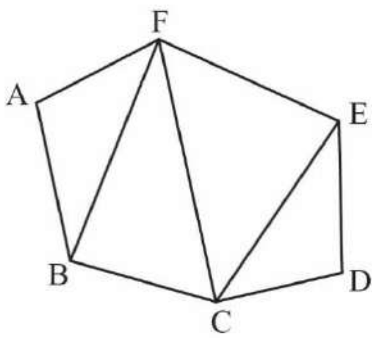
Note: Allow follow through from incorrect values of derivatives.

$y = 2 + 5x - 6x^2 + 20x^3$ ***M1A1***

[12 marks]

Total [17 marks]

3. (a)



A2

[2 marks]

(b) (i) two vertices are of odd degree ***A1***
to have an Eulerian circuit it must have all vertices of even degree ***R1***
hence no Eulerian circuit, but an Eulerian trail ***AG***

(ii) it allows Pauline to go through every door once (provided she starts in room B or room E) ***A1***
and she cannot return to the room in which she started ***A1***

[4 marks]

(c) (i) for example: $A \to F \to E \to D \to C \to B \to A$ ***A2***

Note: Award ***A1*** if the cycle does not return to the start vertex.

(ii) she can visit every room once without repeating and return to the start ***A1***

[3 marks]

(d) $Z \to V \to Y \to X \to U \to W \to Z$ ***A1***
$6 + 4 + 9 + 7 + 10 + 10 = 46$ ***A1***

[2 marks]

continued…

Question 3 continued

(e) attempt to find the minimal spanning tree ***(M1)***
VY
VW
UX
XY ***A2***

> **Note:** Award ***A1*** if one error made.

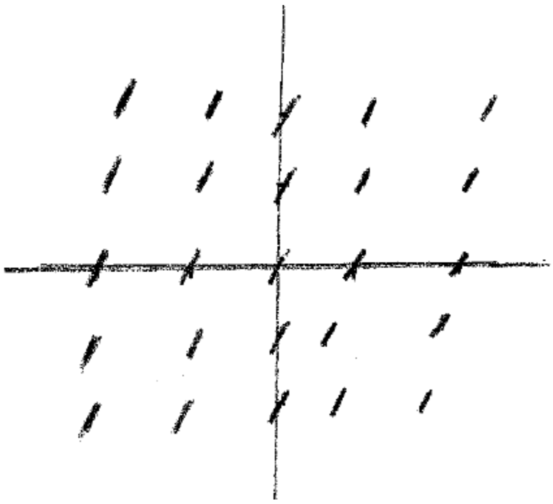
> **Note**: Accept correct drawing of minimal spanning tree.

weight of minimal spanning tree $= 4+5+7+9=25$ ***(A1)***
since Z is removed, we add on VZ and ZY ***(M1)***
hence lower bound for route is $25+13=38$ ***A1***

[6 marks]

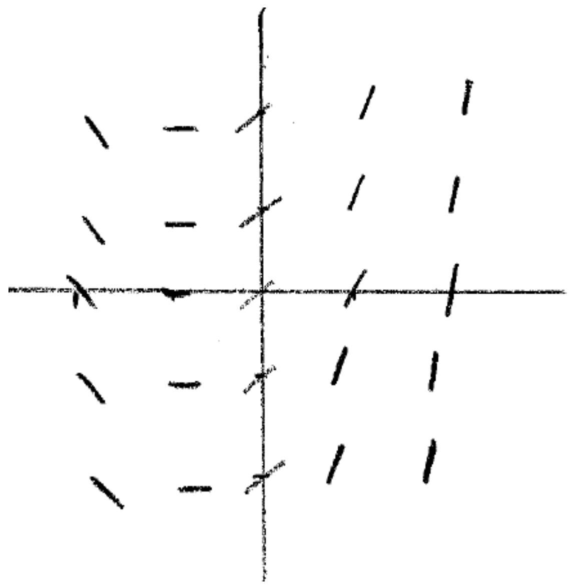
Total [17 marks]

4. (a) (i)



A2

(ii)

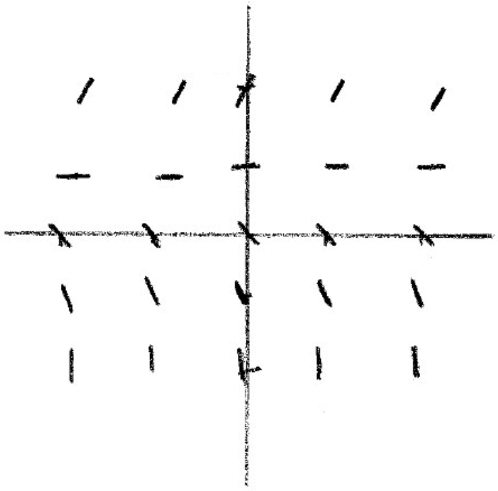


A2

continued…

Question 4 continued

(iii)



A2
[6 marks]

(b) (i) the slope is the same everywhere ***A1***

(ii) all points that have the same x coordinate have the same slope ***A1***
[2 marks]

(c) this is where a straight line appears on the slope field ***A1***
There is no other straight line, all the other solutions are curves ***A1***
[2 marks]

(d) given $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$, the isoclines are $f(x, y) = k$ ***(M1)***

here the isoclines are $y = kx$ (or $x = ky$) ***(A1)***

any two differential equations of the correct form, for example
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{ky}{x}, \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{kx}{y}, \frac{\mathrm{d}y}{\mathrm{d}x} = \sin\left(\frac{y}{x}\right), \frac{\mathrm{d}y}{\mathrm{d}x} = \sin\left(\frac{x}{y}\right)$ ***A1A1***

[4 marks]

Total [14 marks]

5. (a) $A=4\int y\mathrm{d}x$ ***(M1)***

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\Rightarrow$

$y=\frac{b\sqrt{a^2-x^2}}{a}$ ***(A1)***

let $x=a\cos\theta\Rightarrow y=b\sin\theta$ ***M1***

$\frac{dx}{d\theta}=-a\sin\theta$ ***A1***

when $x=0$, $\theta=\frac{\pi}{2}$. When $x=a$, $\theta=0$ ***A1***

$\Rightarrow A=4\int_{\frac{\pi}{2}}^{0}b\sin\theta\left(-a\sin\theta\right)\mathrm{d}\theta$ ***M1***

$\Rightarrow A=-4ab\int_{\frac{\pi}{2}}^{0}\sin^2\theta\mathrm{d}\theta$

$\Rightarrow A=-2ab\int_{\frac{\pi}{2}}^{0}\left(1-\cos 2\theta\right)\mathrm{d}\theta$ ***M1***

$\Rightarrow A=-2ab\left[\theta-\frac{\sin 2\theta}{2}\right]_{\frac{\pi}{2}}^{0}$ ***A1***

$\Rightarrow A=-2ab\left[0-0-\left(\frac{\pi}{2}-0\right)\right]$ ***M1***

$\Rightarrow A=\pi ab$ ***AG***

[9 marks]

(b) (i) $b=2$

hence $2\pi a=8\pi\Rightarrow a=4$ ***A1***

hence major axis lies along the x-axis ***A1***

(ii) $b^2=a^2\left(1-e^2\right)$ ***(M1)***

$4=16\left(1-e^2\right)\Rightarrow e=\frac{\sqrt{3}}{2}$ ***A1***

(iii) coordinates of foci are $\left(\pm ae,\,0\right)=\left(2\sqrt{3},\,0\right),\left(-2\sqrt{3},\,0\right)$ ***A1A1***

(iv) equations of directrices are $x=\pm\frac{a}{e}=\frac{8}{\sqrt{3}},-\frac{8}{\sqrt{3}}$ ***A1A1***

[8 marks]

continued…

Question 5 continued

(c) $a=\frac{3}{2}, b=\frac{5}{2}$ ***(A1)***

hence equation is $\frac{4}{9}(x-2)^2+\frac{4}{25}(y-1)^2=1$ ***M1A1***

[3 marks]

Total [20 marks]

6. (a) $a+e+20=a(\text{mod}100)$ ***(M1)***

$e=-20(\text{mod}100)$ ***(A1)***

$e=80$ ***A1***

[3 marks]

(b) $a+a^{-1}+20=80(\text{mod}100)$ ***(M1)***

inverse of a is $60-a(\text{mod}100)$ ***A1***

[2 marks]

(c) 30 and 80 ***A1A1***

[2 marks]

(d) $a\circ(b*c)=a\circ(b+c+20)(\text{mod}100)$

$=a+(b+c+20)-20(\text{mod}100)$ ***(M1)***

$=a+b+c(\text{mod}100)$ ***A1***

$(a\circ b)*(a\circ c)=(a+b-20)*(a+c-20)(\text{mod}100)$ ***M1***

$=a+b-20+a+c-20+20(\text{mod}100)$

$=2a+b+c-20(\text{mod}100)$ ***A1***

hence we have shown that $a\circ(b*c)\neq(a\circ b)*(a\circ c)$ ***R1***

hence the operation $\circ$ is not distributive over $*$ ***AG***

Note: Accept a counterexample.

[5 marks]

(e) $\{0,5,10,15...\}$ ***A1***

$\{1,4,11,14...\}$ ***A1***

$\{2,3,12,13...\}$ ***A1***

$\{6,9,16,19...\}$ ***A1***

$\{7,8,17,18...\}$ ***A1***

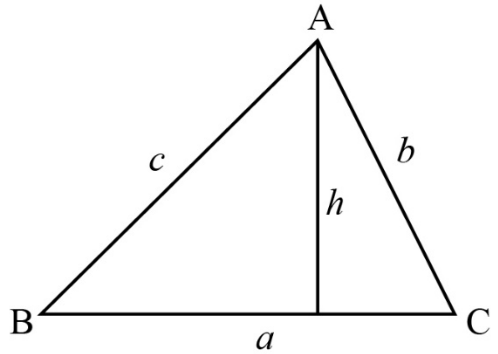
[5 marks]

(f) for example 10 and 50, 20 and 40, 0 and 60… ***A2***

[2 marks]

Total [19 marks]

7. (a) (i)



$\sin \mathrm{B} = \frac{h}{c}$ and $\sin \mathrm{C} = \frac{h}{b}$ ***M1A1***

hence $h = c \sin \mathrm{B} = b \sin \mathrm{C}$ ***A1***

by dropping a perpendicular from B, in exactly the same way we find

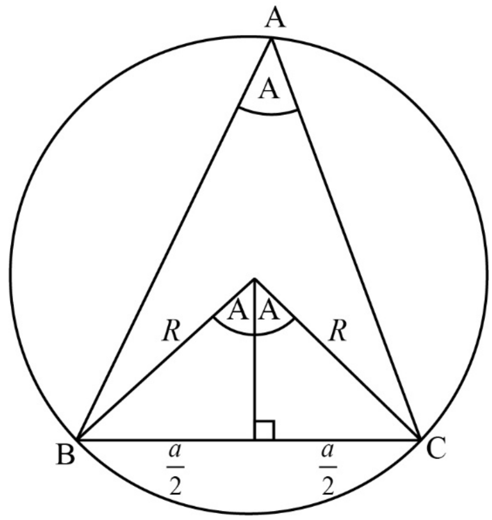
$c \sin\mathrm{A} = a \sin \mathrm{C}$ ***R1***

hence $\frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$

(ii) area $= \frac{1}{2} ah$ ***M1A1***

$= \frac{1}{2} ab \sin \mathrm{C}$ ***AG***

(iii)



since the angle at the centre of circle is twice the angle at the circumference $\sin \mathrm{A} = \frac{a}{2R}$ ***M1A1***

hence $\frac{a}{\sin \mathrm{A}} = 2R$ and therefore $\frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}} = 2R$ ***AG***

continued…

Question 7 continued

(iv) area of the triangle is $\frac{1}{2}ab\sin\mathrm{C}$ ***M1***

since $\sin\mathrm{C}=\frac{c}{2R}$ ***A1***

area of the triangle is $\frac{1}{2}ab\frac{c}{2R}=\frac{abc}{4R}$ ***AG***

[10 marks]

(b) (i) area of the triangle is $\frac{\pi R^2}{6}$ ***(M1)A1***

$(\mathrm{DE})^2+(\mathrm{EF})^2=4R^2$ ***M1***

$(\mathrm{DE})^2=4R^2-(\mathrm{EF})^2$

$\frac{1}{2}(\mathrm{DE})(\mathrm{EF})=\frac{\pi R^2}{6}\Rightarrow(\mathrm{EF})=\frac{\pi R^2}{3(\mathrm{DE})}$ ***M1A1***

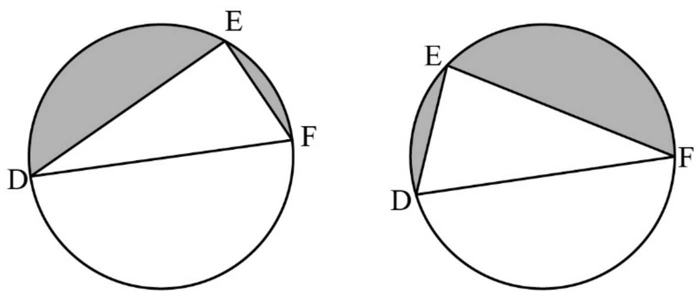
$(\mathrm{DE})^2=4R^2-\frac{\pi^2R^4}{9(\mathrm{DE})^2}$ ***A1***

$9(\mathrm{DE})^4-36(\mathrm{DE})^2R^2+\pi^2R^4=0$ ***A1***

$(\mathrm{DE})^2=\frac{36R^2\pm\sqrt{1296R^4-36\pi^2R^4}}{18}$ ***M1***

$(\mathrm{DE})^2=\frac{36R^2\pm6R^2\sqrt{36-\pi^2}}{18}\left(=\frac{6R^2\pm R^2\sqrt{36-\pi^2}}{3}\right)$ ***A1***

(ii)



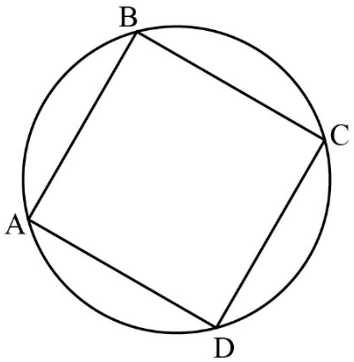
A1A1

[11 marks]

continued…

Question 7 continued

(c)



$\hat{A}+\hat{C}=180^\circ$ (cyclic quadrilateral) ***R1***

however $\hat{A}=\hat{C}$ (ABCD is a parallelogram) ***R1***

$\hat{A}=\hat{C}=90^\circ$ ***A1***

similarly $\hat{B}=\hat{D}=90^\circ$

hence ABCD is a rectangle ***AG***

[3 marks]

Total [24 marks]

8. (a) (i) $\mathrm{P}(X=x)=pq^{x-1}$ for $x=1,2...$

$$G(t)=\sum_{x=1}^{\infty}t^x pq^{x-1}$$ ***M1***

$$=pt\sum_{x=1}^{\infty}(tq)^{x-1}$$ ***A1***

$$=pt(1+tq+(tq)^2...)$$ ***M1***

$$=\frac{pt}{1-tq}$$ ***AG***

(ii) $$G'(t)=\frac{(1-tq)p-pt(-q)}{(1-tq)^2}$$ ***M1A1***

$$\mathrm{E}(X)=G'(1)$$ ***M1***

$$=\frac{(1-q)p+pq}{(1-q)^2}$$ ***A1***

$$=\frac{1}{p}$$ ***AG***

[7 marks]

continued…

Question 8 continued

(b) after 6 serves (3 serves each) we have $ABBAAB$

A serves	B serves		
3 wins	0 losses	$p_1 = {}^3C_3 p_A^3 q_A^0 \, {}^3C_0 p_B^3 q_B^0$	***M1A1***
2 wins	1 loss	$p_2 = {}^3C_2 p_A^2 q_A^1 \, {}^3C_1 p_B^2 q_B^1$	***A1***
1 win	2 losses	$p_3 = {}^3C_1 p_A^1 q_A^2 \, {}^3C_2 p_B^1 q_B^2$	***A1***
0 wins	3 losses	$p_4 = {}^3C_0 p_A^0 q_A^3 \, {}^3C_3 p_B^0 q_B^3$	***A1***

since ${}^3C_0 = {}^3C_3$, ${}^3C_1 = {}^3C_2$

$\sum_{x=0}^{x=3} \binom{3}{x}^2 (p_A)^x (p_B)^x (q_A)^{3-x} (q_B)^{3-x}$ ***AG***

[5 marks]

(c) for $N = 2$ serves are B, A respectively

$\mathrm{P}(N = 2) = \mathrm{P}(B \text{ wins twice}) + \mathrm{P}(A \text{ wins twice})$ ***(M1)***

$= 0.6 \times 0.3 + 0.4 \times 0.7$ ***A1***

$= 0.46$ ***A1***

[3 marks]

(d) for $M = \frac{1}{2}N$

$\mathrm{P}(M = 1) = \mathrm{P}(N = 2) = p_M$ ***M1***

$\mathrm{P}(M = 2) = \mathrm{P}(N = 4)$

$= \mathrm{P}\begin{pmatrix}\text{game does not end after} \\ \text{first two serves}\end{pmatrix} \times \mathrm{P}\begin{pmatrix}\text{game ends after} \\ \text{next two serves}\end{pmatrix} = (1 - p_M)\, p_M$ ***A1***

similarly $\mathrm{P}(M = 3) = (1 - p_M)^2\, p_M$ ***(A1)***

hence $\mathrm{P}(M = r) = (1 - p_M)^{r-1}\, p_M$ ***A1***

hence M has a geometric distribution ***AG***

$\mathrm{P}(M = 1) = \mathrm{P}(N = 2) = p_M = 0.46$ ***A1***

hence $\mathrm{E}(M) = \frac{1}{p} = \frac{1}{0.46} = 2.174$

$\mathrm{E}(N) = \mathrm{E}(2M) = 2\mathrm{E}(M)$ ***M1***

$= 4.35$ ***A1***

[7 marks]

Total [22 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Monday 12 November 2018 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider two events, A and B, such that $\mathrm{P}(A) = \mathrm{P}(A' \cap B) = 0.4$ and $\mathrm{P}(A \cap B) = 0.1$.

(a) By drawing a Venn diagram, or otherwise, find $\mathrm{P}(A \cup B)$. [3]

(b) Show that the events A and B are not independent. [3]

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2. [Maximum mark: 5]

A team of four is to be chosen from a group of four boys and four girls.

(a) Find the number of different possible teams that could be chosen. [3]

(b) Find the number of different possible teams that could be chosen, given that the team must include at least one girl and at least one boy. [2]

3. [Maximum mark: 7]

Consider the function $g(x) = 4\cos x + 1$, $a \le x \le \frac{\pi}{2}$ where $a < \frac{\pi}{2}$.

(a) For $a = -\frac{\pi}{2}$, sketch the graph of $y = g(x)$. Indicate clearly the maximum and minimum values of the function. [3]

(b) Write down the least value of a such that g has an inverse. [1]

(c) For the value of a found in part (b),

(i) write down the domain of g^{-1};

(ii) find an expression for $g^{-1}(x)$. [3]

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4. [Maximum mark: 7]

Consider the following system of equations where $a \in \mathbb{R}$.

$$\begin{gathered} 2x + 4y - z = 10 \\ x + 2y + az = 5 \\ 5x + 12y = 2a. \end{gathered}$$

(a) Find the value of a for which the system of equations does not have a unique solution. [2]

(b) Find the solution of the system of equations when $a = 2$. [5]

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5. [Maximum mark: 6]

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined by $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$, where $t \in \mathbb{R}$.

(a) Find and simplify an expression for $\boldsymbol{a} \bullet \boldsymbol{b}$ in terms of t. [2]

(b) Hence or otherwise, find the values of t for which the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is obtuse. [4]

6. [Maximum mark: 6]

Use mathematical induction to prove that $\sum_{r=1}^{n} r(r!) = (n+1)! - 1$, for $n \in \mathbb{Z}^+$.

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7. [Maximum mark: 6]

Consider the curves C_1 and C_2 defined as follows

$$C_1 : xy = 4 \, , \, x > 0$$
$$C_2 : y^2 - x^2 = 2 \, , \, x > 0$$

(a) Using implicit differentiation, or otherwise, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for each curve in terms of x and y. [4]

Let $\mathrm{P}(a\, ,\, b)$ be the unique point where the curves C_1 and C_2 intersect.

(b) Show that the tangent to C_1 at P is perpendicular to the tangent to C_2 at P. [2]

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8. [Maximum mark: 7]

Consider the equation $z^4 + az^3 + bz^2 + cz + d = 0$, where $a, b, c, d \in \mathbb{R}$ and $z \in \mathbb{C}$.
Two of the roots of the equation are $\log_2 6$ and $i\sqrt{3}$ and the sum of all the roots is $3 + \log_2 3$.
Show that $6a + d + 12 = 0$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider a triangle OAB such that O has coordinates $(0, 0, 0)$, A has coordinates $(0, 1, 2)$ and B has coordinates $(2b, 0, b-1)$ where $b < 0$.

(a) Find, in terms of b, a Cartesian equation of the plane Π containing this triangle. [5]

Let M be the midpoint of the line segment [OB].

(b) Find, in terms of b, the equation of the line L which passes through M and is perpendicular to the plane Π. [3]

(c) Show that L does not intersect the y-axis for any negative value of b. [7]

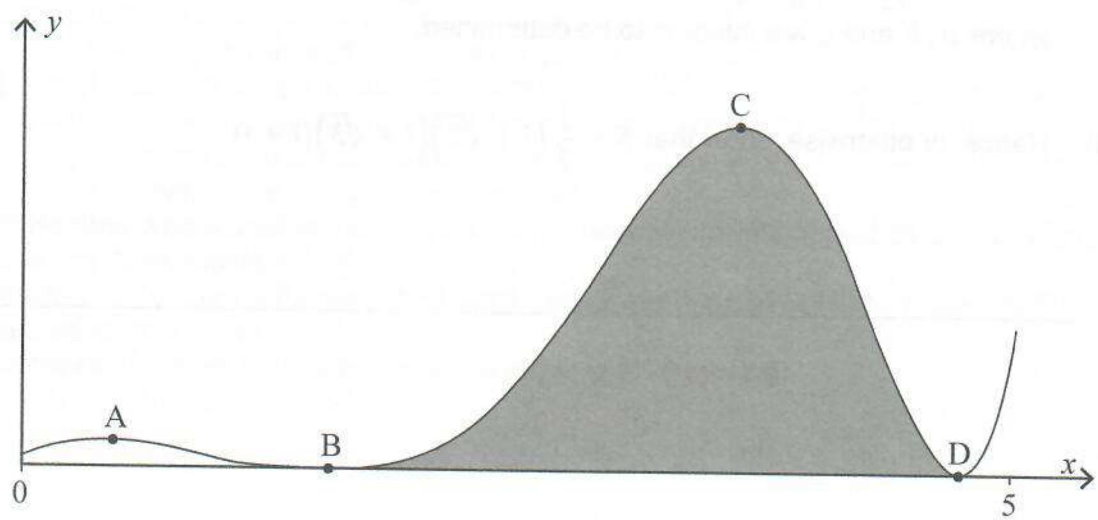
Do **not** write solutions on this page.

10. [Maximum mark: 19]

(a) Use integration by parts to show that $\int e^x \cos 2x dx = \frac{2e^x}{5}\sin 2x + \frac{e^x}{5}\cos 2x + c,\ c \in \mathbb{R}$. [5]

(b) Hence, show that $\int e^x \cos^2 x dx = \frac{e^x}{5}\sin 2x + \frac{e^x}{10}\cos 2x + \frac{e^x}{2} + c,\ c \in \mathbb{R}$. [3]

The function f is defined by $f(x) = e^x \cos^2 x$, where $0 \le x \le 5$. The curve $y = f(x)$ is shown on the following graph which has local maximum points at A and C and touches the x-axis at B and D.



(c) Find the x-coordinates of A and of C, giving your answers in the form $a + \arctan b$, where $a, b \in \mathbb{R}$. [6]

(d) Find the area enclosed by the curve and the x-axis between B and D, as shaded on the diagram. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 16]

(a) Find the roots of $z^{24} = 1$ which satisfy the condition $0 < \arg(z) < \frac{\pi}{2}$, expressing your answers in the form $re^{i\theta}$, where $r, \theta \in \mathbb{R}^+$. [5]

(b) Let S be the sum of the roots found in part (a).

(i) Show that $\operatorname{Re} S = \operatorname{Im} S$.

(ii) By writing $\frac{\pi}{12}$ as $\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$, find the value of $\cos\frac{\pi}{12}$ in the form $\frac{\sqrt{a} + \sqrt{b}}{c}$, where a, b and c are integers to be determined.

(iii) Hence, or otherwise, show that $S = \frac{1}{2}\left(1 + \sqrt{2}\right)\left(1 + \sqrt{3}\right)(1 + i)$. [11]

HL Paper 1, NOV. 2018(Solved by Er. Deepak Online Tutor)

07 January 2019 17:21

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

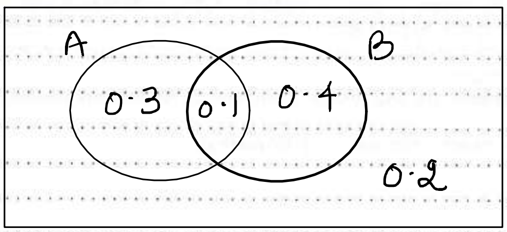
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider two events, A and B, such that $\mathrm{P}(A) = \mathrm{P}(A' \cap B) = 0.4$ and $\mathrm{P}(A \cap B) = 0.1$.

(a) By drawing a Venn diagram, or otherwise, find $\mathrm{P}(A \cup B)$. [3]

(b) Show that the events A and B are not independent. [3]



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$P(A \cup B) = 0\cdot3 + 0\cdot1 + 0\cdot4 = 0\cdot8$

$P(A \cap B) = 0\cdot1$

$P(A) \times P(B) = 0\cdot4 \times 0\cdot5 = 0\cdot2$

$P(A \cap B) \neq P(A) \times P(B)$ so Not Independent

2. [Maximum mark: 5]

A team of four is to be chosen from a group of four boys and four girls.

(a) Find the number of different possible teams that could be chosen. [3]

(b) Find the number of different possible teams that could be chosen, given that the team must include at least one girl and at least one boy. [2]

(a) 4B , 4G

B	G	
0	4	$\rightarrow {}^4C_0 \times {}^4C_4 = 1 \times 1 = 1$
1	3	$\rightarrow {}^4C_1 \times {}^4C_3 = 4 \times 4 = 16$
2	2	$\rightarrow {}^4C_2 \times {}^4C_2 = 6 \times 6 = 36$
3	1	$\rightarrow {}^4C_3 \times {}^4C_1 = 4 \times 4 = 16$
4	0	$\rightarrow {}^4C_4 \times {}^4C_0 = 1 \times 1 = 1$

So total $= 1 + 16 + 36 + 16 + 1 = 70$

(b)

B	G	
1	3	$\rightarrow 16$
2	2	$\rightarrow 36$
3	1	$\rightarrow 16$

$16 + 36 + 16 = 68$

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3. [Maximum mark: 7]

Consider the function $g(x) = 4\cos x + 1$, $a \le x \le \frac{\pi}{2}$ where $a < \frac{\pi}{2}$.

(a) For $a = -\frac{\pi}{2}$, sketch the graph of $y = g(x)$. Indicate clearly the maximum and minimum values of the function. [3]

(b) Write down the least value of a such that g has an inverse. [1]

(c) For the value of a found in part (b),

(i) write down the domain of g^{-1};

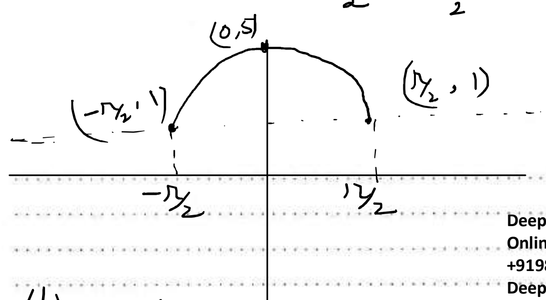
(ii) find an expression for $g^{-1}(x)$. [3]

(a) $g(x) = 4\cos x + 1 \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

min value $\cos x = -1 \Rightarrow [g(x)]_{min} = -4+1 = -3$

max value $\cos x = 1 \Rightarrow [g(x)]_{max} = 4+1 = 5$

principal axis $= \frac{5-3}{2} = \frac{2}{2} = 1$



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(b) $a = 0$

(c) Domain of $g^{-1} = [1,5]$

$y = 4\cos x + 1$

$x \leftrightarrow y \quad x = 4\cos y + 1 \Rightarrow 4\cos y = x-1$

$\cos y = \frac{x-1}{4} \Rightarrow y = \cos^{-1}\left(\frac{x-1}{4}\right)$

$g^{-1}(x) = \cos^{-1}\left(\frac{x-1}{4}\right)$

4. [Maximum mark: 7]

Consider the following system of equations where $a \in \mathbb{R}$.

$$2x + 4y - z = 10$$
$$x + 2y + az = 5$$
$$5x + 12y = 2a.$$

(a) Find the value of a for which the system of equations does not have a unique solution. [2]

(b) Find the solution of the system of equations when $a = 2$. [5]

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(a) M-I

$$\begin{vmatrix} 2 & 4 & -1 \\ 1 & 2 & a \\ 5 & 12 & 0 \end{vmatrix} = 0$$

$$2(0-12a) - 4(0-5a) - 1(12-10) = 0$$

$$-24a + 20a - 2 = 0$$

$$-4a = 2 \Rightarrow a = -\frac{1}{2}$$

M-II

$$\begin{bmatrix} 2 & 4 & -1 \\ 1 & 2 & a \\ 5 & 12 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 2a \end{bmatrix}$$

$R_2 \to 2R_2 - R_1$, $R_3 \to 2R_3 - 5R_1$

$$\begin{bmatrix} 2 & 4 & -1 \\ 0 & 0 & 2a+1 \\ 0 & 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 4a-50 \end{bmatrix}$$

From second row

$$(2a+1)z = 0 \Rightarrow 2a+1 = 0 \Rightarrow a = -\frac{1}{2}$$

(b)

$$a = 2$$

$a = 2$

$$\begin{bmatrix} 2 & 4 & -1 \\ 0 & 0 & 5 \\ 0 & 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -42 \end{bmatrix}$$

$$2x + 4y - z = 10$$

$$5z = 0 \Rightarrow z = 0$$

$$4y + 5z = -42$$

$$4y = -42 \Rightarrow y = -\frac{21}{2}$$

$$2x - 42 - 0 = 10$$

$$2x = 52 \Rightarrow x = 26$$

5. [Maximum mark: 6]

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined by $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$, where $t \in \mathbb{R}$.

(a) Find and simplify an expression for $\boldsymbol{a} \cdot \boldsymbol{b}$ in terms of t. [2]

(b) Hence or otherwise, find the values of t for which the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is obtuse. [4]

$a \cdot b = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$

$= 1 \times 0 + 1 \times (-t) + t \times 4t$

$= 4t^2 - t$

$a \cdot b = |a||b| \cos\theta$

$\theta \rightarrow$ obtuse $\Rightarrow \cos\theta = -ve$

$a \cdot b < 0$

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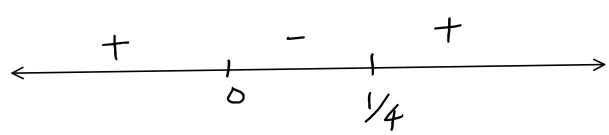
So $4t^2 - t < 0$

$t(4t-1) < 0$



$\boxed{0 < t < \frac{1}{4}}$

6. [Maximum mark: 6]

Use mathematical induction to prove that $\sum_{r=1}^{n} r(r!) = (n+1)! - 1$, for $n \in \mathbb{Z}^+$.

Let $P_n = \sum_{r=1}^{n} r(r!) = (n+1)! - 1$

$n = 1$

$P_1 = \sum_{r=1}^{1} 1(1!) = (1+1)! - 1$

$1 = 2! - 1 = 2 - 1 = 1$ True

$n = k$ $\quad P_k = \sum_{r=1}^{k} r(r!) = (k+1)! - 1$

$n = k+1$

$P_{k+1} = \sum_{r=1}^{k+1} r(r!)$

$= \sum_{r=1}^{k} r(r!) + (k+1)(k+1)!$

$= (k+1)! - 1 + (k+1)(k+1)!$

$= (k+1)![1 + k + 1] - 1$

$= (k+1)!(k+2) - 1$

$= (k+2)! - 1$ = RHS proved

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7. [Maximum mark: 6]

Consider the curves C_1 and C_2 defined as follows

$$C_1 : xy = 4,\ x > 0$$
$$C_2 : y^2 - x^2 = 2,\ x > 0$$

(a) Using implicit differentiation, or otherwise, find $\frac{dy}{dx}$ for each curve in terms of x and y. [4]

Let $P(a, b)$ be the unique point where the curves C_1 and C_2 intersect.

(b) Show that the tangent to C_1 at P is perpendicular to the tangent to C_2 at P. [2]

(a) $C_1 : xy = 4,\ x>0$
differentiating w.r.t x

$x\frac{dy}{dx} + y \cdot 1 = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

$C_2 : y^2 - x^2 = 2,\ x>0$

$2y\frac{dy}{dx} - 2x = 0 \Rightarrow \frac{dy}{dx} = \frac{x}{y}$

(b) $C_1 \equiv \left(\frac{dy}{dx}\right)_{(a,b)} = -\frac{b}{a}$

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$C_2 \equiv \left(\frac{dy}{dx}\right)_{(a,b)} = \frac{a}{b}$

Here $\left(\frac{dy}{dx}\right)_{(a,b)C_1} \times \left(\frac{dy}{dx}\right)_{(a,b)C_2} = -\frac{b}{a} \times \frac{a}{b} = -1$

Here $m_1 m_2 = -1$
So tangents are $\perp$ to each other.

8. [Maximum mark: 7]

Consider the equation $z^4 + az^3 + bz^2 + cz + d = 0$, where $a, b, c, d \in \mathbb{R}$ and $z \in \mathbb{C}$.
Two of the roots of the equation are $\log_2 6$ and $i\sqrt{3}$ and the sum of all the roots is $3 + \log_2 3$.
Show that $6a + d + 12 = 0$.

Sum of the roots $= -a$

One root is $i\sqrt{3}$ so other should be $-i\sqrt{3}$
fourth root $= P$

$\log_2 6 + \cancel{i\sqrt{3}} - \cancel{i\sqrt{3}} + P = 3 + \log_2 3$

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$P = 3 + \log_2 3 - \log_2 6$

$= 3 + \log_2\left(\frac{3}{6}\right) = 3 + \log_2 (2)^{-1}$

$= 3 - \log_2 2 = 3 - 1 = 2$

Product of the roots $= d$

$(\log_2 6) \times i\sqrt{3} \times (-i\sqrt{3}) \times 2 = d$

$3 \times 2 \times \log_2 6 = d \Rightarrow d = 6\log_2 6$

So $6a + d + 12$

$= 6(-3 - \log_2 3) + 6\log_2 6 + 12$

$= -18 - 6\log_2 3 + 6\log_2 6 + 12$

$= -6 - 6(\log_2 3 - \log_2 6)$

$= -6 - 6(\log_2 (2)^{-1}) = -6 + 6 = 0$

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider a triangle OAB such that O has coordinates $(0, 0, 0)$, A has coordinates $(0, 1, 2)$ and B has coordinates $(2b, 0, b-1)$ where $b<0$.

(a) Find, in terms of b, a Cartesian equation of the plane Π containing this triangle. [5]

Let M be the midpoint of the line segment [OB].

(b) Find, in terms of b, the equation of the line L which passes through M and is perpendicular to the plane Π. [3]

(c) Show that L does not intersect the y-axis for any negative value of b. [7]

(a) $O(0,0,0)$, $A(0,1,2)$, $B(2b,0,b-1)$ $\quad b<0$

$$\vec{OA} = \begin{pmatrix}0\\1\\2\end{pmatrix}, \quad \vec{OB} = \begin{pmatrix}2b\\0\\b-1\end{pmatrix}$$

$$\vec{n} = \vec{OA}\times\vec{OB} = \begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 2b & 0 & b-1 \end{vmatrix}$$

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$$= i(b-1-0) - j(0-4b) + k(0-2b)$$
$$= (b-1)i + 4bj - 2bk$$

plane $\pi \Rightarrow$ $\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}b-1\\4b\\-2b\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}\cdot\begin{pmatrix}b-1\\4b\\-2b\end{pmatrix}$$

$$(b-1)x + 4by - 2bz = 0$$

(b) $M \rightarrow$ mid point of OB

$$\left(\frac{2b}{2}, \frac{0}{2}, \frac{b-1}{2}\right) = \left(b, 0, \frac{b-1}{2}\right)$$

Line $L \rightarrow$

$$\frac{x-b}{b-1} = \frac{y-0}{4b} = \frac{z-\left(\frac{b-1}{2}\right)}{-2b}$$

for y axis $\quad x = z = 0$

$$\frac{0-b}{b-1} = \frac{0+\ b-1}{4b}$$

$$(b-1)^2 = -4b^2 \Rightarrow (b-1)^2 + 4b^2 = 0$$

$$(b-1)^2 + (2b)^2 = 0$$

M–II $\quad b^2 - 2b + 1 + 4b^2 = 0$

$5b^2 - 2b + 1 = 0$

$$(b-1)^2+(2b)^2=0$$

The sum of two square quantity can be zero when both are zero simultaneously

$b-1=0 \Rightarrow b=1$
$2b=0 \Rightarrow b=0$ } Not possible

$5b^2-2b+1=0$

$b=\frac{2\pm\sqrt{4-20}}{10}$

here b is Imaginary

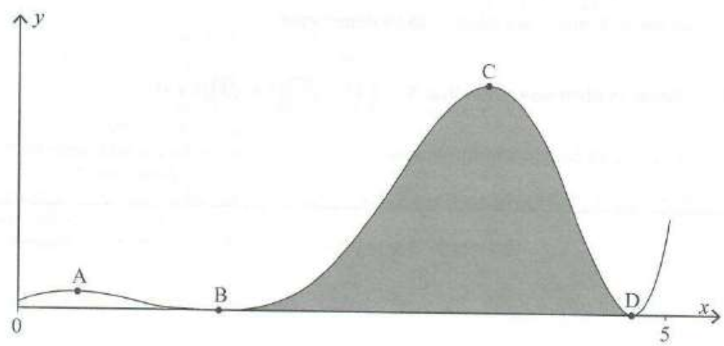
Do **not** write solutions on this page.

10. [Maximum mark: 19]

(a) Use integration by parts to show that $\int e^x \cos 2x\,dx = \frac{2e^x}{5}\sin 2x + \frac{e^x}{5}\cos 2x + c,\ c\in\mathbb{R}$. [5]

(b) Hence, show that $\int e^x \cos^2 x\,dx = \frac{e^x}{5}\sin 2x + \frac{e^x}{10}\cos 2x + \frac{e^x}{2} + c,\ c\in\mathbb{R}$. [3]

The function f is defined by $f(x)=e^x\cos^2 x$, where $0\le x\le 5$. The curve $y=f(x)$ is shown on the following graph which has local maximum points at A and C and touches the x-axis at B and D.



(c) Find the x-coordinates of A and of C, giving your answers in the form $a+\arctan b$, where $a, b\in\mathbb{R}$. [6]

(d) Find the area enclosed by the curve and the x-axis between B and D, as shaded on the diagram. [5]

(a) $\int e^x \cos 2x\,dx$

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$\int u\,dv = uv-\int v\,du$

$u=\cos 2x,\ dv=e^x dx$

$du=-2\sin 2x,\ v=e^x$

So $\int e^x\cos 2x\,dx = e^x\cos 2x + 2\int e^x \sin 2x\,dx$

$\int e^x \sin 2x\,dx$

$u=\sin 2x,\ dv=e^x dx$

$du=2\cos 2x,\ v=e^x$

$$\int e^x\cos 2x\,dx = e^x\cos 2x + 2\left[e^x\sin 2x - 2\int e^x\cos 2x\,dx\right]$$

$$\int e^x \cos 2x\,dx = e^x\cos 2x + 2e^x \sin 2x - 4\int e^x \cos 2x\,dx$$

$$5\int e^x \cos 2x = e^x \cos 2x + 2e^x \sin 2x$$

$$\int e^x \cos 2x = \frac{e^x}{5}\cos 2x + \frac{2e^x}{5}\sin 2x + C$$

(b) $$\int e^x \cos^2 x\,dx = \int e^x\left(\frac{1+\cos 2x}{2}\right)dx$$

$$\cos(2x) = 2\cos^2 x - 1$$
$$2\cos^2 x = 1+\cos(2x)$$
$$\cos^2 x = \frac{1+\cos(2x)}{2}$$

$$= \frac{1}{2}\int e^x dx + \frac{1}{2}\int e^x \cos 2x\,dx$$

$$= \frac{1}{2}e^x + \frac{1}{2}\left[\frac{e^x}{5}\cos 2x + \frac{2e^x}{5}\sin 2x\right] + C$$

$$= \frac{e^x}{5}\sin 2x + \frac{e^x}{10}\cos 2x + \frac{e^x}{2} + C$$

(c) $$f(x) = e^x \cos^2 x \qquad 0 \le x \le 5$$

$$f'(x) = e^x(2\cos x)(-\sin x) + \cos^2 x \cdot e^x$$

$$= e^x \cos x\left[-2\sin x + \cos x\right]$$

for maxima $e^x \cos x\left[-2\sin x + \cos x\right] = 0$

$$-2\sin x + \cos x = 0 \Rightarrow \tan x = \frac{1}{2}$$

$$x = \tan^{-1}\left(\frac{1}{2}\right) \text{ and } x = \pi + \tan^{-1}\left(\frac{1}{2}\right) \text{ as } 0 \le x \le 5$$

(d) for point B and D

$\underline{\underline{M-I}}$

$$f(x) = 0$$
$$e^x \cos^2 x = 0$$
$$\cos x = 0$$
$$x = \frac{\pi}{2},\ \frac{3\pi}{2}$$

$\underline{M-II}$

$$f'(x) = 0$$
$$e^x \cos x\,(-2\sin x + \cos x) = 0$$
$$\cos x = 0$$
$$x = \frac{\pi}{2},\ \frac{3\pi}{2}$$

$$A = \int_{\pi/2}^{3\pi/2} e^x \cos^2 x\,dx = \left[\frac{e^x}{5}\sin 2x + \frac{e^x}{10}\cos 2x + \frac{e^x}{2}\right]_{\pi/2}^{3\pi/2}$$

$$= \left[\frac{e^{3\pi/2}}{5}\sin 3\pi + \frac{e^{3\pi/2}}{10}\cos 3\pi + \frac{e^{3\pi/2}}{2}\right] - \left[\frac{e^{\pi/2}}{5}\sin \pi + \frac{e^{\pi/2}}{10}\cos \pi + \frac{e^{\pi/2}}{2}\right]$$

$$= \left[0 - \frac{e^{3\pi/2}}{10} + \frac{e^{3\pi/2}}{2}\right] - \left[0 - \frac{e^{\pi/2}}{10} + \frac{e^{\pi/2}}{2}\right]$$

$$= e^{3\pi/2}\left[\frac{1}{2} - \frac{1}{10}\right] - e^{\pi/2}\left[\frac{1}{2} - \frac{1}{10}\right]$$

$$= e^{3\pi/2}\left[\frac{1}{2}-\frac{1}{10}\right] - e^{\pi/2}\left[\frac{1}{2}-\frac{1}{10}\right]$$

$$= \frac{2}{5}\left[e^{3\pi/2} - e^{\pi/2}\right]$$

Do **not** write solutions on this page.

11. [Maximum mark: 16]

(a) Find the roots of $z^{24} = 1$ which satisfy the condition $0 < \arg(z) < \frac{\pi}{2}$, expressing your answers in the form $re^{i\theta}$, where $r, \theta \in \mathbb{R}^+$. [5]

(b) Let S be the sum of the roots found in part (a).

(i) Show that $\operatorname{Re} S = \operatorname{Im} S$.

(ii) By writing $\frac{\pi}{12}$ as $\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$, find the value of $\cos\frac{\pi}{12}$ in the form $\frac{\sqrt{a}+\sqrt{b}}{c}$, where a, b and c are integers to be determined.

(iii) Hence, or otherwise, show that $S = \frac{1}{2}\left(1+\sqrt{2}\right)\left(1+\sqrt{3}\right)(1+i)$. [11]

(a) $z^{24} = 1 = \cos(2k\pi) + i\sin(2k\pi)$

$z = \left[\cos(2k\pi) + i\sin(2k\pi)\right]^{1/24}$

$z = \cos\left(\frac{k\pi}{12}\right) + i\sin\left(\frac{k\pi}{12}\right) = e^{i\frac{k\pi}{12}}$

$k=1 \quad z_1 = e^{i\pi/12}$, $k=2 \rightarrow z_2 = e^{i2\pi/12}$

$k=3 \quad z_3 = e^{i3\pi/12}$, $k=4 \rightarrow z_4 = e^{i4\pi/12}$

$k=5, \quad z_5 = e^{i5\pi/12}$

$S = e^{i\pi/12} + e^{i2\pi/12} + e^{i3\pi/12} + e^{i4\pi/12} + e^{i5\pi/12}$

$S = \left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right) + \left(\cos\frac{2\pi}{12} + i\sin\frac{2\pi}{12}\right) + \left(\cos\frac{3\pi}{12} + i\sin\frac{3\pi}{12}\right)$
$+ \left(\cos\frac{4\pi}{12} + i\sin\frac{4\pi}{12}\right) + \left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

$\operatorname{Re}(S) = \cos\left(\frac{\pi}{12}\right) + \cos\left(\frac{2\pi}{12}\right) + \cos\left(\frac{3\pi}{12}\right) + \cos\left(\frac{4\pi}{12}\right) + \cos\left(\frac{5\pi}{12}\right)$

$\operatorname{Im}(S) = \sin\left(\frac{\pi}{12}\right) + \sin\left(\frac{2\pi}{12}\right) + \sin\left(\frac{3\pi}{12}\right) + \sin\left(\frac{4\pi}{12}\right) + \sin\left(\frac{5\pi}{12}\right)$

Here $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{6\pi}{12} - \frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{2} - \frac{5\pi}{12}\right) = \sin\frac{5\pi}{12}$

$$\cos\left(\frac{2\pi}{12}\right) = \cos\left(\frac{6\pi}{12} - \frac{4\pi}{12}\right) = \cos\left(\frac{\pi}{2} - \frac{4\pi}{12}\right) = \sin\frac{4\pi}{12}$$

$$\cos\left(\frac{3\pi}{12}\right) = \cos\left(\frac{6\pi}{12} - \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{2} - \frac{3\pi}{12}\right) = \sin\frac{3\pi}{12}$$

$$\cos\left(\frac{4\pi}{12}\right) = \cos\left(\frac{6\pi}{12} - \frac{2\pi}{12}\right) = \cos\left(\frac{\pi}{2} - \frac{2\pi}{12}\right) = \sin\frac{2\pi}{12}$$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{6\pi}{12} - \frac{\pi}{12}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{12}\right) = \sin\frac{\pi}{12}$$

So $Re(S) = Im(S)$

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(ii) $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

$a = 2,\ b = 6,\ c = 4$

(iii) $Im(S) = Re(S)$

$$S = (1+i)\left[\cos\left(\frac{\pi}{12}\right) + \cos\left(\frac{2\pi}{12}\right) + \cos\left(\frac{3\pi}{12}\right) + \cos\left(\frac{4\pi}{12}\right) + \cos\left(\frac{5\pi}{12}\right)\right]$$

$$= (1+i)\left[\frac{1+\sqrt{3}}{2\sqrt{2}} + \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} + \frac{1}{2} + \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)\right]$$

$$= (1+i)\left[\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} + \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} + \frac{1}{2} + \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)\right]$$

$$= (1+i)\left[\cancel{\frac{1}{2\sqrt{2}}} + \frac{\sqrt{3}}{2\sqrt{2}} + \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \cancel{\frac{1}{\sqrt{2}} \times \frac{1}{2}}\right]$$

$$= (1+i)\left[\frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} + \frac{1}{2}\right]$$

$$= (1+i)\frac{1}{2}\left[\sqrt{6} + \sqrt{3} + \sqrt{2} + 1\right]$$

$$= (1+i)\frac{1}{2}\left[\sqrt{3}[\sqrt{2}+1] + 1[\sqrt{2}+1]\right] = \frac{1}{2}(1+\sqrt{2})(1+\sqrt{3})(1+i)$$

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Tuesday 13 November 2018 (morning)

2 hours

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

8818–7202

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider a geometric sequence with a first term of 4 and a fourth term of -2.916.

(a) Find the common ratio of this sequence. [3]

(b) Find the sum to infinity of this sequence. [2]

2. [Maximum mark: 7]

A function f satisfies the conditions $f(0) = -4$, $f(1) = 0$ and its second derivative is $f''(x) = 15\sqrt{x} + \frac{1}{(x+1)^2}$, $x \geq 0$.

Find $f(x)$.

. . . .

3. [Maximum mark: 8]

It is known that 56% of Infiglow batteries have a life of less than 16 hours, and 94% have a life less than 17 hours. It can be assumed that battery life is modelled by the normal distribution $\mathrm{N}(\mu, \sigma^2)$.

(a) Find the value of μ and the value of σ. [6]

(b) Find the probability that a randomly selected Infiglow battery will have a life of at least 15 hours. [2]

4. [Maximum mark: 5]

Find the value of the constant term in the expansion of $x^4\left(x+\dfrac{3}{x^2}\right)^5$.

5. [Maximum mark: 5]

Differentiate from first principles the function $f(x) = 3x^3 - x$.

6. [Maximum mark: 6]

Let $P(x) = 2x^4 - 15x^3 + ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$.

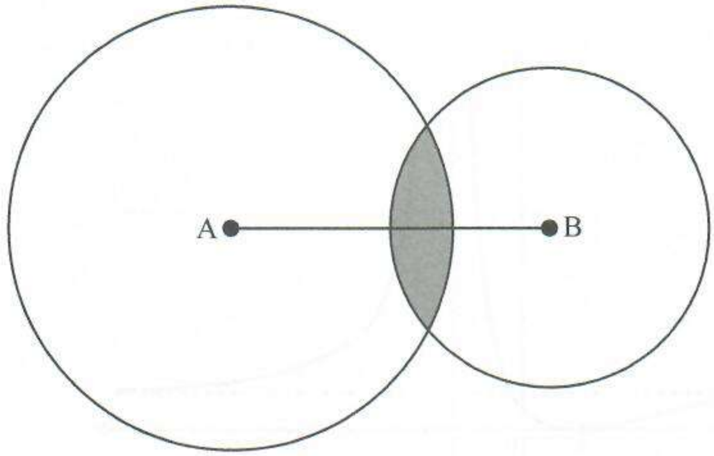
(a) Given that $(x-5)$ is a factor of $P(x)$, find a relationship between a, b and c. [2]

(b) Given that $(x-5)^2$ is a factor of $P(x)$, write down the value of $P'(5)$. [1]

(c) Given that $(x-5)^2$ is a factor of $P(x)$, and that $a = 2$, find the values of b and c. [3]

7. [Maximum mark: 6]

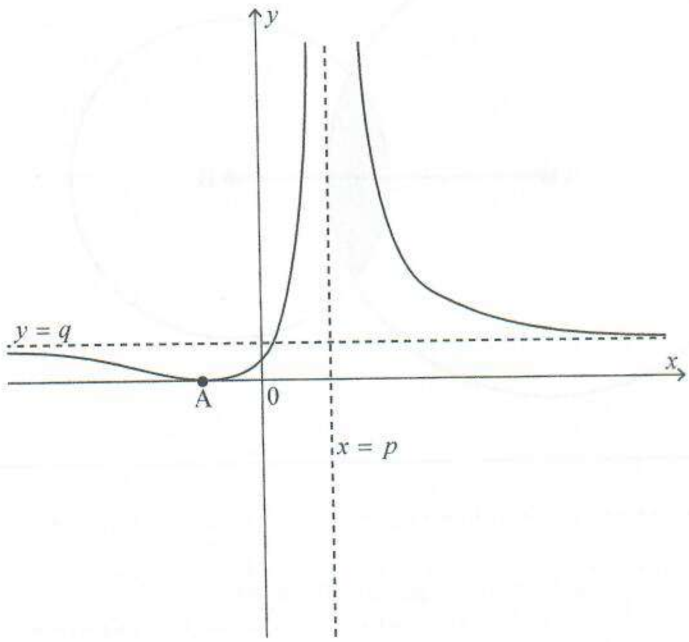
Boat A is situated 10km away from boat B, and each boat has a marine radio transmitter on board. The range of the transmitter on boat A is 7km, and the range of the transmitter on boat B is 5km. The region in which both transmitters can be detected is represented by the shaded region in the following diagram. Find the area of this region.



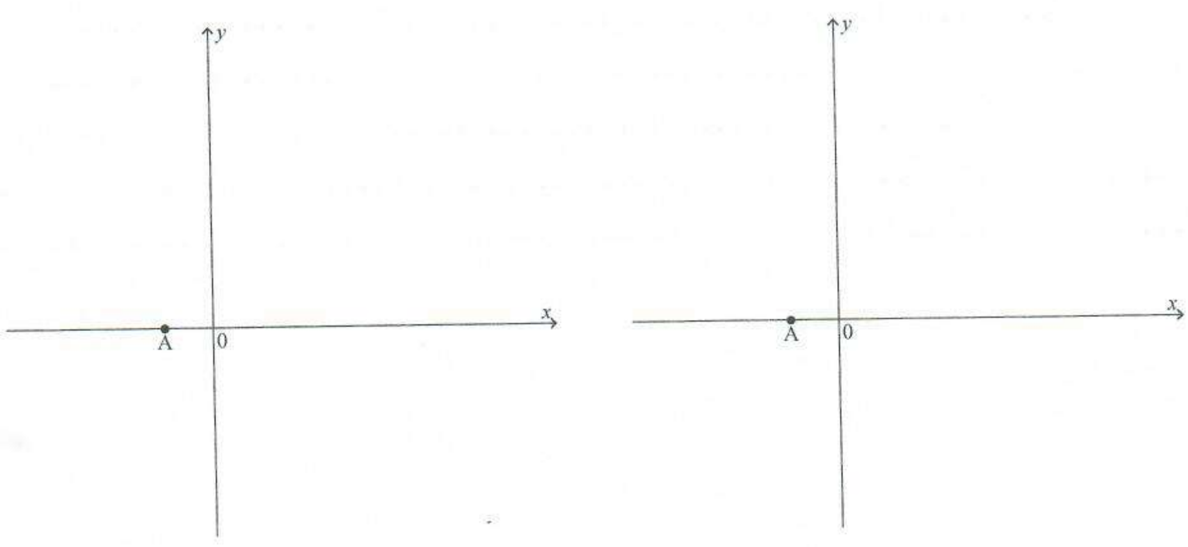
8. [Maximum mark: 8]

Consider the function $f(x)=\dfrac{ax+1}{bx+c}$, $x\neq-\dfrac{c}{b}$, where $a, b, c\in\mathbb{Z}$.

The following graph shows the curve $y=(f(x))^2$. It has asymptotes at $x=p$ and $y=q$ and meets the x-axis at A.



(a) On the following axes, sketch the two possible graphs of $y=f(x)$ giving the equations of any asymptotes in terms of p and q. [4]



(This question continues on the following page)

(Question 8 continued)

(b) Given that $p = \frac{4}{3}$, $q = \frac{4}{9}$ and A has coordinates $\left(-\frac{1}{2}, 0\right)$, determine the possible sets of values for a, b and c. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 19]

The function f is defined by $f(x) = \dfrac{2\ln x + 1}{x - 3}$, $0 < x < 3$.

(a) Find $f'(x)$. [4]

(b) Hence, or otherwise, find the coordinates of the point of inflexion on the graph of $y = f(x)$. [4]

(c) Draw a set of axes showing x and y values between -3 and 3. On these axes

(i) sketch the graph of $y = f(x)$, showing clearly any axis intercepts and giving the equations of any asymptotes.

(ii) sketch the graph of $y = f^{-1}(x)$, showing clearly any axis intercepts and giving the equations of any asymptotes. [8]

(d) Hence, or otherwise, solve the inequality $f(x) > f^{-1}(x)$. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 18]

Willow finds that she receives approximately 70 emails per working day.
She decides to model the number of emails received per working day using the random variable X, where X follows a Poisson distribution with mean 70.

(a) Using this distribution model, find

(i) $\mathrm{P}(X < 60)$

(ii) the standard deviation of X. [4]

(b) In order to test her model, Willow records the number of emails she receives per working day over a period of 6 months. The results are shown in the following table.

Number of emails received (x)	Number of days
$40 \le x \le 49$	2
$50 \le x \le 59$	15
$60 \le x \le 69$	40
$70 \le x \le 79$	53
$80 \le x \le 89$	0
$90 \le x \le 99$	1
$100 \le x \le 109$	3
$110 \le x \le 119$	6

From the table, calculate

(i) an estimate for the mean number of emails received per working day;

(ii) an estimate for the standard deviation of the number of emails received per working day. [5]

(c) Give one piece of evidence that suggests Willow's Poisson distribution model is not a good fit. [1]

Archie works for a different company and knows that he receives emails according to a Poisson distribution, with a mean of λ emails per day.

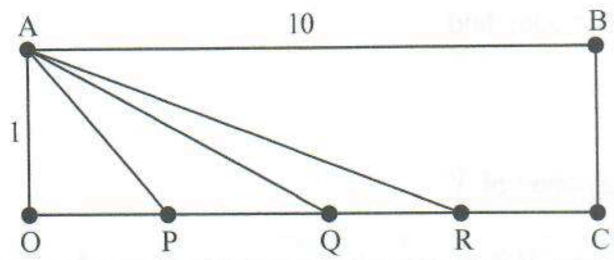
(d) Suppose that the probability of Archie receiving more than 10 emails in total on any one day is 0.99. Find the value of λ. [3]

(e) Now suppose that Archie received exactly 20 emails in total in a consecutive two day period. Show that the probability that he received exactly 10 of them on the first day is independent of λ. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 13]

Consider the rectangle OABC such that $AB = OC = 10$ and $BC = OA = 1$, with the points P, Q and R placed on the line OC such that $OP = p$, $OQ = q$ and $OR = r$, such that $0 < p < q < r < 10$.



Let θ_p be the angle APO, θ_q be the angle AQO and θ_r be the angle ARO.

(a) Find an expression for θ_p in terms of p. [3]

Consider the case when $\theta_p = \theta_q + \theta_r$ and $QR = 1$.

(b) Show that $p = \dfrac{q^2 + q - 1}{2q + 1}$. [6]

(c) By sketching the graph of p as a function of q, determine the range of values of p for which there are possible values of q. [4]

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider a geometric sequence with a first term of 4 and a fourth term of -2.916.

(a) Find the common ratio of this sequence. [3]

(b) Find the sum to infinity of this sequence. [2]

$u_1 = 4, \quad u_4 = u_1 r^3 = -2.916$

$\frac{u_4}{u_1} = r^3 = \frac{-2.916}{4}$

$r = \left(\frac{-2.916}{4}\right)^{1/3}$

$= -0.9$

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$S_\infty = \frac{u_1}{1-r} = \frac{4}{1-(-0.9)} = \frac{4}{1.9} = 2.11$

2. [Maximum mark: 7]

A function f satisfies the conditions $f(0) = -4$, $f(1) = 0$ and its second derivative is $f''(x) = 15\sqrt{x} + \frac{1}{(x+1)^2}$, $x \geq 0$.

Find $f(x)$.

$$f'(x) = \int \left(15\sqrt{x} + \frac{1}{(x+1)^2}\right) dx$$

$$= \frac{15x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{(x+1)^{-2+1}}{-2+1} + C_1$$

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$$= 15 \times \frac{2}{3} x^{3/2} - \frac{1}{x+1} + C_1$$

$$= 10x^{3/2} - \frac{1}{x+1} + C_1$$

$$f(x) = \int f'(x)\,dx = \int \left(10x^{3/2} - \frac{1}{x+1} + C_1\right) dx$$

$$f(x) = 10\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \ln(x+1) + C_1 x + C_2$$

$$f(x) = 10 \times \frac{2}{5} x^{5/2} - \ln(x+1) + C_1 x + C_2$$

$$f(x) = 4x^{5/2} - \ln(x+1) + C_1 x + C_2$$

$$f(0) = -4$$

$$-4 = 0 - \ln(1) + 0 + C_2 \Rightarrow C_2 = -4$$

$$f(x) = 4x^{5/2} - \ln(x+1) + C_1 x - 4$$

$$f(1) = 0$$

$$0 = \cancel{4} - \ln(2) + C_1 - \cancel{4} \Rightarrow C_1 = \ln(2)$$

$$f(x) = 4x^{5/2} - \ln(x+1) + x\ln(2) - 4$$

3. [Maximum mark: 8]

It is known that 56 % of Infiglow batteries have a life of less than 16 hours, and 94 % have a life less than 17 hours. It can be assumed that battery life is modelled by the normal distribution $N(\mu, \sigma^2)$.

(a) Find the value of μ and the value of σ. [6]

(b) Find the probability that a randomly selected Infiglow battery will have a life of at least 15 hours. [2]

(a) $P(X<16) = .56$, $P(X<17) = 0.94$

$\frac{16-\mu}{\sigma} = \text{InvNormal}(.56)$ $\quad$ $\frac{17-\mu}{\sigma} = \text{InvNormal}(.94)$

$\frac{16-\mu}{\sigma} = .151$ $\quad$ $\frac{17-\mu}{\sigma} = 1.555$

$\mu = 16 - .151\sigma$ $\quad$ $\mu = 17 - 1.555\sigma$

$16 - .151\sigma = 17 - 1.555\sigma$

$(1.555 - .151)\sigma = 1$

$\sigma = .712$

$\mu = 15.89$

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(b) $P(X<15) = \text{Normalcdf}(15, 15.89, .712) = .1056$

4. [Maximum mark: 5]

Find the value of the constant term in the expansion of $x^4\left(x+\dfrac{3}{x^2}\right)^5$.

$$x^4\left(x+\frac{3}{x^2}\right)^5$$

for constant term Power of x from this expansion should be x^{-4}

So

$$T_{r+1} = \binom{5}{r}(x)^{5-r}\left(\frac{3}{x^2}\right)^r$$

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$$= \binom{5}{r}(x)^{5-r}(3)^r(x)^{-2r}$$

$$= \binom{5}{r}(3)^r x^{5-3r}$$

$$5-3r = -4$$

$$3r = 9 \Rightarrow r = 3$$

So Constant term $\rightarrow \binom{5}{3}(3)^3 = 10\times 27$

$$= 270$$

5. [Maximum mark: 5]

Differentiate from first principles the function $f(x) = 3x^3 - x$.

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$$f(x) = 3x^3 - x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^3 - (x+h) - 3x^3 + x}{h}$$

$$= \lim_{h \to 0} \frac{3(\cancel{x^3} + 3hx^2 + 3xh^2 + h^3) - \cancel{x} - h - \cancel{3x^3} + \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(9x^2 + 9hx + 3h^2 - 1)}{\cancel{h}}$$

$$= 9x^2 - 1$$

[redacted] [redacted]

6. [Maximum mark: 6]

Let $P(x) = 2x^4 - 15x^3 + ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$.

(a) Given that $(x-5)$ is a factor of $P(x)$, find a relationship between a, b and c. [2]

(b) Given that $(x-5)^2$ is a factor of $P(x)$, write down the value of $P'(5)$. [1]

(c) Given that $(x-5)^2$ is a factor of $P(x)$, and that $a = 2$, find the values of b and c. [3]

$P(x) = 2x^4 - 15x^3 + ax^2 + bx + c$

(a) $P(5) = 0$

$2 \times 5^4 - 15 \times 5^3 + a \times 5^2 + 5b + c = 0$

$25a + 5b + c = 625$ — ①

(b) $p'(5) = 0$

$p'(x) = 8x^3 - 45x^2 + 2ax + b$

$p'(5) = 8 \times 5^3 - 45 \times 5^2 + 10a + b = 0$

$10a + b = 125$ — ②

(c) $a = 2 \Rightarrow 10 \times 2 + b = 125$

$b = 105$ — ③

from ①, ② and ③

$25 \times 2 + 5 \times 105 + c = 625$

$c = 625 - 50 - 525 = 50$

So $a = 2$, $b = 105$, $c = 50$

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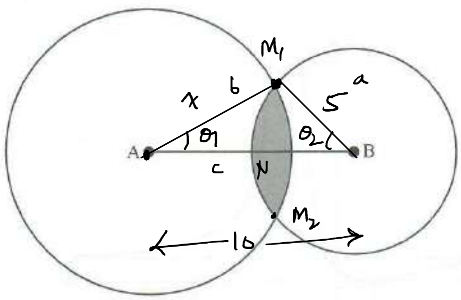
7. [Maximum mark: 6]

Boat A is situated 10km away from boat B, and each boat has a marine radio transmitter on board. The range of the transmitter on boat A is 7km, and the range of the transmitter on boat B is 5km. The region in which both transmitters can be detected is represented by the shaded region in the following diagram. Find the area of this region.

$\frac{1}{2}r^2\theta$

8.8464



Area = Sector $AM_1M_2 - \Delta AM_1M_2$
$+$ Sector $BM_1M_2 - \Delta BM_1M_2$

$= \frac{1}{2}\times 7^2\times 2\theta_1 - \frac{1}{2}\times 7^2\times \sin 2\theta_1$
$+ \frac{1}{2}\times 5^2\times 2\theta_2 - \frac{1}{2}\times 5^2 \sin 2\theta_2$

$= \frac{49}{2}(2\theta_1 - \sin 2\theta_1) + \frac{25}{2}[2\theta_2 - \sin 2\theta_2]$

$= 8.84 \text{ km}^2$

$a^2 = b^2 + c^2 - 2bc\cos\theta_1$

$5^2 = 7^2 + 10^2 - 2\times 7\times 10\times\cos\theta_1$

$\cos\theta_1 = \frac{7^2+10^2-5^2}{2\times 7\times 10}$

$\theta_1 = .4828^c$

$\theta_2 = .7074^c$

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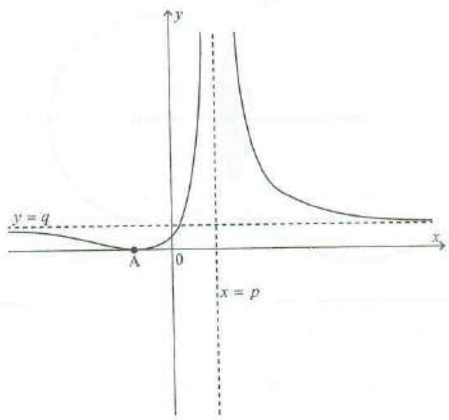
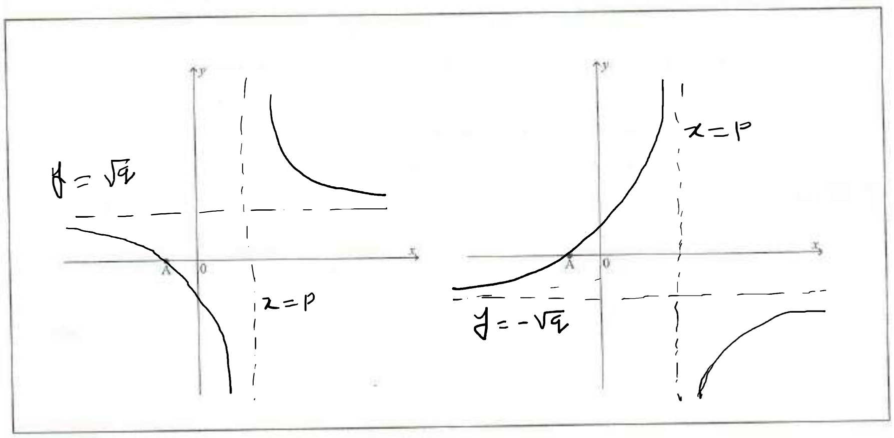
8. [Maximum mark: 8]

Consider the function $f(x) = \dfrac{ax+1}{bx+c}$, $x \neq -\dfrac{c}{b}$, where $a, b, c \in \mathbb{Z}$.

The following graph shows the curve $y = (f(x))^2$. It has asymptotes at $x = p$ and $y = q$ and meets the x-axis at A.



(a) On the following axes, sketch the two possible graphs of $y = f(x)$ giving the equations of any asymptotes in terms of p and q. [4]



(This question continues on the following page)

[redacted]

(Question 8 continued)

(b) Given that $p = \frac{4}{3}$, $q = \frac{4}{9}$ and A has coordinates $\left(-\frac{1}{2}, 0\right)$, determine the possible sets of values for a, b and c. [4]

$f(x) = \dfrac{ax+1}{bx+c}$

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$f(x)^2 = \left(\dfrac{ax+1}{bx+c}\right)^2$

Vt Asympt $x = -\dfrac{c}{b} = \dfrac{4}{3} \Rightarrow c = -\dfrac{4}{3}b$

x intercept $x = -\frac{1}{2}, y = 0 \Rightarrow 0 = \left(-\frac{1}{2}a + 1\right)^2$

$a = 2$

Hz Asymptote $y = \dfrac{a^2}{b^2} = \dfrac{4}{9} \Rightarrow b = \dfrac{3}{2}a = 3$

$c = -\dfrac{4}{3} \times 3 = -4$

$a = 2,\ b = 3,\ c = -4$

[redacted] [redacted]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 19]

The function f is defined by $f(x) = \dfrac{2\ln x + 1}{x - 3}$, $0 < x < 3$.

(a) Find $f'(x)$. [4]

(b) Hence, or otherwise, find the coordinates of the point of inflexion on the graph of $y = f(x)$. [4]

(c) Draw a set of axes showing x and y values between -3 and 3. On these axes

(i) sketch the graph of $y = f(x)$, showing clearly any axis intercepts and giving the equations of any asymptotes.

(ii) sketch the graph of $y = f^{-1}(x)$, showing clearly any axis intercepts and giving the equations of any asymptotes. [8]

(d) Hence, or otherwise, solve the inequality $f(x) > f^{-1}(x)$. [3]

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(a) $$f'(x) = \frac{(x-3)\left(\frac{2}{x}\right) - (2\ln x + 1)\cdot 1}{(x-3)^2}$$

$$= \frac{2x - 6 - 2x\ln x - x}{x(x-3)^2}$$

$$= \frac{x - 6 - 2x\ln(x)}{x(x-3)^2}$$

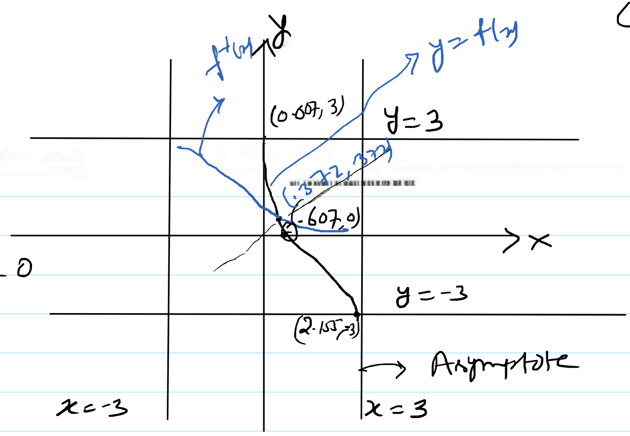
(b) $x = 0.899$

(c) (i)

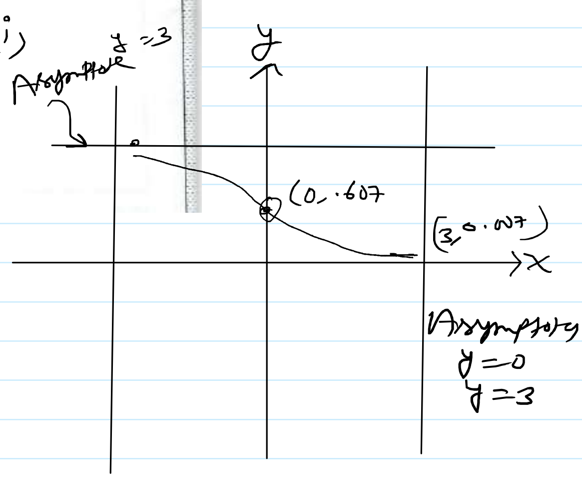
$y = f(x)$

Asymptotes

$x = 3,\ x = 0$



(ii)



Asymptotes
$y = 0$
$y = 3$

(d) $0 < x < 0.372$

Do **not** write solutions on this page.

10. [Maximum mark: 18]

Willow finds that she receives approximately 70 emails per working day. She decides to model the number of emails received per working day using the random variable X, where X follows a Poisson distribution with mean 70.

(a) Using this distribution model, find

(i) $P(X < 60)$

(ii) the standard deviation of X. [4]

(b) In order to test her model, Willow records the number of emails she receives per working day over a period of 6 months. The results are shown in the following table.

Number of emails received (x)	Number of days
$40 \le x \le 49$	2
$50 \le x \le 59$	15
$60 \le x \le 69$	40
$70 \le x \le 79$	53
$80 \le x \le 89$	0
$90 \le x \le 99$	1
$100 \le x \le 109$	3
$110 \le x \le 119$	6

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From the table, calculate

(i) an estimate for the mean number of emails received per working day;

(ii) an estimate for the standard deviation of the number of emails received per working day. [5]

(c) Give one piece of evidence that suggests Willow's Poisson distribution model is not a good fit. [1]

Archie works for a different company and knows that he receives emails according to a Poisson distribution, with a mean of λ emails per day.

(d) Suppose that the probability of Archie receiving more than 10 emails in total on any one day is 0.99. Find the value of λ. [3]

(e) Now suppose that Archie received exactly 20 emails in total in a consecutive two day period. Show that the probability that he received exactly 10 of them on the first day is independent of λ. [5]

[redacted] [redacted]

(a) $\lambda = 70$

(i) $P(X<60) = P(x \le 59)$

$= \cdot 102$

(ii) for poisson distribution

variance $= \lambda$

$S \cdot D = \sqrt{\lambda}$

$= \sqrt{70} = 8 \cdot 37$

(b) (i) mean $= 71 \cdot 08$

(ii) $S \cdot D = 13 \cdot 87$

(c) mean and S.D. are different

(d) $P(X > 10) = 0.99$

$1 - P(X \le 10) = 0.99$

$P(X \le 10) = \cdot 01$

$e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{\underline{|2}} + \cdots \frac{\lambda^{10}}{\underline{|10}}\right] = \cdot 01$

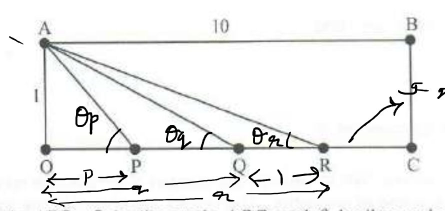
$\lambda = 20$

=

Do **not** write solutions on this page.

11. [Maximum mark: 13]

Consider the rectangle OABC such that $AB = OC = 10$ and $BC = OA = 1$, with the points P, Q and R placed on the line OC such that $OP = p$, $OQ = q$ and $OR = r$, such that $0 < p < q < r < 10$.



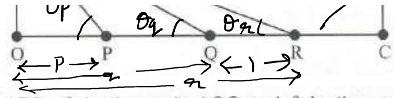
(a) $\tan\theta_p = \frac{1}{p}$

$\theta_p = \tan^{-1}\left(\frac{1}{p}\right)$

(b) $\tan(\theta_p) = \frac{\tan\theta_q + \tan\theta_r}{1 - \tan\theta_q \tan\theta_r}$

$\frac{1}{p} = \frac{\frac{1}{q} + \frac{1}{q+1}}{1 - \frac{1}{q} \times \frac{1}{q+1}}$

$\frac{1}{p} = \frac{q+1+q}{q^2+q-1}$



(a) $\theta_p = \tan^{-1}\left(\frac{1}{p}\right)$

Let θ_p be the angle APO, θ_q be the angle AQO and θ_r be the angle ARO.

(a) Find an expression for θ_p in terms of p. [3]

$q < 9$

Consider the case when $\theta_p = \theta_q + \theta_r$ and $QR = 1$.

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(b) Show that $p = \dfrac{q^2+q-1}{2q+1}$. [6]

(c) By sketching the graph of p as a function of q, determine the range of values of p for which there are possible values of q. [4]

$$\frac{1}{p} = \frac{q+1+q}{q^2+q-1}$$

$$p = \frac{q^2+q-1}{2q+1}$$

(c) $0 < p < 4.684$

[redacted]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 1

Monday 12 November 2018 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

8818–7401

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. The volume of a hemisphere, V, is given by the formula

$$V = \sqrt{\frac{4S^3}{243\pi}},$$

where S is the total surface area.

The total surface area of a given hemisphere is $350\,\text{cm}^2$.

(a) Calculate the volume of this hemisphere in cm^3.
Give your answer correct to **one decimal place**. [3]

(b) Write down your answer to part (a) correct to the nearest integer. [1]

(c) Write down your answer to **part (b)** in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$. [2]

Working:

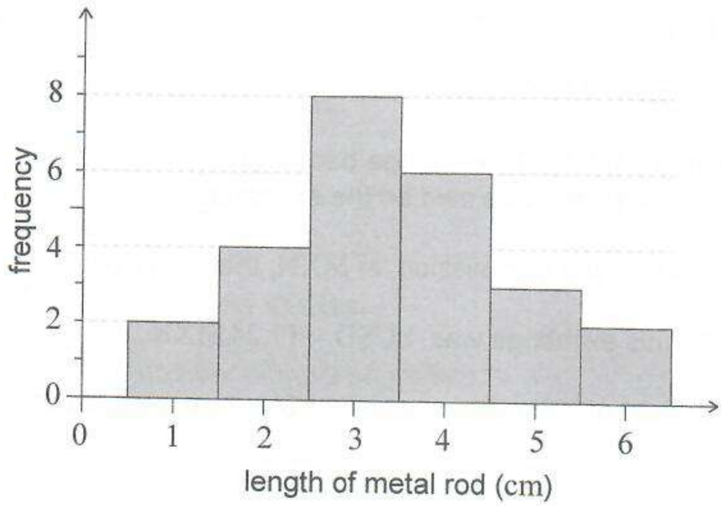
Answers:

(a) . . . .

(b) . . . .

(c) . . . .

2. The histogram shows the lengths of 25 metal rods, each measured correct to the nearest cm.



(a) Write down the modal length of the rods. [1]

(b) Find the median length of the rods. [3]

The upper quartile is $4\,\mathrm{cm}$.

(c) Calculate

(i) the lower quartile;

(ii) the interquartile range. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) (i) . . . .

(ii) . . . .

3. Harry travelled from the USA to Mexico and changed 700 dollars (USD) into pesos (MXN).

The exchange rate was $1\,\mathrm{USD} = 18.86\,\mathrm{MXN}$.

(a) Calculate the amount of MXN Harry received. [2]

On his return, Harry had 2400 MXN to change back into USD.
There was a 3.5% commission to be paid on the exchange.

(b) Calculate the value of the commission, in MXN, that Harry paid. [2]

The exchange rate for this exchange was $1\,\mathrm{USD} = 17.24\,\mathrm{MXN}$.

(c) Calculate the amount of USD Harry received. Give your answer correct to the nearest cent. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

4. Abhinav carries out a χ^2 test at the 1% significance level to determine whether a person's gender impacts their chosen professional field: engineering, medicine or law.
He surveyed 220 people and the results are shown in the table.

	Engineering	**Medicine**	**Law**
Male	55	30	25
Female	35	45	30

(a) State the null hypothesis, H_0, for this test. [1]

(b) Calculate the expected number of male engineers. [2]

(c) Find the p-value for this test. [2]

Abhinav rejects H_0.

(d) State a reason why Abhinav is incorrect in doing so. [1]

Working:

Answers:

(a) . . . .

. . . .

. . . .

(b) . . . .

(c) . . . .

(d) . . . .

. . . .

5. The table shows the first five terms of three sequences: u_n, v_n and w_n.

	n				
	1	**2**	**3**	**4**	**5**
u_n	10	20	40	80	160
v_n	10	20	30	60	100
w_n	10	20	30	40	50

(a) State which sequence is

(i) arithmetic;

(ii) geometric. [2]

(b) Find the exact value of the 11th term of the geometric sequence. [2]

(c) Find the sum of the first 20 terms of the arithmetic sequence. [2]

Working:

Answers:

(a) (i) . . . .

(ii) . . . .

(b) . . . .

(c) . . . .

6. (a) Complete the following truth table. [4]

p	q	$p \vee q$	$\neg q$	$p \wedge \neg q$	$\neg(p \vee q)$	$(p \wedge \neg q) \Rightarrow \neg(p \vee q)$
T	T	T				
T	F	T				
F	T	T				
F	F	F				

(b) State whether the statement $(p \wedge \neg q) \Rightarrow \neg(p \vee q)$ is a contradiction, a tautology or neither. Give a reason for your answer. [2]

Working:

Answer:

(b) . . . .

. . . .

. . . .

. . . .

7. Nick has $\$150\,000$ in a trust fund. Each year he donates $8\,\%$ of the money remaining in his trust fund to charity.

(a) Determine the maximum number of years Nick can donate to charity while keeping at least $\$50\,000$ in the trust fund. [3]

Louise invests $\$200\,000$ in a bank account that pays a nominal interest rate of $5\,\%$, **compounded quarterly**, for eight years.

(b) Calculate the value of Louise's investment at the end of this time. Give your answer correct to the nearest cent. [3]

Working:

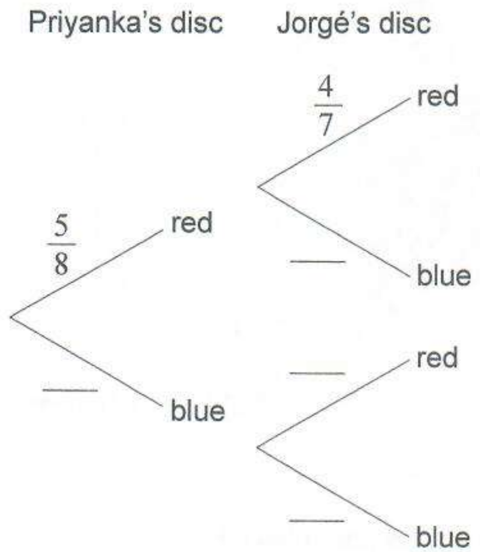
Answers:

(a) . . . .

(b) . . . .

8. A bag contains 5 red and 3 blue discs, all identical except for the colour. First, Priyanka takes a disc at random from the bag and then Jorgé takes a disc at random from the bag.

(a) Complete the tree diagram. [3]



(b) Find the probability that Jorgé chooses a red disc. [3]

Working:

Answer:

(b) . . . .

9. A solid glass paperweight consists of a hemisphere of diameter $6\,\text{cm}$ on top of a cuboid with a square base of length $6\,\text{cm}$, as shown in the diagram.

diagram not to scale

The height of the cuboid, $x\,\text{cm}$, is equal to the height of the hemisphere.

(a) (i) Write down the value of x.

(ii) Calculate the volume of the paperweight. [4]

$1\,\text{cm}^3$ of glass has a mass of 2.56 grams.

(b) Calculate the mass, in grams, of the paperweight. [2]

Working:

Answers:

(a) (i) . . . .

(ii) . . . .

(b) . . . .

10. Consider the following statements.

p: it can go wrong
q: it does go wrong

(a) Write down in symbolic form:

If it does not go wrong then it cannot go wrong. [2]

(b) Write down in words the argument $p \Rightarrow q$. [2]

(c) Write down in words the inverse of $p \Rightarrow q$. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

. . . .

. . . .

(c) . . . .

. . . .

. . . .

11. Consider the curve $y = 5x^3 - 3x$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

The curve has a tangent at the point $\mathrm{P}(-1, -2)$.

(b) Find the gradient of this tangent at point P. [2]

(c) Find the equation of this tangent. Give your answer in the form $y = mx + c$. [2]

Working:

Answers:

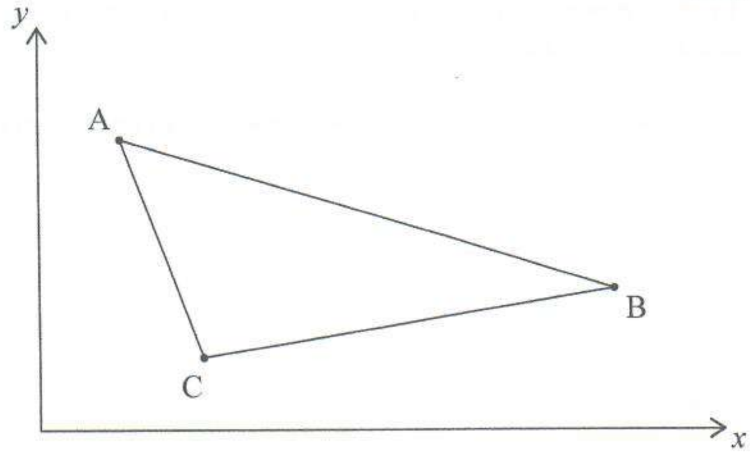
(a) . . . .

(b) . . . .

(c) . . . .

12. The diagram shows a triangle defined by the points $\mathrm{A}(3, 9)$, $\mathrm{B}(15, 6)$ and $\mathrm{C}(5, 3)$.

diagram not to scale



(a) Calculate the gradient of the line AC. [2]

(b) Determine, giving a reason, whether angle $\mathrm{A\hat{C}B}$ is a right angle. [2]

The straight line, L, is parallel to BC and passes through A.

(c) Find the equation of L.
Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

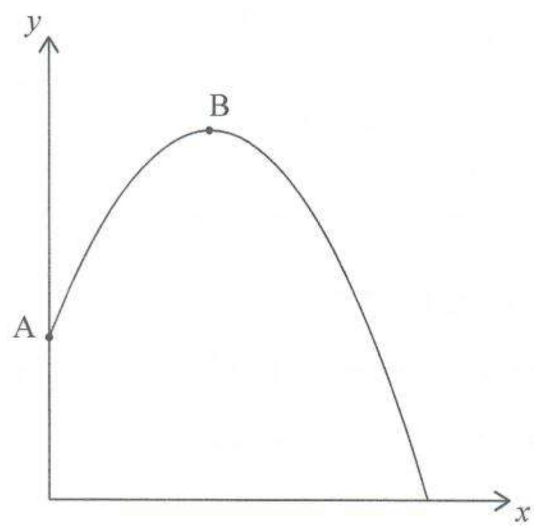
. . . .

(c) . . . .

13. Bella throws a ball from the top of a wall onto flat horizontal ground.

The path of the ball is modelled by the quadratic curve $y = 3 + 4x - x^2$, where x represents the horizontal distance the ball is thrown and y represents the height of the ball above the ground. All distances are measured in metres.

The wall lies along the y-axis. The curve intersects the y-axis at point A and has its vertex at point B.



(a) Write down the height in metres from which the ball was thrown. [1]

(b) Calculate the maximum height, above the ground, reached by the ball. [3]

(c) Find the horizontal distance from the base of the wall to the point at which the ball hits the ground. [2]

(This question continues on the following page)

14. The marks achieved by students taking a college entrance test follow a normal distribution with mean 300 and standard deviation 100.

In this test, 10% of the students achieved a mark greater than k.

(a) Find the value of k. [2]

Marron College accepts only those students who achieve a mark of at least 450 on the test.

(b) Find the probability that a randomly chosen student will be accepted by Marron College. [2]

(c) Given that Naomi attends Marron College, find the probability that she achieved a mark of at least 500 on the test. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

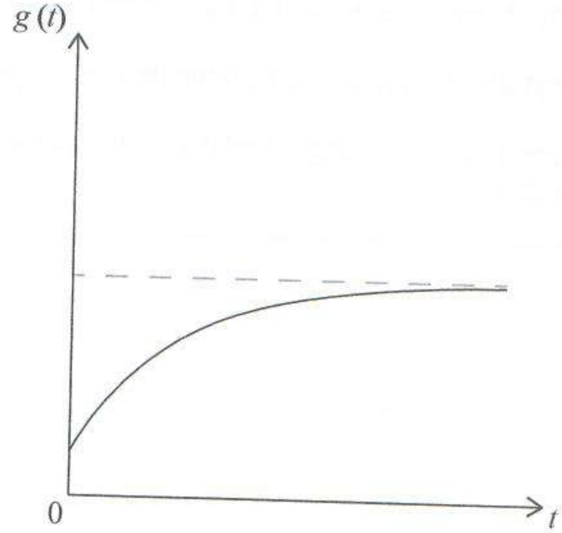
(c) . . . .

15. The amount of yeast, g grams, in a sugar solution can be modelled by the function,

$$g(t) = 10 - k(c^{-t}) \text{ for } t \geq 0$$

where t is the time in minutes.

The graph of $g(t)$ is shown.



The initial amount of yeast in this solution is 2 grams.

(a) Find the value of k. [2]

The amount of yeast in this solution after 3 minutes is 9 grams.

(b) Find the value of c. [3]

(c) Write down the maximum amount of yeast in this solution. [1]

(This question continues on the following page)

N18/5/MATSD/SP1/SPA/TZ0/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Estudios matemáticos
Nivel medio
Prueba 1

Lunes 12 de noviembre de 2018 (tarde)

Número de convocatoria del alumno

1 hora 30 minutos

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Para esta prueba, se necesita una copia sin anotaciones del **cuadernillo de fórmulas de estudios matemáticos NM**.
- Conteste todas las preguntas.
- Escriba sus respuestas en las casillas provistas a tal efecto.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán darse como valores exactos o con una aproximación de tres cifras significativas.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

19 páginas

8818 – 7409

Se otorgará la máxima puntuación a las respuestas correctas. Aun cuando una respuesta sea incorrecta, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Escriba sus respuestas en las casillas provistas a tal efecto. Para los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido hasta su obtención. Por ejemplo, cuando deba utilizar un gráfico de una calculadora de pantalla gráfica para hallar soluciones, deberá dibujar aproximadamente esas gráficas en su respuesta.

1. El volumen, V, de una semiesfera viene dado por la fórmula

$$V = \sqrt{\frac{4S^3}{243\pi}},$$

donde S es el área total de la superficie.

El área total de la superficie de una semiesfera dada es igual a $350\,\text{cm}^2$.

(a) Calcule el volumen en cm^3 de esta semiesfera.
Dé la respuesta redondeando a **un lugar decimal**. [3]

(b) Escriba la respuesta dada en el apartado (a) redondeando al número entero más próximo. [1]

(c) Escriba la respuesta dada en el **apartado (b)** en la forma $a \times 10^k$, donde $1 \le a < 10$ y $k \in \mathbb{Z}$. [2]

Operaciones:

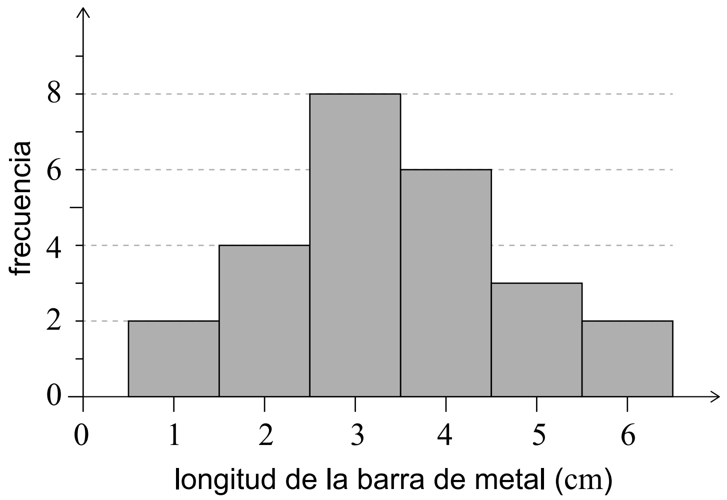
Respuesta:

(a) . . . .

(b) . . . .

(c) . . . .

2. El siguiente histograma muestra las longitudes de 25 barras de metal, cada una de las cuales se ha medido redondeando al número de cm más próximo.



(a) Escriba la moda de las longitudes de las barras. [1]

(b) Halle la mediana de las longitudes de las barras. [3]

El tercer cuartil es $4\,\mathrm{cm}$.

(c) Calcule

(i) el primer cuartil;

(ii) el rango intercuartil. [2]

Operaciones:

Respuesta:

(a) . . . .

(b) . . . .

(c) (i) . . . .

(ii) . . . .

3. Harry viajó desde EE. UU. a México y cambió 700 dólares (USD) a pesos (MXN).

El tipo de cambio aplicado fue $1\,\mathrm{USD} = 18{,}86\,\mathrm{MXN}$.

(a) Calcule cuántos MXN recibió Harry. [2]

A la vuelta, Harry tenía aún $2400\,\mathrm{MXN}$ y los quiso volver a cambiar a USD.
Había que pagar una comisión del $3{,}5\,\%$ por el cambio de divisas.

(b) Calcule en MXN, el valor de la comisión que pagó Harry. [2]

El tipo de cambio que se aplicó a este cambio de divisas fue $1\,\mathrm{USD} = 17{,}24\,\mathrm{MXN}$.

(c) Calcule cuántos USD recibió Harry. Dé la respuesta redondeando al número de céntimos más próximo. [2]

Operaciones:

Respuesta:

(a) . . . .

(b) . . . .

(c) . . . .

4. Abhinav lleva a cabo una prueba de χ^2 a un nivel de significación del 1% para determinar si el sexo de las personas influye en el campo profesional que escogen: ingeniería, medicina o derecho.
Hizo una encuesta en la que participaron 220 personas. Los resultados se muestran en la siguiente tabla.

	Ingeniería	**Medicina**	**Derecho**
Hombres	55	30	25
Mujeres	35	45	30

(a) Indique la hipótesis nula, H_0, para esta prueba. [1]

(b) Calcule el número esperado de ingenieros hombres. [2]

(c) Halle el valor del parámetro p para esta prueba. [2]

Abhinav rechaza H_0.

(d) Indique una razón que explique por qué Abhinav se está equivocando al hacer eso. [1]

Operaciones:

Respuesta:

(a) . . . .

. . . .

. . . .

(b) . . . .

(c) . . . .

(d) . . . .

. . . .

5. La siguiente tabla muestra los cinco primeros términos de tres progresiones: u_n, v_n y w_n.

	n				
	1	**2**	**3**	**4**	**5**
u_n	10	20	40	80	160
v_n	10	20	30	60	100
w_n	10	20	30	40	50

(a) Indique cuál de estas progresiones es

(i) aritmética;

(ii) geométrica. [2]

(b) Halle el valor exacto del $11.^{\text{o}}$ término de la progresión geométrica. [2]

(c) Halle la suma de los 20 primeros términos de la progresión aritmética. [2]

Operaciones:

Respuesta:

(a) (i) . . . .

(ii) . . . .

(b) . . . .

(c) . . . .

6. (a) Complete la siguiente tabla de verdad. [4]

p	q	$p \vee q$	$\neg q$	$p \wedge \neg q$	$\neg(p \vee q)$	$(p \wedge \neg q) \Rightarrow \neg(p \vee q)$
V	V	V				
V	F	V				
F	V	V				
F	F	F				

(b) Indique si el enunciado $(p \wedge \neg q) \Rightarrow \neg(p \vee q)$ es una contradicción lógica, una tautología o ninguna de las dos cosas. Dé una razón que justifique su respuesta. [2]

Operaciones:

Respuesta:

(b) . . . .
. . . .
. . . .
. . . .

7. Nick tiene $150\,000$ \$ en un fideicomiso. Todos los años, dona el 8% del dinero que queda en el fideicomiso.

(a) Determine el máximo número de años que Nick puede donar dinero, manteniendo por lo menos $50\,000$ \$ en el fideicomiso. [3]

Louise invierte $200\,000$ \$ en una cuenta bancaria que paga un tipo de interés nominal del 5%, **compuesto trimestralmente**, durante ocho años.

(b) Calcule el valor de la inversión de Louise cuando finalice este periodo. Dé la respuesta redondeando al número de céntimos más próximo. [3]

Operaciones:

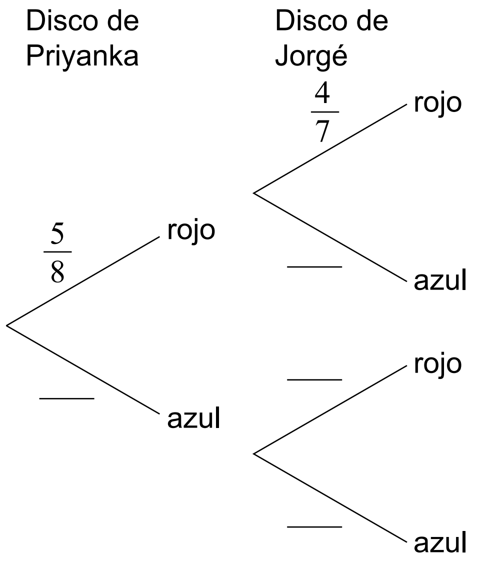
Respuesta:

(a) . . . .

(b) . . . .

8. Una bolsa contiene 5 discos rojos y 3 azules. Todos ellos son idénticos excepto por el color. En primer lugar, Priyanka coge al azar un disco de la bolsa y, a continuación, Jorgé coge al azar un disco de la bolsa.

(a) Complete el diagrama de árbol. [3]



(b) Halle la probabilidad de que Jorgé saque un disco rojo. [3]

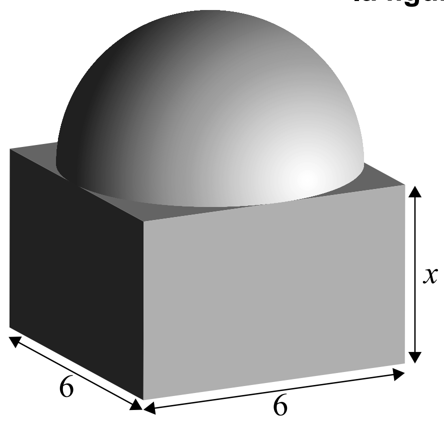
Operaciones:

Respuesta:

(b) . . . .

9. Un pisapapeles de cristal sólido consta de una semiesfera de $6\,\text{cm}$ de diámetro situada encima de un ortoedro de base cuadrada de $6\,\text{cm}$ de longitud, tal y como se muestra en la figura.

la figura no está dibujada a escala



La altura del ortoedro, $x\,\text{cm}$, es igual a la altura de la semiesfera.

(a) (i) Escriba el valor de x.

(ii) Calcule el volumen del pisapapeles. [4]

$1\,\text{cm}^3$ de cristal tiene una masa de $2{,}56$ gramos.

(b) Calcule la masa, en gramos, del pisapapeles. [2]

Operaciones:

Respuesta:

(a) (i) . . . .

(ii) . . . .

(b) . . . .

10. Considere las siguientes proposiciones.

p: puede salir mal
q: sale mal

(a) Escriba en forma simbólica:

Si no sale mal, entonces no puede salir mal. [2]

(b) Escriba con palabras el argumento $p \Rightarrow q$. [2]

(c) Escriba con palabras la proposición contraria de $p \Rightarrow q$. [2]

Operaciones:

Respuesta:

(a) . . . .

(b) . . . .

. . . .

. . . .

(c) . . . .

. . . .

. . . .

11. Considere la curva $y = 5x^3 - 3x$.

(a) Halle $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

La curva tiene una tangente en el punto $\mathrm{P}(-1, -2)$.

(b) Halle la pendiente de esta recta tangente en el punto P. [2]

(c) Halle la ecuación de esta tangente. Dé la respuesta en la forma $y = mx + c$. [2]

Operaciones:

Respuesta:

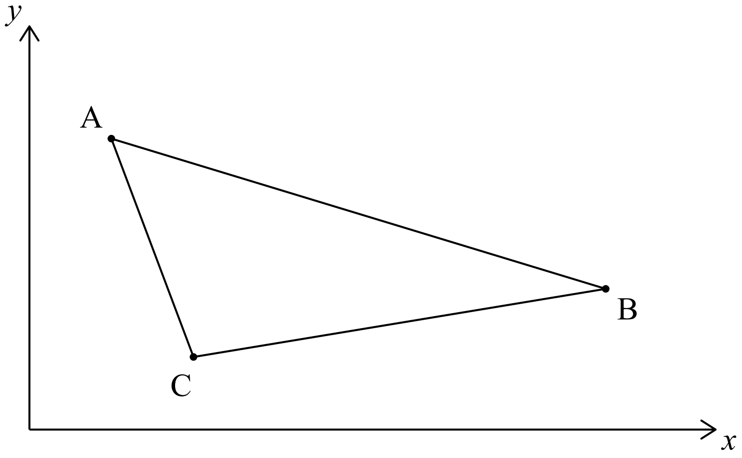
(a) . . . .

(b) . . . .

(c) . . . .

12. La figura muestra un triángulo que está definido por los puntos $\mathrm{A}(3, 9)$, $\mathrm{B}(15, 6)$ y $\mathrm{C}(5, 3)$.

la figura no está dibujada a escala



(a) Calcule la pendiente de la recta AC. [2]

(b) Determine, dando una razón, si el ángulo $\mathrm{A\hat{C}B}$ es un ángulo recto. [2]

La recta L es paralela a BC y pasa por A.

(c) Halle la ecuación de L.
Dé la respuesta en la forma $ax + by + d = 0$, donde a, b y d son números enteros. [2]

Operaciones:

Respuesta:

(a) . . . .

(b) . . . .

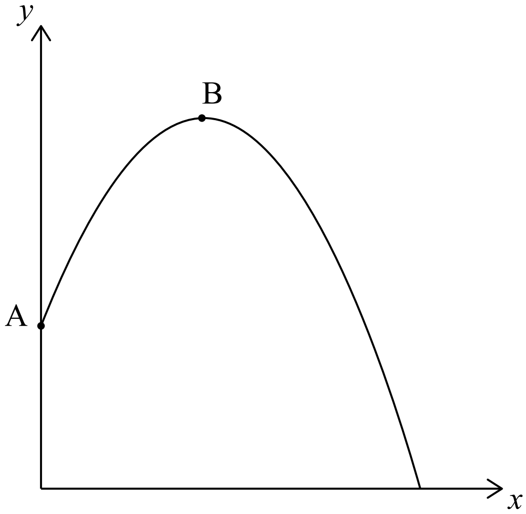
. . . .

(c) . . . .

13. Bella lanza una pelota desde lo alto de un muro y la pelota cae al suelo, que es horizontal y plano.

La trayectoria que sigue la pelota está modelizada por la curva cuadrática $y = 3 + 4x - x^2$, donde x representa la distancia horizontal de la pelota e y representa la altura de la pelota respecto al suelo. Todas las distancias se miden en metros.

La posición del muro coincide con el eje y. La curva corta el eje y en el punto A y tiene el vértice en el punto B.



(a) Escriba la altura en metros desde la cual se lanzó la pelota. [1]

(b) Calcule la altura máxima, respecto al suelo, que alcanza la pelota. [3]

(c) Halle la distancia horizontal desde la base del muro hasta el punto en el que la pelota toca el suelo. [2]

(Esta pregunta continúa en la página siguiente)

(Pregunta 13: continuación)

Operaciones:

Respuesta:

(a) . . . .

(b) . . . .

(c) . . . .

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

14. En un examen de acceso a la universidad, las notas que sacaron un grupo de estudiantes siguen una distribución normal de media 300 y desviación típica 100.

En este examen, el 10% de los estudiantes sacaron una nota mayor que k.

(a) Halle el valor de k. [2]

La Universidad Marron admite únicamente a aquellos estudiantes que, en este examen, sacan una nota de al menos 450.

(b) Halle la probabilidad de que un estudiante elegido al azar sea admitido en la Universidad Marron. [2]

(c) Sabiendo que Naomi estudia en la Universidad Marron, halle la probabilidad de que en este examen haya sacado una nota de al menos 500. [2]

Operaciones:

Respuesta:

(a) . . . .

(b) . . . .

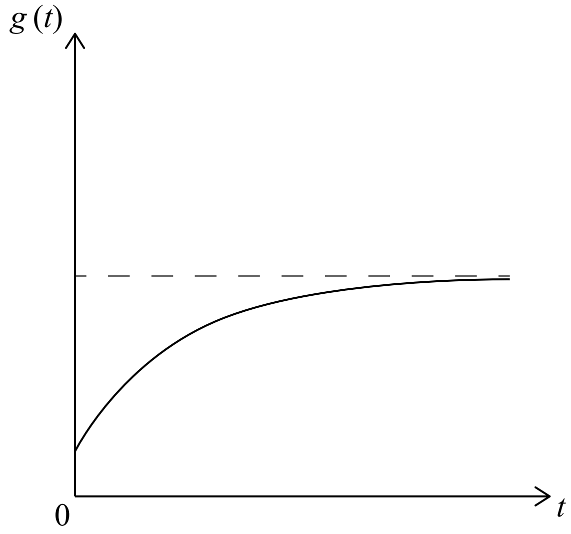
(c) . . . .

15. La cantidad de levadura, g gramos, en una disolución de azúcar se puede modelizar por la función

$$g(t) = 10 - k(c^{-t}) \text{ para } t \geq 0,$$

donde t es el tiempo en minutos.

A continuación se muestra el gráfico de $g(t)$.



En esta disolución, la cantidad inicial de levadura es igual a 2 gramos.

(a) Halle el valor de k. [2]

La cantidad de levadura que hay en esta disolución al cabo de 3 minutos es igual a 9 gramos.

(b) Halle el valor de c. [3]

(c) Escriba la cantidad máxima de levadura en esta disolución. [1]

(Esta pregunta continúa en la página siguiente)

(Pregunta 15: continuación)

Operaciones:

Respuesta:

(a) . . . .

(b) . . . .

(c) . . . .

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

N18/5/MATSD/SP1/SPA/TZ0/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Esquema de calificación

Noviembre de 2018

Estudios matemáticos

Nivel medio

Prueba 1

27 páginas

Esquema de calificación de la Prueba 1
Instrucciones para los examinadores

Observaciones: **En caso de dudas sobre estas instrucciones o temas relacionados con la asignación de puntuaciones, póngase en contacto con su jefe de equipo.**

La puntuación máxima para cada pregunta es 6.

1 Siglas

En el esquema de calificación pueden aparecer las siguientes siglas:

M Puntos otorgados por el **método**

A Puntos otorgados por una **respuesta** o por la **precisión**

C Puntos otorgados por respuestas **correctas** (independientemente del trabajo mostrado)

R Puntos otorgados por un **razonamiento** claro

ft Se pueden otorgar puntos por la **coherencia** con resultados previos en la pregunta

2 Método para corregir

(a) Todas las correcciones deben realizarse en RM Assessor, utilizando las anotaciones de Estudios Matemáticos y de acuerdo con el documento vigente de orientaciones para la corrección electrónica de Estudios Matemáticos NM. Es fundamental leer el documento antes de empezar a corregir.

(b) Si el alumno obtiene la puntuación máxima en una pregunta utilice la anotación ***C6***, si lo ha intentado pero obtiene cero puntos utilice ***C0***. Si no lo ha intentado utilice la tecla de No respuesta. Si un alumno no ha obtenido ni la puntuación máxima ni cero puntos, entonces se **DEBEN** mostrar todas las anotaciones.

(c) En esta prueba, **si aparece la respuesta correcta en la línea de respuesta** se otorgará la puntuación máxima. **¡No es necesario comprobar el trabajo! Otorgue puntos C y siga adelante.**

(d) Si la respuesta no aparece en la línea de respuesta, pero la respuesta correcta se encuentra en el cuadro de operaciones sin trabajo posterior, otorgue la puntuación máxima.

(e) Si la **respuesta es incorrecta**, se deben otorgar puntos por el trabajo realizado, de acuerdo con el esquema de calificación.

(f) No se debe otorgar ningún punto al trabajo tachado por el alumno. Si el alumno ha dado dos respuestas a una pregunta, solo se debe corregir la primera respuesta.

(g) Una respuesta correcta en el cuadro de trabajo transcrita erróneamente a la línea de respuesta puede recibir la puntuación máxima.

(e) Si el procedimiento adecuado lleva a la respuesta correcta, pero esta va seguida de más desarrollos que indican una falta de comprensión matemática, entonces **no** se puede otorgar la puntuación máxima. En la mayoría de estos casos se perderá solo el último punto por respuesta. Una excepción podría ser las respuestas numéricas, donde una respuesta exacta que es correcta está seguida de una expresión decimal incorrecta.

Ejemplo:

	Se ve la respuesta correcta	**Hay más desarrollo**	**Acción**
1.	$8\sqrt{2}$	$5,65685...$ *(valor decimal incorrecto)*	Otorgue el ultimo ***(A1)*** *(ignore el desarrollo posterior)*
2.	$(x-6)\,(x+1)$	$x=6$ y -1	**No** otorgue el último ***(A1)*** *(vea el siguiente ejemplo)*

Ejemplo: Factorice $x^2 - 5x - 6$

Esquema de calificación	Examen del alumno	Corrección
$(x-6)(x+1)$ ***(A1)(A1)***	(i) Línea de respuesta: $(x+6)(x+1)$	***(A0)(A1)***
	(ii) Cuadro de operaciones: $(x-6)(x+1)$	***(A1)***
	seguido de $x=6$ y -1, o simplemente $6, -1$ bien en el cuadro de trabajo o en la línea de respuesta.	***(A0)***

3 Puntos por la coherencia (ft)

En cualquier paso de una resolución, un error puede afectar a todo el trabajo posterior. Para limitar la gravedad de la penalización, se pueden otorgar **puntos por la coherencia**. Los esquemas de calificación indicarán, con **"(ft)"**, dónde es adecuado aplicar el criterio de coherencia en una pregunta.

(a) La coherencia se aplica solo de un apartado de una pregunta a otro apartado posterior de la pregunta. No se aplica dentro del mismo apartado.

(b) Si una respuesta que es consecuencia de un procedimiento de coherencia es muy poco realista (por ejemplo, distancias negativas o de un orden de magnitud excesivo) entonces el punto ***A*** final no se debe otorgar.

(c) Si, debido a un error, una pregunta queda transformada en otra **distinta**, **mucho más sencilla**, entonces el criterio de coherencia podría no ser aplicado.

(d) Para otorgar puntos por coherencia en un apartado de una pregunta, **es necesario que haya un trabajo explícito relacionado con ese apartado**. Una respuesta aislada, obtenida por coherencia, pero sin un desarrollo explícito, se considera incorrecta, y no recibirá ningún punto **aunque se aproxime a la respuesta correcta**.

(e) Una excepción a lo anterior sería una pregunta donde lo que se evalúa es el manejo del alumno en el uso de la calculadora de pantalla gráfica, y donde no se pretende que se muestre ningún desarrollo escrito. **En el esquema de calificación vendrá indicado claramente cuando sea el caso.**

(f) El uso inadvertido de radianes será penalizado la primera vez que aparezca. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez un punto por el uso de radianes.

Ejemplo: Cálculo de ángulos y longitudes en trigonometría

Esquema de calificación		Examen del alumno	Corrección
(a) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$	***(M1)(A1)***	(a) $\frac{\text{sen } A}{4} = \frac{\text{sen } 30}{3}$	***(M1)(A0)*** *(uso del teorema del seno, pero con valores incorrectos)*
$A = 22{,}0^\circ\ (22{,}0243\ldots)$	***(A1)***	$A = 41{,}8^\circ$	***(A0)*** ***(Nota:*** *Aquí, el segundo* ***(A1)*** *no ha sido corregido como* **(ft)** *y no se puede otorgar porque existe un error previo en el* ***mismo*** *apartado de la pregunta.)*
(b) $x = 7\text{tg}\,(22{,}0243\ldots^\circ)$	***(M1)***	(b) caso (i) $x = 7\text{tg}\,41{,}8^\circ$	***(M1)***
$= 2{,}83\ (2{,}831639\ldots)$	***(A1)*(ft)**	$= 6{,}26$	***(A1)*(ft)**
		pero caso (ii) $6{,}26$	***(C0)*** *pues no aparece un desarrollo explícito*

4 Uso del Esquema de calificación

(a) Los puntos ***A*** **dependen** de los puntos ***M*** obtenidos previamente, **no** es posible otorgar ***(M0)(A1)***. Una vez que se otorga un ***(M0)***, se pierden todos los puntos ***A*** que le siguen en ese apartado de la pregunta, incluso si los cálculos son correctos, hasta el siguiente punto ***M***.
La única excepción a lo anterior se hará en el caso de una respuesta donde la precisión se especifique en la pregunta (ver apartado 5).

(b) Los puntos ***A*** **dependen** de los puntos ***R*** obtenidos, **no** es posible otorgar ***(A1)(R0)***. Así pues, el ***(A1)*** no se otorga a una respuesta correcta si no aparece el razonamiento, o este es incorrecto.

(c) Los **métodos alternativos** no siempre se incluyen. Así, si una respuesta es errónea, entonces se debe analizar cuidadosamente el procedimiento, de modo que se asignen puntos por un método distinto de forma coherente con el esquema de calificación.
Cuando en el esquema de calificación se incluyen métodos alternativos para una pregunta, ello viene indicado mediante "**O**" *etc*.

(d) A menos que en la pregunta se especifique lo contrario, se aceptan **expresiones equivalentes**. Por ejemplo: $\frac{\text{sen}\,\theta}{\cos\theta}$ por $\text{tg}\,\theta$. En el esquema de calificación, estas expresiones algebraica o numéricamente equivalentes aparecerán a veces escritas entre paréntesis junto a la respuesta requerida.
Cuando se requieran respuestas numéricas como respuesta final de un apartado de una pregunta, en el esquema de calificación se mostrará en este orden;
la respuesta con 3 cifras significativas a partir de la solución completa de la calculadora;
el valor exacto (por ejemplo $\sqrt{3}$ si corresponde);
la solución completa de la calculadora en la forma $2{,}83163\ldots$ como en el ejemplo anterior.
Cuando se dan las respuestas con 3 cifras significativas y se utilizan después en apartados posteriores de la pregunta llevando a una solución con 3 cifras significativas **distinta**, también se darán estas soluciones.

(e) Dado que este es un examen internacional, se aceptarán todas las **formas de notación alternativas**. Algunos ejemplos:

Comas decimales: $1{,}7$; $1'7$; $1\cdot 7$; $1;7$.
Los números decimales menores que 1 podrían aparecer escritos sin un cero en el frente: 0.49 o $.49$.
Descripciones de un intervalo: $3 < x < 5$; $(3;\,5)$; $]\,3;\,5\,[$.
Distintas formas de notación de las propiedades de los conjuntos (por ejemplo complementario): A'; $\overline{A}$; A^c; $U - A$;$(A\,;U \setminus A$.
Distintas formas de notación lógica: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p, -p$.
$p \Rightarrow q$; $p \rightarrow q$.
El nivel de significación podría escribirse como α .

(f) Puntos discrecionales: habrá ocasiones excepcionales en las que el esquema de calificación no cubra el trabajo que aparece. En estos casos se utilizará la nota DM para indicar que el examinador ha utilizado su criterio. La discrecionalidad debe utilizarse con moderación, y si existe duda se debe plantear una excepción a través de RM Assessor al jefe del equipo.

A partir de noviembre de 2011 ya no se aplicarán las penalizaciones AP, FP y UP. La precisión y las unidades serán evaluados en preguntas específicas y los puntos se otorgarán de acuerdo a las reglas dadas en los apartados 5, 6 y 7.

5 Precisión de las respuestas

Instrucciones: Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas correctamente a 3 cifras significativas.

1. Si el alumno muestra la solución a 4 o más cifras significativas, la cual redondeada a 3 cifras significativas daría la respuesta requerida entonces otorgue ***(A1)*** e ignore cualquier redondeo posterior. **Nota:** La solución sin redondear puede estar tanto en la línea de respuesta como en el cuadro de operaciones.

2. Si el alumno **no** muestra la solución sin redondear, entonces otorgue ***(A1)*** si la solución dada está aproximada correctamente a 2 o más cifras significativas, y ***(A0)*** en cualquier otro caso.

3. Si una solución aproximada a 2 cifras significativas se utiliza en apartados posteriores, entonces para otorgar más puntos debe aparecer el desarrollo. (Este criterio es el mismo que en el que se aplica coherencia a partir de una respuesta errónea.)

Estos 3 casos (vea los superíndices) se han resumido en la siguiente tabla e ilustrado después con ejemplos.

<table>
<tr><td></td><td colspan="5">Si la solución final de alumno se expresa…</td></tr>
<tr><td></td><td>Exacta o a 4 o más cifras significativas (y que redondeada a 3 cifras significativas daría la respuesta correcta)</td><td>Aproximada incorrectamente a 3 cifras significativas</td><td>Aproximada correctamente a 2 cifras significativas[3]</td><td>Aproximada incorrectamente a 2 cifras significativas</td><td>Aproximada correcta o incorrectamente a 1 cifra significativa</td></tr>
<tr><td>Aparece la solución sin redondear[1]</td><td colspan="5">Otorgue (A1) independientemente del redondeo correcto o incorrecto</td></tr>
<tr><td>No aparece la solución sin redondear[2]</td><td>(A1)</td><td>(A0)</td><td>(A1)</td><td>(A0)</td><td>(A0)</td></tr>
<tr><td>Criterio para los apartados posteriores</td><td>Como en MS</td><td colspan="4">Como en el criterio de coherencia, solo si se muestra el desarrollo[3]</td></tr>
</table>

Ejemplos:

Esquema de calificación	Examen del alumno	Corrección
9,43 (9,43398…) ***(A1)***	(i) 9,43398… se muestra en el cuadro de trabajo seguido de 9; 9,4; 9,43; 9,434 (redondeados correctamente)	***(A1)***
	(ii) 9,43398… se muestra en el cuadro de trabajo seguido de 9,433; 9,44, *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 9,4	***(A1)***
	(iv) 9	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 9,3	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 9,44	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Esquema de calificación	Examen del alumno	Corrección
7,44 (7,43798…) ***(A1)***	(i) 7,43798… se muestra en el cuadro de trabajo seguido de 7; 7,4; 7,44; 7,438 *etc.* (redondeados correctamente)	***(A1)***
	(ii) 7,43798… se muestra en el cuadro de trabajo seguido de 7,437 ó 7,43 *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 7,4	***(A1)***
	(iv) 7	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 7,5	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 7,43	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Ejemplo: ABC es un triángulo rectángulo con el ángulo $\text{ABC} = 90^\circ$, $\text{AC} = 32 \text{ cm}$ y $\text{AB} = 30 \text{ cm}$. Halle (a) la longitud de BC, (b) el área del triángulo ABC.

Esquema de calificación	Examen del alumno	Corrección
(a) $\text{BC} = \sqrt{32^2 - 30^2}$ ***(M1)*** *Otorgue **(M1)** por la sustitución correcta en el teorema de Pitágoras* $= 11{,}1 \ \left(\sqrt{124}\ 11{,}1355...\right)(\text{cm})$ ***(A1)***	(a) $\text{BC} = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$ *(solo se muestra la aproximación a 2 cifras significativas, pero correcta)*	***(M1)*** ***(A1)***
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11{,}1355...$ ***(M1)*** *Otorgue **(M1)** por la sustitución correcta en la fórmula del área de un triángulo* $= 167 \ \ (167{,}032...)(\text{cm}^2)$ ***(A1)*(ft)**	(b) caso (i) $\text{Área} = \frac{1}{2} \times 30 \times 11$ *(se muestra el desarrollo)* $= 165\,(\text{cm}^2)$ caso (ii) $= 165\,(\text{cm}^2)$ *(No se muestra el desarrollo, la solución 11 se trata como procedimiento de coherencia, por lo que no se deben otorgar puntos en este caso)*	***(M1)*** ***(A1)*(ft)** ***(M0)(A0)*(ft)**

La aproximación a 3 cifras significativas de una solución exacta **se debe aceptar si se realiza correctamente**.

Las soluciones exactas del tipo $\frac{1}{4}$ se pueden escribir como decimales con menos de 3 cifras significativas si el resultado sigue siendo exacto. La simplificación de una fracción a su expresión irreducible **no** es imprescindible, sin embargo cuando una respuesta se simplifica a un entero, esto es lo esperado. Fracciones que incluyen un decimal en el numerador y/o en el denominador son aceptadas para demostrar una substitución correcta, sin embargo, no como respuesta final.

Razones de π y respuestas con expresiones de raíces cuadradas de enteros o cualquier potencia racional de un entero (por ejemplo, $\sqrt{13}, 2^{2/3}, \sqrt[4]{5}$), se pueden aceptar como respuestas exactas. Todas las otras potencias (por ejemplo de no enteros) y valores de funciones transcendentes tales como seno y coseno se deben evaluar.

Si el nivel de precisión viene especificado en la pregunta, se asignarán puntos por dar la respuesta con la precisión requerida. En **todos** estos casos no se obtiene el punto final si el redondeo no sigue las instrucciones dadas en la pregunta. El punto por la precisión especificada se puede considerar como punto por coherencia **(ft)** con independencia de un ***(M0)*** inmediatamente anterior.

6 Nivel de precisión en las preguntas sobre cuestiones financieras

El nivel de precisión de las respuestas se especificará en todas las preguntas que se relacionen con valores en dinero. Generalmente será a la unidad más cercana o de dos cifras decimales. La primera respuesta que no venga dada con el nivel de precisión especificado no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez la un punto por la precisión incorrecta en una pregunta sobre cuestiones financieras.

Ejemplo: En una pregunta sobre cuestiones financieras se pide una precisión de dos cifras decimales.

Esquema de calificación	Examen del alumno	Corrección
$\$231,62$ $(231,6189)$ ***(A1)***	(i) $231,6$	***(A0)***
	(ii) 232	***(A0)***
	(Aproximación correcta con un nivel de precisión distinto del requerido)	
	(iii) $231,61$	***(A0)***
	(iv) $232,00$	***(A0)***
	(Aproximación incorrecta al nivel de precisión requerido)	

7 Unidades de medida en las respuestas

En determinadas preguntas será necesario expresar las unidades de medida, y esto vendrá claramente indicado en el esquema de calificación. La primera respuesta correcta pero sin unidades o con unidades incorrectas no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la falta de unidades o unidades incorrectas. Las unidades de medida se tendrán en cuenta solamente cuando se ha otorgado ***(A1)*** a la respuesta numérica siguiendo las indicaciones sobre la precisión dadas en el apartado 5.

Esquema de calificación	Examen del alumno	Corrección
(a) 37000 m^2 ***(A1)***	(a) 36000 m^2	***(A0)***
	(respuesta incorrecta, por lo que no se consideran las unidades)	
(b) 3200 m^3 ***(A1)***	(b) 3200 m^2	***(A0)***
	(unidades incorrectas)	

8 Calculadoras de pantalla gráfica

Con frecuencia los alumnos obtienen las soluciones directamente de la calculadora. Deben utilizar la notación matemática y no la notación de la calculadora. No se puede otorgar puntos por método a respuestas incorrectas basadas únicamente en notación de calculadora. No se puede otorgar puntos por método al comentario “usé la calculadora de pantalla gráfica”.

1. (a) $\sqrt{\dfrac{4(350)^3}{243\pi}}$ **O BIEN** $\sqrt{\dfrac{171500000}{763,407...}}$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido 350 en la fórmula del volumen.

$= 473,973\ldots$ ***(A1)***

$= 474,0\ (\text{cm}^3)$ ***(A1)*(ft)** ***(C3)***

> **Nota:** El ***(A1)(ft)*** se otorga por redondear **su** respuesta a 1 lugar decimal, siempre y cuando la respuesta sin redondear se vea.

[3 puntos]

(b) $474\ (\text{cm}^3)$ ***(A1)*(ft)** ***(C1)***

> **Nota:** Arrastre de error (ft) desde (a).

[1 punto]

(c) $4,74\times10^2\ (\text{cm}^3)$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

> **Nota:** Error de arrastre **solo** de la parte **(b).**
> Otorgue ***(A0)(A0)*** por respuestas del tipo $0,474\times10^3$.

[2 puntos]

Total [6 puntos]

2. (a) 3 ***(A1)*** ***(C1)***

[1 punto]

(b) la mediana corresponde a la $13.^{\text{a}}$ posición. ***(M1)***
frecuencias acumuladas: $2,\ 6,\ 14,\ 20,\ 23,\ 25$ ***(M1)***

mediana $= 3$ ***(A1)*** ***(C3)***

[3 puntos]

(c) (i) $2{,}5$ ***(A1)***

(ii) $1{,}5$ ***(A1)*****(ft)*** ***(C2)***

Nota: Otorgue ***(A1)*****(ft)** si la suma de **sus** partes (c)(i) y (c)(ii) es 4.

[2 puntos]

Total [6 puntos]

3. (a) $700 \times 18,86$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por la multiplicación por $18,86$.

$= 13\,200 \quad (13\,202)$ (MXN) ***(A1)*** ***(C2)***

[2 puntos]

(b) $2400 \times 0,035$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por la multiplicación por $0,035$.

$= 84$ (MXN) ***(A1)*** ***(C2)***

[2 puntos]

(c) $\dfrac{2400 - \text{la respuesta de (b)}}{17,24}$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por dividir 2400 menos el valor obtenido en (b), entre $17,24$.
> Arrastre de error (ft) desde el apartado (b).

$= 134,34$ (USD) ***(A1)*****(ft)** ***(C2)***

> **Nota:** Conceda a lo más ***(M1)(A0)*** si la respuesta final no está redondeada al centavo.

[2 puntos]

Total [6 puntos]

4. (a) El género y la profesión elegida son independientes. ***(A1)*** ***(C1)***

Nota: Acepte que "no existe asociación entre la profesión elegida y el sexo de la persona". Acepte "no son dependientes". No acepte "no están relacionadas" ni "no están correlacionadas" ni "no influye" ni "no afecta".

[1 punto]

(b) $\frac{110}{220}\times\frac{90}{220}\times 220 \quad \left(\frac{110\times 90}{220}\right)$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula del valor esperado.

$= 45$ ***(A1)*** ***(C2)***

[2 puntos]

(c) $0{,}0193 \quad (0{,}0192644\ldots)$ ***(A2)*** ***(C2)***

[2 puntos]

(d) $0{,}0193 > 0{,}01 \quad (1\ \%)$ ***(A1)*(ft)**

O BIEN

El valor del parámetro p es mayor que el nivel de significación $(1\ \%)$ ***(A1)*(ft)** ***(C1)***

Nota: Tiene que haber escrito un valor numérico en el apartado (c) para poder concederle aquí el ***(A1)*(ft)**. Arrastre de error (ft) desde (c), solo si $> 0{,}01$. Acepte una respuesta correcta que sea el resultado de haber comparado el valor numérico del estadístico χ^2 **y también** el valor numérico del valor crítico: $7{,}89898\ldots < 9{,}21$.

[1 punto]

Total [6 puntos]

5. (a) (i) w_n ***(A1)***

(ii) u_n ***(A1)*** ***(C2)***

[2 puntos]

(b) $10(2)^{11-1}$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula de la progresión geométrica.

$= 10\,240$ ***(A1)*(ft)** ***(C2)***

Nota: Acepte solo la respuesta exacta. Las reglas sobre precisión no se aplican en esta parte, no acepte la respuesta a 3 cs de $10\,200$.

[2 puntos]

(c) $\frac{20\left[2(10)+(19)(10)\right]}{2}$ **O BIEN** $\frac{20(10+200)}{2}$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula de la serie aritmética.

$= 2100$ ***(A1)*(ft)** ***(C2)***

[2 puntos]

Total [6 puntos]

6. (a)

p	q	$p \vee q$	$\neg q$	$p \wedge \neg q$	$\neg(p \vee q)$	$(p \wedge \neg q) \Rightarrow \neg(p \vee q)$
V	V	V	**F**	**F**	**F**	**V**
V	F	V	**V**	**V**	**F**	**F**
F	V	V	**F**	**F**	**F**	**V**
F	F	F	**V**	**F**	**V**	**V**

(A1)(A1)*(ft)*(A1)(A1)*(ft)** ***(C4)

Nota: Conceda ***(A1)*** por cada columna que sea correcta, habiendo aplicado arrastre de error (ft) desde las columnas respectivas.

[4 puntos]

(b) ninguna de las dos cosas ***(A1)*(ft)**
dado que en la última columna ni todos los resultados son “verdadero” y ni tampoco son todos “falso". ***(R1)*** ***(C2)***

Nota: No conceda ***(A1)(R0)***. Arrastre de error (ft) desde una tabla de verdad incorrecta, pero solo si su razonamiento es coherente con la última columna de la tabla**.**

Conceda ***(R1)*** solo si la columna final está **claramente** identificada en la justificación.

[2 puntos]

Total [6 puntos]

7. (a) $150\,000\left(1-\frac{8}{100}\right)^{n}=50\,000$ ***(A1)(M1)***

Nota: Conceda ***(A1)*** por haber sustituido correctamente los datos en la fórmula del interés compuesto **o** por haber sustituido correctamente en la fórmula de la progresión geométrica (con $r=0{,}92$); conceda ***(M1)*** por haber igualado a $50\,000$ la fórmula del interés compuesto o la fórmula de un término de una sucesión geométrica(en la que se habían sustituido previamente los datos).

O BIEN

$\text{I}\,\%=-8$ ***(A1)(M1)***
$\text{VP}=\pm 150\,000$
$\text{VF}=\mp 50\,000$
$\text{P/A}=1$
$\text{C/A}=1$

Nota: Conceda ***(A1)*** por haber escrito $\text{C/A}=1$; conceda ***(M1)*** por otras expresiones correctas. VF y VP deben tener signo opuesto.

O BIEN

$138\,000,\ 126\,960,\ 116\,803.20,\ 107\,458.94,\ 98\,862.23,\ \ldots$ ***(M1)***

$t_{13}=50\,737.96,\ t_{14}=46\,678.92$ ***(A1)***

Nota: Conceda ***(M1)*** por una lista de al menos 5 términos correctos comenzando con $138\,000$, ***(A1)*** por identificar t_{13} y t_{14}.

$n=13{,}1757$
13 años ***(A1)*** ***(C3)***

Nota: La respuesta ha de ser un número entero.

[3 puntos]

Continúa en la página siguiente...

Continuación de la Pregunta 7

(b) $200\,000\left(1+\dfrac{5}{4(100)}\right)^{8\times4}$ ***(M1)(A1)***

Nota: Conceda ***(M1)*** por haber sustituido los datos en la fórmula del interés compuesto; conceda ***(A1)*** si los ha sustituido correctamente.

O BIEN

$\mathrm{N}=8$ ***(A1)(M1)***
$\mathrm{I\%}=5$
$\mathrm{VP}=\pm 200\,000$
$\mathrm{P/A}=1$
$\mathrm{C/A}=4$

Nota: Conceda ***(A1)*** por haber escrito $\mathrm{C/A}=4$; conceda ***(M1)*** por otras expresiones correctas.

O BIEN

$\mathrm{N}=32$ ***(A1)(M1)***
$\mathrm{I\%}=5$
$\mathrm{VP}=\pm 200\,000$
$\mathrm{P/A}=4$
$\mathrm{C/A}=4$

Nota: Conceda ***(A1)*** por haber escrito $\mathrm{C/A}=4$; ***(M1)*** por otras expresiones correctas.

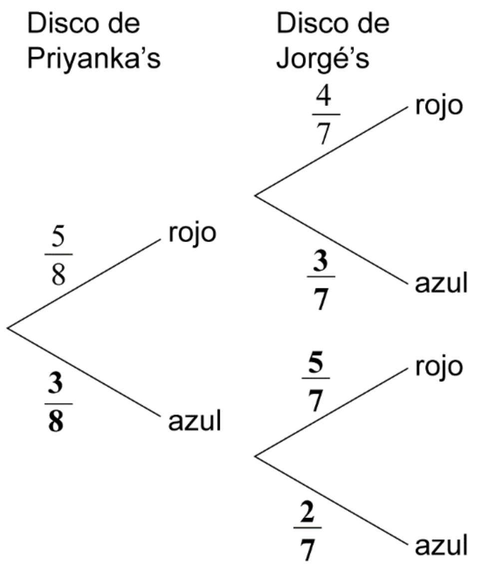
$=297\,626{,}10\ (\$)$ ***(A1)*** ***(C3)***

Nota: La respuesta se ha de dar redondeando a dos lugares decimales.

[3 puntos]

Total [6 puntos]

8. (a)



(A1)(A1)(A1) ***(C3)***

Nota:	Conceda ***(A1)*** por cada par de ramas que sea correcto.

[3 puntos]

(b) $\frac{5}{8}\times\frac{4}{7}+\frac{3}{8}\times\frac{5}{7}$ ***(A1)*(ft)*(M1)***

Nota:	Conceda ***(A1)*(ft)** por haber escrito **sus** dos productos correctos con valores procedentes de su diagrama de árbol. Arrastre de error (ft) desde el apartado (a); conceda ***(M1)*** por haber sumado los dos productos. Conceda ***(M0)*** si se suman productos adicionales.

$=\frac{5}{8}\ \left(\frac{35}{56};\ 0{,}625;\ 62{,}5\%\right)$ ***(A1)*(ft)** ***(C3)***

Nota:	Arrastre de error (ft) desde el diagrama de árbol, solo si las probabilidades están es el intervalo $[0,1]$.

[3 puntos]

Total [6 puntos]

9. Es necesario que haya especificado las unidades en el apartado (a)(ii)

(a) (i) $3\,(\text{cm})$ ***(A1)***

(ii) $\frac{1}{2} \times \frac{4\pi \times (3)^3}{3} + 3 \times (6)^2$ ***(M1)(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente **sus** datos en la formula del volumen de la esfera y haberla dividido en 2, ***(M1)*** por haber sustituido correctamente **sus** datos en la formula del volumen del ortoedro.

$= 165\,\text{cm}^3 \quad (164{,}548\ldots)$ ***(A1)*(ft)** ***(C4)***

Nota: La respuesta es $165\,\text{cm}^3$; es necesario que haya incluido las unidades. Arrastre de error (ft) desde el apartado (a)(i).

[4 puntos]

(b) el valor dado en a(ii) $164{,}548\ldots \times 2{,}56$ ***(M1)***

Nota: Conceda ***(M1)*** por haber multiplicado la respuesta obtenida en el apartado (a)(ii) por $2{,}56$.

$= 421(\text{g}) \quad \left(421{,}244\ldots(\text{g})\right)$ ***(A1)*(ft)** ***(C2)***

Nota: Arrastre de error (ft) desde el apartado (a)(ii).

[2 puntos]

Total [6 puntos]

10. (a) $\neg q \Rightarrow \neg p$ ***(A1)(A1)*** ***(C2)***

> **Nota:** Conceda ***(A1)*** por haber escrito dos negaciones; conceda ***(A1)*** por haber escrito un antecedente correcto y un consecuente correcto a cada lado de la implicación.

[2 puntos]

(b) Si puede salir mal, entonces sale mal ***(A1)(A1)*** ***(C2)***

> **Nota:** Conceda ***(A1)*** por haber escrito una expresión del tipo "si…entonces…"; conceda ***(A1)*** por haber escrito un antecedente y un consecuente correctos.

[2 puntos]

(c) Si no puede salir mal, entonces no sale mal. ***(A1)(A1)*****(ft)** ***(C2)***

> **Nota:** Conceda ***(A1)*** por haber escrito una expresión del tipo "si…entonces…"; conceda ***(A1)*****(ft)** por haber escrito un antecedente y un consecuente correctos. Arrastre de error (ft) desde el apartado (b).

[2 puntos]

Total [6 puntos]

11. (a) $15x^2 - 3$ ***(A1)(A1)*** ***(C2)***

Nota: Conceda ***(A1)*** por haber escrito $15x^2$; ***(A1)*** por haber escrito -3. Conceda como mucho ***(A1)(A0)*** si en su respuesta aparecen términos adicionales.

[2 puntos]

(b) $15(-1)^2 - 3$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido -1 en la expresión de $\frac{\mathrm{d}y}{\mathrm{d}x}$ que obtuvo en (a).

$= 12$ ***(A1)*(ft)** ***(C2)***

Nota: Arrastre de error (ft) desde el apartado (a).

[2 puntos]

(c) $(y - (-2)) = 12(x - (-1))$ ***(M1)***

O BIEN

$-2 = 12(-1) + c$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente el punto dado **y** su pendiente en la ecuación de la recta.

$y = 12x + 10$ ***(A1)*(ft)** ***(C2)***

Nota: Arrastre de error (ft) desde el apartado (b).

[2 puntos]

Total [6 puntos]

12. (a) $\frac{3-9}{5-3}$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido correctamente los datos en la fórmula de la pendiente.

$=-3$ ***(A1)*** ***(C2)***

[2 puntos]

(b) La pendiente de $\mathrm{CB}=0,3$ ***(A1)***

y dado que $0,3 \neq -\frac{1}{(-3)}$, el ángulo ACB no es un ángulo recto. ***(A1)*****(ft)** ***(C2)***

> **Nota:** Conceda el ***(A1)*****(ft)** únicamente si el alumno ha dado una justificación numérica. Arrastre de error (ft) desde el apartado (a).
>
> Si la formula de la distancia y el teorema de Pitágoras son usados: $\mathrm{AB}=12{,}3693\ldots$, $\mathrm{AC}=6{,}32455\ldots$, $\mathrm{BC}=10{,}4403\ldots$. Conceda ***(M1)*** por sumar los cuadrados de **sus** AC y BC, ***(A1)*** por **sus** tres longitudes correctas, **además** de mostrar que $149\,(\mathrm{AC}^2+\mathrm{BC}^2)\neq 153\,(\mathrm{AB}^2)$.
>
> Si se utilizaron la formula de la distancia y la ley del coseno, conceda ***(M1)*** por sustituir correctamente **sus** lados en la ley del coseno, ***(A1)*** por los tres lados correctos **y** mostrar que $\mathrm{A\hat{C}B}=91{,}7357...(^\circ)\neq 90(^\circ)$.

[2 puntos]

(c) $y-9=0,3(x-3)$ **O BIEN** $9=0,3(3)+c$ ***(A1)*****(ft)**

> **Nota:** Conceda error de arrastre de la parte (b). Conceda ***(A1)*****(ft)** por haber sustituido correctamente su pendiente y el punto dado en la ecuación de la recta. Si la pendiente BC no se calculó para usarla en la parte (b), entonces la pendiente sustituida en esta parte debe ser $0{,}3$ para obtener ***(A1).***

$3x-10y+81=0$ (u otros múltiplos enteros) ***(A1)*****(ft)** ***(C2)***

> **Nota:** Conceda ***(A1)*****(ft)** por escribir **su** ecuación en la forma $ax+by+d=0$ con coeficientes enteros. Error de arrastre de la parte (c).

[2 puntos]

Total [6 puntos]

13. (a) $3\,(\text{m})$ ***(A1)*** ***(C1)***

[1 punto]

(b) $(x=)\ \dfrac{-4}{2(-1)}$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente los datos en la fórmula del vértice.

O BIEN

$0=4-2x$ ***(M1)***

Nota: Conceda ***(M1)*** por haber derivado e igualado correctamente la expresión a cero.

$x=2$ ***(A1)***

$(y=)\ 7\,(\text{m})$ ***(A1)*(ft)*** ***(C3)***

Nota: El ***(A1)*(ft)** final proviene del valor x que haya indicado, pero solo si su $y>$ que su valor de la parte (a).

[3 puntos]

(c) $3+4x-x^2=0$ ***(M1)***

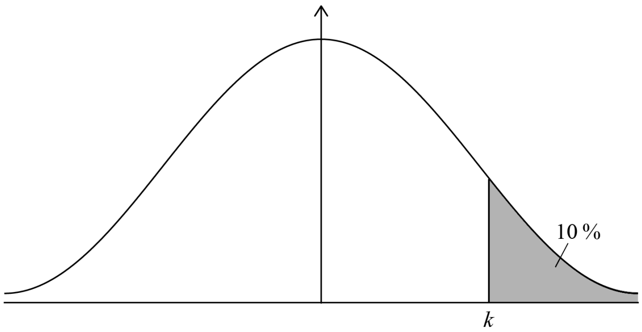
Nota: Conceda ***(M1)*** por haber sustituido correctamente $y=0$ en la ecuación de la curva.

$x=4{,}65\,(\text{m})\ \ (4{,}64575\ldots)$ ***(A1)*** ***(C2)***

[2 puntos]

Total [6 puntos]

14. (a)



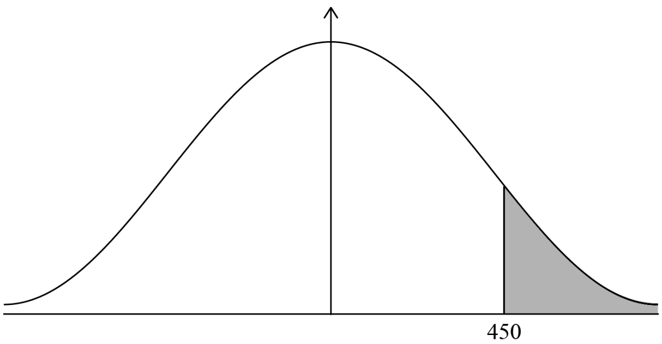
(M1)

Nota:	Conceda ***(M1)*** por incluir un diagrama que muestre el área sombreada y el valor porcentual correctos, k tiene que ser mayor que la media. **O BIEN** Conceda ***(M1)*** por haber escrito $\mathrm{P}(\text{nota} > k) = 0{,}1$ o $\mathrm{P}(\text{nota} \le k) = 0{,}9$.

$428 \quad (428{,}155\ldots)$ ***(A1)*** ***(C2)***

[2 puntos]

(b)



(M1)

Nota:	Conceda ***(M1)*** por incluir un diagrama que muestre el área sombreada correcta y el valor 450 puesto a la derecha de la media. **O BIEN** Conceda ***(M1)*** por haber escrito $\mathrm{P}(\text{nota} \ge 450)$.

$0{,}0668 \quad (0{,}0668072\ldots;\ 6.68\%,\ 6{,}68072\ldots\%)$ ***(A1)*** ***(C2)***

[2 puntos]

Continúa en la página siguiente...

Continuación de la Pregunta 14

(c) $\frac{0,0228}{0,0668}\ \left(\frac{0,0227500\ldots}{0,0668072\ldots}\right)$ ***(M1)***

Nota:	Conceda ***(M1)*** por $0,0228$ ($0,0227500\ldots$) es visto. Acepte $1-0,97725$.

$=0,341\ \left(0,340532\ldots;\ 34,1\%;\ 34,0532\ldots\%\right)$ ***(A1)*(ft)** ***(C2)***

Nota:	Arrastre de error (ft) desde el apartado (b) siempre y cuando la respuesta esté entre cero y 1.

[2 puntos]

Total [6 puntos]

15. (a) $2 = 10 - k(c^0)$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido 2 y 0 en la función.

$(k =)\ 8$ ***(A1)*** ***(C2)***

[2 puntos]

(b) $9 = 10 - 8(c^{-3})$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido su k, 9, y 3 en la función.

$c^{-3} = 0.125$ **O BIEN** $c^3 = 8$ **O BIEN** $c^{-3} = \frac{1}{8}$ ***(M1)***

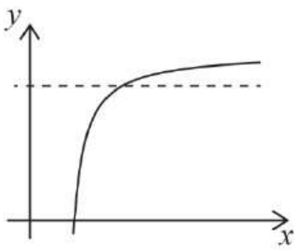
Nota: Conceda ***(M1)*** por haber despejado “c^3” o “c^{-3}” del resto de las constantes.

O BIEN

$0 = 9 - 10 + 8(c^{-3})$ ***(M1)***

Nota: Conceda ***(M1)*** igualar a 0 un miembro de su ecuación.

O BIEN



(M1)

Nota: Conceda ***(M1)*** por un bosquejo del gráfico de su ecuación.

$(c =)\ 2$ ***(A1)*(ft)** ***(C3)***

Nota: Arrastre de error (ft) desde el apartado (a).

[3 puntos]

(c) 10 (gramos) ***(A1)*** ***(C1)***

[1 punto]

Total [6 puntos]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Tuesday 13 November 2018 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

8818 – 7402

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 14]

The marks obtained by nine Mathematical Studies SL students in their projects (x) and their final IB examination scores (y) were recorded. These data were used to determine whether the project mark is a good predictor of the examination score. The results are shown in the table.

Project mark (x)	10	18	14	18	10	18	6	18	14
Examination score (y)	47	61	61	47	54	68	47	54	47

(a) Use your graphic display calculator to write down

(i) $\overline{x}$, the mean project mark;

(ii) $\overline{y}$, the mean examination score;

(iii) r, Pearson's product–moment correlation coefficient. [4]

The equation of the regression line y on x is $y = mx + c$.

(b) (i) Find the exact value of m and of c for these data.

(ii) Show that the point $\text{M}(\overline{x}, \overline{y})$ lies on the regression line y on x. [4]

A tenth student, Jerome, obtained a project mark of 17.

(c) (i) Use the regression line y on x to estimate Jerome's examination score.

(ii) Justify whether it is valid to use the regression line y on x to estimate Jerome's examination score. [4]

In his final IB examination Jerome scored 65.

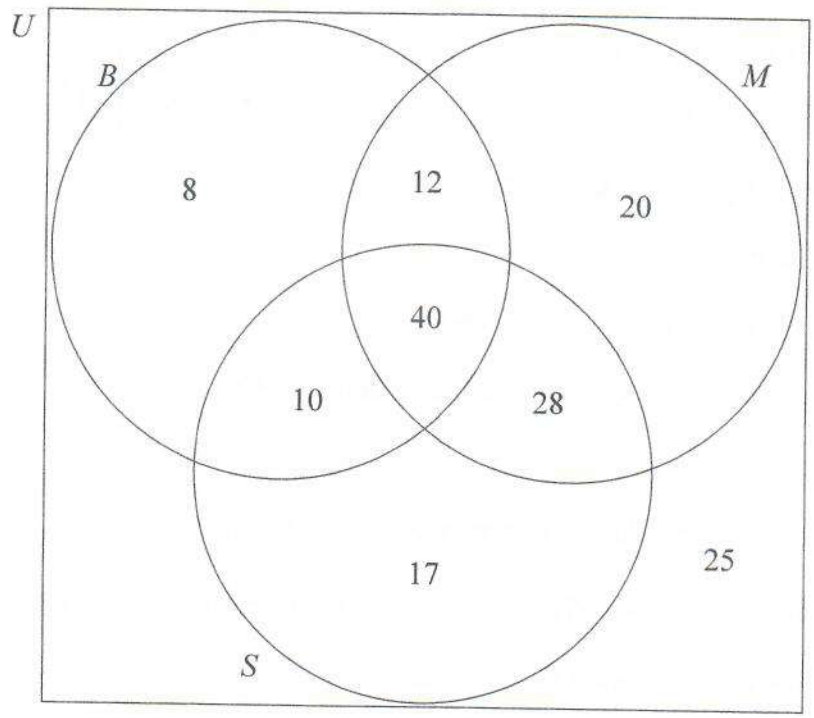
(d) Calculate the percentage error in Jerome's estimated examination score. [2]

2. [Maximum mark: 14]

160 students attend a dual language school in which the students are taught only in Spanish or taught only in English.

A survey was conducted in order to analyse the number of students studying Biology or Mathematics. The results are shown in the Venn diagram.

Set S represents those students who are **taught** in Spanish.
Set B represents those students who **study** Biology.
Set M represents those students who **study** Mathematics.



(a) Find the number of students in the school that

(i) are taught in Spanish;

(ii) study Mathematics in English;

(iii) study both Biology and Mathematics. [6]

(This question continues on the following page)

(Question 2 continued)

(b) Write down

(i) $n(S \cap (M \cup B))$;

(ii) $n(B \cap M \cap S')$. [2]

A student from the school is chosen at random.

(c) Find the probability that this student

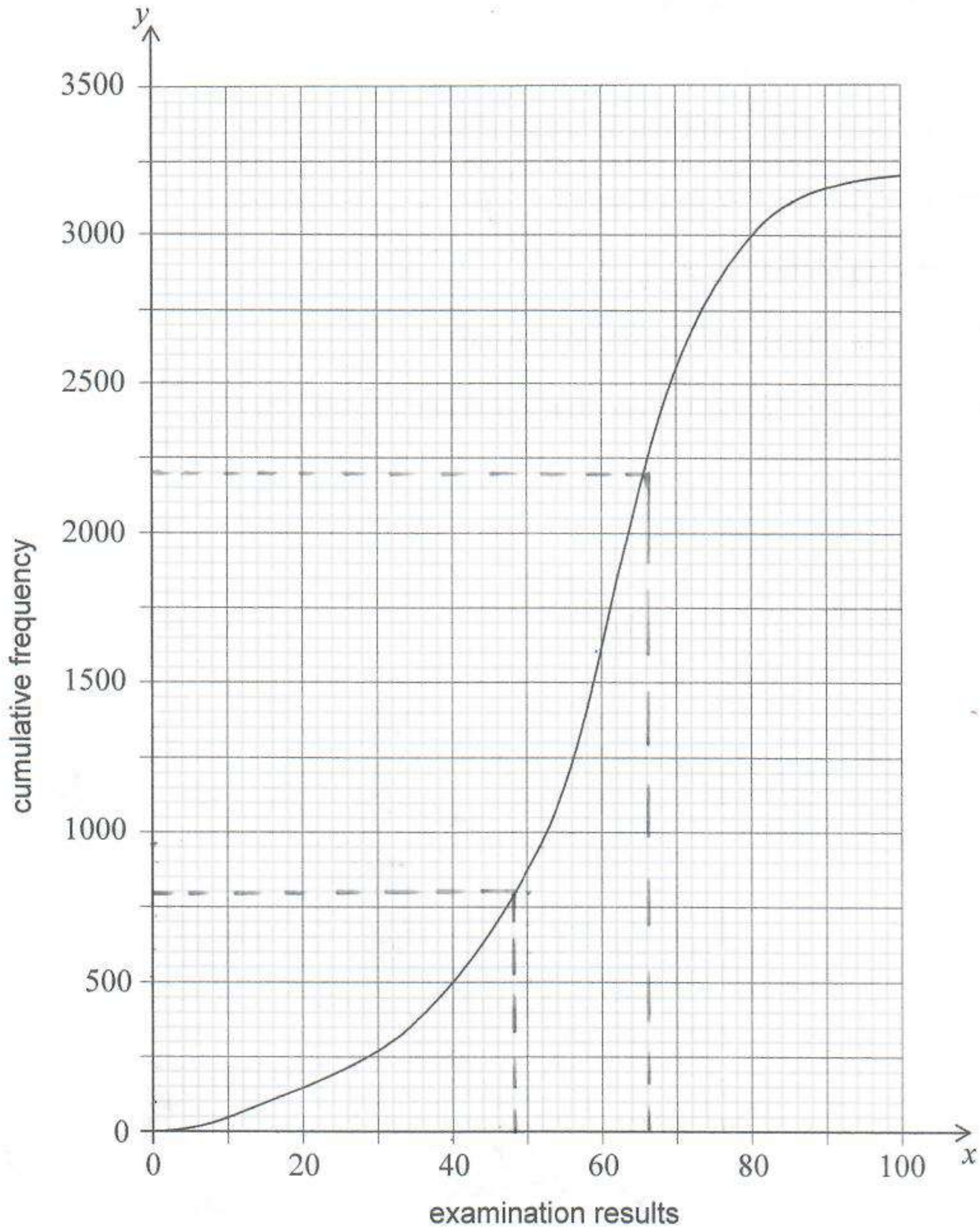
(i) studies Mathematics;

(ii) studies neither Biology nor Mathematics;

(iii) is taught in Spanish, given that the student studies Biology. [6]

3. [Maximum mark: 16]

The final examination results obtained by a group of 3200 Biology students are summarized on the cumulative frequency graph.



(a) Find

(i) the median of the examination results;

(ii) the interquartile range. [5]

350 of the group obtained the highest possible grade in the examination.

(b) Find the final examination result required to obtain the highest possible grade. [2]

(This question continues on the following page)

(Question 3 continued)

The grouped frequency table summarizes the examination results of this group of students.

Examination result (x)	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Frequency	150	350	1100	1400	200

(c) Write down

(i) the modal class;

(ii) the mid-interval value of the modal class. [3]

(d) Calculate an estimate of

(i) the mean examination result;

(ii) the standard deviation, giving your answer correct to **three decimal places**. [3]

The teacher sets a grade boundary that is one standard deviation below the mean.

(e) Use the cumulative frequency graph to estimate the number of students whose final examination result was below this grade boundary. [3]

4. [Maximum mark: 13]

Consider the function $f(x) = \frac{27}{x^2} - 16x$, $x \neq 0$.

(a) Sketch the graph of $y = f(x)$, for $-4 \leq x \leq 3$ and $-50 \leq y \leq 100$. [4]

(b) Use your graphic display calculator to find

(i) the zero of $f(x)$;

(ii) the coordinates of the local minimum point;

(iii) the equation of the tangent to the graph of $y = f(x)$ at the point $(-2, 38.75)$. Give your answer in the form $y = mx + c$. [5]

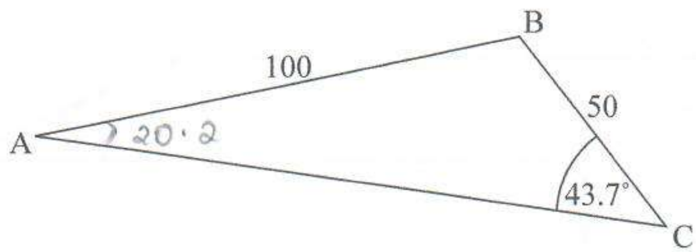
(c) Sketch the graph of the function $g(x) = 10x + 40$ on the same axes. [2]

(d) Solve the equation $f(x) = g(x)$. [2]

5. [Maximum mark: 15]

A flat horizontal area, ABC, is such that $\text{AB} = 100\,\text{m}$, $\text{BC} = 50\,\text{m}$ and angle $\text{A}\hat{\text{C}}\text{B} = 43.7^\circ$ as shown in the diagram.

diagram not to scale



(a) Show that the size of angle $\text{B}\hat{\text{A}}\text{C}$ is 20.2°, correct to 3 significant figures. [3]

(b) Calculate the area of triangle ABC. [4]

(c) Find the length of AC. [3]

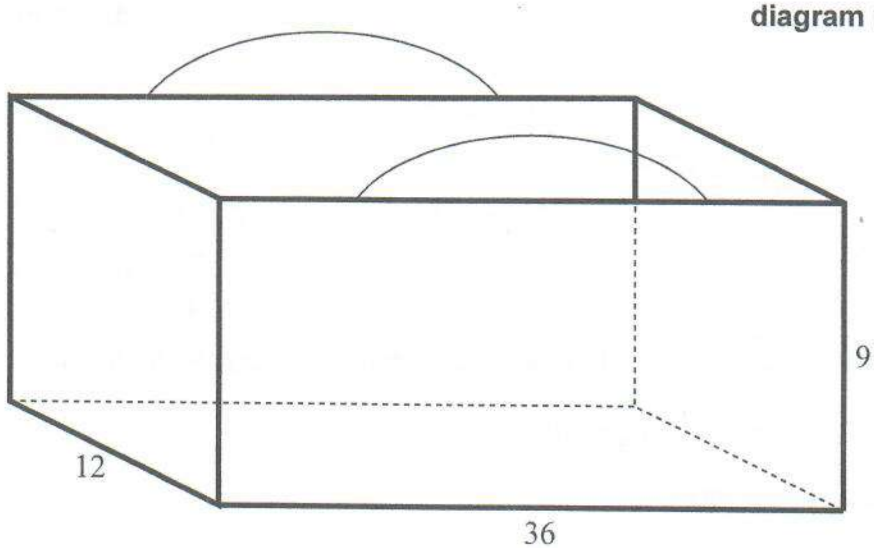
A vertical pole, TB, is constructed at point B and has height $25\,\text{m}$.

(d) Calculate the angle of elevation of T from, M, the midpoint of the side AC. [5]

6. [Maximum mark: 18]

Haruka has an eco-friendly bag in the shape of a cuboid with width $12\,\text{cm}$, length $36\,\text{cm}$ and height of $9\,\text{cm}$. The bag is made from five rectangular pieces of cloth and is open at the top.

diagram not to scale



(a) Calculate the area of cloth, in cm^2, needed to make Haruka's bag. [2]

(b) Calculate the volume, in cm^3, of the bag. [2]

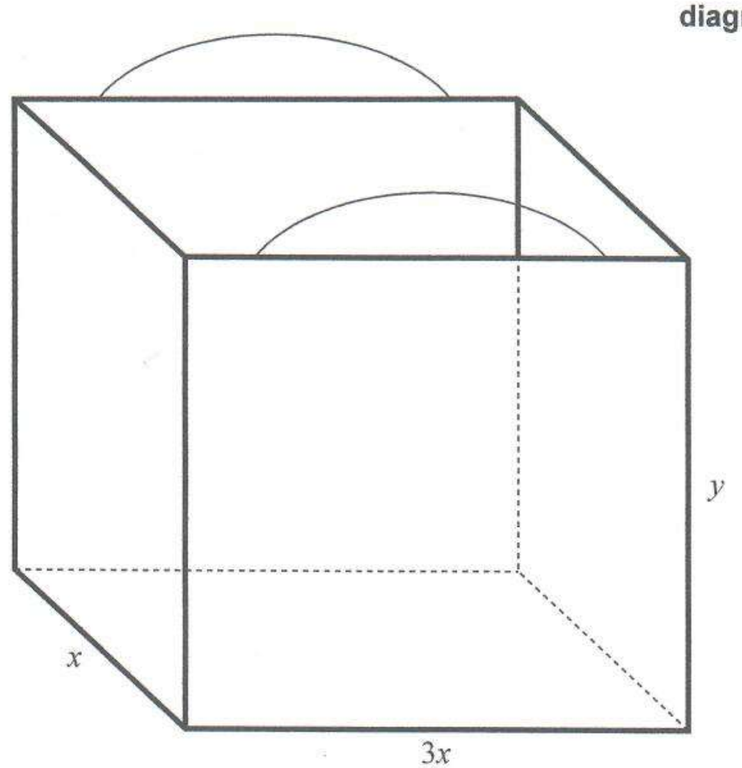
(This question continues on the following page)

(Question 6 continued)

Nanako decides to make her own eco-friendly bag in the shape of a cuboid such that the surface area is minimized.

The width of Nanako's bag is x cm, its length is three times its width and its height is y cm.

diagram not to scale



The volume of Nanako's bag is $3888\,\text{cm}^3$.

(c) Use this value to write down, and simplify, the equation in x and y for the volume of Nanako's bag. [2]

(d) Write down and simplify an expression in x and y for the area of cloth, A, used to make Nanako's bag. [2]

(e) Use your answers to parts (c) and (d) to show that

$$A = 3x^2 + \frac{10368}{x}.$$ [2]

(f) Find $\frac{\mathrm{d}A}{\mathrm{d}x}$. [3]

(g) Use your answer to part (f) to show that the width of Nanako's bag is $12\,\text{cm}$. [3]

The cloth used to make Nanako's bag costs 4 Japanese Yen (JPY) per cm^2.

(h) Find the cost of the cloth used to make Nanako's bag. [2]

Diploma Programme
Programme du diplôme
Programa del Diploma

Estudios matemáticos
Nivel medio
Prueba 2

Martes 13 de noviembre de 2018 (mañana)

1 hora 30 minutos

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Para esta prueba, se necesita una copia sin anotaciones del **cuadernillo de fórmulas de estudios matemáticos NM**.
- Conteste todas las preguntas en el cuadernillo de respuestas provisto.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán darse como valores exactos o con una aproximación de tres cifras significativas.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

8818 – 7410

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Página en blanco

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta. Se recomienda que muestre todos los cálculos, siempre que sea posible. Cuando la respuesta sea incorrecta se otorgarán algunos puntos siempre que aparezca el método empleado y éste sea correcto. Para los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el proceso seguido hasta su obtención. Por ejemplo, cuando deba utilizar un gráfico de una calculadora de pantalla gráfica para hallar soluciones, deberá dibujar esas gráficas en su respuesta.

1. [Puntuación máxima: 14]

Se recopilaron las notas obtenidas por nueve alumnos de Estudios Matemáticos NM en el proyecto (x) y en el examen final del IB (y). Estos datos se utilizaron para determinar si la nota del proyecto es un buen indicador de la nota del examen. Los resultados se muestran en la siguiente tabla.

Nota del proyecto (x)	10	18	14	18	10	18	6	18	14
Nota del examen (y)	47	61	61	47	54	68	47	54	47

(a) Utilice la calculadora de pantalla gráfica para escribir

(i) $\bar{x}$, la media de las notas obtenidas en el proyecto;

(ii) $\bar{y}$, la media de las notas obtenidas en el examen;

(iii) r, el coeficiente de correlación momento-producto de Pearson. [4]

La ecuación de la recta de regresión de y sobre x es $y = mx + c$.

(b) (i) Halle el valor exacto de m y de c para estos datos.

(ii) Muestre que el punto $\mathrm{M}(\bar{x}, \bar{y})$ pertenece a la recta de regresión de y sobre x. [4]

Un décimo estudiante, Jerome, obtuvo en el proyecto una nota de 17.

(c) (i) Utilice la recta de regresión de y sobre x para estimar la nota que obtendrá Jerome en el examen.

(ii) Justifique si es válido utilizar la recta de regresión de y sobre x para estimar la nota que obtendrá Jerome en el examen. [4]

En el examen final del IB Jerome sacó un 65.

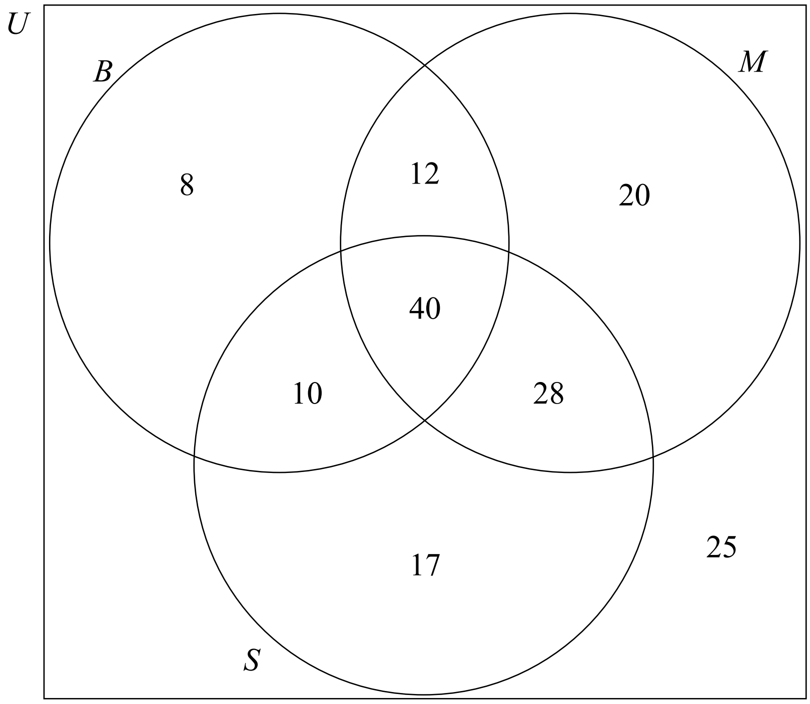
(d) Calcule el porcentaje de error de la nota estimada del examen de Jerome. [2]

2. [Puntuación máxima: 14]

Hay 160 alumnos que asisten a un colegio bilingüe, en el que los alumnos reciben las clases o solo en español o solo en inglés.

Se llevó a cabo una encuesta para averiguar cuántos alumnos estaban estudiando Biología o Matemáticas. Los resultados se muestran en el siguiente diagrama de Venn.

El conjunto S representa a aquellos alumnos que **reciben las clases** en español.
El conjunto B representa a aquellos alumnos que **estudian** Biología.
El conjunto M representa a aquellos alumnos que **estudian** Matemáticas.



(a) Halle el número de alumnos del colegio que

(i) reciben las clases en español;

(ii) estudian Matemáticas en inglés;

(iii) estudian Biología y también Matemáticas. [6]

(Esta pregunta continúa en la página siguiente)

(Pregunta 2: continuación)

(b) Escriba

(i) $n\left(S \cap (M \cup B)\right)$;

(ii) $n\left(B \cap M \cap S'\right)$. [2]

Se escoge al azar a un alumno del colegio.

(c) Halle la probabilidad del este alumno

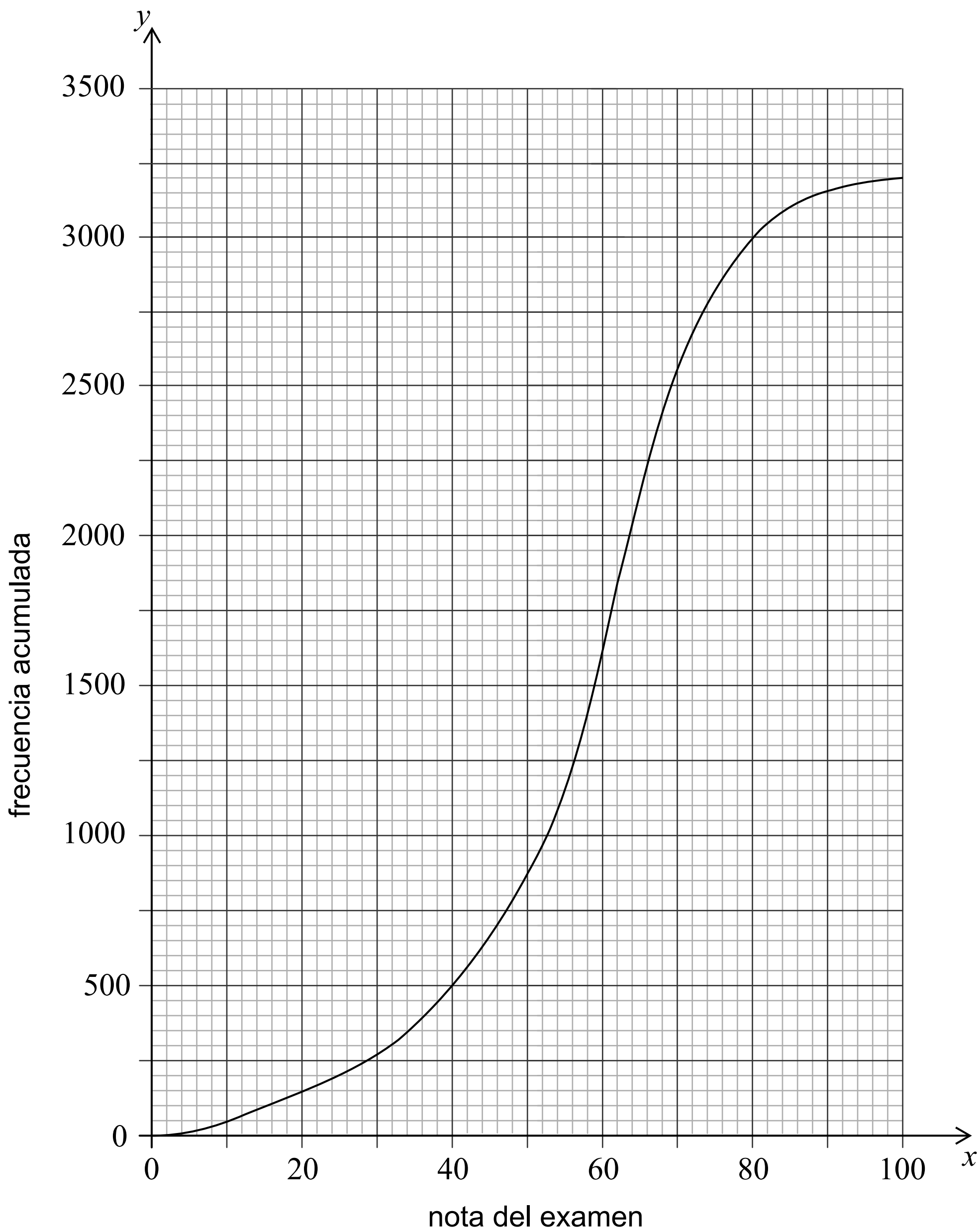
(i) estudie Matemáticas;

(ii) no estudie ni Biología ni Matemáticas;

(iii) reciba las clases en español, sabiendo que dicho alumno estudia Biología. [6]

3. [Puntuación máxima: 16]

En el siguiente gráfico de frecuencias acumuladas se resumen las notas que obtuvieron en el examen final un grupo de 3200 alumnos de Biología.



(a) Halle

(i) la mediana de las notas del examen;

(ii) el rango intercuartil. [5]

En ese grupo hubo 350 alumnos que obtuvieron en el examen la máxima calificación posible.

(b) Halle la nota que se necesita en el examen final para lograr la máxima calificación posible. [2]

(Esta pregunta continúa en la página siguiente)

(Pregunta 3: continuación)

La siguiente tabla de frecuencias agrupadas resume las notas que obtuvieron en el examen este grupo de alumnos.

Nota del examen (x)	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Frecuencia	150	350	1100	1400	200

(c) Escriba

(i) la clase modal;

(ii) el valor central del intervalo correspondiente a la clase modal. [3]

(d) Calcule una estimación de

(i) la media de las notas del examen;

(ii) la desviación típica, redondeando la respuesta a **tres lugares decimales**. [3]

El profesor establece una nota límite entre calificaciones que se encuentra a una desviación típica debajo de la media.

(e) Utilice el gráfico de frecuencias acumuladas para estimar el número de alumnos que sacaron en el examen una nota por debajo de esta nota límite. [3]

4. [Puntuación máxima: 13]

Considere la función $f(x) = \dfrac{27}{x^2} - 16x$, $x \neq 0$.

(a) Dibuje aproximadamente el gráfico de $y = f(x)$, para $-4 \leq x \leq 3$ e $-50 \leq y \leq 100$. [4]

(b) Utilice la calculadora de pantalla gráfica para hallar

(i) el cero de $f(x)$;

(ii) las coordenadas del punto mínimo local;

(iii) la ecuación de la tangente al gráfico de $y = f(x)$ en el punto $(-2\,;\,38{,}75)$.
Dé la respuesta en la forma $y = mx + c$. [5]

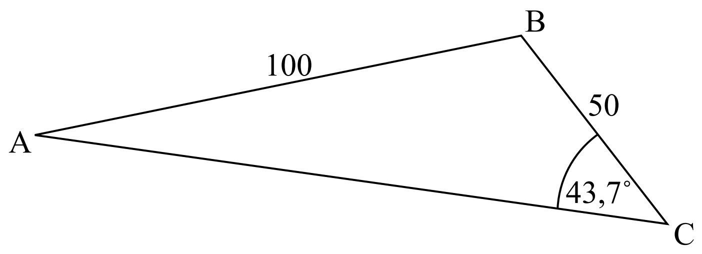
(c) Sobre los mismos ejes de coordenadas dibuje aproximadamente el gráfico de la función $g(x) = 10x + 40$. [2]

(d) Resuelva la ecuación $f(x) = g(x)$. [2]

5. [Puntuación máxima: 15]

La superficie plana y horizontal ABC es tal que $\mathrm{AB} = 100\,\mathrm{m}$, $\mathrm{BC} = 50\,\mathrm{m}$ y el ángulo $\mathrm{A\hat{C}B} = 43{,}7^\circ$ tal y como se muestra en la figura.

la figura no está dibujada a escala



(a) Muestre que el valor del ángulo $\mathrm{B\hat{A}C}$ es $20{,}2^\circ$, redondeado a 3 cifras significativas. [3]

(b) Calcule el área de triángulo ABC. [4]

(c) Halle la longitud de AC. [3]

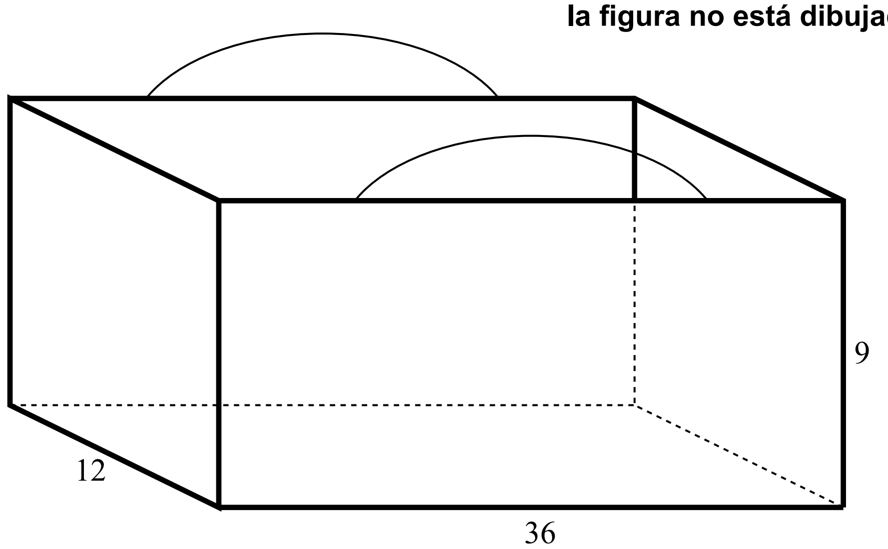
El poste vertical TB se coloca en el punto B y tiene una altura de $25\,\mathrm{m}$.

(d) Calcule el ángulo de elevación de T visto desde M, el punto medio del lado AC. [5]

6. [Puntuación máxima: 18]

Haruka tiene un bolso ecológico que tiene forma de ortoedro de $12\,\text{cm}$ de ancho, $36\,\text{cm}$ de longitud y $9\,\text{cm}$ de altura. El bolso está hecho con cinco piezas de tela rectangulares y está abierto por la parte de arriba.

la figura no está dibujada a escala



(a) Calcule el área de tela, en cm^2, que se necesita para hacer el bolso de Haruka. [2]

(b) Calcule el volumen del bolso, en cm^3. [2]

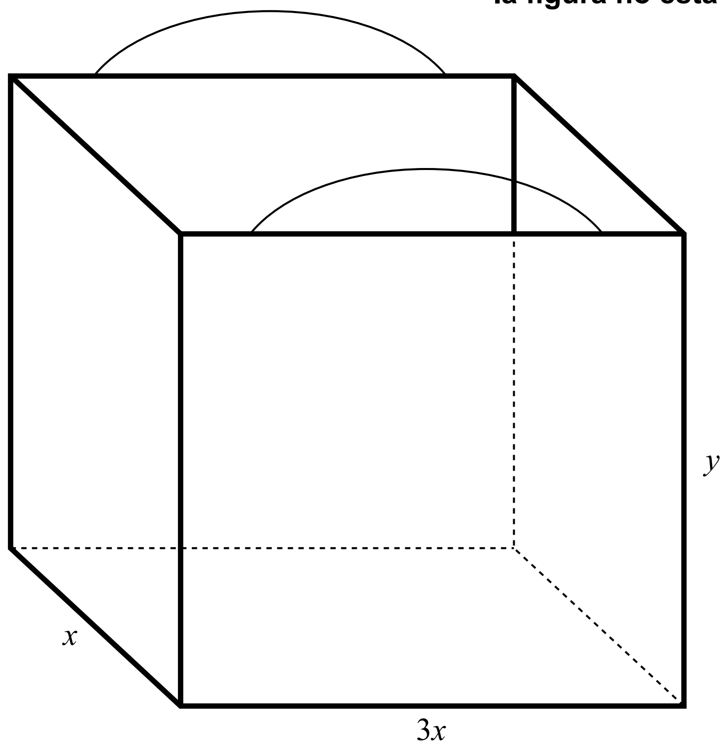
(Esta pregunta continúa en la página siguiente)

(Pregunta 6: continuación)

Nanako decide fabricar su propio bolso ecológico en forma de ortoedro de modo tal que se minimice el área de su superficie.

El bolso de Nanako tiene una anchura de x cm, una longitud que es el triple de su anchura y una altura igual a y cm.

la figura no está dibujada a escala



El volumen del bolso de Nanako es igual a $3888\ \text{cm}^3$.

(c) Utilice este valor para escribir, y simplificar, una ecuación en x e y para el volumen del bolso de Nanako. [2]

(d) Escriba y simplifique una expresión en x e y para el área de tela, A, que se ha utilizado para fabricar el bolso de Nanako. [2]

(e) Utilice las respuestas obtenidas en los apartados (c) y (d) para mostrar que

$$A = 3x^2 + \frac{10368}{x}.$$ [2]

(f) Halle $\frac{\mathrm{d}A}{\mathrm{d}x}$. [3]

(g) Utilice la respuesta obtenida en el apartado (f) para mostrar que la anchura del bolso de Nanako es igual a 12 cm. [3]

La tela que se utiliza para fabricar el bolso de Nanako cuesta 4 yenes japoneses (JPY) por cm^2.

(h) Halle el costo de la tela que se ha utilizado para fabricar el bolso de Nanako. [2]

N18/5/MATSD/SPA/SP2/TZ0/XX/M

Esquema de calificación

Noviembre de 2018

Estudios matemáticos

Nivel medio

Prueba 2

22 páginas

Instrucciones para los examinadores

Observaciones: En caso de dudas sobre estas instrucciones o temas relacionados con la asignación de puntuaciones, póngase en contacto con su jefe de equipo.

1 Siglas

M Puntos otorgados por el **método**
A Puntos otorgados por una **respuesta** o por la **precisión**
R Puntos otorgados por un **razonamiento** claro
G Puntos otorgados por soluciones correctas obtenidas mediante la **calculadora de pantalla gráfica**, independientemente del trabajo mostrado.
AG **Respuesta incluida** en la pregunta y, en consecuencia; no se otorgan puntos.
ft Se pueden otorgar puntos por la **coherencia** con resultados previos en la pregunta.

2 Método para corregir

(a) Todas las correcciones deben realizarse en RM Assessor, utilizando las anotaciones de Estudios Matemáticos y de acuerdo con el documento vigente de orientaciones para la corrección electrónica de Estudios Matemáticos NM. Es fundamental leer el documento antes de empezar a corregir.

(b) Si un apartado de una pregunta es del todo correcto use las anotaciones tic con números para otorgar la puntuación máxima. Si un apartado es completamente erróneo use la nota ***A0***, de lo contrario se deben mostrar todas las anotaciones.

(c) No se debe otorgar ningún punto al trabajo tachado por el alumno.

(d) Si el alumno ha dado dos respuestas a una pregunta, solo se debe corregir la primera respuesta.

(e) Si el procedimiento adecuado lleva a la respuesta correcta, pero esta va seguida de más desarrollos que indican una falta de comprensión matemática, entonces **no** se puede otorgar la puntuación máxima. En la mayoría de estos casos se perderá solo el último punto por respuesta. Una excepción podría ser las respuestas numéricas, donde una respuesta exacta que es correcta está seguida de una expresión decimal incorrecta.

Ejemplo:

	Se ve la respuesta correcta	**Hay más desarrollo**	**Acción**
1.	$8\sqrt{2}$	$5{,}65685...$ *(valor decimal incorrecto)*	Otorgue el ultimo ***(A1)*** *(ignore el desarrollo posterior)*
2.	$(x-6)(x+1)$	$x=6$ and -1	**No** otorgue el último ***(A1)***

Ejemplo: Calcule la pendiente de la recta que pasa por los puntos $(5;3)$ y $(0;9)$.

Esquema de calificación	**Examen del alumno**	**Corrección**
$\frac{9-3}{0-5}$ ***(M1)*** *Otorgue (M1) por la sustitución correcta en la fórmula de la pendiente* $=-\frac{6}{5}$ ***(A1)***	(i) $\frac{9-3}{0-5}=-\frac{6}{5}$	***(M1)***
	La pendiente es $=-\frac{6}{5}$	***(A1)***
	(Existe una comprensión clara de la pendiente.)	
	$y=-\frac{6}{5}x+9$	
	(ii) $\frac{9-3}{0-5}=-\frac{6}{5}$	***(M1)***
	$y=-\frac{6}{5}x+9$	***(A0)***
	(Existe confusión sobre lo requerido.)	

3 Puntos por la coherencia (ft)

En cualquier paso de una resolución, un error puede afectar a todo el trabajo posterior. Para limitar la gravedad de la penalización, se pueden otorgar puntos por la **coherencia (ft)**. Los esquemas de calificación indicarán, con **"(ft)"**, dónde es adecuado aplicar el criterio de coherencia en una pregunta.

(a) La coherencia se aplica solo de un apartado de una pregunta a otro apartado posterior de la pregunta. No se aplica dentro del mismo apartado.

(b) Si una respuesta que es consecuencia de un procedimiento de coherencia es muy poco realista (por ejemplo, distancias negativas o de un orden de magnitud excesivo) entonces el punto ***A*** final no se debe otorgar.

(c) Si, debido a un error, una pregunta queda transformada en otra **distinta**, **mucho más sencilla**, entonces el criterio de coherencia podría no ser aplicado.

(d) Para otorgar puntos por coherencia en un apartado de una pregunta, **es necesario que haya un trabajo explícito relacionado con ese apartado**. Una respuesta aislada, obtenida por coherencia, pero sin un desarrollo explícito, se considera incorrecta, y no recibirá ningún punto **aunque se aproxime a la respuesta correcta**.

(e) Una excepción a lo anterior sería una pregunta donde lo que se evalúa es el manejo del alumno en el uso de la calculadora de pantalla gráfica, y donde no se pretende que se muestre ningún desarrollo escrito. **En el esquema de calificación vendrá indicado claramente cuando sea el caso.**

(f) El uso inadvertido de radianes será penalizado la primera vez que aparezca. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo un punto por el uso de radianes.

Ejemplo: Cálculo de ángulos y longitudes en trigonometría

Esquema de calificación	Examen del alumno	Corrección
(a) $\frac{\text{sen } A}{3}=\frac{\text{sen } 30}{4}$ ***(M1)(A1)*** *Otorgue **(M1)** por la sustitución en el teorema del seno, **(A1)** por las sustituciones correctas.*	(a) $\frac{\text{sen } A}{4}=\frac{\text{sen } 30}{3}$	***(M1)(A0)*** *(uso del teorema del seno, pero con valores incorrectos)*
$A=22{,}0^\circ\ (22{,}0243\ldots)$ ***(A1)(G2)***	$A=41{,}8^\circ$	***(A0)*** *(**Observación**: Aquí, el segundo **(A1)** no ha sido corregido como* **(ft)** *y no se puede otorgar porque existe un error previo en el **mismo** apartado de la pregunta.)*
(b) $x=7\text{tg}\,(22{,}0243\ldots^\circ)$ ***(M1)*** $=2{,}83\ (2{,}83163\ldots)$ ***(A1)*(ft)**	(b) caso (i) $x=7\text{tg}\,41{,}8^\circ$ $=6{,}26$ ***pero*** caso (ii) $6{,}26$	***(M1)*** ***(A1)*(ft)** ***(G0)*** *pues no aparece un desarrollo explícito*

4 Uso del Esquema de calificación

(a) Los puntos ***A* dependen** de los puntos ***M*** obtenidos previamente, **no** es posible otorgar ***(M0)(A1)***. Una vez que se otorga un ***(M0)***, se pierden todos los puntos ***A*** que le siguen en ese apartado de la pregunta, incluso si los cálculos son correctos, hasta el siguiente punto ***M***.
La única excepción a lo anterior se hará en el caso de una respuesta donde la precisión se especifique en la pregunta (ver apartado 5).

(b) Los puntos ***A* dependen** de los puntos ***R*** obtenidos, **no** es posible otorgar ***(A1)(R0)***. Así pues, el ***(A1)*** no se puede otorgar a una respuesta que sea correcta cuando no aparece el razonamiento, o este es incorrecto.

(c) En la prueba 2 se espera que los alumnos demuestren su destreza en la comunicación matemática mediante el uso de desarrollos adecuados. Las respuestas que sean correctas, pero no se basen en un desarrollo adecuado **no siempre recibirán la puntuación máxima**. Estas respuestas sin desarrollo que las sustente vienen designadas por ***G*** en el esquema de calificación, como una alternativa a la puntuación máxima. Ejemplo ***(M1)(A1)(A1)(G2).***

Ejemplo: Uso de la trigonometría para el cálculo de un ángulo de un triángulo.

Esquema de calificación	Examen del alumno	Corrección
(a) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ ***(M1)(A1)*** *Otorgue **(M1)** por la sustitución en el teorema del seno, **(A1)** por las sustituciones correctas.* $A = 22{,}0^\circ\ (22{,}0243\ldots)$ ***(A1)(G2)***	(i) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ $A = 22{,}0^\circ$ (ii) $A = 22{,}0^\circ$ ***Observación:*** *Los puntos **G** se utilizan solo si no se muestra ningún desarrollo, pero la respuesta es correcta.*	***(M1)(A1)*** ***(A1)*** ***(G2)***

(d) Los **métodos alternativos** no siempre se incluyen. Así, si una respuesta es errónea, entonces se debe analizar cuidadosamente el procedimiento, de modo que se asignen puntos por un método distinto de forma coherente con el esquema de calificación.
Cuando en el esquema de calificación se incluyen métodos alternativos para una pregunta, ello viene indicado mediante "**O**" *etc*.

(e) A menos que en la pregunta se especifique lo contrario, se aceptan **expresiones equivalentes**.
Por ejemplo: $\frac{\text{sen } \theta}{\cos \theta}$ por $\text{tg } \theta$.

En el esquema de calificación, estas expresiones algebraica o numéricamente equivalentes aparecerán a veces escritas entre paréntesis junto a la respuesta requerida.
Cuando se requieran respuestas numéricas como respuesta final de un apartado de una pregunta, en el esquema de calificación se mostrará en este orden:
la respuesta con 3 cifras significativas a partir de la solución completa de la calculadora;
el valor exacto (por ejemplo $\frac{2}{3}$ si corresponde);
la solución completa de la calculadora en la forma $2{,}83163\ldots$ como en el ejemplo anterior.
Cuando se dan las respuestas con 3 cifras significativas y se utilizan después en apartados posteriores de la pregunta llevando a una solución con 3 cifras significativas **distinta**, también se darán estas soluciones.

(f) Dado que este es un examen internacional, se aceptarán todas las **formas de notación alternativas**. Algunos ejemplos:

Comas decimales: 1,7; 1’7; $1\cdot 7$; 1,7.

Los números decimales menores que 1 podrían aparecer escritos sin un cero en el frente: 0.49 o .49.

Distintas descripciones de un intervalo: $3 < x < 5$; (3; 5);] 3; 5 [.

Distintas formas de notación de las propiedades de los conjuntos (por ejemplo complementario): A'; $\overline{A}$; A^c; $U - A$; $(A$; $U \setminus A$.

Distintas formas de notación lógica: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p$.
$p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

El nivel de significación podría escribirse como α .

(g) Puntos discrecionales: habrá ocasiones excepcionales en las que el esquema de calificación no cubra el trabajo que aparece. En estos casos se utilizará la nota DM para indicar que el examinador ha utilizado su criterio. La discrecionalidad debe utilizarse con moderación, y si existe duda se debe plantear una excepción a través de RM Assessor al jefe del equipo.

A partir de noviembre de 2011 no habrá una única penalización por prueba por precisión AP, precisión financiera FP y unidades UP. En lugar de ello, estas destrezas serán evaluadas en preguntas específicas y los puntos se otorgarán de acuerdo a lo especificado en los apartados 5, 6 y 7.

5 Precisión de las respuestas

Una precisión incorrecta debe ser penalizada una sola vez en cada pregunta de acuerdo a las siguientes reglas.

Instrucciones: Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas correctamente a 3 cifras significativas.

1. Si el alumno muestra la solución a 4 o más cifras significativas, la cual redondeada a 3 cifras significativas daría la respuesta requerida entonces otorgue ***(A1)*** e ignore cualquier redondeo posterior.
Observación: La solución sin redondear puede estar tanto en la línea de respuesta como en el cuadro de trabajo.

2. Si el alumno **no** muestra la solución sin redondear, entonces otorgue ***(A1)*** si la solución dada está aproximada **correctamente** a 2 o más cifras significativas, y ***(A0)*** en cualquier otro caso.

3. Si una solución aproximada a 2 cifras significativas se utiliza en apartados posteriores, entonces para otorgar más puntos debe aparecer el desarrollo. (Este criterio es el mismo que en el que se aplica coherencia a partir de una respuesta errónea.)

Estos 3 casos (vea los supra índices) se han resumido en la siguiente tabla e ilustrado después con ejemplos.

<table>
<tr><td rowspan="2"></td><td colspan="5">Si la solución final de alumno se expresa…</td></tr>
<tr><td>Exacta o a 4 o más cifras significativas (y que redondeada a 3 cifras significativas daría la respuesta correcta)</td><td>Aproximada incorrectamente a 3 cifras significativas</td><td>Aproximada correctamente a 2 cifras significativas[3]</td><td>Aproximada incorrectamente a 2 cifras significativas</td><td>Aproximada correcta o incorrectamente a 1 cifra significativa</td></tr>
<tr><td>Aparece la solución sin redondear[1]</td><td colspan="5">Otorgue (A1) independientemente del redondeo correcto o incorrecto</td></tr>
<tr><td>No aparece la solución sin redondear[2]</td><td>(A1)</td><td>(A0)</td><td>(A1)</td><td>(A0)</td><td>(A0)</td></tr>
<tr><td>Criterio para los apartados posteriores</td><td>Como en MS</td><td colspan="4">Como en el criterio de coherencia, solo si se muestra el desarrollo[3]</td></tr>
</table>

Ejemplos:

Esquema de calificación	Examen del alumno	Corrección
$9,43\ (9,43398\ldots)$ ***(A1)***	(i) $9,43398\ldots$ se muestra en el cuadro de trabajo seguido de 9; 9,4; 9,43; 9,434 (redondeados correctamente)	***(A1)***
	(ii) $9,43398\ldots$ se muestra en el cuadro de trabajo seguido de 9,433; 9,44, *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 9,4	***(A1)***
	(iv) 9	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 9,3	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 9,44	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Esquema de calificación	Examen del alumno	Corrección
$7,44\ (7,43798\ldots)$ ***(A1)***	(i) $7,43798\ldots$ se muestra en el cuadro de trabajo seguido de 7; 7,4; 7,44; 7,438 *etc.* (redondeados correctamente)	***(A1)***
	(ii) $7,43798\ldots$ se muestra seguido de 7,437; 7,43 *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 7,4	***(A1)***
	(iv) 7	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 7,5	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 7,43	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Ejemplo: ABC es un triángulo rectángulo con el ángulo $\text{ABC} = 90^\circ$, $\text{AC} = 32\text{ cm}$ y $\text{AB} = 30\text{ cm}$. Halle (a) la longitud de BC, (b) el área del triángulo ABC.

Esquema de calificación	Examen del alumno	Corrección
(a) $\text{BC} = \sqrt{32^2 - 30^2}$ ***(M1)*** *Otorgue **(M1)** por la sustitución correcta en el teorema de Pitágoras* $= 11{,}1\left(\sqrt{124}; 11{,}1355...\right)(\text{cm})$ ***(A1)***	(a) $\text{BC} = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$ *(solo se muestra la aproximación a 2 cifras significativas, pero correcta)*	***(M1)*** ***(A1)***
(b) $\text{Área} = \frac{1}{2} \times 30 \times 11{,}1355...$ ***(M1)*** *Otorgue **(M1)** por la sustitución correcta en la fórmula del área de un triángulo* $= 167(167{,}032...)(\text{cm}^2)$ ***(A1)*(ft)**	(b) caso (i) $\text{Área} = \frac{1}{2} \times 30 \times 11$ *(se muestra el desarrollo)* $= 165\,(\text{cm}^2)$ caso (ii) $= 165\,(\text{cm}^2)$ *(No se muestra el desarrollo, la solución* 11 *se trata como procedimiento de coherencia, por lo que no se deben otorgar puntos en este caso)*	***(M1)*** ***(A1)*(ft)** ***(M0)(A0)*(ft)**

La aproximación a 3 cifras significativas de una solución exacta **se debe aceptar si se realiza correctamente**.

Las soluciones exactas del tipo $\frac{1}{4}$ se pueden escribir como decimales con menos de 3 cifras significativas si el resultado sigue siendo exacto. La simplificación de una fracción a su expresión irreducible **no** es imprescindible. Las fracciones que contienen un numerador y/o un denominador decimal se aceptan para mostrar sustituciones pero no como respuesta final.

Razones de π y respuestas con expresiones de raíces cuadradas de enteros o cualquier potencia racional de un entero (por ejemplo, $\sqrt{13}, 2^{2/3}, \sqrt[4]{5}$), se pueden aceptar como respuestas exactas. Todas las otras potencias (por ejemplo de no enteros) y valores de funciones transcendentes tales como seno y coseno se deben evaluar.

Si el nivel de precisión viene especificado en la pregunta, se asignarán puntos por dar la respuesta con la precisión requerida. En **todos** estos casos no se obtiene el punto final si el redondeo no sigue las instrucciones dadas en la pregunta. El punto por la precisión especificada se puede considerar como punto por coherencia **(ft)** con independencia de un ***(M0)*** inmediatamente anterior.

6 Nivel de precisión en las preguntas sobre cuestiones financieras

El nivel de precisión de las respuestas se especificará en todas las preguntas que se relacionen con valores en dinero. Generalmente será a la unidad más cercana o a dos cifras decimales. La primera respuesta que no venga dada con el nivel de precisión especificado no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la precisión incorrecta en una pregunta sobre cuestiones financieras.

Ejemplo: En una pregunta sobre cuestiones financieras se pide una precisión de 2 cifras decimales.

Esquema de calificación	Examen del alumno	Corrección
$\$231{,}62\,(231{,}6189)$ ***(A1)***	(i) $231{,}6$	***(A0)***
	(ii) 232	***(A0)***
	(Aproximación correcta con un nivel de precisión distinto del requerido)	
	(iii) $231{,}61$	***(A0)***
	(iv) $232{,}00$	***(A0)***
	(Aproximación incorrecta al nivel de precisión requerido)	

7 Unidades de medida en las respuestas

En determinadas preguntas será necesario expresar las unidades de medida, y esto vendrá claramente indicado en el esquema de calificación. La primera respuesta correcta pero sin unidades o con unidades incorrectas no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la falta de unidades o unidades incorrectas.

Las unidades de medida se tendrán en cuenta solamente cuando se ha otorgado ***(A1)*** a la respuesta numérica siguiendo las indicaciones sobre la precisión dadas en el apartado 5.

Ejemplo:

Esquema de calificación	Examen del alumno	Corrección
(a) $37000\ \mathrm{m}^2$ ***(A1)***	(a) $36000\ \mathrm{m}^2$	***(A0)***
	(respuesta incorrecta, por lo que no se consideran las unidades)	
(b) $3200\ \mathrm{m}^3$ ***(A1)***	(b) $3200\ \mathrm{m}^2$	***(A0)***
	(unidades incorrectas)	

8 Calculadoras de pantalla gráfica

Con frecuencia los alumnos van a obtener las soluciones directamente de la calculadora. Deben utilizar la notación matemática y no la notación de la calculadora. No se puede otorgar puntos por método a respuestas incorrectas basadas únicamente en notación de calculadora. No se puede otorgar puntos por método al comentario “usé la calculadora de pantalla gráfica”.

1. (a) (i) 14 ***(G1)***

(ii) 54 ***(G1)***

(iii) $0,5$ ***(G2)***

[4 puntos]

(b) (i) $m = 0,875;\ c = 41,75\ \left(m = \dfrac{7}{8},\ c = \dfrac{167}{4}\right)$ ***(A1)(A1)***

> **Nota:** Conceda ***(A1)*** si ha escrito $0,875$. Conceda ***(A1)*** si ha escrito $41,75$. No conceda ***(A1)*** si redondea $41,75$ a $41,8$.

(ii) $y = 0,875(14) + 41,75$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido correctamente en su recta de regresión. Arrastre de error (ft) desde el apartado (a)(i) y el apartado (b)(i).

$= 54$

y, por lo tanto, el punto medio pertenece a la recta de regresión

(Acepte: 54 es (justamente) $\bar{y}$, el valor medio de los valores de y) ***(A1)***

> **Nota:** No conceda ***(A1)*** a menos que el alumno haya escrito explícitamente la conclusión y haya escrito el 54. El ***(A1)*** se puede conceder **únicamente** si la conclusión a la que llega es coherente con su ecuación y si el punto pertenece a la recta.
> El haber utilizado $41,8$ como valor de c imposibilita la concesión del ***(A1)***.

O BIEN

$54 = 0,875(14) + 41,75$ ***(M1)***

$54 = 54$

> **Nota:** Conceda ***(M1)*** por haber sustituido correctamente los valores en la expresión de su recta de regresión. Arrastre de error (ft) desde el apartado (a)(i) y el apartado (b)(i).

y, por lo tanto, el punto medio pertenece a la recta de regresión ***(A1)***

> **Nota:** No conceda ***(A1)*** a menos que el alumno haya escrito explícitamente la conclusión. Arrastre de error (FT) desde el apartado (a). El haber utilizado $41,8$ como valor de c imposibilita la concesión del ***(A1)***.

[4 puntos]

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Continuación de la Pregunta 1

(c) (i) $y = 0,875(17) + 41,75$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido correctamente el valor en su recta de regresión.

$= 56,6 \quad (56,625)$ ***(A1)*(ft)*(G2)***

> **Nota:** Arrastre de error (ft) desde el apartado (b)(i).

(ii) la estimación es válida ***(A1)***
dado que esto es una interpolación **y** el coeficiente de correlación es suficientemente grande. ***(R1)***

O BIEN

la estimación no es válida ***(A1)***
dado que el coeficiente de correlación no es lo suficientemente grande. ***(R1)***

> **Nota:** No conceda ***(A1)(R0)***. Se puede conceder el ***(R1)*** por un razonamiento basado en la fuerza de la correlación, pero no acepte ni "el coeficiente de correlación no es lo suficientemente fuerte" ni "la correlación no es lo suficientemente grande".
> Conceda ***(A0)(R0)*** si el alumno no dio ninguna respuesta numérica en el apartado (a)(iii).

[4 puntos]

(d) $\left|\frac{56,6-65}{65}\right| \times 100$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula del porcentaje de error. Arrastre de error (ft) desde el apartado (c)(i).

$= 12,9\ (\%) \quad (12,9230\ldots)$ ***(A1)*(ft)*(G2)***

> **Nota:** Arrastre de error (ft) desde el apartado (c)(i). No tenga en cuenta (perdone) el uso del símbolo de porcentaje.
> Conceda ***(G0)*** por dar como respuesta $-12,9$ sin haber incluido nada del desarrollo del ejercicio.

[2 puntos]

Total [14 puntos]

2. (a) (i) $10+40+28+17$ ***(M1)***
$=95$ ***(A1)(G2)***

(ii) $20+12$ ***(M1)***
$=32$ ***(A1)(G2)***

(iii) $12+40$ ***(M1)***
$=52$ ***(A1)(G2)***

Nota: Conceda ***(M1)*** por cada suma correcta (por ejemplo: $10+40+28+17$) que haya escrito.

[6 puntos]

(b) (i) 78 ***(A1)***

(ii) 12 ***(A1)***

[2 puntos]

(c) (i) $\frac{100}{160}\ \left(\frac{5}{8};\ 0{,}625;\ 62{,}5\%\right)$ ***(A1)(A1)(G2)***

(ii) $\frac{42}{160}\ \left(\frac{21}{80};\ 0{,}263\,(0{,}2625);\ 26{,}3\%\,(26{,}25\%)\right)$ ***(A1)(A1) (G2)***

(iii) $\frac{50}{70}\ \left(\frac{5}{7};\ 0{,}714;\ 71{,}4\%\right)$ ***(A1)(A1)(G2)***

Nota: Durante todo el apartado (c), conceda ***(A1)*** si el numerador es correcto, ***(A1)*** si el denominador es correcto. Las respuestas deben ser probabilidades para conceder ***(A1)***.

[6 puntos]

Total [14 puntos]

3. (a) (i) 60 ***(A2)***

(ii) $68-48$ ***(A1)(M1)***

Nota: Conceda ***(A1)*** por haber escrito dos cuartiles correctos, ***(M1)*** por haber hallado la diferencia entre sus dos cuartiles.

$=20$ ***(A1)*****(ft)*****(G3)***

[5 puntos]

(b) $3200-350=2850$ ***(M1)***

Nota: Conceda ***(M1)*** si ha escrito 2850. Arrastre de error a partir de su 3200.

(Nota límite $=$) 76 ***(A1)(G2)***

[2 puntos]

(c) (i) $60 < x \le 80$ ***(A1)(A1)***

Nota: Conceda ***(A1)*** por haber escrito 60, 80; conceda ***(A1)*** por haber escrito correctamente la inecuación estricta ($<$) y la inecuación débil ($\le$).

(ii) 70 ***(A1)*****(ft)**

Nota: Arrastre de error (ft) desde el apartado (c)(i).

[3 puntos]

(d) (i) $57,2 \quad (57,1875)$ ***(A2)*****(ft)**

Nota: Arrastre de error (ft) desde el apartado (c)(ii).

(ii) $18,496$ ***(A1)***

Nota: Conceda ***(A0)*** por $18,499$.

[3 puntos]

(e) $57,2-18,5$ ***(M1)***
$= 38,7 \quad (38,6918\ldots)$ ***(A1)*****(ft)**

Nota: Conceda ***(M1)*** por restar su desviación típica de su media. Arrastre de error (ft) a partir del apartado (d) aún cuando no haya procedimiento presente.

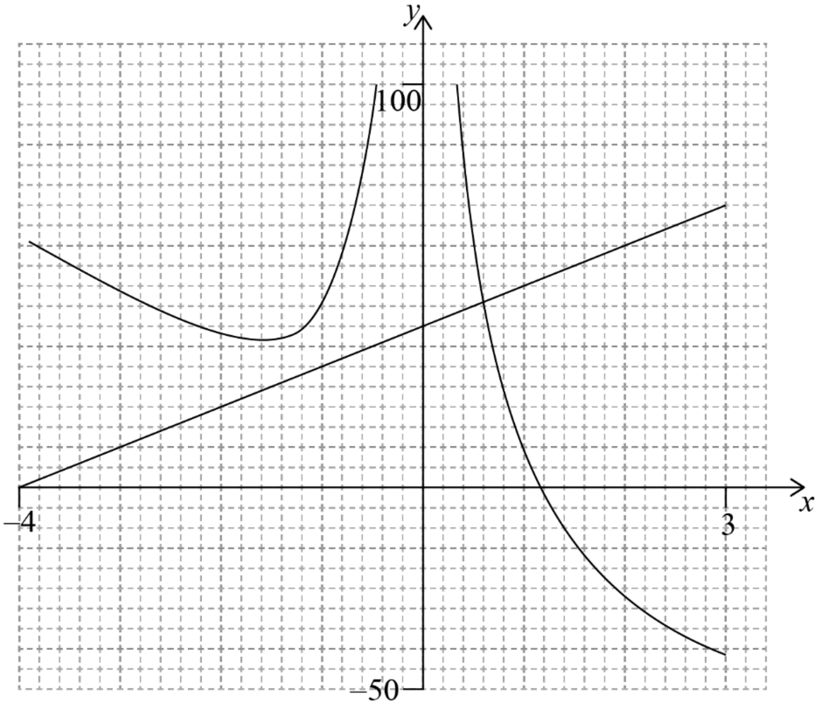
450 alumnos ***(A1)*****(ft)*****(G2)***

Nota: Acepte cualquier respuesta que esté comprendida entre 450 y 475, ambos inclusive. Arrastre de error desde el apartado (d), ajustando un rango aceptable si fuera necesario.

[3 puntos]

Total [16 puntos]

4. (a)



(A1)(A1)(A1)(A1)

Nota:	Conceda ***(A1)*** por incluir rótulos en los ejes y algún indicativo de la escala utilizada; acepte y o $f(x)$. No se requiere el uso de papel milimetrado. Si no se muestra una escala asuma la ventana para el cero y el punto mínimo. Conceda ***(A1)*** por una curva suave cuya forma general sea la correcta. Conceda ***(A1)*** por una *intersección con el eje* x que se encuentre más cerca del eje y que del extremo derecho de la curva dibujada. Conceda ***(A1)*** por un mínimo local correcto cuya coordenada x esté más cerca del eje y que del extremo izquierdo de la curva, y cuya coordenada y sea menor que la mitad de la distancia que hay hasta la parte superior de la curva. Conceda como mucho ***(A1)(A0)(A1)(A1)*** si la curva trazada corta al eje y o si la curva trazada se aleja del eje y cuando x se aproxima a cero.

[4 puntos]

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Continuación de la Pregunta 4

(b) (i) $1,19 \quad (1,19055\ldots)$ ***(A1)***

> **Nota:** Acepte como respuesta $(1,19\ ,\ 0)$.
> No aplique arrastre de error (ft) desde un bosquejo (= curva dibujada por el alumno) que sea incorrecto.

(ii) $(-1,5\ ,\ 36)$ ***(A1)(A1)***

> **Nota:** Conceda ***(A0)(A1)*** si ha omitido los paréntesis.
> Acepte $x=-1,5,\ y=36$.

(iii) $y=-9,25x+20,3 \quad (y=-9,25x+20,25)$ ***(A1)(A1)***

> **Nota:** Conceda ***(A1)*** por el $-9,25x$, conceda ***(A1)*** por el $+20,25$; conceda como máximo ***(A0)(A1)*** si la respuesta dada no es una ecuación.

[5 puntos]

(c) recta correcta: $y=10x+40$, trazada en los mismos ejes de coordenadas que la curva del apartado (a) ***(A1)(A1)***

> **Nota:** Conceda ***(A1)*** por haber trazado una recta de pendiente positiva; conceda ***(A1)*** por una intersección con el eje x y con el eje y situadas aproximadamente en las posiciones correctas. Conceda como mucho ***(A0)(A1)*** si no se ha utilizado la regla para trazar la recta. Si utiliza un nuevo sistema de ejes conceda como mucho ***(A0)(A1)***.

[2 puntos]

(d) $0,684 \quad (0,68362\ldots,)$ ***(G2)***

> **Nota:** Conceda como mucho ***(G1)*** si también ha dado el valor de y $(46,8)$ como parte de la respuesta.
> Conceda ***(G1)*** por $0,683$ según la regla de única penalización por precisión en cada pregunta.

[2 puntos]

Total [13 puntos]

5. Es necesario que haya especificado las unidades en el apartado (b)

(a) $\frac{\operatorname{sen} 43{,}7°}{100} = \frac{\operatorname{sen} \mathrm{BAC}}{50}$ ***(M1)(A1)***

Nota: Conceda ***(M1)*** por haber sustituido en la fórmula del teorema del seno; conceda ***(A1)*** si la sustitución se ha hecho correctamente.

$\mathrm{BAC} = 20{,}2087\ldots = 20{,}2°$ ***(A1)(AG)***

Nota: Conceda ***(A1)*** solamente si el alumno ha escrito los dos valores: la respuesta correcta sin redondear y también la respuesta correcta tras haber aplicado el redondeo.

[3 puntos]

(b) $\frac{1}{2}(100)(50)\operatorname{sen}(116{,}1°)$ ***(A1)(M1)(A1)***

Nota: Conceda ***(A1)*** por haber escrito $116{,}1$ o 116 ; ***(M1)*** por haber sustituido los valores en la fórmula del área de un triángulo y ***(A1)*** si la sustitución se ha hecho correctamente.

$= 2250\ \mathrm{m}^2 \quad \left(2245{,}06\ldots\ \mathrm{m}^2\right)$ ***(A1)(G3)***

Nota: La respuesta es $2250\ \mathrm{m}^2$; es necesario que haya incluido las unidades. El haber usado $20{,}2087\ldots$ lleva a un resultado de $2245{,}23\ldots$

[4 puntos]

Continúa en la página siguiente...

Continuación de la Pregunta 5

(c) $\dfrac{100}{\text{sen } 43,7°} = \dfrac{\text{AC}}{\text{sen } (116,1°)}$ ***(M1)(A1)*(ft)**

Nota:	Conceda ***(M1)*** por haber sustituido en la fórmula del teorema del seno; conceda ***(A1)*(ft)** si la sustitución se hizo correctamente. Arrastre de error a partir de su $116,1$.

$\text{AC} = 130(\text{m}) \ \ (129,982\ldots(\text{m}))$ ***(A1)*(ft)*(G2)***

Nota:	El usar $20,2087\ldots$ lleva a un resultado de $129,992\ldots$

O BIEN

$\text{AC}^2 = 100^2 + 50^2 - 2(100)(50)\cos(116.1)$ ***(M1)(A1)*(ft)**

Nota:	Conceda ***(M1)*** por haber sustituido en la fórmula del teorema del coseno; conceda ***(A1)*(ft)** si la sustitución se hizo correctamente. Arrastre de error a partir de su $116,1$.

$\text{AC} = 130(\text{m}) \ \ (129,997\ldots(\text{m}))$ ***(A1)*(ft)*(G2)***

Nota:	Conceda ***M1*** por haber sustituido en la fórmula del teorema del coseno, ***(A1)ft*** si la sustitución se hizo correctamente.

[3 puntos]

Continúa en la página siguiente...

Continuación de la Pregunta 5

(d) $\text{BM}^2 = 100^2 + 65^2 - 2(100)(65)\cos(20,2^\circ)$

O BIEN

$\text{BM}^2 = 50^2 + 65^2 - 2(50)(65)\cos(43,7^\circ)$ ***(M1)(A1)*****(ft)**

Nota:	Conceda ***(M1)*** por haber sustituido los datos en la fórmula del teorema del coseno; conceda ***(A1)*****(ft)** si la sustitución se ha hecho correctamente incluyendo la mitad de su AC.

$\text{BM} = 45,0\ (44,9954\ldots$ **O BIEN** $45,0079\ldots)$ ***(A1)*****(ft)**

Nota:	El uso de $20,2052\ldots$ conduce a un resultado de 45. Conceda ***(G2)*** por haber escrito $45,0$ sin mostrar procedimiento.

$\tan(\text{T}\hat{\text{M}}\text{B}) = \dfrac{25}{\text{su valor de BM}}$ ***(M1)***

Nota:	Conceda ***(M1)*** por haber sustituido correctamente en la fórmula de la tangente.

$\text{T}\hat{\text{M}}\text{B} = 29,1^\circ\ (29,0546\ldots^\circ)$ ***(A1)*****(ft)*****(G4)***

Nota:	Arrastre de error (ft) dentro del apartado (d), siempre y cuando hayan indicado su valor de BM. El haber usado $44,9954$ conduce a un resultado de $29,0570\ldots$. y el uso de $45,0079\ldots$ conduce a un resultado de $29,0503\ldots$ Arrastre de error a partir de su AC de la parte (c).

[5 puntos]

Total [15 puntos]

6. (a) $36 \times 12 + 2(9 \times 12) + 2(9 \times 36)$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente en la fórmula del área de la superficie del ortoedro.

$= 1300\left(\text{cm}^2\right) \quad \left(1296\left(\text{cm}^2\right)\right)$ ***(A1)(G2)***

[2 puntos]

(b) $36 \times 9 \times 12$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente en la fórmula del volumen del ortoedro.

$= 3890\left(\text{cm}^3\right) \quad \left(3888\left(\text{cm}^3\right)\right)$ ***(A1)(G2)***

[2 puntos]

(c) $3x \times x \times y = 3888$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente en la fórmula del volumen del ortoedro e igualado a 3888.

$x^2 y = 1296$ ***(A1)(G2)***

Nota: Conceda ***(A1)*** por haber escrito correctamente la fórmula completamente simplificada del volumen del ortoedro. Acepte $y = \frac{1296}{x^2}$.

[2 puntos]

(d) $(A =)\ 3x^2 + 2(xy) + 2(3xy)$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente en la fórmula del área de la superficie del ortoedro.

$(A =)\ 3x^2 + 8xy$ ***(A1)(G2)***

Nota: Conceda ***(A1)*** por haber escrito correctamente la fórmula simplificada del área de la superficie del ortoedro.

[2 puntos]

Continúa en la página siguiente...

Continuación de la Pregunta 6

(e) $A = 3x^2 + 8x\left(\frac{1296}{x^2}\right)$ ***(A1)*(ft)*(M1)***

Nota:	Conceda ***(A1)*(ft)** por haber reordenado correctamente su respuesta del apartado (c) (vale también si ya lo reordenaron en el apartado (c)); conceda ***(M1)*** por sustituir su respuesta del apartado (c) en su respuesta al apartado (d), pero solo si eso le condujo a la respuesta dada en el enunciado, que debe escribirse.

$A = 3x^2 + \frac{10\,368}{x}$ ***(AG)***

[2 puntos]

(f) $\frac{\mathrm{d}A}{\mathrm{d}x} = 6x - \frac{10\,368}{x^2}$ ***(A1)(A1)(A1)***

Nota:	Conceda ***(A1)*** por el $6x$, ***(A1)*** por el $-10\,368$ y ***(A1)*** por el x^{-2}. Conceda un máximo de ***(A1)(A1)(A0)*** si se han escrito términos demás.

[3 puntos]

(g) $6x - \frac{10\,368}{x^2} = 0$ ***(M1)***

Nota:	Conceda ***(M1)*** por igualar a cero su expresión de $\frac{\mathrm{d}A}{\mathrm{d}x}$.

$6x^3 = 10\,368$ **O BIEN** $6x^3 - 10\,368 = 0$ **O BIEN** $x^3 - 1728 = 0$ ***(M1)***

Nota:	Conceda ***(M1)*** por haber reordenado correctamente su ecuación de modo tal que se eliminaran las fracciones.

$x = \sqrt[3]{1728}$ ***(A1)***

$x = 12\,(\mathrm{cm})$ ***(AG)***

Nota:	Conceda el último ***(A1)*** solamente si está presente el renglón de ***(AG)***. La sustitución de $x = 12$ invalida el método, conceda un máximo de ***(M1)(M0)(A0)***.

[3 puntos]

Continúa en la página siguiente...

Continuación de la Pregunta 6

(h) $\left(3(12)^2 + \dfrac{10368}{12}\right) \times 4$ ***(M1)***

Nota:	Conceda ***(M1)*** por haber sustituido 12 en la fórmula del área y por haber multiplicado por 4 el resultado de dicha fórmula.

$= 5180$ (JPY) (5184 JPY)) ***(A1)(G2)***

[2 puntos]

Total [18 puntos]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel superior
Prueba 1

Lunes 13 de mayo de 2019 (tarde)

Número de convocatoria del alumno

2 horas

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba no se permite el uso de ninguna calculadora.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[100 puntos]**.

13 páginas

2219 – 7225

No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 4]

En una progresión aritmética, la suma de los términos 3.º y 8.º es igual a 1.
Sabiendo que la suma de los siete primeros términos es 35, determine el primer término y la diferencia común.

2. [Puntuación máxima: 6]

Tres puntos del espacio tridimensional tienen por coordenadas $\mathrm{A}(0, 0, 2)$, $\mathrm{B}(0, 2, 0)$ y $\mathrm{C}(3, 1, 0)$.

(a) Halle el vector

(i) $\vec{\mathrm{AB}}$;

(ii) $\vec{\mathrm{AC}}$. [2]

(b) A partir de lo anterior o de cualquier otro modo, halle el área del triángulo ABC. [4]

3. [Puntuación máxima: 5]

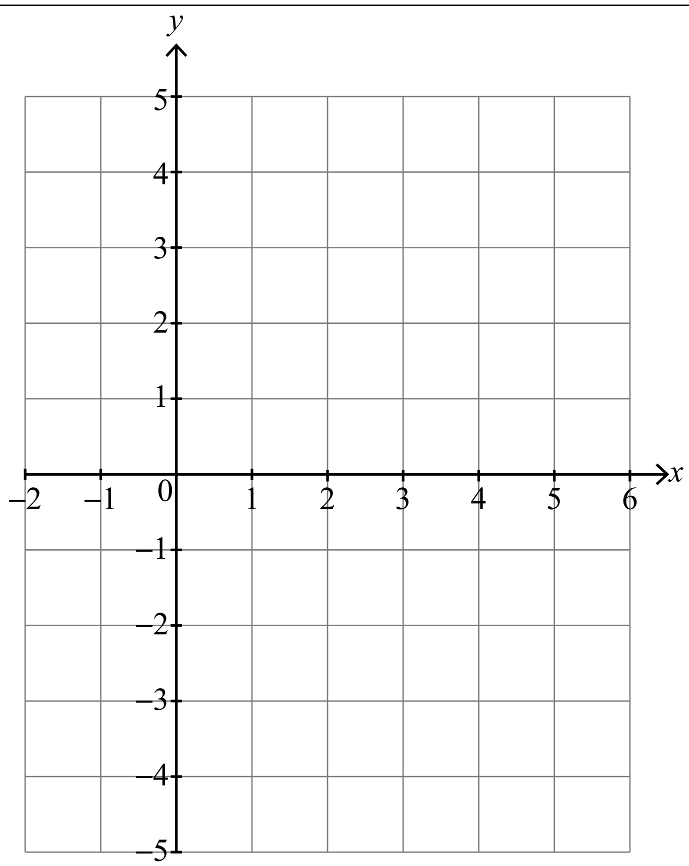
Considere la función $f(x) = x^4 - 6x^2 - 2x + 4$, $x \in \mathbb{R}$.
El gráfico de f se traslada dos unidades hacia la izquierda para dar lugar a la función $g(x)$.
Exprese $g(x)$ de la forma $ax^4 + bx^3 + cx^2 + dx + e$, donde $a, b, c, d, e \in \mathbb{Z}$.

4. [Puntuación máxima: 5]

Utilizando la sustitución $u = \operatorname{sen} x$, halle $\int \frac{\cos^3 x \, dx}{\sqrt{\operatorname{sen} x}}$.

5. [Puntuación máxima: 8]

(a) Dibuje aproximadamente el gráfico de $y = \dfrac{x-4}{2x-5}$, e indique las ecuaciones de todas las asíntotas y las coordenadas de todos los puntos de corte con los ejes. [5]



(b) Considere la función $f : x \to \sqrt{\dfrac{x-4}{2x-5}}$.

Escriba

(i) el mayor dominio posible de f;

(ii) el correspondiente recorrido de f. [3]

6. [Puntuación máxima: 7]

La curva C viene dada por la ecuación $y = x\tan\left(\frac{\pi xy}{4}\right)$.

(a) Muestre que, en el punto $(1\,,\,1)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2+\pi}{2-\pi}$. [5]

(b) A partir de lo anterior, halle la ecuación de la recta normal a C en el punto $(1\,,\,1)$. [2]

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No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

7. [Puntuación máxima: 7]

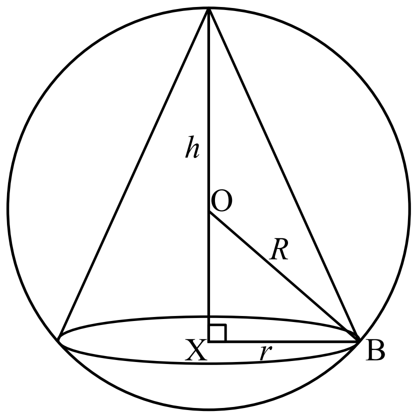
Resuelva el sistema de ecuaciones

$$\log_2 6x = 1 + 2\log_2 y$$
$$1 + \log_6 x = \log_6(15y - 25).$$

. . . .

8. [Puntuación máxima: 8]

Un cono circular recto de radio r está inscrito en una esfera de centro O y radio R, tal y como se muestra en la siguiente figura. La altura perpendicular del cono es h, X denota el centro de la base y B es un punto donde el cono toca la esfera.



(a) Muestre que el volumen del cono se puede expresar mediante $V = \frac{\pi}{3}\left(2Rh^2 - h^3\right)$. [4]

(b) Sabiendo que existe un cono inscrito que tiene volumen máximo, muestre que el volumen de dicho cono es $\frac{32\pi R^3}{81}$. [4]

(Esta pregunta continúa en la página siguiente)

(Pregunta 8: continuación)

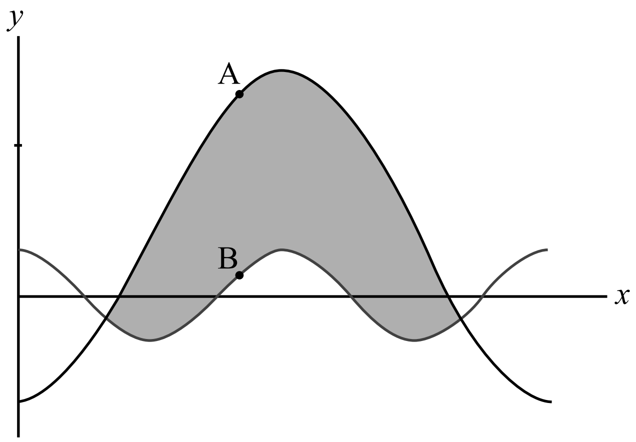
No escriba soluciones en esta página.

Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

9. [Puntuación máxima: 17]

Considere las funciones f y g definidas en el dominio $0 < x < 2\pi$ mediante $f(x) = 3\cos 2x$ y $g(x) = 4 - 11\cos x$.
La siguiente figura muestra el gráfico de $y = f(x)$ y el de $y = g(x)$.



(a) Halle la coordenada x de cada uno de los puntos de intersección de los dos gráficos. [6]

(b) Halle el área exacta de la región sombreada; dé la respuesta de la forma $p\pi + q\sqrt{3}$, donde p, $q \in \mathbb{Q}$. [5]

En los puntos A y B de la figura, los dos gráficos tienen la misma pendiente.

(c) Determine la coordenada y de A, perteneciente al gráfico de g. [6]

No escriba soluciones en esta página.

10. [Puntuación máxima: 16]

La variable aleatoria X tiene función de densidad de probabilidad f, que viene dada por

$$f(x) = \begin{cases} k\left(\pi - \operatorname{arcsen} x\right) & 0 \le x \le 1 \\ 0 & \text{resto de valores} \end{cases}$$, donde k es una constante positiva.

(a) Indique la moda de X. [1]

(b) (i) Halle $\int \operatorname{arcsen} x \, \mathrm{d}x$.

(ii) A partir de lo anterior, muestre que $k = \dfrac{2}{2+\pi}$. [6]

(c) Sabiendo que $y = \left(\dfrac{x^2}{2}\right)\operatorname{arcsen} x - \left(\dfrac{1}{4}\right)\operatorname{arcsen} x + \left(\dfrac{x}{4}\right)\sqrt{1-x^2}$, muestre que

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x \operatorname{arcsen} x$;

(ii) $\mathrm{E}\left(X\right) = \dfrac{3\pi}{4\left(\pi+2\right)}$. [9]

11. [Puntuación máxima: 17]

Considere las funciones f y g definidas mediante $f(x) = \ln|x|$, $x \in \mathbb{R} \setminus \{0\}$ y $g(x) = \ln|x+k|$, $x \in \mathbb{R} \setminus \{-k\}$, donde $k \in \mathbb{R}$, $k > 2$.

(a) Describa la transformación mediante la cual $f(x)$ se transforma en $g(x)$. [1]

(b) Indique el recorrido de g. [1]

(c) En un mismo sistema de ejes de coordenadas, dibuje aproximadamente los gráficos de $y = f(x)$ e $y = g(x)$, indicando claramente todos los puntos de corte con los ejes. [6]

Los gráficos de f y g se cortan en el punto P.

(d) Halle las coordenadas de P. [2]

La tangente a $y = f(x)$ en P pasa por el origen $(0, 0)$.

(e) Determine el valor de k. [7]

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

No escriba en esta página.

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Las respuestas que se escriban en
esta página no serán corregidas.

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2019

Mathematics

Higher level

Paper 1

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. attempting to form two equations involving u_1 and d ***M1***

$(u_1+2d)+(u_1+7d)=1$ and $\frac{7}{2}[2u_1+6d]=35$

$2u_1+9d=1$

$14u_1+42d=70\ (2u_1+6d=10)$ ***A1***

Note: Award ***A1*** for any two correct equations

attempting to solve their equations: ***M1***

$u_1=14$, $d=-3$ ***A1***

[4 marks]

2. (a) (i) $\vec{\mathrm{AB}}=\begin{pmatrix}0\\2\\-2\end{pmatrix}$ ***A1***

(ii) $\vec{\mathrm{AC}}=\begin{pmatrix}3\\1\\-2\end{pmatrix}$ ***A1***

Note: Accept row vectors or equivalent.

[2 marks]

(b) **METHOD 1**

attempt at vector product using $\vec{\mathrm{AB}}$ and $\vec{\mathrm{AC}}$. ***(M1)***

$\pm(2\boldsymbol{i}+6\boldsymbol{j}+6\boldsymbol{k})$ ***A1***

attempt to use area $=\frac{1}{2}\left|\vec{\mathrm{AB}}\times\vec{\mathrm{AC}}\right|$ ***M1***

$=\frac{\sqrt{76}}{2}\left(=\sqrt{19}\right)$ ***A1***

[4 marks]

continued…

Question 2 continued

METHOD 2

attempt to use $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}} = \left|\overrightarrow{\mathrm{AB}}\right|\left|\overrightarrow{\mathrm{AC}}\right|\cos\theta$ ***M1***

$$\begin{pmatrix}0\\2\\-2\end{pmatrix}.\begin{pmatrix}3\\1\\-2\end{pmatrix} = \sqrt{0^2+2^2+(-2)^2}\sqrt{3^2+1^2+(-2)^2}\cos\theta$$

$6 = \sqrt{8}\sqrt{14}\cos\theta$ ***A1***

$$\cos\theta = \frac{6}{\sqrt{8}\sqrt{14}} = \frac{6}{\sqrt{112}}$$

attempt to use area $= \frac{1}{2}\left|\overrightarrow{\mathrm{AB}}\right|\left|\overrightarrow{\mathrm{AC}}\right|\sin\theta$ ***M1***

$$= \frac{1}{2}\sqrt{8}\sqrt{14}\sqrt{1-\frac{36}{112}}\left(= \frac{1}{2}\sqrt{8}\sqrt{14}\sqrt{\frac{76}{112}}\right)$$

$= \frac{\sqrt{76}}{2}\left(=\sqrt{19}\right)$ ***A1***

[4 marks]

Total [6 marks]

3. $g(x) = f(x+2)\left(=(x+2)^4 - 6(x+2)^2 - 2(x+2) + 4\right)$ ***M1***

attempt to expand $(x+2)^4$ ***M1***

$(x+2)^4 = x^4 + 4(2x^3) + 6(2^2x^2) + 4(2^3x) + 2^4$ ***(A1)***

$= x^4 + 8x^3 + 24x^2 + 32x + 16$ ***A1***

$g(x) = x^4 + 8x^3 + 24x^2 + 32x + 16 - 6(x^2 + 4x + 4) - 2x - 4 + 4$

$= x^4 + 8x^3 + 18x^2 + 6x - 8$ ***A1***

Note: For correct expansion of $f(x-2) = x^4 - 8x^3 + 18x^2 - 10x$ award max ***M0M1(A1)A0A1***.

[5 marks]

4. $u = \sin x \Rightarrow \mathrm{d}u = \cos x \mathrm{d}x$ ***(A1)***

valid attempt to write integral in terms of u and $\mathrm{d}u$ ***M1***

$\int \frac{\cos^3 x \ \mathrm{d}x}{\sqrt{\sin x}} = \int \frac{(1-u^2)\ \mathrm{d}u}{\sqrt{u}}$ ***A1***

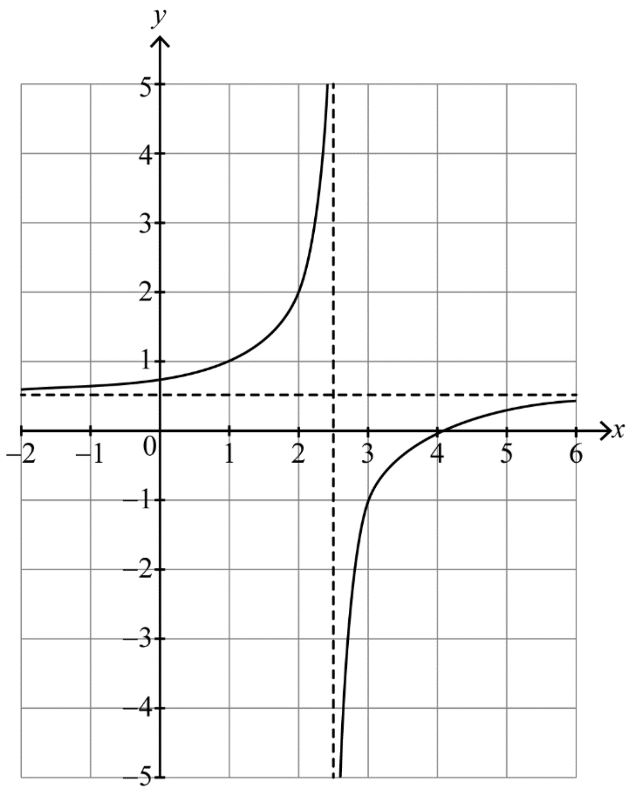
$= \int \left(u^{-\frac{1}{2}} - u^{\frac{3}{2}} \right) \mathrm{d}u$

$= 2u^{\frac{1}{2}} - \frac{2u^{\frac{5}{2}}}{5} (+c)$ ***(A1)***

$= 2\sqrt{\sin x} - \frac{2\left(\sqrt{\sin x}\right)^5}{5} (+c)$ or equivalent ***A1***

[5 marks]

5. (a)



correct shape: two branches in correct quadrants with asymptotic behaviour ***A1***

crosses at $(4, 0)$ and $\left(0, \frac{4}{5}\right)$ ***A1A1***

asymptotes at $x = \frac{5}{2}$ and $y = \frac{1}{2}$ ***A1A1***

[5 marks]

continued…

Question 5 continued

(b) (i) $x < \frac{5}{2}, x \geq 4$ ***A1A1***

(ii) $f(x) \geq 0, f(x) \neq \frac{1}{\sqrt{2}} \left(f(x) \in \mathbb{R}\right)$ ***A1***

Note: Follow through from their graph, as long as it is a rectangular hyperbola.

Note: Allow range expressed in terms of y.

[3 marks]

Total [8 marks]

6. (a) attempt to differentiate implicitly ***M1***

$$\frac{dy}{dx} = x\sec^2\left(\frac{\pi xy}{4}\right)\left[\frac{\pi}{4}x\frac{dy}{dx} + \frac{\pi}{4}y\right] + \tan\left(\frac{\pi xy}{4}\right)$$ ***A1A1***

Note: Award ***A1*** for each term.

attempt to substitute $x = 1,\ y = 1$ into their equation for $\frac{dy}{dx}$ ***M1***

$$\frac{dy}{dx} = \frac{\pi}{2}\frac{dy}{dx} + \frac{\pi}{2} + 1$$

$$\frac{dy}{dx}\left(1 - \frac{\pi}{2}\right) = \frac{\pi}{2} + 1$$ ***A1***

$$\frac{dy}{dx} = \frac{2+\pi}{2-\pi}$$ ***AG***

[5 marks]

(b) attempt to use gradient of normal $= \frac{-1}{\frac{dy}{dx}}$ ***(M1)***

$$= \frac{\pi - 2}{\pi + 2}$$

so equation of normal is $y - 1 = \frac{\pi-2}{\pi+2}(x-1)$ or $y = \frac{\pi-2}{\pi+2}x + \frac{4}{\pi+2}$ ***A1***

[2 marks]

Total [7 marks]

7. use of at least one "log rule" applied correctly for the first equation ***M1***

$\log_2 6x = \log_2 2 + 2\log_2 y$

$= \log_2 2 + \log_2 y^2$

$= \log_2 \left(2y^2\right)$

$\Rightarrow 6x = 2y^2$ ***A1***

use of at least one "log rule" applied correctly for the second equation ***M1***

$\log_6 \left(15y - 25\right) = 1 + \log_6 x$

$= \log_6 6 + \log_6 x$

$= \log_6 6x$

$\Rightarrow 15y - 25 = 6x$ ***A1***

attempt to eliminate x (or y) from their two equations ***M1***

$2y^2 = 15y - 25$

$2y^2 - 15y + 25 = 0$

$\left(2y-5\right)\left(y-5\right) = 0$

$x = \frac{25}{12}$, $y = \frac{5}{2}$, ***A1***

or $x = \frac{25}{3}$, $y = 5$ ***A1***

Note: x, y values do not have to be "paired" to gain either of the final two ***A*** marks.

[7 marks]

8. (a) attempt to use Pythagoras in triangle OXB ***M1***

$\Rightarrow r^2 = R^2 - \left(h - R\right)^2$ ***A1***

substitution of their r^2 into formula for volume of cone $V = \frac{\pi r^2 h}{3}$ ***M1***

$= \frac{\pi h}{3}\left(R^2 - \left(h - R\right)^2\right)$

$= \frac{\pi h}{3}\left(R^2 - \left(h^2 + R^2 - 2hR\right)\right)$ ***A1***

Note: This **A** mark is independent and may be seen anywhere for the correct expansion of $\left(h - R\right)^2$.

$= \frac{\pi h}{3}\left(2hR - h^2\right)$

$= \frac{\pi}{3}\left(2Rh^2 - h^3\right)$ ***AG***

[4 marks]

continued…

Question 8 continued

(b) at max, $\frac{\mathrm{d}V}{\mathrm{d}h} = 0$ ***R1***

$$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{\pi}{3}\left(4Rh - 3h^2\right)$$

$$\Rightarrow 4Rh = 3h^2$$

$$\Rightarrow h = \frac{4R}{3} \text{ (since } h \neq 0\text{)}$$ ***A1***

EITHER

$V_{\max} = \frac{\pi}{3}\left(2Rh^2 - h^3\right)$ from part (a)

$$= \frac{\pi}{3}\left(2R\left(\frac{4R}{3}\right)^2 - \left(\frac{4R}{3}\right)^3\right)$$ ***A1***

$$= \frac{\pi}{3}\left(2R\frac{16R^2}{9} - \left(\frac{64R^3}{27}\right)\right)$$ ***A1***

OR

$$r^2 = R^2 - \left(\frac{4R}{3} - R\right)^2$$

$$r^2 = R^2 - \frac{R^2}{9} = \frac{8R^2}{9}$$ ***A1***

$$\Rightarrow V_{\max} = \frac{\pi r^2}{3}\left(\frac{4R}{3}\right)$$

$$= \frac{4\pi R}{9}\left(\frac{8R^2}{9}\right)$$ ***A1***

THEN

$$= \frac{32\pi R^3}{81}$$ ***AG***

[4 marks]

Total [8 marks]

Section B

9. (a) $3\cos 2x = 4 - 11\cos x$

attempt to form a quadratic in $\cos x$ ***M1***

$3\left(2\cos^2 x - 1\right) = 4 - 11\cos x$ ***A1***

$\left(6\cos^2 x + 11\cos x - 7 = 0\right)$

valid attempt to solve their quadratic ***M1***

$(3\cos x + 7)(2\cos x - 1) = 0$

$\cos x = \frac{1}{2}$ ***A1***

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ ***A1A1***

Note: Ignore any "extra" solutions.

[6 marks]

(b) consider $(\pm)\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (4 - 11\cos x - 3\cos 2x)\,\mathrm{d}x$ ***M1***

$= (\pm)\left[4x - 11\sin x - \frac{3}{2}\sin 2x\right]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$ ***A1***

Note: Ignore lack of or incorrect limits at this stage.

attempt to substitute their limits into their integral ***M1***

$= \frac{20\pi}{3} - 11\sin\frac{5\pi}{3} - \frac{3}{2}\sin\frac{10\pi}{3} - \left(\frac{4\pi}{3} - 11\sin\frac{\pi}{3} - \frac{3}{2}\sin\frac{2\pi}{3}\right)$

$= \frac{16\pi}{3} + \frac{11\sqrt{3}}{2} + \frac{3\sqrt{3}}{4} + \frac{11\sqrt{3}}{2} + \frac{3\sqrt{3}}{4}$

$= \frac{16\pi}{3} + \frac{25\sqrt{3}}{2}$ ***A1A1***

[5 marks]

(c) attempt to differentiate both functions and equate ***M1***

$-6\sin 2x = 11\sin x$ ***A1***

attempt to solve for x ***M1***

$11\sin x + 12\sin x\cos x = 0$

$\sin x(11 + 12\cos x) = 0$

$\cos x = -\frac{11}{12}$ $\left(\text{or } \sin x = 0\right)$ ***A1***

$\Rightarrow y = 4 - 11\left(-\frac{11}{12}\right)$ ***M1***

$y = \frac{169}{12}\left(= 14\frac{1}{12}\right)$ ***A1***

[6 marks]

Total [17 marks]

10. (a) mode is 0 ***A1***

[1 mark]

(b) (i) attempt at integration by parts ***(M1)***

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}, dv = dx$$

$$= x\arcsin x - \int \frac{x dx}{\sqrt{1-x^2}}$$ ***A1***

$$= x\arcsin x + \sqrt{1-x^2}\ (+c)$$ ***A1***

(ii) $$\int_0^1 (\pi - \arcsin x)\, dx = \left[\pi x - x\arcsin x - \sqrt{1-x^2}\right]_0^1$$ ***A1***

$$= \left(\pi - \frac{\pi}{2} - 0\right) - (0 - 0 - 1) = \frac{\pi}{2} + 1$$

$$= \frac{\pi + 2}{2}$$ ***A1***

$$\int_0^1 k(\pi - \arcsin x)\, dx = 1$$ ***(M1)***

Note: This line can be seen (or implied) anywhere.

Note: Do not allow ***FT A*** marks from bi to bii.

$$k\left(\frac{\pi+2}{2}\right) = 1$$

$$\Rightarrow k = \frac{2}{2+\pi}$$ ***AG***

[6 marks]

(c) (i) attempt to use product rule to differentiate ***M1***

$$\frac{dy}{dx} = x\arcsin x + \frac{x^2}{2\sqrt{1-x^2}} - \frac{1}{4\sqrt{1-x^2}} - \frac{x^2}{4\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{4}$$ ***A2***

Note: Award ***A2*** for all terms correct, ***A1*** for 4 correct terms.

$$= x\arcsin x + \frac{2x^2}{4\sqrt{1-x^2}} - \frac{1}{4\sqrt{1-x^2}} - \frac{x^2}{4\sqrt{1-x^2}} + \frac{1-x^2}{4\sqrt{1-x^2}}$$ ***A1***

Note: Award ***A1*** for equivalent combination of correct terms over a common denominator.

$$= x\arcsin x$$ ***AG***

continued…

Question 10 continued

(ii) $\mathrm{E}(X) = k\int_0^1 x(\pi - \arcsin x)\,\mathrm{d}x$ ***M1***

$= k\int_0^1 (\pi x - x\arcsin x)\,\mathrm{d}x$

$= k\left[\frac{\pi x^2}{2} - \frac{x^2}{2}\arcsin x + \frac{1}{4}\arcsin x - \frac{x}{4}\sqrt{1-x^2}\right]_0^1$ ***A1A1***

Note: Award ***A1*** for first term, ***A1*** for next 3 terms.

$= k\left[\left(\frac{\pi}{2} - \frac{\pi}{4} + \frac{\pi}{8}\right) - (0)\right]$ ***A1***

$= \left(\frac{2}{2+\pi}\right)\frac{3\pi}{8}$ ***A1***

$= \frac{3\pi}{4(\pi+2)}$ ***AG***

[9 marks]

Total [16 marks]

11. (a) translation k units to the left (or equivalent) ***A1***

[1 mark]

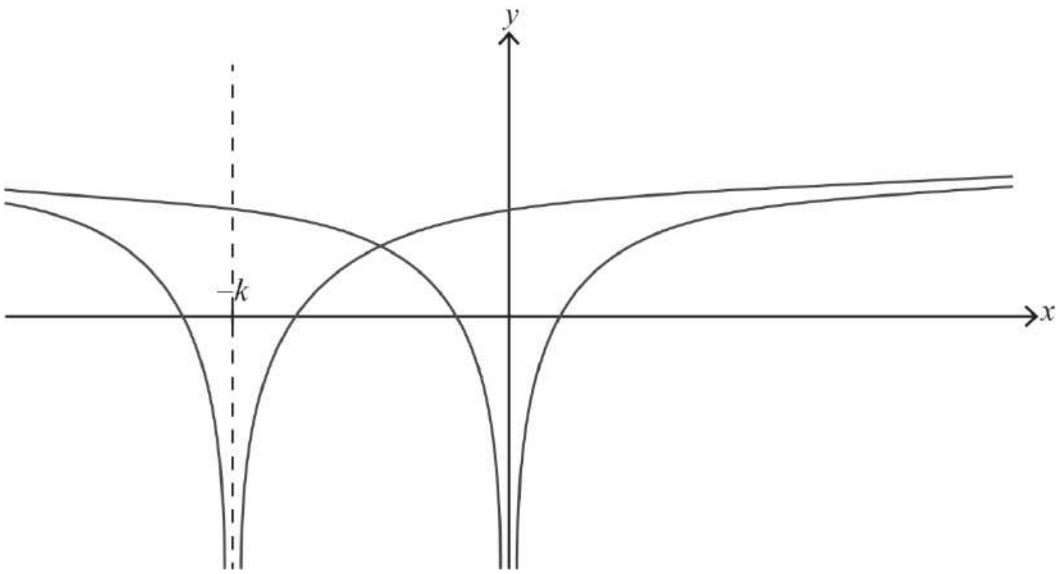
(b) range is $(g(x) \in)\mathbb{R}$ ***A1***

[1 mark]

continued…

Question 11 continued

(c)



correct shape of $y = f(x)$ ***A1***

their $f(x)$ translated k units to left (possibly shown by $x = -k$ marked on x-axis) ***A1***

asymptote included and marked as $x = -k$ ***A1***

$f(x)$ intersects x-axis at $x = -1,\ x = 1$ ***A1***

$g(x)$ intersects x-axis at $x = -k-1,\ x = -k+1$ ***A1***

$g(x)$ intersects y-axis at $y = \ln k$ ***A1***

Note: Do not penalise candidates if their graphs "cross" as $x \to \pm\infty$.

Note: Do not award ***FT*** marks from the candidate's part (a) to part (c).

[6 marks]

(d) at P $\ln(x+k) = \ln(-x)$

attempt to solve $x + k = -x$ (or equivalent) ***(M1)***

$x = -\frac{k}{2} \Rightarrow y = \ln\left(\frac{k}{2}\right)$ (or $y = \ln\left|\frac{k}{2}\right|$) ***A1***

$\mathrm{P}\left(-\frac{k}{2}, \ln\frac{k}{2}\right)$ (or $\mathrm{P}\left(-\frac{k}{2}, \ln\left|\frac{k}{2}\right|\right)$)

[2 marks]

continued…

Question 11 continued

(e) attempt to differentiate $\ln(-x)$ or $\ln|x|$ ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$ ***A1***

at P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{k}$ ***A1***

recognition that tangent passes through origin $\Rightarrow \frac{y}{x} = \frac{\mathrm{d}y}{\mathrm{d}x}$ ***(M1)***

$$\frac{\ln\left(\frac{k}{2}\right)}{-\frac{k}{2}} = \frac{-2}{k}$$ ***A1***

$$\ln\left(\frac{k}{2}\right) = 1$$ ***(A1)***

$\Rightarrow k = 2\mathrm{e}$ ***A1***

[7 marks]

Note:	For candidates who explicitly differentiate $\ln(x)$ (rather than $\ln(-x)$ or $\ln\|x\|$, award ***M0A0A1M1A1A1A1***.

Total [17 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel superior
Prueba 2

Martes 14 de mayo de 2019 (mañana)

Número de convocatoria del alumno

2 horas

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[100 puntos]**.

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No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 5]

En el triángulo ABC, $\mathrm{AB} = 5$, $\mathrm{BC} = 14$ y $\mathrm{AC} = 11$.
Halle todos los ángulos interiores del triángulo. Dé las respuestas en grados, con una aproximación de una cifra decimal.

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2. [Puntuación máxima: 5]

Timmy tiene una tienda. Sus ingresos diarios, procedentes de la venta de sus artículos, se pueden modelizar por una distribución normal, siendo la media de los ingresos diarios igual a $820 y la desviación típica igual a $230. Para poder tener beneficios, los ingresos diarios de Timmy tienen que ser mayores que $1000.

(a) Calcule la probabilidad de que, en un día elegido al azar, Timmy tenga beneficios. [2]

La tienda abre 24 días al mes.

(b) Calcule la probabilidad de que, en un mes elegido al azar, Timmy tenga beneficios entre 5 y 10 días (ambos inclusive). [3]

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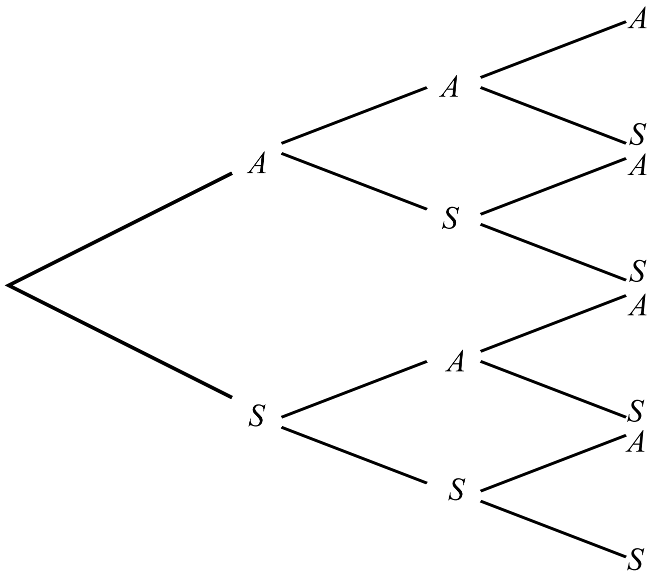
3. [Puntuación máxima: 8]

Iqbal, para practicar, decide hacer tres exámenes de muestra de matemáticas. La probabilidad de que apruebe el primer examen es igual a 0,6. Cuando aprueba un examen, Iqbal gana confianza en sí mismo, de modo que la probabilidad de que apruebe el siguiente examen aumenta en 0,1. Cuando suspende un examen, la probabilidad de que apruebe el siguiente examen es igual a 0,6.

(a) Complete el siguiente diagrama de árbol de probabilidades para los tres exámenes de muestra que hace Iqbal, rotulando cada rama con la probabilidad correcta. [3]

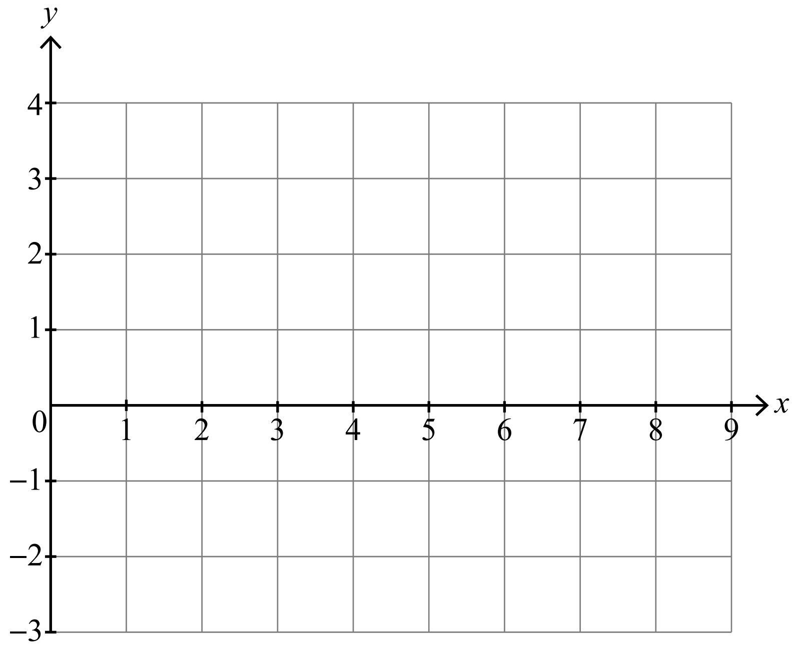


(b) Calcule la probabilidad de que Iqbal apruebe al menos dos de los exámenes. [2]

(c) Halle la probabilidad de que Iqbal apruebe el tercer examen, sabiendo que solo ha aprobado uno de los exámenes anteriores. [3]

4. [Puntuación máxima: 6]

(a) Dibuje aproximadamente los gráficos de $y = \operatorname{sen}^3 x + \ln x$ y de $y = 1 + \cos x$ en los siguientes ejes de coordenadas, para $0 < x \leq 9$. [2]



(b) A partir de lo anterior, resuelva $\operatorname{sen}^3 x + \ln x - \cos x - 1 < 0$ en el intervalo $0 < x \leq 9$. [4]

5. [Puntuación máxima: 6]

(a) Demuestre la identidad $\dfrac{1+\operatorname{sen} 2x}{\cos 2x} \equiv \dfrac{1+\tan x}{1-\tan x}$. [4]

(b) Resuelva la ecuación $\dfrac{1+\operatorname{sen} 2x}{\cos 2x} = \sqrt{3}$ para $0 \le x < 2\pi$. [2]

6. [Puntuación máxima: 6]

Una partícula se mueve a lo largo de una recta horizontal, de modo tal que en el instante t segundos, $t \geq 0$, su aceleración a viene dada por $a = 2t - 1$. Cuando $t = 6$, su desplazamiento s respecto a un origen fijo O es igual a $18{,}25$ m. Cuando $t = 15$, el desplazamiento respecto a O es igual a $922{,}75$ m. Halle una expresión para s en función de t.

7. [Puntuación máxima: 7]

Suponga que u_1 es el primer término de una serie geométrica de razón r.
Demuestre mediante inducción matemática que la suma de los n primeros términos, S_n, viene dada por $S_n = \dfrac{u_1\left(1-r^n\right)}{1-r}$, donde $n \in \mathbb{Z}^+$.

8. [Puntuación máxima: 7]

(a) Halle las raíces de la ecuación $w^3 = 8\mathrm{i}$, $w \in \mathbb{C}$. Dé las respuestas en forma cartesiana. [4]

Una de las raíces, w_1, satisface la condición $\mathrm{Re}(w_1) = 0$.

(b) Sabiendo que $w_1 = \dfrac{z}{z - \mathrm{i}}$, exprese z en la forma $a + b\mathrm{i}$ donde $a, b \in \mathbb{Q}$. [3]

No escriba soluciones en esta página.

Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

9. [Puntuación máxima: 15]

Considere el polinomio $P(z) \equiv z^4 - 6z^3 - 2z^2 + 58z - 51$, $z \in \mathbb{C}$.

(a) Exprese $P(z)$ en la forma $(z^2 + az + b)(z^2 + cz + d)$ donde $a, b, c, d \in \mathbb{R}$. [7]

(b) Dibuje aproximadamente el gráfico de $y = x^4 - 6x^3 - 2x^2 + 58x - 51$, indicando claramente las coordenadas de todos los máximos, mínimos y cortes con los ejes. [6]

(c) A partir de lo anterior o de cualquier otro modo, indique qué condición tiene que cumplir $k \in \mathbb{R}$ para que todas las raíces de la ecuación $P(z) = k$ sean reales. [2]

10. [Puntuación máxima: 16]

Steffi la gata callejera acude con frecuencia a casa de Will en busca de comida. Sea X la variable aleatoria discreta “el número de veces al día que Steffi acude a casa de Will”. La variable aleatoria X se puede modelizar por una distribución de Poisson de media 2,1.

(a) Halle la probabilidad de que un día elegido al azar, Steffi no acuda a casa de Will. [2]

Sea Y la variable aleatoria discreta “el número de veces al día que a Steffi le dan de comer en casa de Will”. Cada día, a Steffi solo le dan de comer las cuatro primeras veces que acude a la casa.

(b) Copie y complete la siguiente tabla de distribución de probabilidades correspondiente a Y. [4]

y	0	1	2	3	4
$\mathrm{P}(Y = y)$					

(c) A partir de lo anterior, halle el número esperado de veces al día que le dan de comer a Steffi en casa de Will. [3]

(d) En un año cualquiera de 365 días, la probabilidad de que Steffi no acuda a casa de Will como mucho n días en total es igual a $0,5$ (aproximada a una cifra decimal). Halle el valor de n. [3]

(e) Muestre que el número esperado de veces al año en las que Steffi acude a casa de Will y no le dan de comer es al menos 30. [4]

No escriba soluciones en esta página.

11. [Puntuación máxima: 19]

El plano Π_1 contiene a los puntos $\mathrm{P}(1, 6, -7)$, $\mathrm{Q}(0, 1, 1)$ y $\mathrm{R}(2, 0, -4)$.

(a) Halle la ecuación cartesiana del plano que contiene a P, Q y R. [6]

La ecuación cartesiana del plano Π_2 es $x - 3y - z = 3$.

(b) Sabiendo que Π_1 y Π_2 se cortan en una recta L, verifique que la ecuación vectorial de L se puede dar mediante $\boldsymbol{r} = \begin{pmatrix} \frac{5}{4} \\ 0 \\ -\frac{7}{4} \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{2} \\ 1 \\ -\frac{5}{2} \end{pmatrix}$. [3]

La ecuación cartesiana del plano Π_3 es $ax + by + cz = 1$.

(c) Sabiendo que Π_3 es paralelo a la recta L, muestre que $a + 2b - 5c = 0$. [1]

Considere el caso en el que Π_3 contiene a L.

(d) (i) Muestre que $5a - 7c = 4$.

(ii) Sabiendo que Π_3 está igual de inclinado hacia Π_1 que hacia Π_2, determine dos ecuaciones cartesianas posibles y distintas para Π_3. [9]

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

Markscheme

May 2019

Mathematics

Higher level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 ***N*** marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. attempt to apply cosine rule ***M1***

$$\cos A = \frac{5^2 + 11^2 - 14^2}{2 \times 5 \times 11} = -0.4545...$$

$\Rightarrow A = 117.03569...^\circ$

$\Rightarrow A = 117.0^\circ$ ***A1***

attempt to apply sine rule or cosine rule: ***M1***

$$\frac{\sin 117.03569...^\circ}{14} = \frac{\sin B}{11}$$

$\Rightarrow B = 44.4153...^\circ$

$\Rightarrow B = 44.4^\circ$ ***A1***

$C = 180^\circ - A - B$

$C = 18.5^\circ$ ***A1***

Note: Candidates may attempt to find angles in any order of their choosing.

[5 marks]

2. (a) $X \sim N\left(820, 230^2\right)$ ***(M1)***

Note: Award ***M1*** for an attempt to use normal distribution. Accept labelled normal graph.

$\Rightarrow P\left(X > 1000\right) = 0.217$ ***A1***

[2 marks]

(b) $Y \sim B\left(24, 0.217...\right)$ ***(M1)***

Note: Award ***M1*** for recognition of binomial distribution with parameters.

$P\left(Y \le 10\right) - P\left(Y \le 4\right)$ ***(M1)***

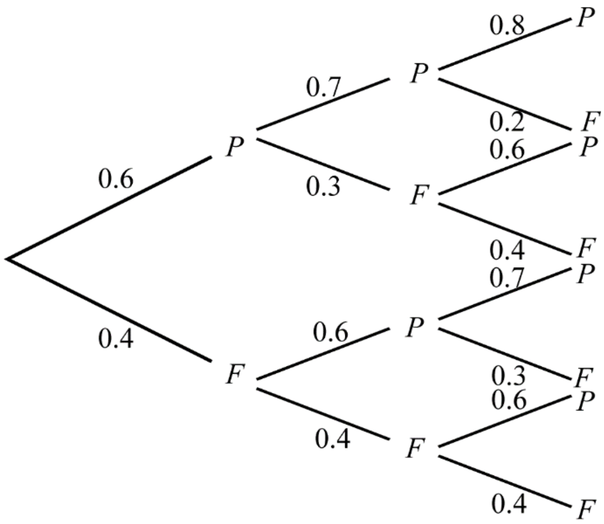
Note: Award ***M1*** for an attempt to find $P(5 \le Y \le 10)$ or $P\left(Y \le 10\right) - P\left(Y \le 4\right)$.

$= 0.613$ ***A1***

[3 marks]

Total [5 marks]

3. (a)



A1A1A1

Note: Award ***A1*** for each correct column of probabilities.

[3 marks]

(b) probability (at least twice) =

EITHER

$(0.6\times0.7\times0.8)+(0.6\times0.7\times0.2)+(0.6\times0.3\times0.6)+(0.4\times0.6\times0.7)$ ***(M1)***

OR

$(0.6\times0.7)\ +\ (0.6\times0.3\times0.6)\ +\ (0.4\times0.6\times0.7)$ ***(M1)***

Note: Award ***M1*** for summing all required probabilities.

THEN

$=0.696$ ***A1***

[2 marks]

(c) P(passes third paper given only one paper passed before)

$=\dfrac{\text{P (passes third AND only one paper passed before)}}{\text{P (passes once in first two papers)}}$ ***(M1)***

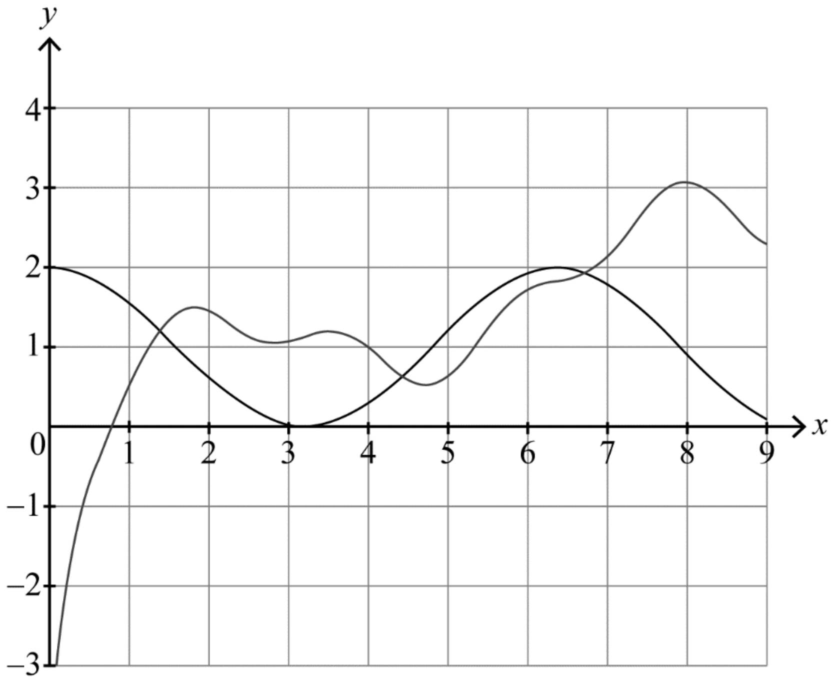
$=\dfrac{(0.6\times0.3\times0.6)+(0.4\times0.6\times0.7)}{(0.6\times0.3)+(0.4\times0.6)}$ ***A1***

$=0.657$ ***A1***

[3 marks]

Total [8 marks]

4. (a)



A1A1

Note: Award ***A1*** for each correct curve, showing all local max & mins.

Note: Award ***A0A0*** for the curves drawn in degrees.

[2 marks]

(b) $x = 1.35, 4.35, 6.64$ ***(M1)***

Note: Award ***M1*** for attempt to find points of intersections between two curves.

$0 < x < 1.35$ ***A1***

Note: Accept $x < 1.35$.

$4.35 < x < 6.64$ ***A1A1***

Note: Award ***A1*** for correct endpoints, ***A1*** for correct inequalities.

Note: Award ***M1FTA1FTA0FTA0FT*** for $0 < x < 7.31$.

Note: Accept $x < 7.31$.

[4 marks]

Total [6 marks]

5. (a) **METHOD 1**

$$\text{LHS} = \frac{1+\sin 2x}{\cos 2x} = \frac{1+2\sin x\cos x}{\cos^2 x-\sin^2 x}$$ ***M1***

$$= \frac{\left(\cos^2 x+\sin^2 x\right)+2\sin x\cos x}{\cos^2 x-\sin^2 x}$$ ***M1***

$$= \frac{\left(\cos x+\sin x\right)^2}{\left(\cos x+\sin x\right)\left(\cos x-\sin x\right)}$$ ***A1***

$$= \frac{\cos x+\sin x}{\cos x-\sin x}$$

$$= \frac{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}$$ ***A1***

$$= \frac{1+\tan x}{1-\tan x}$$ ***AG***

Note: Candidates may start with RHS, apply MS in reverse.

[4 marks]

METHOD 2

$$\text{LHS} = \frac{1+\sin 2x}{\cos 2x} = \frac{1+2\sin x\cos x}{\cos^2 x-\sin^2 x}$$ ***M1***

dividing numerator and denominator by $\cos^2 x$ ***M1***

$$= \frac{\sec^2 x+2\tan x}{1-\tan^2 x}$$

$$= \frac{1+\tan^2 x+2\tan x}{1-\tan^2 x}$$ ***A1***

$$= \frac{(\tan x+1)^2}{(1-\tan x)(1+\tan x)}$$ ***A1***

$$= \frac{1+\tan x}{1-\tan x}$$ ***AG***

Note: Candidates may start with RHS; apply MS in reverse.

[4 marks]

(b) valid attempt to solve $\frac{1+\tan x}{1-\tan x} = \sqrt{3}$ ***(M1)***

$$\tan x = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$x = 0.262\left(=\frac{\pi}{12}\right),\ x = 3.40\left(=\frac{13\pi}{12}\right)$$ ***A1***

Note: Award ***M1A0*** if only one correct solution is given.

[2 marks]

Total [6 marks]

6. attempt to integrate a to find v ***M1***

$v = \int a \, \mathrm{d}t = \int (2t-1) \, \mathrm{d}t$

$= t^2 - t + c$ ***A1***

$s = \int v \, \mathrm{d}t = \int (t^2 - t + c) \, \mathrm{d}t$

$= \frac{t^3}{3} - \frac{t^2}{2} + ct + d$ ***A1***

attempt at substitution of given values ***(M1)***

at $t=6$, $18.25 = 72 - 18 + 6c + d$

at $t=15$, $922.75 = 1125 - 112.5 + 15c + d$

solve simultaneously: ***(M1)***

$c = -6$; $d = 0.25$ ***A1***

$\Rightarrow s = \frac{t^3}{3} - \frac{t^2}{2} - 6t + \frac{1}{4}$

[6 marks]

7. $n=1 \Rightarrow S_1 = u_1$, so true for $n=1$ ***R1***

assume true for $n=k$, ie. $S_k = \frac{u_1(1-r^k)}{1-r}$ ***M1***

Note: Award ***M0*** for statements such as "let $n=k$ ".

Note: Subsequent marks after the first ***M1*** are independent of this mark and can be awarded.

$S_{k+1} = S_k + u_1 r^k$ ***M1***

$S_{k+1} = \frac{u_1(1-r^k)}{1-r} + u_1 r^k$ ***A1***

$S_{k+1} = \frac{u_1(1-r^k)}{1-r} + \frac{u_1 r^k (1-r)}{1-r}$

$S_{k+1} = \frac{u_1 - u_1 r^k + u_1 r^k - r u_1 r^k}{1-r}$ ***A1***

$S_{k+1} = \frac{u_1(1-r^{k+1})}{1-r}$ ***A1***

true for $n=1$ and if true for $n=k$ then true for $n=k+1$,
the statement is true for any positive integer (or equivalent). ***R1***

Note: Award the final ***R1*** mark provided at least four of the previous marks are gained.

[7 marks]

8. (a) **METHOD 1**

$w^3 = 8\mathrm{i}$

writing $8\mathrm{i} = 8\left(\cos\left(\frac{\pi}{2}+2\pi k\right)+\mathrm{i}\sin\left(\frac{\pi}{2}+2\pi k\right)\right)$ ***(M1)***

Note: Award ***M1*** for an attempt to find cube roots of w using modulus-argument form.

cube roots $w = 2\left(\cos\left(\frac{\frac{\pi}{2}+2\pi k}{3}\right)+\mathrm{i}\sin\left(\frac{\frac{\pi}{2}+2\pi k}{3}\right)\right)$ ***(M1)***

ie. $w=\sqrt{3}+\mathrm{i},-\sqrt{3}+\mathrm{i},-2\mathrm{i}$ ***A2***

Note: Award ***A2*** for all 3 correct, ***A1*** for 2 correct.

Note: Accept $w=1.73+\mathrm{i}$ and $w=-1.73+\mathrm{i}$.

[4 marks]

METHOD 2

$w^3+(2\mathrm{i})^3=0$

$(w+2\mathrm{i})\left(w^2-2w\mathrm{i}-4\right)=0$ ***M1***

$w=\frac{2\mathrm{i}\pm\sqrt{12}}{2}$ ***M1***

$w=\sqrt{3}+\mathrm{i},-\sqrt{3}+\mathrm{i},-2\mathrm{i}$ ***A2***

Note: Award ***A2*** for all 3 correct, ***A1*** for 2 correct.

Note: Accept $w=1.73+\mathrm{i}$ and $w=-1.73+\mathrm{i}$.

[4 marks]

(b) $w_1=-2\mathrm{i}$

$\frac{z}{z-\mathrm{i}}=-2\mathrm{i}$ ***M1***

$z=-2\mathrm{i}\left(z-\mathrm{i}\right)$

$z(1+2\mathrm{i})=-2$

$z=\frac{-2}{1+2\mathrm{i}}$ ***A1***

$z=-\frac{2}{5}+\frac{4}{5}\mathrm{i}$ ***A1***

Note: Accept $a=-\frac{2}{5}, b=\frac{4}{5}$.

[3 marks]

Total [7 marks]

Section B

9. (a) **METHOD 1**

attempt to find roots or factors ***(M1)***

roots are -3, 1, $(4+\mathrm{i}),(4-\mathrm{i})$ ***A1A1***

Note: Award ***A1*** for each pair of roots or factors, real and complex.

attempt to form quadratic ***M1***

$(z-1)(z+3)=z^2+2z-3$ ***A1***

$(z-(4+\mathrm{i}))(z-(4-\mathrm{i}))$

$=z^2-(4-\mathrm{i})z-(4+\mathrm{i})z+17$ ***(A1)***

$=z^2-8z+17$ ***A1***

$z^4-6z^3-2z^2+58z-51=(z^2-8z+17)(z^2+2z-3)$

[7 marks]

METHOD 2

attempt to find roots or factors ***(M1)***

real roots are -3, 1 (or real factors $(z+3),(z-1)$) ***A1***

attempt to form quadratic ***M1***

$(z-1)(z+3)=z^2+2z-3$ ***A1***

$z^4-6z^3-2z^2+58z-51=[z^2+2z-3][z^2+kz+17]$

equate coefficients of z^2 ***M1***

$-2=2k-3+17$ ***A1***

solve to give $k=-8$ ***A1***

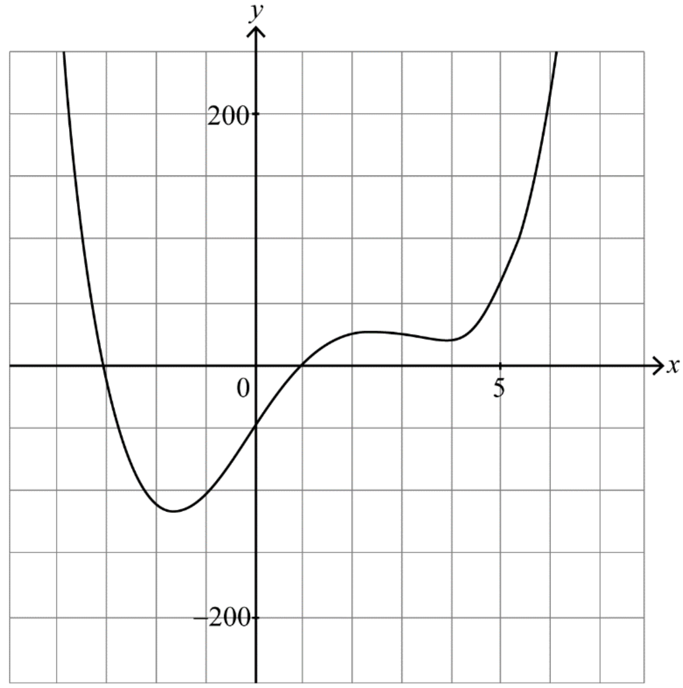
$z^4-6z^3-2z^2+58z-51=(z^2-8z+17)(z^2+2z-3)$

[7 marks]

continued…

Question 9 continued

(b)



shape **A1**

x-axis intercepts at $(-3, 0)$, $(1, 0)$ and y-axis intercept at $(0, -51)$ **A1A1**

minimum points at $(-1.62, -118)$ and $(3.72, 19.7)$ **A1A1**

maximum point at $(2.40, 26.9)$ **A1**

Note: Coordinates may be seen on the graph or elsewhere.

Note: Accept -3, 1 and -51 marked on the axes.

[6 marks]

(c) from graph, $19.7 \le k \le 26.9$ **A1A1**

Note: Award ***A1*** for correct endpoints and ***A1*** for correct inequalities.

[2 marks]

Total [15 marks]

10. (a) $X \sim \mathrm{Po}(2.1)$

$\mathrm{P}(X = 0) = 0.122\left(= \mathrm{e}^{-2.1}\right)$ ***(M1)A1***

[2 marks]

continued…

Question 10 continued

(b)

y	0	1	2	3	4
$\mathrm{P}(Y=y)$	$0.122\ldots$ $\left(=\mathrm{e}^{-2.1}\right)$	$0.257\ldots$ $\left(=\mathrm{e}^{-2.1}2.1\right)$	$0.270\ldots$ $\left(=\frac{\mathrm{e}^{-2.1}2.1^2}{2!}\right)$	$0.189\ldots$ $\left(=\frac{\mathrm{e}^{-2.1}2.1^3}{3!}\right)$	$0.161\ldots$

A1A1A1A1

Note: Award ***A1*** for each correct probability for $Y=1, 2, 3, 4$. Accept 0.162 for $\mathrm{P}(Y=4)$.

[4 marks]

(c) $\mathrm{E}(Y)=\sum y\mathrm{P}(Y=y)$ ***(M1)***

$=1\times 0.257\ldots+2\times 0.270\ldots+3\times 0.189\ldots+4\times 0.161\ldots$ ***(A1)***

$=2.01$ ***A1***

[3 marks]

(d) let T be the no of days per year that Steffi does not visit

$T\sim \mathrm{B}(365,\ 0.122\ldots)$ ***(M1)***

require $0.45\le \mathrm{P}(T\le n)<0.55$ ***(M1)***

$\mathrm{P}(T\le 44)=0.51$

$n=44$ ***A1***

[3 marks]

(e) **METHOD 1**

let V be the discrete random variable "number of times Steffi is not fed per day"

$\mathrm{E}(V)=1\times\mathrm{P}(X=5)+2\times\mathrm{P}(X=6)+3\times\mathrm{P}(X=7)+\cdots$ ***M1***

$=1\times 0.0416\ldots+2\times 0.0145\ldots+3\times 0.00437\ldots+\cdots$ ***A1***

$=0.083979\ldots$ ***A1***

expected no of occasions per year $>0.083979\ldots\times 365=30.7$ ***A1***

hence Steffi can expect not to be fed on at least 30 occasions ***AG***

Note: Candidates may consider summing more than three terms in their calculation for $\mathrm{E}(V)$.

[4 marks]

METHOD 2

$\mathrm{E}(X)-\mathrm{E}(Y)=0.0903\ldots$ ***M1A1***

$0.0903\ldots\times 365$ ***M1***

$=33.0>30$ ***A1AG***

[4 marks]

Total [16 marks]

11. (a) **METHOD 1**

for example

$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} -1 \\ -5 \\ 8 \end{pmatrix}, \ \overrightarrow{\mathrm{PR}} = \begin{pmatrix} 1 \\ -6 \\ 3 \end{pmatrix}$ ***A1A1***

$\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}} = 33\boldsymbol{i} + 11\boldsymbol{j} + 11\boldsymbol{k}$ ***(M1)A1***

$\boldsymbol{r.n} = \boldsymbol{a.n}$

$33x + 11y + 11z = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 33 \\ 11 \\ 11 \end{pmatrix} = 22$ ***(M1)***

$\Rightarrow 3x + y + z = 2$ or equivalent ***A1***

[6 marks]

METHOD 2

assume plane can be written as $ax + by + cz = 1$ ***M1***

substituting each set of coordinates gives the system of equations:

$a + 6b - 7c = 1$

$0a + b + c = 1$

$2a + 0b - 4c = 1$ ***A1***

solving by GDC ***(M1)***

$a = \frac{3}{2}, \ b = \frac{1}{2}, \ c = \frac{1}{2}$ ***A1A1A1***

$\Rightarrow \frac{3}{2}x + \frac{1}{2}y + \frac{1}{2}z = 1$ or equivalent

[6 marks]

(b) **METHOD 1**

substitution of equation of line into both equations of planes ***M1***

$3\left(\frac{5}{4} + \frac{\lambda}{2}\right) + \lambda + \left(-\frac{7}{4} - \frac{5\lambda}{2}\right) = 2$ ***A1***

$\left(\frac{5}{4} + \frac{\lambda}{2}\right) - 3\lambda - \left(-\frac{7}{4} - \frac{5\lambda}{2}\right) = 3$ ***A1***

[3 marks]

continued…

Question 11 continued

METHOD 2

adding Π_1 and Π_2 gives $4x - 2y = 5$ ***M1***

given $y = \lambda \Rightarrow x = \frac{5}{4} + \frac{\lambda}{2}$ ***A1***

$z = 2 - y - 3x = -\frac{7}{4} - \frac{5\lambda}{2}$ ***A1***

$$\Rightarrow \boldsymbol{r} = \begin{pmatrix} \frac{5}{4} \\ 0 \\ -\frac{7}{4} \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{2} \\ 1 \\ -\frac{5}{2} \end{pmatrix}$$ ***AG***

[3 marks]

METHOD 3

$$\boldsymbol{n}_1 \times \boldsymbol{n}_2 = \begin{pmatrix} 2 \\ 4 \\ -10 \end{pmatrix}$$ ***A1***

$$= 4\begin{pmatrix} \frac{1}{2} \\ 1 \\ -\frac{5}{2} \end{pmatrix}$$ ***R1***

common point $\frac{5}{4} - 3(0) - \left(-\frac{7}{4}\right) = 3$ and $-3\left(\frac{5}{4}\right) - 0 - \left(-\frac{7}{4}\right) = -2$ ***A1***

[3 marks]

(c) normal to Π_3 is perpendicular to direction of L

$$\Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} = 0$$ ***A1***

$\Rightarrow a + 2b - 5c = 0$ ***AG***

[1 mark]

continued…

Question 11 continued

(d) (i) substituting $\begin{pmatrix} \frac{5}{4} \\ 0 \\ -\frac{7}{4} \end{pmatrix}$ into Π_3: ***M1***

$\frac{5a}{4} - \frac{7c}{4} = 1$ ***A1***

$5a - 7c = 4$ ***AG***

(ii) attempt to find scalar products for Π_1 and Π_3, Π_2 and Π_3 and equating ***M1***

$$\frac{3a+b+c}{\sqrt{11}\sqrt{a^2+b^2+c^2}} = \frac{a-3b-c}{\sqrt{11}\sqrt{a^2+b^2+c^2}}$$ ***M1***

Note: Accept $3a+b+c = a-3b-c$.

$\Rightarrow a+2b+c=0$ ***A1***

attempt to solve $a+2b+c=0$, $a+2b-5c=0$, $5a-7c=4$ ***M1***

$\Rightarrow a = \frac{4}{5}$, $b = -\frac{2}{5}$, $c = 0$ ***A1***

hence equation is $\frac{4x}{5} - \frac{2y}{5} = 1$

for second equation:

$$\frac{3a+b+c}{\sqrt{11}\sqrt{a^2+b^2+c^2}} = -\frac{a-3b-c}{\sqrt{11}\sqrt{a^2+b^2+c^2}}$$ ***(M1)***

$\Rightarrow 2a-b=0$

attempt to solve $2a-b=0$, $a+2b-5c=0$, $5a-7c=4$

$\Rightarrow a=-2$, $b=-4$, $c=-2$ ***A1***

hence equation is $-2x-4y-2z=1$

[9 marks]

Total [19 marks]

Matemáticas
Nivel superior
Prueba 3 – Análisis

Miércoles 15 de mayo de 2019 (mañana)

1 hora

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

2219 – 7228

Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 7]

Un modelo sencillo para predecir la población mundial se plantea del siguiente modo: en el tiempo t años, la población mundial es x, que se puede suponer que es una variable continua. La razón a la que aumenta x debido a los nacimientos es $0{,}056x$ y la razón a la que disminuye x debido a los fallecimientos es $0{,}035x$.

(a) Muestre que $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 0{,}021x$. [1]

(b) Halle el número de años que tendrán que transcurrir para que la población mundial se duplique. [6]

2. [Puntuación máxima: 9]

(a) Muestre que $1 - x^2 + x^4 - x^6 + \ldots = \dfrac{1}{1+x^2}$, donde $|x| < 1$. [1]

(b) A partir de lo anterior, escriba los cuatro primeros términos no nulos de la serie de potencias para $f(x) = \dfrac{1}{1+4x^2}$. [2]

(c) Utilizando el resultado de (b), halle los cuatro primeros términos no nulos de la serie de potencias para $f(x) = \arctan 2x$. [6]

3. [Puntuación máxima: 9]

Considere la serie $\displaystyle\sum_{n=1}^{\infty} \frac{A \times 8^n}{3^{2n+1}}$.

(a) Sabiendo que $A = \dfrac{1}{n}$, utilice el criterio de comparación para mostrar que la serie converge. [4]

(b) Sabiendo que $A = n$, determine si la serie converge o diverge. [5]

4. [Puntuación máxima: 9]

Utilizando la regla de L'Hôpital, halle $\displaystyle\lim_{x \to 0} \left(\frac{\tan 3x - 3\tan x}{\operatorname{sen} 3x - 3\operatorname{sen} x} \right)$. [9]

5. [Puntuación máxima: 16]

Considere la ecuación diferencial $2xy\dfrac{\mathrm{d}y}{\mathrm{d}x} = y^2 - x^2$, donde $x > 0$.

(a) Resuelva la ecuación diferencial y muestre que una solución general de dicha ecuación es $x^2 + y^2 = cx$, donde c es una constante positiva. [11]

(b) Demuestre que la curva de esta solución general tiene dos rectas tangentes horizontales e indique la ecuación de cada una de ellas, en función de c. [5]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2019

Calculus

Higher level

Paper 3

14 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685\ldots$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets** eg **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT***.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) $\frac{dx}{dt} = 0.056x - 0.035x$ ***A1***

$\frac{dx}{dt} = 0.021x$ ***AG***

[1 mark]

(b) **METHOD 1**

$\frac{dx}{dt} = 0.021x$

attempt to separate variables ***M1***

$\int \frac{1}{x} dx = \int 0.021 dt$ ***A1***

$\ln x = 0.021t(+c)$ ***A1***

EITHER

$x = Ae^{0.021t}$

$\Rightarrow 2A = Ae^{0.021t}$ ***A1***

Note: This ***A1*** is independent of the following marks.

OR

$t = 0, x = x_0 \Rightarrow c = \ln x_0$

$\Rightarrow \ln 2x_0 = 0.021t + \ln x_0$ ***A1***

Note: This ***A1*** is independent of the following marks.

THEN

$\Rightarrow \ln 2 = 0.021t$ ***(M1)***

$\Rightarrow t = 33$ years ***A1***

Note: If a candidate writes $t = 33.007$, so $t = 34$ then award the final ***A1***.

[6 marks]

continued…

Question 1 continued

METHOD 2

$$\frac{dx}{dt} = 0.021x$$

attempt to separate variables **M1**

$$\int_A^{2A} \frac{1}{x}dx = \int_0^t 0.021du$$ **A1A1**

> **Note:** Award ***A1*** for correct integrals and ***A1*** for correct limits seen anywhere. Do not penalize use of t in place of u.

$$[\ln x]_A^{2A} = [0.021u]_0^t$$ **A1**

$$\Rightarrow \ln 2 = 0.021t$$ ***(M1)***

$$\Rightarrow t = 33$$ **A1**

[6 marks]

Total [7 marks]

2. (a) $S = 1 - x^2 + x^4 - x^6 + \ldots$

recognition of GP $u_1 = 1,\ r = -x^2$ **M1**

$$S_\infty = \frac{1}{1+x^2}$$ **AG**

> **Note:** Accept a correct algebraic method such as $(1+x^2)(1-x^2+x^4-x^6+...) = 1+x^2-x^2-x^4+x^4+... = 1$.

> **Note:** Accept finding the Maclaurin series for $\frac{1}{(1+x^2)}$ only if the first four derivatives and their values at $x=0$ are shown.

> **Note:** Accept a correct argument based on using the Maclaurin series for $\arctan x$.

[1 mark]

(b) $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \ldots$

attempt to substitute $2x$ ***(M1)***

$$f(x) = \frac{1}{1+4x^2} = 1 - 4x^2 + 16x^4 - 64x^6 + \ldots$$ **A1**

> **Note:** Accept use of a GP with $r = -4x^2$.

[2 marks]

continued…

Question 2 continued

(c) **EITHER**

$$\int \frac{1}{1+4x^2}\,\mathrm{d}x = \frac{1}{2}\arctan 2x\,(+c)$$ ***M1A1***

OR

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arctan 2x) = \frac{2}{1+4x^2}$$ ***M1A1***

THEN

$$\frac{1}{2}\arctan 2x\,(+c) = \int\left(1-4x^2+16x^4-64x^6+\ldots\right)\mathrm{d}x$$ ***(M1)***

$$= x - \frac{4x^3}{3} + \frac{16x^5}{5} - \frac{64x^7}{7} + \ldots$$ ***(A1)***

when $x=0$ $\arctan 2x = 0 \Rightarrow c = 0$ ***R1***

$$(\arctan 2x) = 2x - \frac{8x^3}{3} + \frac{32x^5}{5} - \frac{128x^7}{7}$$ ***A1***

Note: No accuracy marks should be lost due to absence of c.

[6 marks]

Total [9 marks]

3. (a) attempt to use the comparison test with any convergent series ***M1***

$\frac{8^n}{n3^{2n+1}} < \frac{8^n}{3^{2n}}$ ***(A1)***

Note: Award ***A0*** for comparing series, *eg*, $\sum_{n=1}^{\infty}\frac{8^n}{n3^{2n+1}} < \sum_{n=1}^{\infty}\frac{8^n}{3^{2n}}$. However, subsequent marks may still be awarded.

$< \left(\frac{8}{9}\right)^n$ ***A1***

Note: Award ***A1*** for recognition of a geometric series with $r = \frac{8}{9}$.

$\sum_{n=1}^{\infty}\left(\frac{8}{9}\right)^n$ converges (as it is a geometric series with common ratio $|r| < 1$) ***R1***

Note: Award ***R0*** for a statement such as "$\left(\frac{8}{9}\right)^n$ converges".

series converges by comparison test ***AG***

Note: Award a maximum of ***M0A0A0R1***, if the limit comparison test is used instead of the comparison test.

[4 marks]

(b) attempt to use the ratio test ***M1***

$\frac{a_{n+1}}{a_n} = \frac{(n+1)8^{n+1}}{3^{2n+3}} \times \frac{3^{2n+1}}{n8^n}$ ***A1***

$= \frac{8(n+1)}{9n}\left(= \frac{8}{9}\left(1 + \frac{1}{n}\right)\right)$ ***(M1)***

$\to \frac{8}{9}$ $(\text{as } n \to \infty)$ ***A1***

since $\frac{8}{9} < 1$ series converges (by the ratio test) ***R1***

Note: Award ***R1*** for comparing their limit to 1 and stating a consistent conclusion. Award ***R0*** if their limit equals 1.

[5 marks]

Total [9 marks]

4. $\lim\limits_{x\to 0}\left(\dfrac{\tan 3x - 3\tan x}{\sin 3x - 3\sin x}\right)$

$\lim\limits_{x\to 0}\left(\dfrac{3\sec^2 3x - 3\sec^2 x}{3\cos 3x - 3\cos x}\right) \quad \left(= \lim\limits_{x\to 0}\left(\dfrac{\sec^2 3x - \sec^2 x}{\cos 3x - \cos x}\right)\right)$ ***M1A1A1***

Note: Award ***M1*** for attempt at differentiation using l'Hopital's rule, ***A1*** for numerator, ***A1*** for denominator.

METHOD 1

using l'Hopital's rule again

$= \lim\limits_{x\to 0}\left(\dfrac{18\sec^2 3x\tan 3x - 6\sec^2 x\tan x}{-9\sin 3x + 3\sin x}\right)\left(= \lim\limits_{x\to 0}\left(\dfrac{6\sec^2 3x\tan 3x - 2\sec^2 x\tan x}{-3\sin 3x + \sin x}\right)\right)$ ***A1A1***

EITHER

$= \lim\limits_{x\to 0}\left(\dfrac{108\sec^2 3x\tan^2 3x + 54\sec^4 3x - 12\sec^2 x\tan^2 x - 6\sec^4 x}{-27\cos 3x + 3\cos x}\right)$ ***A1A1***

Note: Not all terms in numerator need to be written in final fraction. Award ***A1*** for $54\sec^4 3x + ... - 6\sec^4 x - ....$ However, if the terms are written, they must be correct to award ***A1***.

attempt to substitute $x = 0$ ***M1***

$= \dfrac{48}{-24}$

OR

$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(18\sec^2 3x\tan 3x - 6\sec^2 x\tan x\right)\Big|_{x=0} = 48$ ***(M1)A1***

$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(-9\sin 3x + 3\sin x\right)\Big|_{x=0} = -24$ ***A1***

THEN

$\left(\lim\limits_{x\to 0}\left(\dfrac{\tan 3x - 3\tan x}{\sin 3x - 3\sin x}\right)\right) = -2$ ***A1***

continued…

Question 4 continued

METHOD 2

$$= \lim_{x \to 0} \left(\frac{\frac{3}{\cos^2 3x} - \frac{3}{\cos^2 x}}{3\cos 3x - 3\cos x} \right)$$ ***M1***

$$= \lim_{x \to 0} \left(\frac{\cos^2 x - \cos^2 3x}{\cos^2 3x \cos^2 x \left(\cos 3x - \cos x \right)} \right)$$ ***A1***

$$= \lim_{x \to 0} \left(\frac{\cos x + \cos 3x}{-\cos^2 3x \cos^2 x} \right)$$ ***M1A1***

attempt to substitute $x = 0$ ***M1***

$$= \frac{2}{-1}$$

$$= -2$$ ***A1***

Total [9 marks]

5. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 - x^2}{2xy}$

let $y = vx$ ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = v + x\frac{\mathrm{d}v}{\mathrm{d}x}$ ***(A1)***

$v + x\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{v^2x^2 - x^2}{2vx^2}$ ***(M1)***

$v + x\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{v^2 - 1}{2v} \quad \left(= \frac{v}{2} - \frac{1}{2v}\right)$ ***(A1)***

Note: Or equivalent attempt at simplification.

$x\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{-v^2 - 1}{2v} \quad \left(= -\frac{v}{2} - \frac{1}{2v}\right)$ ***A1***

$\frac{2v}{1+v^2}\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{1}{x}$ ***(M1)***

$\int \frac{2v}{1+v^2}\mathrm{d}v = \int -\frac{1}{x}\mathrm{d}x$ ***(A1)***

$\ln\left(1+v^2\right) = -\ln x + \ln c$ ***A1A1***

Note: Award ***A1*** for LHS and ***A1*** for RHS and a constant.

$\ln\left(1+\left(\frac{y}{x}\right)^2\right) = -\ln x + \ln c$ ***M1***

Note: Award ***M1*** for substituting $v = \frac{y}{x}$. May be seen at a later stage.

$1+\left(\frac{y}{x}\right)^2 = \frac{c}{x}$ ***A1***

Note: Award ***A1*** for any correct equivalent equation without logarithms.

$x^2 + y^2 = cx$ ***AG***

[11 marks]

continued…

Question 5 continued

(b) **METHOD 1**

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 - x^2}{2xy}$

(for horizontal tangents) $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ ***M1***

$\left(\Rightarrow y^2 = x^2\right) \Rightarrow y = \pm x$

EITHER

using $x^2 + y^2 = cx \Rightarrow 2x^2 = cx$ ***M1***

$2x^2 - cx = 0 \Rightarrow x = \frac{c}{2}$ ***A1***

Note: Award ***M1A1*** for $2y^2 = \pm cy$.

OR

using implicit differentiation of $x^2 + y^2 = cx$

$2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = c$ ***M1***

Note: Accept differentiation of $y = \sqrt{cx - x^2}$.

$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow x = \frac{c}{2}$ ***A1***

THEN

tangents at $y = \frac{c}{2},\ y = -\frac{c}{2}$ ***A1A1***

hence there are two tangents ***AG***

METHOD 2

$x^2 + y^2 = cx$

$\left(x - \frac{c}{2}\right)^2 + y^2 = \frac{c^2}{4}$ ***M1A1***

this is a circle radius $\frac{c}{2}$ centre $\left(\frac{c}{2}, 0\right)$ ***A1***

hence there are two tangents ***AG***

tangents at $y = \frac{c}{2}, y = -\frac{c}{2}$ ***A1A1***

[5 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel superior
Prueba 3 – Matemáticas discretas

Miércoles 15 de mayo de 2019 (mañana)

1 hora

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

2219 – 7227

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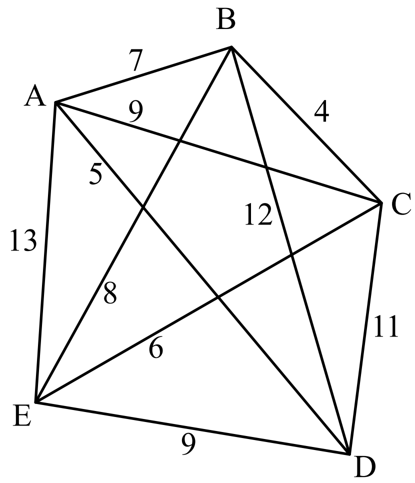
1. [Puntuación máxima: 14]

En un complejo residencial de viviendas hay una empresa de TV que está instalando cable de fibra óptica en 5 viviendas: A, B, C, D y E.
Las rutas que pueden seguir los cables se representan en el siguiente grafo G, donde cada vértice representa una vivienda y cada arista representa una posible ruta para los cables. El peso de las aristas representa el coste, en miles de dólares, de la instalación de cables entre las viviendas.



La empresa quiere hallar el coste mínimo que puede tener una instalación que incluya a todas las viviendas y vuelva al inicio del trazado.

(a) Empezando en A, utilice el algoritmo de vecino más próximo para hallar un límite superior para ese coste mínimo de instalación. [3]

(b) Eliminando A, utilice el algoritmo de vértice borrado para hallar un límite inferior para ese coste mínimo de instalación. [5]

(c) (i) Indique una ruta que dé lugar a ese límite inferior.

(ii) Indique por qué es esa una solución al problema de hallar el coste mínimo de instalación. [2]

(Este pregunta continúa en la página siguiente)

(Pregunta 1: continuación)

En un segundo complejo residencial de viviendas, el diseño de la instalación de cable de fibra óptica entre las cinco viviendas P, Q, R, S y T se indica en la siguiente tabla de adyacencia.

	P	Q	R	S	T
P	–	1	0	0	1
Q	1	–	1	1	0
R	0	1	–	0	1
S	0	1	0	–	1
T	1	0	1	1	–

En esta tabla de adyacencia el 0 indica que no hay ningún cable que conecta las dos casas y un 1 indica que hay un cable que conecta las dos casas.

(d) Dibuje un grafo H que represente este segundo complejo residencial de viviendas. [2]

(e) Explique por qué el grafo H es bipartito. [2]

2. [Puntuación máxima: 16]

(a) Utilice el algoritmo de Euclides para hallar $\operatorname{mcd}(564, 254)$. [5]

(b) Halle una solución general de la ecuación diofántica lineal $564x + 254y = 94$. [8]

(c) Halle las dos soluciones tales que $x, y \in [-300, 300]$. [3]

3. [Puntuación máxima: 12]

El 1 de marzo, en un país dado, hay 5000 lugares con contaminación medioambiental que hay que limpiar. Para cuando llega el 1 de abril, el 80 % de estos 5000 lugares contaminados ya se han limpiado, pero se detectan 200 lugares nuevos que también habría que limpiar. Se supone que esta situación se va repitiendo cada mes. Jim plantea una relación de recurrencia de primer grado que refleja toda esta información.

(a) (i) Indique la relación de recurrencia de primer grado que plantea Jim para el número de lugares, u_n, que todavía quedan por limpiar transcurridos n meses, en la forma $u_n = Au_{n-1} + B$, donde A y B son constantes no nulas.

(ii) Indique el valor de u_0. [2]

(b) Resuelva la relación de recurrencia de primer grado de Jim. [5]

Jim plantea ahora una relación de recurrencia de segundo grado que ofrece información sobre las labores de limpieza medioambiental que se realizan en otro país distinto. Este segundo modelo es $d_n = 0{,}6d_{n-1} - 0{,}09d_{n-2}$, siendo las condiciones iniciales $d_0 = d_1 = 4000$.

(c) Resuelva la relación de recurrencia de segundo grado que ha planteado Jim. [5]

4. [Puntuación máxima: 8]

(a) Utilizando el pequeño teorema de Fermat, muestre que la congruencia $x^{22} + x^{11} \equiv 2 \pmod{11}$ se puede expresar de la forma $(x+6)^2 - 36 \equiv 2 \pmod{11}$. [4]

(b) A partir de lo anterior, resuelva $x^{22} + x^{11} \equiv 2 \pmod{11}$. [4]

Markscheme

May 2019

Discrete mathematics

Higher level

Paper 3

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685\ldots$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets** eg **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) attempt at nearest neighbour algorithm ***M1***

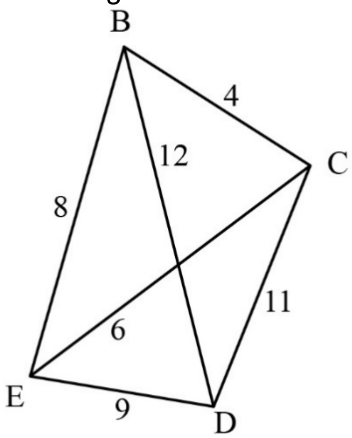
AD
DE
EC
CB
BA ***A1***

Note: Award ***M1*** for a route that begins with AD and then DE.

upper bound $= 5 + 9 + 6 + 4 + 7$
$= 31 (= \$31000)$ ***A1***

[3 marks]

(b) removing vertex A and its adjacent edges produces



minimal spanning tree is

BC	(4)
CE	(6)
ED	(9)

A1A1A1

Note: Accept a correct drawing of the minimal spanning tree.

adding in the weights of 2 deleted edges of least weight AB and AD ***(M1)***
lower bound $= 19 + 7 + 5$
$= 31 (= \$31000)$ ***A1***

[5 marks]

continued…

Question 1 continued

(c) (i) *eg.* ABCEDA ***A1***

Note: Accept any starting vertex.

(ii) **EITHER**

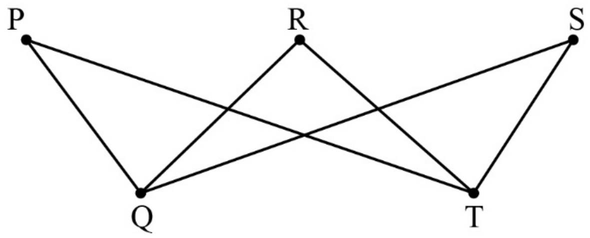
because the upper bound and lower bound are equal (and the lower bound is a cycle) ***R1***

OR

because the route which gives the lower bound is a (Hamiltonian) cycle ***R1***

[2 marks]

(d)



A2

[2 marks]

(e) **EITHER**

the vertices are in two (disjoint) sets where every edge connects a vertex in one set with a vertex in the other set ***A1***

OR

the vertices are in two sets such that no two vertices in the same set are adjacent

THEN

$\{P,R,S\}$ and $\{Q,T\}$ ***A1***

Note: Award the above ***A1*** for $\{P,R,S\}$ and $\{Q,T\}$ stated here or seen clearly as two disjoint sets in part (d).

[2 marks]

Total [14 marks]

2. (a) $564 = 2(254) + 56$ ***M1***

$254 = 4(56) + 30$ ***A1***

$56 = 1(30) + 26$ ***M1***

$30 = 1(26) + 4$

$26 = 6(4) + 2$ ***A1***

$4 = 2(2) + 0$

$\Rightarrow \gcd(564, 254) = 2$ ***A1***

[5 marks]

(b) **METHOD 1**

$2 = 26 - 6(4)$ ***(M1)***

$= 26 - 6(30 - 26) = 7(26) - 6(30)$ ***(A1)***

$= 7(56 - 30) - 6(30) = 7(56) - 13(30)$ ***(A1)***

$= 7(56) - 13(254 - 4(56)) = 59(56) - 13(254)$

$2 = 59(564 - 2(254)) - 13(254) = 59(564) - 131(254)$ ***(A1)***

$94 = 2773(564) - 6157(254)$ ***(M1)***

a solution is $x_0 = 2773$ and $y_0 = -6157$ (or equivalent) ***(A1)***

$x = 2773 + \left(\frac{254}{2}\right)t$ $(x = 2773 + 127t)$ (or equivalent) $(t \in \mathbb{Z})$ ***A1***

$y = -6157 - \left(\frac{564}{2}\right)t$ $(y = -6157 - 282t)$ (or equivalent) $(t \in \mathbb{Z})$ ***A1***

Note: Award ***A1FTA1FT*** for a candidate's $x = x_0 + \left(\frac{254}{2}\right)t$ $(x = x_0 + 127t)$ and $y = y_0 - \left(\frac{564}{2}\right)t$ $(y = y_0 - 282t)$.

Note: Accept the tracking of linear combinations when applying the Euclidean algorithm (could be displayed in part (a)).

[8 marks]

continued…

Question 2 continued

METHOD 2

$94 = 3(30) + 4$ ***(M1)***

$= 3(1(26) + 4) + 4 = 3(26) + 4(4)$ ***(A1)***

$= 3(26) + 4(30 - 1(26)) = 4(30) - 1(26)$ ***(A1)***

$= 4(30) - 1(56 - 1(30)) = 5(30) - 1(56)$

$= 5(254 - 4(56)) - 1(56) = 5(254) - 21(56)$ ***(A1)***

$= 5(254) - 21(564 - 2(254))$

$94 = -21(564) + 47(254)$ ***(M1)***

$x_0 = -21$ and $y_0 = 47$ ***(A1)***

$x = -21 + \left(\frac{254}{2}\right)t \ (x = -21 + 127t)$ (or equivalent) $(t \in \mathbb{Z})$ ***A1***

$y = 47 - \left(\frac{564}{2}\right)t \ (y = 47 - 282t)$ (or equivalent) $(t \in \mathbb{Z})$ ***A1***

Note: Award ***A1FTA1FT*** for a candidate's $x = x_0 + \left(\frac{254}{2}\right)t \ (x = x_0 + 127t)$ and $y = y_0 - \left(\frac{564}{2}\right)t \ (y = y_0 - 282t)$.

Note: Accept the tracking of linear combinations when applying the Euclidean algorithm (could be displayed in part (a)).

[8 marks]

(c) attempt to find a value of t to give (x, y) in range ***(M1)***

$x = -21, 106$ ***A1***

$y = 47, -235$ ***A1***

Note: x, y- values do not have to be "paired" to gain either of the final two ***A*** marks.

Note: Award ***A1A0*** for one correct (x, y).

[3 marks]

Total [16 marks]

3. (a) (i) $u_n = 0.2u_{n-1} + 200$ ***A1***

(ii) $u_0 = 5000$ ***A1***

[2 marks]

Note: In part (b), award full ***FT*** for $u_n = 4000(0.8)^n + 1000$ which is obtained from $u_n = 0.8u_{n-1} + 200$ from part (a) (i).

(b) **METHOD 1**

solution is of the form $u_n = C(0.2)^n + D$ ***M1***

attempt to use both initial conditions ($u_0 = 5000$ and $u_1 = 1200$) ***M1***

$5000 = C + D$; $1200 = C(0.2) + D$ ***A1***

attempt to solve equations simultaneously

$C = 4750, D = 250$

$u_n = 4750(0.2)^n + 250$ ***A1***

[5 marks]

METHOD 2

solution is of the form $u_n = C(0.2)^n + D$ ***M1***

attempt to substitute u_n into their recurrence relation ***M1***

$C(0.2)^n + D = 0.2\left(C(0.2)^{n-1} + D\right) + 200$

$D = 0.2D + 200 \Rightarrow D = 250$ ***M1***

($u_0 = 5000 \Rightarrow$) $5000 = C + 250$ ***A1***

$C = 4750$

$u_n = 4750(0.2)^n + 250$ ***A1***

[5 marks]

METHOD 3

solution is of the form $u_n = cr^n + b\left(\frac{r^n - 1}{}\right)$ ***(M1)***

recognising that $u_0 = 5000$, $r = 0.2$ and $b = 200$ ***(M1)***

$u_n = 5000(0.2)^n + 200\left(\frac{0.2^n - 1}{0.2 - 1}\right)$ ***A1***

$u_n = 5000(0.2)^n - 250(0.2^n - 1)$ ***(M1)***

$u_n = 4750(0.2)^n + 250$ ***A1***

[5 marks]

continued…

Question 3 continued

METHOD 4

$u_{n+1} = 0.2u_n + 200;\ u_n = 0.2u_{n-1} + 200$

$u_{n+1} - u_n = 0.2u_n - 0.2u_{n-1}$ and so $u_{n+1} - 1.2u_n + 0.2u_{n-1} = 0$

auxiliary equation is $\lambda^2 - 1.2\lambda + 0.2 = 0$

$\lambda = 0.2, 1$ and so $u_n = C(1)^n + D(0.2)^n$ ***M1***

attempt to use both initial conditions ($u_0 = 5000$ and $u_1 = 1200$) ***M1***

$5000 = C + D$; $1200 = C(0.2) + D$ ***A1***

attempt to solve equations simultaneously ***(M1)***

$C = 4750, D = 250$

$u_n = 4750(0.2)^n + 250$ ***A1***

[5 marks]

METHOD 5

let $u_n = h_n + p_n$

$h_n = C(0.2)^n$ ***M1***

there is a constant $p_n = D$ that satisfies the recurrence relation ***M1***

$D = 0.2D + 200 \Rightarrow p_n = D = 250$ ***M1***

($u_0 = 5000 \Rightarrow$) $5000 = C + 250$ ***A1***

$C = 4750$

$u_n = 4750(0.2)^n + 250$ ***A1***

[5 marks]

METHOD 6

$u_n = 0.2u_{n-1} + 200$

$u_n = 0.2(0.2u_{n-2} + 200) + 200 = 0.2^2 u_{n-2} + 0.2 \times 200 + 200$ ***(M1)***

$u_n = 0.2^2(0.2u_{n-3} + 200) + 0.2 \times 200 + 200 = 0.2^3 u_{n-3} + 0.2^2 \times 200 + 0.2 \times 200 + 200$

(M1)

$u_n = 0.2^{n-1}(0.2u_0 + 200) + 200\left(0.2^{n-2} + 0.2^{n-3} + \ldots + 0.2 + 1\right)$

$u_n = 0.2^n u_0 + 200\left(0.2^{n-1} + 0.2^{n-2} + 0.2^{n-3} + \ldots + 0.2 + 1\right)$ ***(A1)***

$u_n = 5000\left(0.2^n\right) + 200\left(\dfrac{1 - 0.2^n}{1 - 0.2}\right)$ ***M1***

$u_n = 4750(0.2)^n + 250$ ***A1***

[5 marks]

continued…

Question 3 continued

(c) $d_n = 0.6d_{n-1} - 0.09d_{n-2}$

auxiliary equation is $\lambda^2 - 0.6\lambda + 0.09 = 0$ (or equivalent *eg*, $(\lambda - 0.3)^2 = 0$) ***A1***

$\lambda = 0.3$ ***(A1)***

form of solution is $d_n = A(0.3)^n + Bn(0.3)^n$ $\left(d_n = (A + Bn)(0.3)^n\right)$ ***M1***

attempt to use both initial conditions ($d_0 = 4000$ and $d_1 = 4000$) ***M1***

Note: Award this ***M1*** for using initial conditions on an incorrect solution form.

$d_0 = 4000 \Rightarrow A = 4000$

$d_1 = 4000 = 4000(0.3) + B(0.3) \Rightarrow B = \dfrac{28000}{3}$

$d_n = 4000(0.3)^n + \dfrac{28000n(0.3)^n}{3}$ $\left(d_n = \left(4000 + \dfrac{28000n}{3}\right)(0.3)^n\right)$ ***A1***

[5 marks]

Total [12 marks]

4. (a) $x^{22} + x^{11} \equiv 2 \pmod{11}$

attempting to use Fermat's little theorem or its corollary (can be seen anywhere) ***M1***

eg, $x^{11} \equiv x \pmod{11}$

or $x^{10} \equiv 1 \pmod{11}$ (as x cannot be a multiple of 11)

$\Rightarrow x^{22} + x^{11} \equiv \left(x^2 + x^1\right) \pmod{11}$ ***A1***

$\equiv \left(x^2 + 12x\right) \pmod{11}$ ***A1***

$\equiv \left(x^2 + 12x + 36 - 36\right) \pmod{11}$ ***A1***

$\Rightarrow (x+6)^2 - 36 \equiv 2 \pmod{11}$ ***AG***

Note: Condone the absence of $\bmod 11$ in the lines associated with ***A*** marks.

[4 marks]

(b) **METHOD 1**

$(x+6)^2 \equiv 5 \pmod{11}$ (or equivalent) ***A1***

for example, attempting a (non-exhaustive) tabular solution ***(M1)***

EITHER

$(x+6)$	$(x+6)^2 \pmod{11}$
4	5
7	5

$x + 6 \equiv 4 \pmod{11}$ or $x + 6 \equiv 7 \pmod{11}$

OR

x	$(x+6)^2 \pmod{11}$
1	5
9	5

continued…

Question 4 continued

THEN

$x \equiv 9(\text{mod}\,11)$ or $x \equiv 1(\text{mod}\,11)$ ***A1A1***

Note: Condone the absence of $\text{mod}\,11$ in the working but not in the final answers.

Note: Accept $x \equiv -2(\text{mod}\,11)$.

Note: Accept $x = 1 + 11k$, $x = 9 + 11k$ (where $k \in \mathbb{Z}$).

[4 marks]

METHOD 2

$x^2 + x \equiv 2(\text{mod}\,11)$

$x^2 + x - 2 \equiv 0(\text{mod}\,11)$ ***A1***

$(x+2)(x-1) \equiv 0(\text{mod}\,11)$ ***(M1)***

$x + 2 \equiv 0(\text{mod}\,11)$ or $x - 1 \equiv 0(\text{mod}\,11)$

$x \equiv 9(\text{mod}\,11)$ or $x \equiv 1(\text{mod}\,11)$ ***A1A1***

Note: Condone the absence of $\text{mod}\,11$ in the working but not in the final answers.

Note: Accept $x \equiv -2(\text{mod}\,11)$.

Note: Accept $x = 1 + 11k$, $x = 9 + 11k$ (where $k \in \mathbb{Z}$).

[4 marks]

Total [8 marks]

Matemáticas
Nivel superior
Prueba 3 – Conjuntos, relaciones y grupos

Miércoles 15 de mayo de 2019 (mañana)

1 hora

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

2219 – 7229

Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 9]

La relación R se define sobre $\mathbb{N}$ mediante $xRy \Leftrightarrow x^2 + y^2 \equiv 0 \pmod 2$.

(a) Muestre que R es una relación de equivalencia. [7]

(b) Determine las clases de equivalencia. [2]

2. [Puntuación máxima: 14]

Las operaciones binarias $\circ$ y $*$ se definen sobre el conjunto de los números complejos, tal que $z_1 \circ z_2 = A(z_1 + z_2)$ y $z_1 * z_2 = Bz_1z_2$ donde A y B son constantes reales no nulas.

(a) Determine el elemento neutro con respecto a $*$. [2]

(b) Muestre que todo elemento del conjunto de los números complejos, exceptuando $z = 0$, tiene un simétrico con respecto a $*$. [3]

(c) Halle el valor de A para el cual la operación $\circ$ es asociativa. [4]

(d) Muestre que $*$ es distributiva respecto a $\circ$ para todos los valores de A y de B. [5]

3. [Puntuación máxima: 9]

Un grupo no abeliano $\{G, \circ\}$ consta de ocho elementos $e, a, a^2, a^3, b, ab, a^2b, a^3b,$ donde e es el elemento neutro. La operación binaria es $\circ$ y, por ejemplo, $a \circ b$ se representa como ab. El elemento a es de orden 4, el elemento b es de orden 2 y $ba = a^3b$.

(a) Demuestre que

(i) $ba^2 = a^2b$;

(ii) $ba^3 = ab$. [4]

(b) El grupo $\{G, \circ\}$ tiene la siguiente tabla de Cayley. Halle los elementos $\boldsymbol{P, Q, R, S, T, U, V, W}$ de este grupo.

$\circ$	e	a	a^2	a^3	b	ab	a^2b	a^3b
e	e	a	a^2	a^3	b	ab	a^2b	a^3b
a	a	a^2	a^3	e	ab	a^2b	a^3b	b
a^2	a^2	a^3	e	a	a^2b	a^3b	b	ab
a^3	a^3	e	a	a^2	a^3b	b	ab	a^2b
b	b	a^3b	a^2b	ab	e	a^3	a^2	a
ab	ab	b	a^3b	a^2b	a	e	$\boldsymbol{R}$	$\boldsymbol{U}$
a^2b	a^2b	ab	b	$\boldsymbol{P}$	a^2	a	$\boldsymbol{S}$	$\boldsymbol{V}$
a^3b	a^3b	a^2b	ab	$\boldsymbol{Q}$	a^3	a^2	$\boldsymbol{T}$	$\boldsymbol{W}$

[4]

(c) Indique por qué $\{G, \circ\}$ no es isomorfo al grupo $\{\{0, 1, 2, 3, 4, 5, 6, 7\}, +_8\}$, donde $+_8$ representa la suma módulo 8. [1]

4. [Puntuación máxima: 7]

Sea S el conjunto de números reales de la forma $a + b\sqrt{5}$, donde $a, b \in \mathbb{Q}$, a^2 no puede ser igual a $5b^2$, y a, b no son simultáneamente iguales a cero. Muestre que S es un grupo con respecto a la operación de la multiplicación. Puede suponer que se cumple la propiedad asociativa. [7]

5. [Puntuación máxima: 11]

Considere la función $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ que se define mediante $f(x, y) = (x + y, x - y)$.

(a) Demuestre que la función f es biyectiva. [9]

Considere la función $g: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ que se define mediante $g(x, y) = (x + y, x - y)$.

(b) Explique por qué la función g no es una función biyectiva. [2]

Markscheme

May 2019

Sets, relations and groups

Higher level

Paper 3

10 pages

Instructions to Examiners

Abbreviations

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  Examples

	Correct answer seen	**Further working seen**	**Action**
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1. (a) $xRx \Rightarrow x^2 + x^2 = 2x^2$ ***M1***

since this is equal to $0 \pmod 2$ it is reflexive ***R1***

$xRy \Leftrightarrow x^2 + y^2 \equiv 0 \pmod 2$

$\Rightarrow y^2 + x^2 \equiv 0 \pmod 2$ ***M1***

$\Rightarrow yRx$, hence symmetric ***R1***

$xRy \Leftrightarrow x^2 + y^2 \equiv 0 \pmod 2 (= 2m)$ and $yRz \Leftrightarrow y^2 + z^2 \equiv 0 \pmod 2 (= 2n)$ ***M1***

adding and recognising $2y^2 = 0 \pmod 2$ $\Rightarrow x^2 + y^2 + y^2 + z^2 = x^2 + 0 (\mathrm{mod}\,2) + z^2$

M1

$\Rightarrow x^2 + z^2 = 2m + 2n = 2(m+n) \equiv 0 \pmod 2$

$\Rightarrow xRz$ and hence transitive ***R1***

[7 marks]

(b) one consists of even natural numbers and the other consists of odd natural numbers ***A1A1***

[2 marks]

Total [9 marks]

2. (a) $z_1 * e = Bz_1e (= z_1)$ ***(M1)***

$\Rightarrow e = \frac{1}{B}$ ***A1***

[2 marks]

(b) $z_1 * z_1^{-1} = \frac{1}{B}$ ***M1***

$\Rightarrow Bz_1z_1^{-1} = \frac{1}{B}$ ***(A1)***

$\Rightarrow z_1^{-1} = \frac{1}{B^2 z_1}$ ***A1***

which is a member of $\mathbb{C}$ (provided $z_1 \neq 0$)

hence inverse exists for all $\mathbb{C}$ (except $z_1 = 0$) ***AG***

Note: Candidates may use $a + \mathrm{i}b$ in place of z_1.

[3 marks]

continued…

Question 2 continued

(c) attempt to show that $(z_1 \circ z_2) \circ z_3 = z_1 \circ (z_2 \circ z_3)$ ***(M1)***

$LHS = (z_1 \circ z_2) \circ z_3 = A(z_1 + z_2) \circ z_3$

$\Rightarrow A(Az_1 + Az_2 + z_3)$ ***A1***

$RHS = z_1 \circ (z_2 \circ z_3) = z_1 \circ A(z_2 + z_3)$

$\Rightarrow A(z_1 + Az_2 + Az_3)$ ***A1***

$LHS = RHS$ when $A = 1$ ***A1***

Note: If a candidate "guesses" $A = 1$ and then verifies the result works award ***M1A0A0A1***.

[4 marks]

(d) attempt to show that $z_1 * (z_2 \circ z_3) = (z_1 * z_2) \circ (z_1 * z_3)$ ***M1***

$LHS = z_1 * (z_2 \circ z_3) = z_1 * A(z_2 + z_3)$ ***(A1)***

$= ABz_1(z_2 + z_3)(= ABz_1z_2 + ABz_1z_3)$ or equivalent ***A1***

$RHS = (z_1 * z_2) \circ (z_1 * z_3) = (Bz_1z_2) \circ (Bz_1z_3)$ ***A1***

$= AB(z_1z_2 + z_1z_3)$

$= ABz_1(z_2 + z_3)(= ABz_1z_2 + ABz_1z_3)$ or equivalent ***A1***

hence $*$ is distributive over $\circ$ for all values of A and B ***AG***

Note: Any reasonable attempt at recognising the distributive property should be given ***M1***. This could include only one side being correct.

[5 marks]

Total [14 marks]

3. (a) (i) $ba^2 = baa = a^3ba$ ***M1***

$a^3ba = a^3a^3b = a^4a^2b = ea^2b$ ***M1***

$= a^2b$ ***AG***

(ii) $ba^3 = baa^2 = a^3ba^2$ ***M1***

$= a^3baa = a^3a^3ba = a^3a^3a^3b = (a^4)^2 ab = (e)^2 ab$ ***M1***

$= ab$ ***AG***

[4 marks]

continued…

Question 3 continued

(b) $P = a^3 b$
$Q = b$
$R = a^3$
$S = e$
$T = a$
$U = a^2$
$V = a^3$
$W = e$ ***A4***

Note: Award ***A3*** for 6 or 7 correct, ***A2*** for 4 or 5 correct, ***A1*** for 2 or 3 correct and ***A0*** otherwise.

[4 marks]

(c) since one group is Abelian and the other is not then they are not isomorphic ***R1***

Note: Award ***R1*** for any valid reason – for example "one group commutative".

[1 mark]

Total [9 marks]

4. closure: Consider $\left(m+n\sqrt{5}\right)\left(p+q\sqrt{5}\right)$ ***M1***

$= mp + 5nq + \sqrt{5}\left(np + mq\right)$ ***A1***

this is of the form $a+b\sqrt{5}$ and since neither $m+n\sqrt{5}$ nor $p+q\sqrt{5} = 0+0\sqrt{5}$, the product cannot be zero

(also noting $p^2 \neq 5q^2 : \left(mp+5nq\right)^2 = 5\left(np+mq\right)^2 \Rightarrow \left(m^2-5n^2\right)\left(p^2-5q^2\right)=0$

therefore $\left(mp+5nq\right)^2 = 5\left(np+mq\right)^2$ iff $\left(m^2-5n^2\right)=0$ or $\left(p^2-5q^2\right)=0$

hence closed) ***R1***

identity: $1+0\sqrt{5}$ ***A1***

inverse: Consider $\dfrac{1}{a+b\sqrt{5}}$ ***M1***

$= \dfrac{a-b\sqrt{5}}{\left(a+b\sqrt{5}\right)\left(a-b\sqrt{5}\right)}$ ***(M1)***

$= \dfrac{a}{a^2-5b^2} - \sqrt{5}\left(\dfrac{b}{a^2-5b^2}\right)$ ***A1***

(since a, b cannot simultaneously be zero and a^2 cannot equal $5b^2$, all elements have an inverse)
(associativity can be assumed)
hence S is a group under the operation of multiplication ***AG***

Total [7 marks]

5. (a) for bijection required to show injective and surjective ***(M1)***

suppose $f(x,y)=f(u,v)$ ***(M1)***

$\Rightarrow x+y=u+v \quad (i)\,,\ x-y=u-v \quad (ii)$ ***A1***

$(i)+(ii)\ 2x=2u \Rightarrow x=u\ ,\ (i)-(ii)\ 2y=2v \Rightarrow y=v$ ***(M1)A1***

$(x,y)=(u,v)$

since $f(x,y)=f(u,v) \Rightarrow (x,y)=(u,v)$ ($\Rightarrow f$ is injective) ***R1***

let $f(x,y)=(s,t)$ ***(M1)***

$\Rightarrow x+y=s,\ x-y=t$

$\Rightarrow x=\dfrac{s+t}{2},\ y=\dfrac{s-t}{2}$ ***A1***

$\dfrac{s+t}{2}$ and $\dfrac{s-t}{2}$ are both real ($\Rightarrow f$ is surjective) ***R1***

hence f is a bijection ***AG***

[9 marks]

(b) consider surjectivity ***(M1)***

since $x=\dfrac{s+t}{2}, y=\dfrac{s-t}{2}$, do not necessarily belong to $\mathbb{Z}$ (if s, t are both odd or s, t are both even the expressions belong to $\mathbb{Z}$ but if s is even and t is odd or vice versa this is no longer true) ***R1***

hence the function g is not surjective and hence not a bijection ***AG***

Note: Accept using a counter example.

[2 marks]

Total [11 marks]

Matemáticas
Nivel superior
Prueba 3 – Estadística y probabilidad

Miércoles 15 de mayo de 2019 (mañana)

1 hora

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NS y de ampliación de matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

2219 – 7230

Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 16]

La variable aleatoria continua X tiene la siguiente función densidad de probabilidad:

$$f(x) = \begin{cases} kx & 0 \le x < 1 \\ kx^2 & 1 \le x \le 2 \\ 0 & \text{resto de valores} \end{cases}.$$

(a) Muestre que $k = \frac{6}{17}$. [4]

(b) Halle la función de distribución de probabilidad acumulada de X. [6]

(c) Halle la mediana, m, de X. [3]

(d) Halle $\mathrm{P}(|X - m| < 0{,}75)$. [3]

2. [Puntuación máxima: 12]

En el departamento de atención al cliente de una empresa hay empleados que atienden el teléfono. El tiempo que tarda un empleado en resolver la consulta de un cliente es una variable aleatoria que se puede modelizar por una distribución normal de media 150 segundos y desviación típica 45 segundos.

(a) Halle la probabilidad de que el tiempo que tarda un empleado en resolver la consulta de un cliente elegido al azar sea mayor que 180 segundos. [2]

(b) Halle la probabilidad de que el tiempo que tarda un empleado en resolver las consultas de los tres clientes que tiene haciendo cola sea inferior a nueve minutos. [4]

Al comienzo de la jornada Amanda, una de las empleadas, tiene a cuatro clientes haciendo cola. Un segundo empleado, Brian, tiene a tres clientes haciendo cola. Puede suponer que estos dos empleados trabajan de manera independiente.

(c) Halle la probabilidad de que Amanda termine de resolver las consultas de su cola antes de que Brian termine de resolver las consultas de la suya. [6]

3. [Puntuación máxima: 10]

En una población grande de gallinas, los pesos de dichas gallinas siguen una distribución normal de media μ kg y desviación típica σ kg. Se toma de esta población una muestra aleatoria compuesta por 100 gallinas.
Sea $\bar{X}$ la media de los pesos correspondiente a esta muestra.

(a) Indique cómo es la distribución de $\bar{X}$ y dé su media y su varianza. [1]

Los valores de la muestra se resumen mediante $\sum x = 199,8$ y $\sum x^2 = 407,8$, donde x kg es el peso de una gallina.

(b) Halle una estimación sin sesgo de μ. [1]

(c) Halle una estimación sin sesgo de σ^2. [2]

(d) Halle un intervalo de confianza del 90 % para μ. [3]

(e) Se halla que $\sigma = 0,27$. Se decide contrastar, a un nivel de significación del 1 %, la hipótesis nula $\mu = 1,95$ frente a la hipótesis alternativa $\mu > 1,95$.

(i) Halle el valor del parámetro p correspondiente a este contraste.

(ii) Escriba la conclusión a la que haya llegado. [3]

4. [Puntuación máxima: 12]

Se sabe que X, Y, Z son variables aleatorias y que c es una constante.

(a) Muestre que $\mathrm{Cov}(X + c, Y) = \mathrm{Cov}(X, Y)$. [3]

(b) Muestre que $\mathrm{Cov}(X + Y, Z) = \mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z)$. [3]

Se sabe que S y T son dos variables independientes que siguen una distribución normal de media 0 y varianza 1.

(c) Utilizando los resultados de (a) y (b), halle el valor de $\mathrm{Cov}\left(1 + S, S + ST^2\right)$. [6]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2019

Statistics and probability

Higher level

Paper 3

11 pages

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$= k\left[\frac{x^2}{2}\right]_0^1 + k\left[\frac{x^3}{3}\right]_1^2$ ***(A1)***

$= k\left[\frac{1}{2} + \left(\frac{8}{3} - \frac{1}{3}\right)\right]$ ***A1***

put expression equal to 1 ***M1***

$k = \frac{6}{17}$ ***AG***

[4 marks]

(b) $\int_0^x \frac{6}{17}t\mathrm{d}t = \frac{3x^2}{17}$ ***(A1)***

$F(x) = \frac{3x^2}{17}$, $0 \le x < 1$ ***A1***

$\int_1^x \frac{6}{17}t^2\mathrm{d}t = \frac{2x^3}{17} - \frac{2}{17}$ ***A1***

$F(x) = \frac{2x^3}{17} - \frac{2}{17} + F(1), 1 \le x \le 2$ ***M1***

$= \frac{2x^3}{17} + \frac{1}{17}$ ***A1***

$F(x) = 0$, $x < 0$ and $F(x) = 1$, $x > 2$ ***A1***

Note: Condone the use of x as the variable of integration.

Note: Accept the use of k in lines 1 and 3.

Note: Allow either weak or strong inequalities.

[6 marks]

(c) recognition that the median lies between 1 and 2 ***(M1)***

$F(m) = 0.5 \Rightarrow 0.5 = \frac{2}{17}m^3 + \frac{1}{17}$ ***(M1)***

$\Rightarrow m = 1.55$ ***A1***

Note: ***FT*** their $F(x)$ from (b) if possible.

[3 marks]

continued…

Question 1 continued

(d) $\mathrm{P}\left(-0.75 \le X - 1.55 \le 0.75\right)$

$= \mathrm{P}\left(0.8 \le X \le 2.3\right)$ ***(M1)***

$= F\left(2.3\right) - F\left(0.8\right)$ ***(M1)***

$1 - \frac{3}{17}(0.8036...)^2$

$= 0.886$ ***A1***

Note: Accept all answers that round to 0.89.

Note: ***FT*** their m from (c).

[3 marks]

Total [16 marks]

2.

Note: In question 2, accept answers that round correctly to 2 significant figures.

(a) $X \sim \mathrm{N}\left(150, 45^2\right)$

$\mathrm{P}\left(X > 180\right) = 0.252$ ***(M1)A1***

[2 marks]

(b) required to find $\mathrm{P}(X_1 + X_2 + X_3 < 540)$

let $S = X_1 + X_2 + X_3$

$\mathrm{E}(S) = 450$ ***(A1)***

$\mathrm{Var}(S) = 3\mathrm{Var}(X)$ ***(M1)***

$= 3 \times 45^2 \left(\Rightarrow \sigma = 45\sqrt{3}\right)$ $(= 6075)$ ***(A1)***

$\mathrm{P}(S < 540) = 0.876$ ***A1***

Note: In (b) and (c) condone incorrect notation, *eg*, $3X$ for $X_1 + X_2 + X_3$.

[4 marks]

(c) let $Y = \left(X_1 + X_2 + X_3 + X_4\right) - \left(X_5 + X_6 + X_7\right)$ ***(M1)***

$\mathrm{E}\left(Y\right) = \mathrm{E}\left(X\right) = 150$ ***(A1)***

$\mathrm{Var}\left(Y\right) = 4\mathrm{Var}\left(X\right) + 3\mathrm{Var}\left(X\right)\left(= 7\mathrm{Var}\left(X\right)\right)$ ***(M1)***

$= 14175$ ***(A1)***

required to find $\mathrm{P}(Y < 0)$ ***(M1)***

$= 0.104$ ***A1***

[6 marks]

Total [12 marks]

3. (a) $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{100}\right)$ ***A1***

Note: Accept n in place of 100.

[1 mark]

(b) $\hat{\mu} = \frac{\sum x}{n} = \frac{199.8}{100} = 1.998$ ***A1***

Note: Accept 2.00, 2.0 and 2.

[1 mark]

(c) $s_{n-1}{}^2 = \frac{n}{n-1}\left(\frac{\sum x^2}{n} - \bar{x}^2\right) = \frac{100}{99}\left(\frac{407.8}{100} - 1.998^2\right)$ ***(M1)***

$= 0.086864$

unbiased estimate for σ^2 is 0.0869 ***A1***

Note: Accept any answer which rounds to 0.087.

[2 marks]

(d) 90% confidence interval is $1.998 \pm 1.660\sqrt{\frac{0.0869}{100}}$ ***(M1)***

$= (1.95, 2.05)$ ***A1A1***

Note: ***FT*** their σ from (c).

Note: Condone the use of the z-value 1.645 since n is large.

Note: Accept any values that round to 1.95 and 2.05.

[3 marks]

(e) (i) p-value is 0.0377 ***A2***

Note: Award ***A1*** for the 2-tail value 0.0754.

Note: Award ***A2*** for 0.0377 and ***A1*** for any other value that rounds to 0.038.

Note: ***FT*** their estimated mean from (b), note that 2 gives $p = 0.032(0)$.

(ii) accept the null hypothesis ***A1***

Note: ***FT*** their p-value.

[3 marks]

Total [10 marks]

4. (a) **METHOD 1**

$\mathrm{Cov}(X+c,Y)=\mathrm{E}([X+c]Y)-\mathrm{E}(X+c)\mathrm{E}(Y)$ ***M1***

$=\mathrm{E}(XY+cY)-\mathrm{E}(X)\mathrm{E}(Y)-c\mathrm{E}(Y)$ ***A1***

$=\mathrm{E}(XY)+\mathrm{E}(cY)-\mathrm{E}(X)\mathrm{E}(Y)-c\mathrm{E}(Y)$

$=\mathrm{E}(XY)+c\mathrm{E}(Y)-\mathrm{E}(X)\mathrm{E}(Y)-c\mathrm{E}(Y)$ ***A1***

$=\mathrm{Cov}(X,Y)$ ***AG***

METHOD 2

$\mathrm{Cov}(X+c,Y)=\mathrm{E}[(X+c-\mathrm{E}(X+c))(Y-\mathrm{E}(Y))]$ ***M1***

$=\mathrm{E}[(X+c-\mathrm{E}(X)-\mathrm{E}(c))(Y-\mathrm{E}(Y))]$ ***A1***

$=\mathrm{E}[(X+c-\mathrm{E}(X)-c)(Y-\mathrm{E}(Y))]$ ***A1***

$=\mathrm{Cov}(X,Y)$ ***AG***

[3 marks]

(b) **METHOD 1**

$\mathrm{Cov}(X+Y,Z)=\mathrm{E}([X+Y]Z)-\mathrm{E}(X+Y)\mathrm{E}(Z)$ ***M1***

$=\mathrm{E}(XZ+YZ)-(\mathrm{E}(X)+\mathrm{E}(Y))\mathrm{E}(Z)$ ***A1***

$=\mathrm{E}(XZ)+\mathrm{E}(YZ)-\mathrm{E}(X)\mathrm{E}(Z)-\mathrm{E}(Y)\mathrm{E}(Z)$ ***A1***

$=\mathrm{Cov}(X,Z)+\mathrm{Cov}(Y,Z)$ ***AG***

METHOD 2

$\mathrm{Cov}(X+Y,Z)=\mathrm{E}[(X+Y-\mathrm{E}(X+Y))(Z-\mathrm{E}(Z))]$ ***M1***

$=\mathrm{E}[(X+Y-\mathrm{E}(X)-\mathrm{E}(Y))(Z-\mathrm{E}(Z))]$

$=\mathrm{E}[(X-\mathrm{E}(X)+Y-\mathrm{E}(Y))(Z-\mathrm{E}(Z))]$ ***A1***

$=\mathrm{E}[(X-\mathrm{E}(X))(Z-\mathrm{E}(Z))]+\mathrm{E}[(Y-\mathrm{E}(Y))(Z-\mathrm{E}(Z))]$ ***A1***

$=\mathrm{Cov}(X,Z)+\mathrm{Cov}(Y,Z)$ ***AG***

[3 marks]

continued…

Question 4 continued

(c) $\text{Cov}\left(1+S, S+ST^2\right)$

$= \text{Cov}\left(S, S+ST^2\right)$ (from a) ***M1***

$= \text{Cov}\left(S, S\right) + \text{Cov}\left(S, ST^2\right)$ (from b) ***M1***

METHOD 1

$= \text{Var}\left(S\right) + \text{E}\left(S^2T^2\right) - \text{E}\left(S\right)\text{E}\left(ST^2\right)$ ***A1***

$= \text{Var}\left(S\right) + \text{E}\left(S^2\right)\text{E}\left(T^2\right) - \text{E}\left(S\right)\text{E}\left(ST^2\right)$ ***A1***

$= \text{Var}\left(S\right) + \text{Var}\left(S\right)\text{Var}\left(T\right) - \text{E}\left(S\right)\text{E}\left(ST^2\right)$ ***(A1)***

$= 1 + 1 - 0$

$= 2$ ***A1***

METHOD 2

$= \text{Var}(S) + \text{E}[(S - \text{E}(S))(ST^2 - \text{E}(ST^2))]$ ***A1***

$= \text{Var}(S) + \text{E}(S \times ST^2)$ ***A1***

$= \text{Var}(S) + \text{Var}(S)\text{Var}(T)$ ***(A1)***

$= 1 + 1 + 0$

$= 2$ ***A1***

[6 marks]

Total [12 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel medio
Prueba 1

Lunes 13 de mayo de 2019 (tarde)

1 hora 30 minutos

Número de convocatoria del alumno

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba no se permite el uso de ninguna calculadora.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NM** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

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No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 6]

La siguiente tabla muestra la distribución de probabilidad de una variable aleatoria discreta X.

$\boldsymbol{X}$	**0**	**1**	**2**	**3**
$\mathrm{P}(X=x)$	$\frac{3}{13}$	$\frac{1}{13}$	$\frac{4}{13}$	k

(a) Halle el valor de k. [3]

(b) Halle $\mathrm{E}(X)$. [3]

2. [Puntuación máxima: 6]

Considere los vectores $\boldsymbol{a} = \begin{pmatrix} 0 \\ 3 \\ p \end{pmatrix}$ y $\boldsymbol{b} = \begin{pmatrix} 0 \\ 6 \\ 18 \end{pmatrix}$.

Halle el valor de p para el cual $\boldsymbol{a}$ y $\boldsymbol{b}$ son

(a) paralelos; [2]

(b) perpendiculares. [4]

3. [Puntuación máxima: 6]

Considere la función $f(x) = \dfrac{3x+1}{x-2}$, $x \neq 2$.

(a) Para el gráfico de f,

(i) escriba la ecuación de la asíntota vertical;

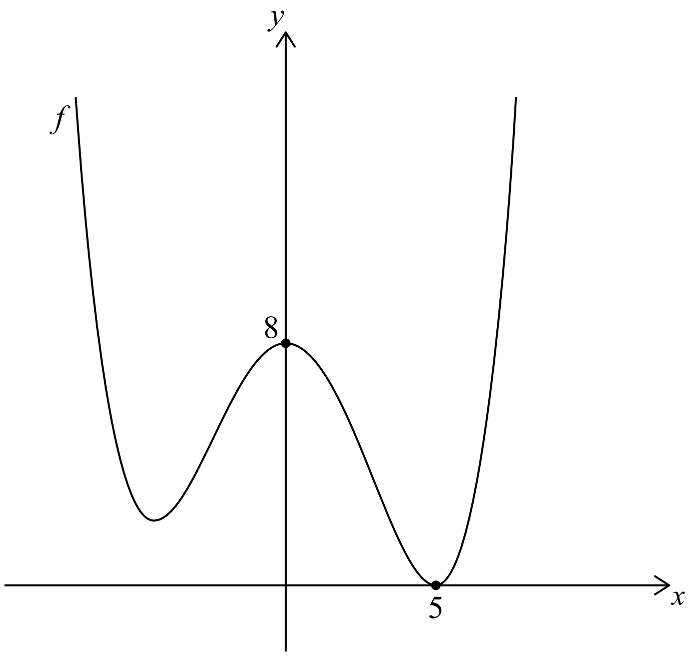
(ii) halle la ecuación de la asíntota horizontal. [3]

Sea $g(x) = x^2 + 4$, $x \in \mathbb{R}$.

(b) Halle $(f \circ g)(1)$. [3]

4. [Puntuación máxima: 7]

La siguiente figura muestra una parte del gráfico de f. La intersección con el eje x está en $(5, 0)$ y la intersección con el eje y está en $(0, 8)$.



(a) Halle la intersección con el eje y del gráfico de

(i) $f(x) + 3$;

(ii) $f(4x)$. [3]

(b) Halle la intersección con el eje x del gráfico de $f(2x)$. [2]

(c) Describa la transformación $f(x + 1)$. [2]

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5. [Puntuación máxima: 6]

Considere la función $f(x) = (1 - k)x^2 + x + k$, $x \in \mathbb{R}$. Halle el valor de k para el cual $f(x)$ tiene dos raíces reales iguales.

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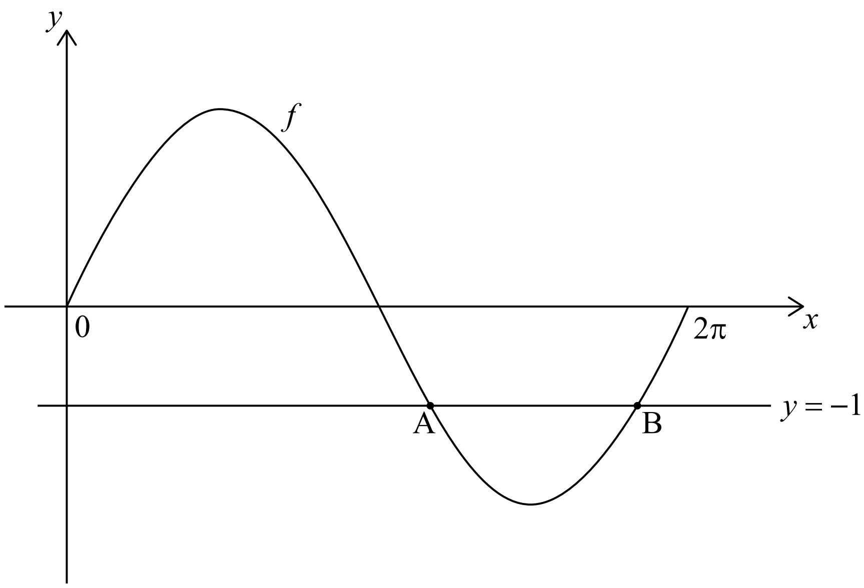
6. [Puntuación máxima: 7]

Resuelva $\log_4(2-x) = \log_{16}(13-4x)$.

7. [Puntuación máxima: 7]

Considere el gráfico de la función $f(x) = 2\operatorname{sen} x$, $0 \le x < 2\pi$. El gráfico de f corta a la recta $y = -1$ exactamente dos veces: en el punto A y en el punto B. Esta información aparece reflejada en la siguiente figura.



(a) Halle la coordenada x de A y de B. [4]

Considere el gráfico de $g(x) = 2\operatorname{sen} px$, $0 \le x < 2\pi$, donde $p > 0$.

(b) Halle el mayor valor de p tal que el gráfico de g no corta a la recta $y = -1$. [3]

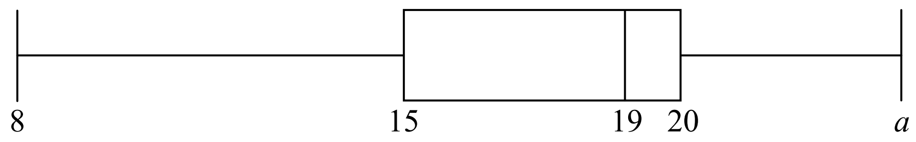
No escriba soluciones en esta página.

Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

8. [Puntuación máxima: 16]

Un grupo de 10 chicas fueron anotando durante toda una semana el número de horas que veían la televisión. Los resultados se resumen en el siguiente diagrama de caja y bigotes.



(a) El rango de los datos es 16. Halle el valor de a. [2]

(b) Halle el valor del rango intercuartil. [2]

Este grupo de chicas, en total, vieron 180 horas de televisión.

(c) Halle la media del número de horas que las chicas de este grupo vieron la televisión esa semana. [2]

Un grupo de 20 chicos también anotaron el número de horas que vieron la televisión esa misma semana. Los resultados se resumen en la siguiente tabla.

$\bar{x} = 21$	$\sigma = 3$

(d) (i) Halle el número total de horas que este grupo de chicos estuvieron viendo la televisión esa semana.

(ii) Halle la media del número de horas que **todos esos 30** chicos y chicas estuvieron viendo la televisión esa semana. [5]

La semana siguiente el grupo de los chicos tuvieron exámenes. Durante esta semana de exámenes los chicos estuvieron viendo la televisión la mitad del tiempo que la semana anterior.

(e) Para esta semana de exámenes, halle

(i) la media del número de horas que el grupo de chicos estuvieron viendo la televisión;

(ii) la varianza del número de horas que el grupo de chicos estuvieron viendo la televisión. [5]

No escriba soluciones en esta página.

9. [Puntuación máxima: 15]

Sea θ un ángulo **obtuso** tal que $\operatorname{sen}\theta = \frac{3}{5}$.

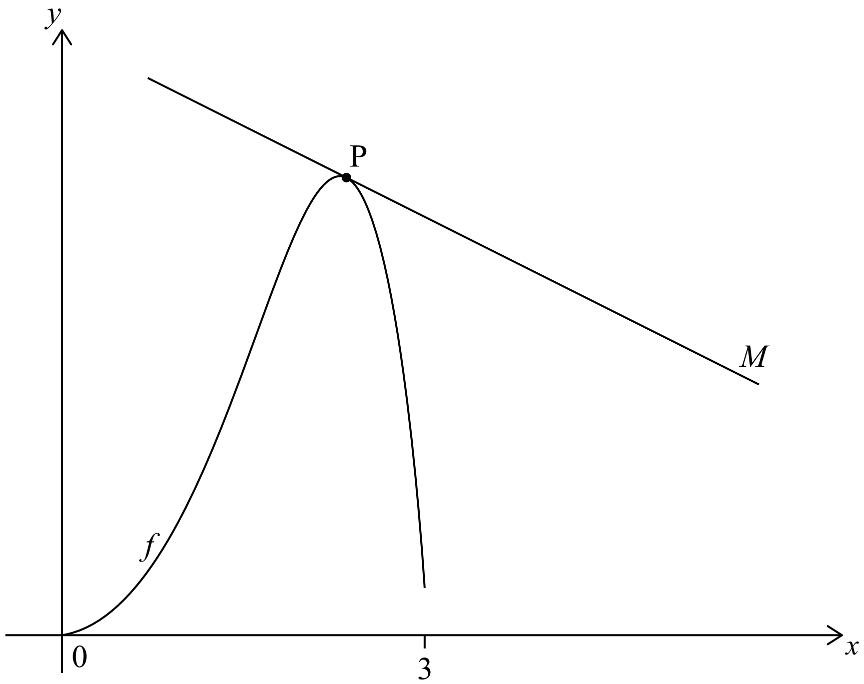
(a) Halle el valor de $\tan\theta$. [4]

(b) La recta L pasa por el origen y su pendiente es igual a $\tan\theta$. Halle la ecuación de L. [2]

Sea $f(x) = \mathrm{e}^x \operatorname{sen} x - \frac{3x}{4}$.

(c) Halle la derivada de f. [5]

La siguiente figura muestra el gráfico de f para $0 \le x \le 3$. La recta M es tangente al gráfico de f en el punto P.



(d) Sabiendo que M es paralela a L, halle la coordenada x de P. [4]

No escriba soluciones en esta página.

10. [Puntuación máxima: 14]

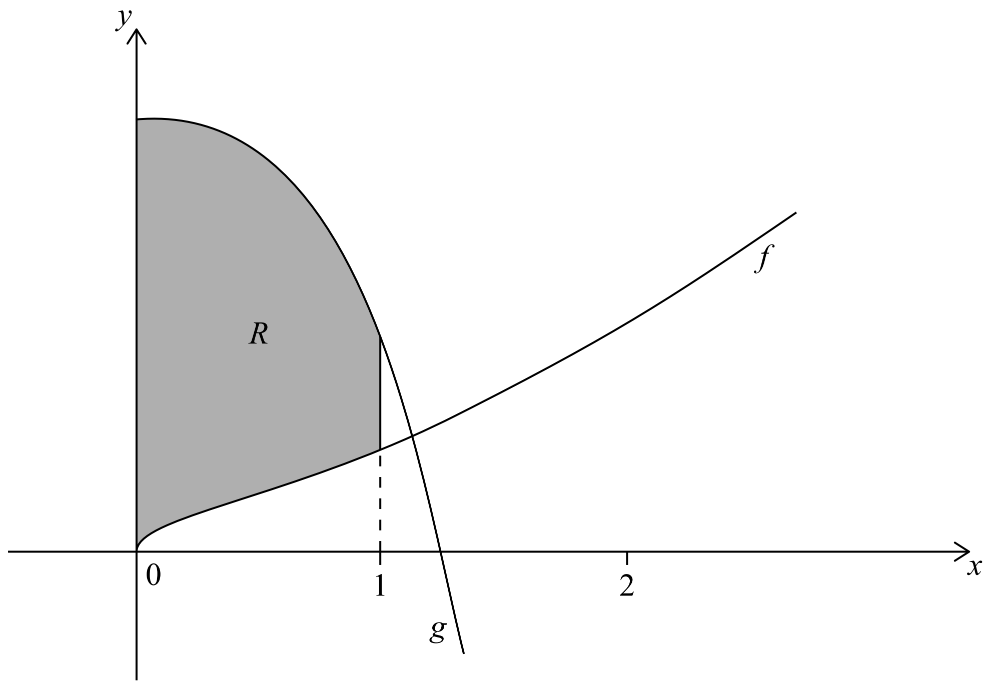
Sea $y = \left(x^3 + x\right)^{\frac{3}{2}}$.

(a) Halle $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) A partir de lo anterior, halle $\int \left(3x^2 + 1\right)\sqrt{x^3 + x}\ \mathrm{d}x$. [3]

Considere las funciones $f(x) = \sqrt{x^3 + x}$ y $g(x) = 6 - 3x^2\sqrt{x^3 + x}$, para $x \geq 0$.

En la siguiente figura se muestran los gráficos de f y g.



La región sombreada R está delimitada por los gráficos de f y g, la recta $x = 1$ y el eje y.

(c) Escriba una expresión para el área de R. [2]

(d) A partir de lo anterior, halle el área exacta de R. [6]

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2019

Mathematics

Standard level

Paper 1

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*
- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer.

Section A

1. (a) evidence of using $\sum p = 1$ ***(M1)***

correct working ***(A1)***

eg $\frac{3}{13}+\frac{1}{13}+\frac{4}{13}+k=1,\ 1-\frac{8}{13}$

$k=\frac{5}{13}$ ***A1*** ***N2***

[3 marks]

(b) valid approach to find $\mathrm{E}(X)$ ***(M1)***

eg $1\times\frac{1}{13}+2\times\frac{4}{13}+3\times k$, $0\times\frac{3}{13}+1\times\frac{1}{13}+2\times\frac{4}{13}+3\times\frac{5}{13}$

correct working ***(A1)***

eg $\frac{1}{13}+\frac{8}{13}+\frac{15}{13}$

$\mathrm{E}(X)=\frac{24}{13}$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

2. (a) valid approach ***(M1)***

eg $\boldsymbol{b}=2\boldsymbol{a}$, $\boldsymbol{a}=k\boldsymbol{b}$, $\cos\theta=1$, $\boldsymbol{a}\bullet\boldsymbol{b}=-|\boldsymbol{a}||\boldsymbol{b}|$, $2p=18$

$p=9$ ***A1*** ***N2***

[2 marks]

(b) evidence of scalar product ***(M1)***

eg $\boldsymbol{a}\bullet\boldsymbol{b}$, $(0)(0)+(3)(6)+p(18)$

recognizing $\boldsymbol{a}\bullet\boldsymbol{b}=0$ (seen anywhere) ***(M1)***

correct working ***(A1)***

eg $18+18p=0$, $18p=-18$

$p=-1$ ***A1*** ***N3***

[4 marks]

Total [6 marks]

3. (a) (i) $x=2$ (must be an equation) ***A1*** ***N1***

(ii) valid approach ***(M1)***

eg $3+\dfrac{7}{x-2}$, $x\to\infty$, $\dfrac{3x}{x}$, $\dfrac{3}{1}$, $\dfrac{3+\frac{1}{x}}{1-\frac{2}{x}}$, $\dfrac{3(x-2)+7}{x-2}$

$y=3$ (must be an equation) ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to substitute 1 into $g(x)$ or $f(x)$ ***(M1)***

eg 1^2+4, $\dfrac{3+1}{1-2}$

$g(1)=5$ ***(A1)***

$(f\circ g)(1)=\dfrac{16}{3}$ ***A1*** ***N2***

METHOD 2

attempt to form composite function (in any order) ***(M1)***

eg $\dfrac{3\left(x^2+4\right)+1}{x^2+4-2}$, $\left(\dfrac{3x+1}{x-2}\right)^2+4$

correct substitution ***(A1)***

eg $\dfrac{3(5)+1}{5-2}$

$(f\circ g)(1)=\dfrac{16}{3}$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

4. (a) (i) y-intercept is 11 (accept $(0, 11)$) ***A1*** ***N1***

(ii) valid approach ***(M1)***

eg $f(4\times 0) = f(0)$, recognizing stretch of $\frac{1}{4}$ in x-direction

y-intercept is 8 (accept $(0, 8)$) ***A1*** ***N2***

[3 marks]

(b) x-intercept is $\frac{5}{2}$ $(=2.5)$ (accept $\left(\frac{5}{2}, 0\right)$ or $(2.5, 0)$) ***A2*** ***N2***

[2 marks]

(c) correct name, correct magnitude **and** direction ***A1A1*** ***N2***

eg name: translation, (horizontal) shift (do not accept move)

eg magnitude and direction: 1 unit to the left, $\begin{pmatrix}-1\\0\end{pmatrix}$, horizontal by -1

[2 marks]

Total [7 marks]

5. correct substitution into discriminant (do not accept only in quadratic formula) ***(A1)***

eg $1-4(1-k)k$

correct expansion of discriminant (do not accept only in quadratic formula) ***A1***

eg $1-4k+4k^2$, $4k^2-4k=-1$

recognizing discriminant equals 0 (seen anywhere) ***M1***

eg $\Delta = 0$, $b^2-4ac=0$

valid attempt to solve **their** quadratic in k ***(M1)***

eg factorizing equation, use of quadratic formula, completing the square, recognizing vertex on x-axis

correct working ***(A1)***

eg $(2k-1)^2$, $\frac{-(-4)\pm\sqrt{16-4(4)(1)}}{2(4)}$, $\left(k-\frac{1}{2}\right)^2=0$, $k=\frac{-(-4)}{2(4)}$

$k=\frac{1}{2}$ ***A1*** ***N2***

[6 marks]

6.

Note: The first three ***A*** marks are awarded for correct application of log properties, including with incorrect expressions, and in any order.

correct application of change of base (accept any base) ***(A1)***

eg $\frac{\log_4(13-4x)}{\log_4 16}$, $\frac{\log_{16}(2-x)}{\log_{16} 4}$, $\frac{\log_2(2-x)}{\log_2 4}$, $\frac{\log(13-4x)}{\log 16}$

correct numerical value ***(A1)***

eg $\log_4 16 = 2$, $\log_{16} 4 = \frac{1}{2}$

correct application of $r\log_c a = \log_c a^r$ ***(A1)***

eg $\log_4(2-x)^2$

correct equation without logs ***A1***

eg $(2-x)^2 = 13-4x$, $(2-x)^4 = (13-4x)^2$, $4-4x+x^2 = 13-4x$

correct working ***A1***

eg $x^2 = 9$

$x = -3$ ***A2*** ***N2***

[7 marks]

7. (a) correct equation ***(A1)***

eg $2\sin x = -1$, $\sin x = -\frac{1}{2}$

one correct value for $\sin^{-1}\left(\frac{1}{2}\right)$ or $\sin^{-1}\left(-\frac{1}{2}\right)$ (seen anywhere) ***(A1)***

eg $\frac{\pi}{6}$, $\frac{5\pi}{6}$, 30°, 150°, 210°, 330°

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\left(\text{accept } \left(\frac{7\pi}{6}, -1\right), \left(\frac{11\pi}{6}, -1\right)\right)$ ***A1A1*** ***N1N1***

Note: Award ***A1A1A1A0*** if more solutions given in addition to both correct answers.

[4 marks]

(b) recognizing period of g is larger than the period of f ***(M1)***

eg sketch of g with larger period (may be seen on diagram), A at $x = 2\pi$,

image of A when $x > 2\pi$, $\frac{7\pi}{6} \to 2\pi$, $2\sin(2\pi p) = -1$, $\frac{7\pi}{6} \times k = 2\pi$

correct working ***(A1)***

eg $\frac{7\pi}{6} \cdot \frac{1}{p} = 2\pi$, $2\pi p = \frac{7\pi}{6}$, $\frac{12}{7}$

$p = \frac{7}{12}$ $\left(\text{accept } p < \frac{7}{12} \text{ or } p \le \frac{7}{12}\right)$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

Section B

8. (a) valid approach *(M1)*
eg $16+8$, $a-8$

24 (hours) ***A1*** ***N2***
[2 marks]

(b) valid approach *(M1)*
eg $20-15$, Q_3-Q_1, $15-20$

IQR $=5$ ***A1*** ***N2***
[2 marks]

(c) correct working *(A1)*
eg $\frac{180}{10}, \frac{180}{n}, \frac{\sum x}{10}$

mean $=18$ (hours) ***A1*** ***N2***
[2 marks]

(d) (i) attempt to find total hours for group B *(M1)*
eg $\bar{x}\times n$

group B total hours $=420$ (seen anywhere) ***A1*** ***N2***

(ii) attempt to find sum for combined group (may be seen in working) *(M1)*
eg $180+420$, 600

correct working *(A1)*

eg $\frac{180+420}{30}, \frac{600}{30}$

mean $=20$ (hours) ***A1*** ***N2***

[5 marks]

continued…

Question 8 continued

(e) (i) valid approach to find the new mean **(M1)**

eg $\frac{1}{2}\mu$, $\frac{1}{2}\times 21$

mean $=\frac{21}{2}(=10.5)$ (hours) **A1** **N2**

(ii) variance = σ^2 (seen anywhere) **(A1)**

eg $\sigma^2 = 9$, $3^2 = 9$, $\left(\frac{3}{2}\right)^2$, 3^2

valid attempt to find new standard deviation or variance **(M1)**

eg $\frac{1}{4}\times 3^2$, $\frac{1}{2}\times 3$, $\frac{3}{2}$

variance = $\frac{9}{4}$ $(=2.25)$ (hours) **A1** **N2**

[5 marks]

Total [16 marks]

9. (a) evidence of valid approach ***(M1)***

eg sketch of triangle with sides 3 and 5, $\cos^2\theta = 1 - \sin^2\theta$

correct working ***(A1)***

eg missing side is 4 (may be seen in sketch), $\cos\theta = \frac{4}{5}$, $\cos\theta = -\frac{4}{5}$

$\tan\theta = -\frac{3}{4}$ ***A2*** ***N4***

[4 marks]

(b) correct substitution of either gradient **or** origin into equation of line (do not accept $y = mx + b$) ***(A1)***

eg $y = x\tan\theta$, $y - 0 = m(x - 0)$, $y = mx$

$y = -\frac{3}{4}x$ ***A1*** ***N2***

Note: Award ***A1A0*** for $L = -\frac{3}{4}x$.

[2 marks]

(c) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{-3x}{4}\right) = -\frac{3}{4}$ (seen anywhere, including answer) ***A1***

choosing product rule ***(M1)***

eg $uv' + vu'$

correct derivatives (must be seen in a correct product rule) ***A1A1***

eg $\cos x$, e^x

$f'(x) = \mathrm{e}^x\cos x + \mathrm{e}^x\sin x - \frac{3}{4}$ $\left(= \mathrm{e}^x(\cos x + \sin x) - \frac{3}{4}\right)$ ***A1*** ***N5***

[5 marks]

continued…

Question 9 continued

(d) valid approach to equate **their** gradients ***(M1)***

eg $f' = \tan\theta$, $f' = -\frac{3}{4}$, $e^x \cos x + e^x \sin x - \frac{3}{4} = -\frac{3}{4}$,

$e^x(\cos x + \sin x) - \frac{3}{4} = -\frac{3}{4}$

correct equation without e^x ***(A1)***

eg $\sin x = -\cos x$, $\cos x + \sin x = 0$, $\frac{-\sin x}{\cos x} = 1$

correct working ***(A1)***

eg $\tan\theta = -1$, $x = 135^\circ$

$x = \frac{3\pi}{4}$ (do not accept 135°) ***A1*** ***N1***

Note: Do not award the final ***A1*** if additional answers are given.

[4 marks]

Total [15 marks]

10. (a) evidence of choosing chain rule ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\times\frac{\mathrm{d}u}{\mathrm{d}x}$, $u=x^3+x$, $u'=3x^2+1$

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2}\left(x^3+x\right)^{\frac{1}{2}}\left(3x^2+1\right)$ $\left(=\frac{3}{2}\sqrt{x^3+x}\left(3x^2+1\right)\right)$ ***A2*** ***N3***

[3 marks]

(b) integrating by inspection from (a) or by substitution ***(M1)***

eg $\frac{2}{3}\int\frac{3}{2}\left(3x^2+1\right)\sqrt{x^3+x}\,\mathrm{d}x$, $u=x^3+x$, $\frac{\mathrm{d}u}{\mathrm{d}x}=3x^2+1$, $\int u^{\frac{1}{2}}$, $\frac{u^{\frac{3}{2}}}{1.5}$

correct integrated expression in terms of x ***A2*** ***N3***

eg $\frac{2}{3}(x^3+x)^{\frac{3}{2}}+C$, $\frac{(x^3+x)^{1.5}}{1.5}+C$

[3 marks]

(c) integrating and subtracting functions (in any order) ***(M1)***

eg $\int g-f$, $\int f-\int g$

correct integral (including limits, accept absence of $\mathrm{d}x$) ***A1*** ***N2***

eg $\int_0^1\left(g-f\right)\mathrm{d}x$, $\int_0^1 6-3x^2\sqrt{x^3+x}-\sqrt{x^3+x}\,\mathrm{d}x$, $\int_0^1 g(x)-\int_0^1 f(x)$

[2 marks]

continued…

Question 10 continued

(d) recognizing $\sqrt{x^3+x}$ is a common factor (seen anywhere, may be seen in part (c)) ***(M1)***

eg $\left(-3x^2-1\right)\sqrt{x^3+x}$, $\int 6-\left(3x^2+1\right)\sqrt{x^3+x}$, $\left(3x^2-1\right)\sqrt{x^3+x}$

correct integration ***(A1)(A1)***

eg $6x-\frac{2}{3}\left(x^3+x\right)^{\frac{3}{2}}$

> **Note:** Award ***A1*** for $6x$ and award ***A1*** for $-\frac{2}{3}\left(x^3+x\right)^{\frac{3}{2}}$.

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $6-\frac{2}{3}(1^3+1)^{\frac{3}{2}}$, $0-\left[6-\frac{2}{3}(1^3+1)^{\frac{3}{2}}\right]$

correct working ***(A1)***

eg $6-\frac{2}{3}\times 2\sqrt{2}$, $6-\frac{2}{3}\times\sqrt{4}\times\sqrt{2}$

area of $R=6-\frac{4\sqrt{2}}{3}$ $\left(=6-\frac{2}{3}\sqrt{8},\quad 6-\frac{2}{3}\times 2^{\frac{3}{2}},\quad \frac{18-4\sqrt{2}}{3}\right)$ ***A1*** ***N3***

[6 marks]

Total [14 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel medio
Prueba 2

Martes 14 de mayo de 2019 (mañana)

Número de convocatoria del alumno

1 hora 30 minutos

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de matemáticas NM** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

2219 – 7310

No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 6]

Un grupo de 7 hombres adultos querían saber si existía una relación entre su índice de masa corporal (IMC) y el perímetro de su cintura. Para ello, se midió el perímetro de su cintura (en centímetros) y se calculó su IMC. La siguiente tabla muestra los resultados.

Cintura (x cm)	58	63	75	82	93	98	105
IMC (y)	19	20	22	23	25	24	26

La relación que existe entre x e y se puede modelizar mediante la ecuación de regresión $y = ax + b$.

(a) (i) Escriba el valor de a y el de b.

(ii) Halle el coeficiente de correlación. [4]

(b) Utilice la ecuación de regresión para estimar el IMC de un hombre adulto que tenga un perímetro de cintura de $95\,\text{cm}$. [2]

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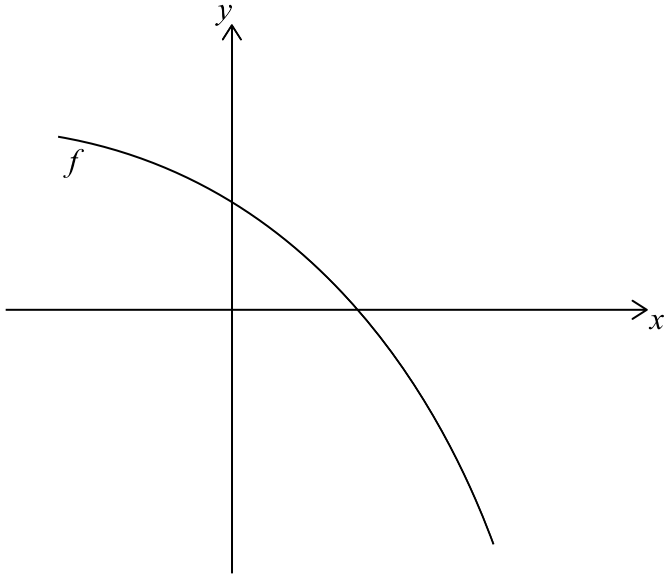
2. [Puntuación máxima: 5]

Sea $f(x) = 4 - 2e^x$. La siguiente figura muestra una parte del gráfico de f.



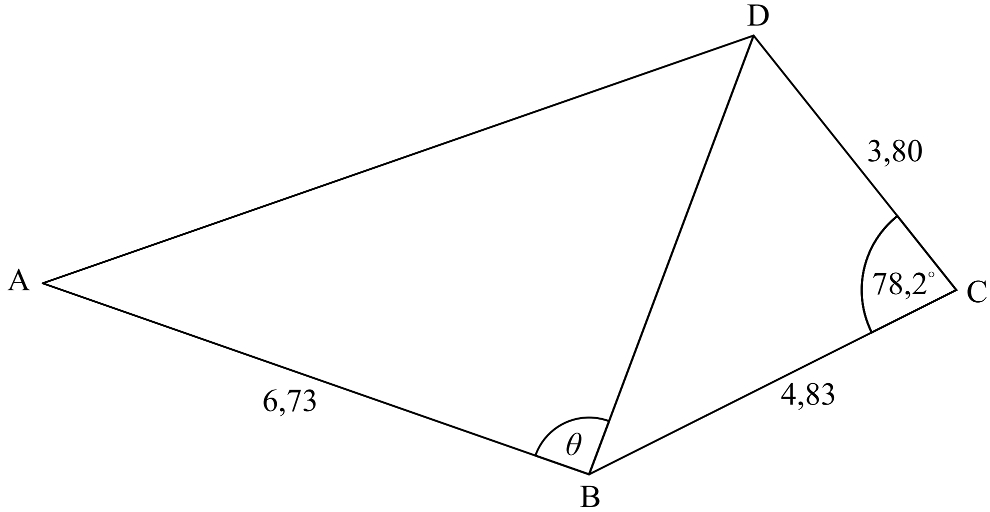
(a) Halle la intersección del gráfico de f con el eje x. [2]

(b) La región delimitada por el gráfico de f, el eje x y el eje y se rota 360° alrededor del eje x. Halle el volumen del sólido así generado. [3]

3. [Puntuación máxima: 7]

La siguiente figura muestra el cuadrilátero ABCD.

la figura no está dibujada a escala



$\mathrm{AB} = 6{,}73\,\mathrm{cm}$, $\mathrm{BC} = 4{,}83\,\mathrm{cm}$, $\mathrm{B\hat{C}D} = 78{,}2^\circ$ y $\mathrm{CD} = 3{,}80\,\mathrm{cm}$.

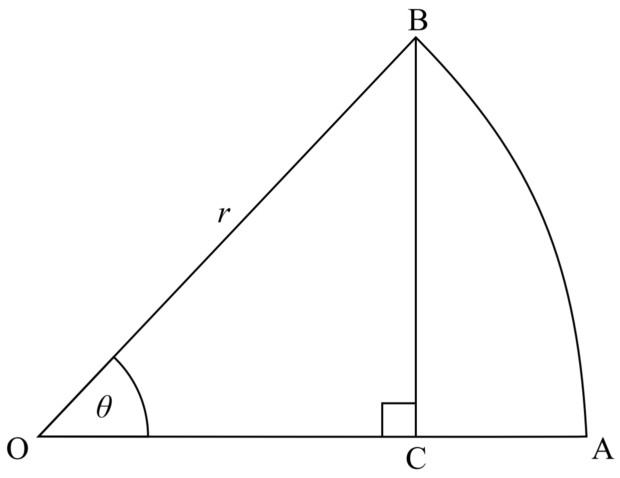
(a) Halle BD. [3]

(b) El área del triángulo ABD es $18{,}5\,\mathrm{cm}^2$. Halle los posibles valores de θ. [4]

4. [Puntuación máxima: 7]

OAB es un sector del círculo que tiene centro O y radio r, tal y como se muestra en la siguiente figura.

la figura no está dibujada a escala



El ángulo AOB mide θ radianes, donde $0 < \theta < \frac{\pi}{2}$.

El punto C pertenece a OA, siendo OA perpendicular a BC.

(a) Muestre que $\mathrm{OC} = r\cos\theta$. [1]

(b) Halle el área del triángulo OBC en función de r y de θ. [2]

(c) Sabiendo que el área del triángulo OBC es igual a $\frac{3}{5}$ del área de sector circular OAB, halle θ. [4]

5. [Puntuación máxima: 6]

La población de peces que hay en un lago está modelizada por la función

$$f(t)=\frac{1000}{1+24e^{-0,2t}},\ 0\le t\le 30, \text{ donde } t \text{ se mide en meses.}$$

(a) Halle la población de peces en $t=10$. [2]

(b) Halle la razón a la que está aumentando la población de peces en $t=10$. [2]

(c) Halle el valor de t en el que la población de peces está aumentando más rápidamente. [2]

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6. [Puntuación máxima: 7]

En el desarrollo de la siguiente expresión, halle el valor exacto del término constante.

$$x^3\left(\frac{1}{2x}+x^2\right)^{15}$$

7. [Puntuación máxima: 6]

La ecuación vectorial de la recta L es $\boldsymbol{r} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + t\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$.

El punto P es el punto de L que más cerca está del origen. Halle las coordenadas de P.

No escriba soluciones en esta página.

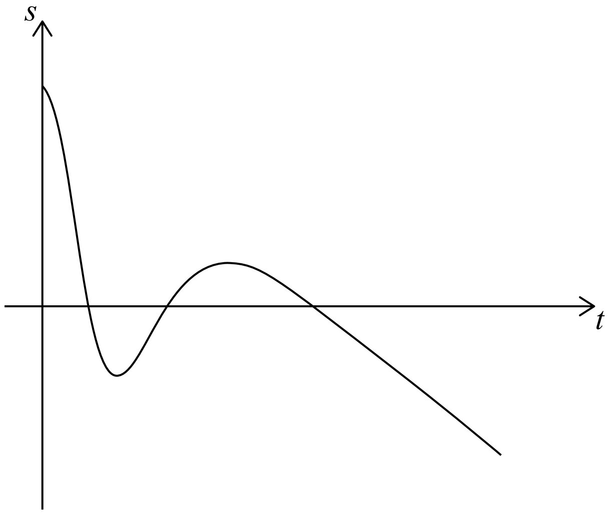
Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

8. [Puntuación máxima: 16]

En esta pregunta, las distancias están en centímetros y el tiempo está en segundos.

La partícula A se está moviendo a lo largo de una línea recta, de manera tal que su desplazamiento respecto a un punto P, en el instante t segundos, viene dado por $s_A = 15 - t - 6t^3 e^{-0,8t}$, $0 \le t \le 25$. Esta información se representa en la siguiente figura.



(a) Halle el desplazamiento inicial de la partícula A respecto al punto P. [2]

(b) Halle el valor de t en el que la partícula A llega al punto P por primera vez. [2]

(c) Halle el valor de t en el que la partícula A cambia de sentido por primera vez. [2]

(d) Halle la distancia total que recorre la partícula A en los primeros 3 segundos. [3]

La partícula B se mueve a lo largo de la misma recta y empieza a moverse al mismo tiempo que la partícula A. La velocidad de la partícula B viene dada por $v_B = 8 - 2t$, $0 \le t \le 25$.

(e) (i) Sabiendo que las partículas A y B parten del mismo punto, halle la función desplazamiento s_B correspondiente a la partícula B.

(ii) Halle el otro valor de t en que las partículas A y B se vuelven a encontrar. [7]

No escriba soluciones en esta página.

9. [Puntuación máxima: 14]

En el aeropuerto de Penna, la probabilidad de que todos los pasajeros lleguen a tiempo para coger un vuelo $\mathrm{P}(A)$ es igual a $0{,}70$. La probabilidad de que el vuelo salga a tiempo $\mathrm{P}(D)$ es igual a $0{,}85$. La probabilidad de que todos los pasajeros lleguen a tiempo para coger un vuelo y este salga a tiempo es igual a $0{,}65$.

(a) Muestre que el suceso A y el suceso D **no** son independientes. [2]

(b) (i) Halle $\mathrm{P}(A \cap D')$.

(ii) Sabiendo que todos los pasajeros de un vuelo han llegado a tiempo, halle la probabilidad de que dicho vuelo **no** salga a tiempo. [5]

El número de horas que los pilotos vuelan a la semana sigue una distribución normal de media 25 horas y desviación típica σ. Un 90% de los pilotos vuelan menos de 28 horas a la semana.

(c) Halle el valor de σ. [3]

(d) En todos los vuelos hay dos pilotos. Halle el porcentaje de vuelos en los que **los dos** pilotos volaron más de 30 horas la semana pasada. [4]

No escriba soluciones en esta página.

10. [Puntuación máxima: 16]

Una progresión aritmética es tal que $u_1 = 1{,}3$, $u_2 = 1{,}4$ y $u_k = 31{,}2$.

(a) Halle el valor de k. [4]

(b) Halle el valor exacto de S_k. [2]

Considere los términos u_n de esta progresión tales que $n \le k$.

Sea F la suma de todos los términos para los cuales n no es un múltiplo de 3.

(c) Muestre que $F = 3240$. [5]

Sea la serie geométrica infinita $S_\infty = a + \dfrac{a}{\sqrt{2}} + \dfrac{a}{2} + \ldots,\ a \in \mathbb{Z}^+$.

(d) Halle el mayor valor de a para el cual $S_\infty < F$. [5]

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

Markscheme

May 2019

Mathematics

Standard level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown **(M, A, R)**.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.

- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have *another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value

the exact value if applicable, the correct 3 sf answer

Section A

1. (a) (i) valid approach *(M1)*

eg correct value for a or b (or for correct r or $r^2 = 0.955631$ seen in (ii))

$0.141120,\ 11.1424$

$a = 0.141,\ b = 11.1$ ***A1A1*** ***N3***

(ii) 0.977563

$r = 0.978$ ***A1*** ***N1***

[4 marks]

(b) correct substitution into **their** regression equation *(A1)*

eg $0.141(95) + 11.1$

24.5488

24.5 ***A1*** ***N2***

[2 marks]

Total [6 marks]

2. (a) valid approach *(M1)*

eg $f(x) = 0,\ 4 - 2e^x = 0$

0.693147

$x = \ln 2$ (exact), 0.693 ***A1*** ***N2***

[2 marks]

(b) attempt to substitute either their correct limits or the function into formula involving f^2 *(M1)*

eg $\int_0^{0.693} f^2, \quad \pi\int\left(4 - 2e^x\right)^2 dx,\ \int_0^{\ln 2}\left(4 - 2e^x\right)^2$

3.42545

volume $= 3.43$ ***A2*** ***N3***

[3 marks]

Total [5 marks]

3. (a) choosing cosine rule *(M1)*

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS *(A1)*

eg $4.83^2 + 3.80^2 - 2 \times 4.83 \times 3.80 \times \cos 78.2$, 30.2622, $4.83^2 + 3.80^2 - 2(4.83)(3.80)\cos 1.36$

5.50111

5.50 (cm) ***A1*** ***N2***

[3 marks]

(b) correct substitution for area of triangle ABD *(A1)*

eg $\frac{1}{2} \times 6.73 \times 5.50111 \sin\theta$

correct equation ***A1***

eg $\frac{1}{2} \times 6.73 \times 5.50111 \sin\theta = 18.5$, $\sin\theta = 0.999393$

88.0023, 91.9976, 1.53593, 1.60566

$\theta = 88.0$ (degrees) or 1.54 (radians)

$\theta = 92.0$ (degrees) or 1.61 (radians)

A1A1 ***N2***

[4 marks]

Total [7 marks]

4. (a) $\cos\theta = \dfrac{\text{OC}}{r}$ ***A1***

$\text{OC} = r\cos\theta$ ***AG*** ***N0***

[1 mark]

(b) valid approach ***(M1)***

eg $\frac{1}{2}\text{OC}\times\text{OB}\sin\theta,\ \text{BC} = r\sin\theta$, $\frac{1}{2}r\cos\theta\times\text{BC}$, $\frac{1}{2}r\sin\theta\times\text{OC}$

area $= \frac{1}{2}r^2\sin\theta\cos\theta \left(= \frac{1}{4}r^2\sin(2\theta)\right)$ (must be in terms of r and θ) ***A1*** ***N2***

[2 marks]

(c) valid attempt to express the relationship between the areas (seen anywhere) ***(M1)***

eg $\text{OCB} = \frac{3}{5}\text{OBA}$, $\frac{1}{2}r^2\sin\theta\cos\theta = \frac{3}{5}\times\frac{1}{2}r^2\theta$, $\frac{1}{4}r^2\sin 2\theta = \frac{3}{10}r^2\theta$

correct equation in terms of θ only ***A1***

eg $\sin\theta\cos\theta = \frac{3}{5}\theta$, $\frac{1}{4}\sin 2\theta = \frac{3}{10}\theta$

valid attempt to solve **their** equation ***(M1)***

eg sketch, -0.830017 , 0

0.830017

$\theta = 0.830$ ***A1*** ***N2***

Note: Do not award final ***A1*** if additional answers given.

[4 marks]

Total [7 marks]

5. (a) valid approach *(M1)*
eg $f(10)$

235.402
235 (fish) (must be an integer) ***A1*** ***N2***
[2 marks]

(b) recognizing rate of change is derivative *(M1)*
eg rate $= f'$, $f'(10)$, sketch of f', 35 (fish per month)

35.9976
36.0 (fish per month) ***A1*** ***N2***
[2 marks]

(c) valid approach *(M1)*
eg maximum of f', $f'' = 0$

15.890
15.9 (months) ***A1*** ***N2***
[2 marks]

Total [6 marks]

6. valid approach for expanding binomial (must have correct substitution for parameters, but accept "r" or an incorrect value for r) ***(M1)***

eg $\binom{15}{r}\left(\frac{1}{2x}\right)^{(15-r)}\left(x^2\right)^r$, $\left(\frac{1}{2x}\right)^{15}\left(x^2\right)^0+15\left(\frac{1}{2x}\right)^{14}\left(x^2\right)^1+\binom{15}{2}\left(\frac{1}{2x}\right)^{13}\left(x^2\right)^2+\ldots$

recognizing need to find the term containing x^{-3} in the expansion of $\left(\frac{1}{2x}+x^2\right)^{15}$ ***(M1)***

correct equation ***(A1)***

eg $\left(x^{-1}\right)^{15-r}\left(x^2\right)^r=x^{-3}$, $\left(x^{-1}\right)^r\left(x^2\right)^{15-r}=x^{-3}$, $-15+r+2r=-3$

identifying the correct term (seen anywhere) ***A1***

eg $r=4$, $r=11$, $n-r=4$

correct working ***(A1)(A1)***

eg $\binom{15}{4}\left(\frac{1}{2x}\right)^{15-4}$, $1365\times\frac{1}{2^{11}}$

Note: Award ***A1*** for each factor.

$\frac{1365}{2048}$ ***A1*** ***N2***

[7 marks]

7. **METHOD 1 (Distance between the origin and P)**

correct position vector for OP *(A1)*

eg $\vec{\mathrm{OP}} = \begin{pmatrix} -1+4t \\ 3+5t \\ 8-t \end{pmatrix}$, $\mathrm{P} = (-1+4t,\, 3+5t,\, 8-t)$

correct expression for OP or OP^2 (seen anywhere) ***A1***

eg $\sqrt{(-1+4t)^2+(3+5t)^2+(8-t)^2}$, $(-1+4x)^2+(3+5x)^2+(8-x)^2$

valid attempt to find the minimum of OP *(M1)*

eg $d'=0$, root on sketch of d', min indicated on sketch of d

$t=-\frac{1}{14}, -0.0714285$ *(A1)*

substitute their value of t into L (only award if there is working to find t) *(M1)*

eg one correct coordinate, $-1+4\left(-\frac{1}{14}\right)$

$(-1.28571,\, 2.64285,\, 8.07142)$

$\left(-\frac{9}{7}, \frac{37}{14}, \frac{113}{14}\right) = (-1.29,\, 2.64,\, 8.07)$ ***A1*** ***N2***

METHOD 2 (Perpendicular vectors)

recognizing that closest implies perpendicular *(M1)*

eg $\vec{\mathrm{OP}} \perp L$ (may be seen on sketch), $a \bullet b = 0$

valid approach involving $\vec{\mathrm{OP}}$ *(M1)*

eg $\vec{\mathrm{OP}} = \begin{pmatrix} -1+4t \\ 3+5t \\ 8-t \end{pmatrix}$, $\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix} \bullet \vec{\mathrm{OP}}$, $\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix} \perp \vec{\mathrm{OP}}$

correct scalar product ***A1***

eg $4(-1+4t)+5(3+5t)-1(8-t)$, $-4+16t+15+25t-8+t=0$, $42t+3$

$t=-\frac{1}{14}, -0.0714285$ *(A1)*

substitute their value of t into L or $\vec{\mathrm{OP}}$ (only award if scalar product used to find t) *(M1)*

eg one correct coordinate, $-1+4\left(-\frac{1}{14}\right)$

$(-1.28571,\, 2.64285,\, 8.07142)$

$\left(-\frac{9}{7}, \frac{37}{14}, \frac{113}{14}\right) = (-1.29,\, 2.64,\, 8.07)$ ***A1*** ***N2***

[6 marks]

Section B

8. (a) valid approach *(M1)*

eg $s_A(0)$, $s(0)$, $t = 0$

15 (cm) ***A1*** ***N2***

[2 marks]

(b) valid approach *(M1)*

eg $s_A = 0$, $s = 0$, 6.79321, 14.8651

2.46941

$t = 2.47$ (seconds) ***A1*** ***N2***

[2 marks]

(c) recognizing when change in direction occurs *(M1)*

eg slope of s changes sign, $s' = 0$, minimum point, 10.0144, (4.08, -4.66)

4.07702

$t = 4.08$ (seconds) ***A1*** ***N2***

[2 marks]

(d) **METHOD 1 (using displacement)**

correct displacement or distance from P at $t = 3$ (seen anywhere) *(A1)*

eg -2.69630, 2.69630

valid approach *(M1)*

eg $15 + 2.69630$, $s(3) - s(0)$, -17.6963

17.6963

17.7 (cm) ***A1*** ***N2***

METHOD 2 (using velocity)

attempt to substitute either limits or the velocity function into distance formula involving $|v|$ *(M1)*

eg $\int_0^3 |v| \,\mathrm{d}t$, $\int \left| -1 - 18t^2 \mathrm{e}^{-0.8t} + 4.8t^3 \mathrm{e}^{-0.8t} \right|$

17.6963

17.7 (cm) ***A2*** ***N2***

[3 marks]

continued…

Question 8 continued

(e) (i) recognize the need to integrate velocity **(M1)**

eg $\int v(t)$

$8t - \frac{2t^2}{2} + c$ (accept x instead of t and missing c) **(A2)**

substituting initial condition into their integrated expression (must have c) **(M1)**

eg $15 = 8(0) - \frac{2(0)^2}{2} + c$, $c = 15$

$s_B(t) = 8t - t^2 + 15$ **A1** **N3**

(ii) valid approach **(M1)**

eg $s_A = s_B$, sketch, $(9.30404, 2.86710)$

9.30404

$t = 9.30$ (seconds) **A1** **N2**

Note: If candidates obtain $s_B(t) = 8t - t^2$ in part (e)(i), there are 2 solutions for part (e)(ii), 1.32463 and 7.79009. Award the last ***A1*** in part (e)(ii) only if both solutions are given.

[7 marks]

Total [16 marks]

9. (a) **METHOD 1**

multiplication of $\mathrm{P}(A)$ and $\mathrm{P}(D)$ ***(A1)***

eg 0.70×0.85, 0.595

correct reasoning for their probabilities ***R1***

eg $0.595 \neq 0.65$, $0.70 \times 0.85 \neq \mathrm{P}(A \cap D)$

A and D are not independent ***AG*** ***N0***

METHOD 2

calculation of $\mathrm{P}(D \mid A)$ ***(A1)***

eg $\frac{13}{14}$, 0.928

correct reasoning for their probabilities ***R1***

eg $0.928 \neq 0.85$, $\frac{0.65}{0.7} \neq \mathrm{P}(D)$

A and D are not independent ***AG*** ***N0***

[2 marks]

(b) (i) correct working ***(A1)***

eg $\mathrm{P}(A) - \mathrm{P}(A \cap D)$, $0.7 - 0.65$, correct shading and/or value on Venn diagram

$\mathrm{P}(A \cap D') = 0.05$ ***A1*** ***N2***

(ii) recognizing conditional probability (seen anywhere) ***(M1)***

eg $\frac{\mathrm{P}(D' \cap A)}{\mathrm{P}(A)}$, $\mathrm{P}(A \mid B)$

correct working ***(A1)***

eg $\frac{0.05}{0.7}$

0.071428

$\mathrm{P}(D' \mid A) = \frac{1}{14}$, 0.0714 ***A1*** ***N2***

[5 marks]

continued…

Question 9 continued

(c) finding standardized value for 28 hours (seen anywhere) ***(A1)***
eg $z = 1.28155$

correct working to find σ ***(A1)***
eg $1.28155 = \dfrac{28-25}{\sigma}$, $\dfrac{28-25}{1.28155}$

2.34091
$\sigma = 2.34$ ***A1*** ***N2***
[3 marks]

(d) $\mathrm{P}(X > 30) = 0.0163429$ ***(A1)***

valid approach (seen anywhere) ***(M1)***
eg $\left[\mathrm{P}\left(X > 30\right)\right]^2$, $(0.01634)^2$, $\mathrm{B}(2,\ 0.0163429)$, $2.67\mathrm{E}\text{-}4$, $2.66\mathrm{E}\text{-}4$

0.0267090
$0.0267\,\%$ ***A2*** ***N3***
[4 marks]

Total [14 marks]

10. (a) attempt to find d ***(M1)***
eg $1.4-1.3$, u_1-u_2, $1.4=1.3+(2-1)d$

$d=0.1$ (may be seen in expression for u_n) ***(A1)***

correct equation ***(A1)***
eg $1.3+(k-1)\times 0.1=31.2$, $0.1k=30$

$k=300$ ***A1*** ***N3***
[4 marks]

(b) correct substitution ***(A1)***
eg $\frac{300}{2}(1.3+31.2)$, $\frac{300}{2}\left[2(1.3)+(300-1)(0.1)\right]$, $\frac{300}{2}\left[2.6+299(0.1)\right]$

$S_k=4875$ ***A1*** ***N2***
[2 marks]

(c) recognizing need to find the sequence of multiples of 3 (seen anywhere) ***(M1)***
eg first term is u_3 $(=1.5)$ (accept notation $u_1=1.5$),
$d=0.1\times 3$ $(=0.3)$, 100 terms (accept $n=100$), last term is 31.2
(accept notation $u_{100}=31.2$), $u_3+u_6+u_9+\ldots$ (accept $F=u_3+u_6+u_9+\ldots$)

correct working for sum of sequence where n is a multiple of 3 ***A2***
$\frac{100}{2}(1.5+31.2)$, $50(2\times 1.5+99\times 0.3)$, 1635

valid approach (seen anywhere) ***(M1)***

eg $S_k-(u_3+u_6+\ldots)$, $S_k-\frac{100}{2}(1.5+31.2)$, S_k-(their sum for $(u_3+u_6+\ldots)$)

correct working (seen anywhere) ***A1***
eg S_k-1635, $4875-1635$

$F=3240$ ***AG*** ***N0***
[5 marks]

continued…

Question 10 continued

(d) attempt to find r ***(M1)***
eg dividing consecutive terms

correct value of r (seen anywhere, including in formula)
eg $\frac{1}{\sqrt{2}}$, $0.707106\ldots$, $\frac{a}{0.293\ldots}$ ***A1***

correct working (accept equation) ***(A1)***
eg $\frac{a}{1-\frac{1}{\sqrt{2}}} < 3240$

correct working ***A1***

METHOD 1 (analytical)

eg $3240\times\left(1-\frac{1}{\sqrt{2}}\right)$, $a < 948.974$, 948.974

METHOD 2 (using table, must find both S_∞ values)

eg when $a = 948$, $S_\infty = 3236.67\ldots$ **AND** when $a = 949$, $S_\infty = 3240.08\ldots$

$a = 948$ ***A1*** ***N2***
[5 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Estudios matemáticos
Nivel medio
Prueba 1

Lunes 13 de mayo de 2019 (tarde)

Número de convocatoria del alumno

1 hora 30 minutos

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Para esta prueba, se necesita una copia sin anotaciones del **cuadernillo de fórmulas de estudios matemáticos NM**.
- Conteste todas las preguntas.
- Escriba sus respuestas en las casillas provistas a tal efecto.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán darse como valores exactos o con una aproximación de tres cifras significativas.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

20 páginas

2219 – 7409

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

Se otorgará la máxima puntuación a las respuestas correctas. Aun cuando una respuesta sea incorrecta, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Escriba sus respuestas en las casillas provistas a tal efecto. Para los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido hasta su obtención. Por ejemplo, cuando deba utilizar un gráfico de una calculadora de pantalla gráfica para hallar soluciones, deberá dibujar aproximadamente esas gráficas en su respuesta.

1. La forma de la luna se puede modelizar mediante una esfera de $3\,474\,000$ metros de diámetro.

(a) Utilice este modelo para calcular la circunferencia de la luna en **kilómetros**. Escriba el valor completo que aparece en la pantalla de la calculadora. [3]

(b) Dé la respuesta que dio en el apartado (a) redondeando a tres cifras significativas. [1]

(c) Escriba la respuesta que dio en el **apartado (b)** en la forma $a \times 10^k$, donde $1 \le a < 10$, $k \in \mathbb{Z}$. [2]

Operaciones:

Respuestas:

(a) . . . .

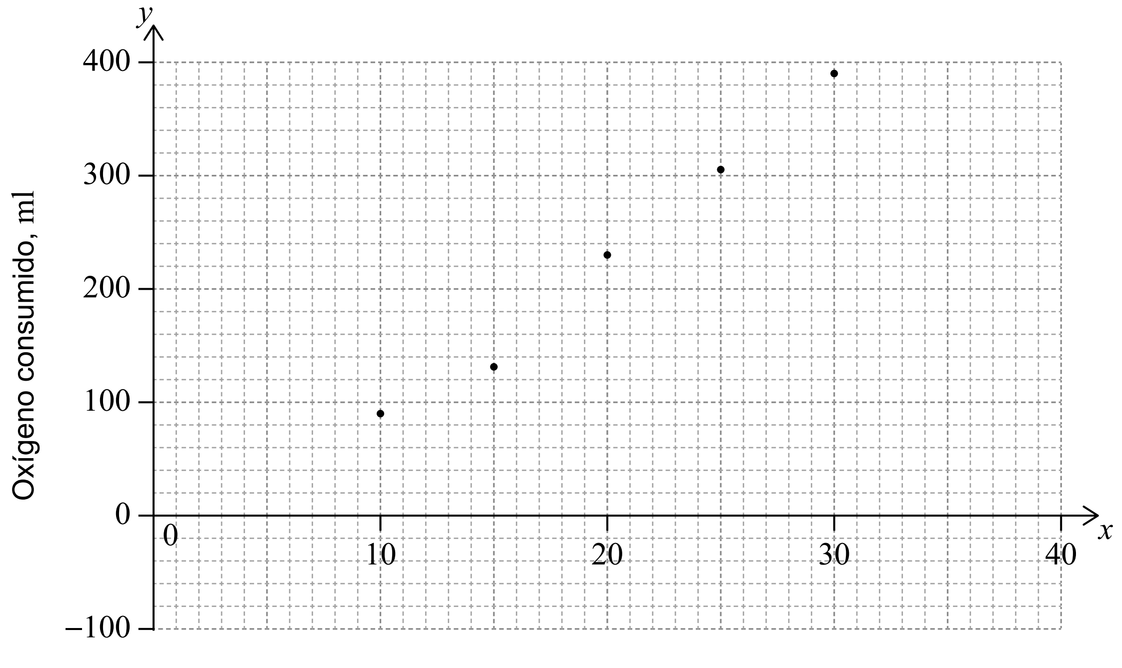
(b) . . . .

(c) . . . .

2. Los escarabajos de Colorado son una plaga que puede provocar importantes daños a los cultivos de patata. La cantidad de oxígeno (en mililitros [ml]) que consume al día uno de estos escarabajos de Colorado aumenta con la temperatura, tal y como se muestra en la siguiente tabla.

Temperatura, °C (x)	10	15	20	25	30
Oxígeno consumido, ml (y)	90	133	230	306	391

Esta información se ha utilizado para elaborar un diagrama de dispersión.



(a) Halle la ecuación de la recta de regresión de y sobre x. [2]

Las coordenadas de la media son $(20, 230)$.

(b) En el mismo diagrama de dispersión, dibuje con precisión la recta de regresión de y sobre x. [2]

Para poder estimar la cantidad de oxígeno consumido, se considera que esta recta de regresión es fiable para temperaturas x tales que $a \le x \le b$.

(c) Escriba el valor de a y el de b . [2]

Operaciones:

Respuestas:

(a) . . . .

(c) . . . .

3. En esta pregunta, dé todas las respuestas redondeando a dos lugares decimales.

Velina viaja de Nueva York a Copenhague y se lleva 1200 dólares estadounidenses (USD). Ella cambia este dinero a coronas danesas (DKK). El tipo de cambio es $1\,\text{USD} = 7{,}0208\,\text{DKK}$.

(a) Calcule cuánto dinero recibe Velina en DKK. [2]

Al final del viaje a Velina le han sobrado $3450\ \text{DKK}$ y decide cambiarlas a USD. El banco cobra una comisión del 5%. El tipo de cambio sigue siendo $1\,\text{USD} = 7{,}0208\,\text{DKK}$.

(b) (i) Calcule la cantidad de DKK que le quedan para cambiar una vez aplicada la comisión.

(ii) A partir de lo anterior, calcule la cantidad de USD que recibirá. [4]

Operaciones:

Respuestas:

(a) . . . .

(b) (i) . . . .

(ii) . . . .

4. Considere las siguientes proposiciones.

p: a es divisible entre 9
q: b es divisible entre 9
s: El producto de a y b es divisible entre 9

(a) Escriba $s \Leftrightarrow (p \wedge q)$ con palabras. [3]

(b) Complete la siguiente tabla de verdad. [2]

p	q	s	$p \wedge q$	$s \Leftrightarrow (p \wedge q)$
V	V	V		
V	V	F		
V	F	V		
V	F	F		
F	V	V		
F	V	F		
F	F	V		
F	F	F		

(c) Justifique por qué $s \Leftrightarrow (p \wedge q)$ no es ni una tautología ni una contradicción. [1]

Operaciones:

Respuestas:

(a) . . . .

. . . .

. . . .

. . . .

(c) . . . .

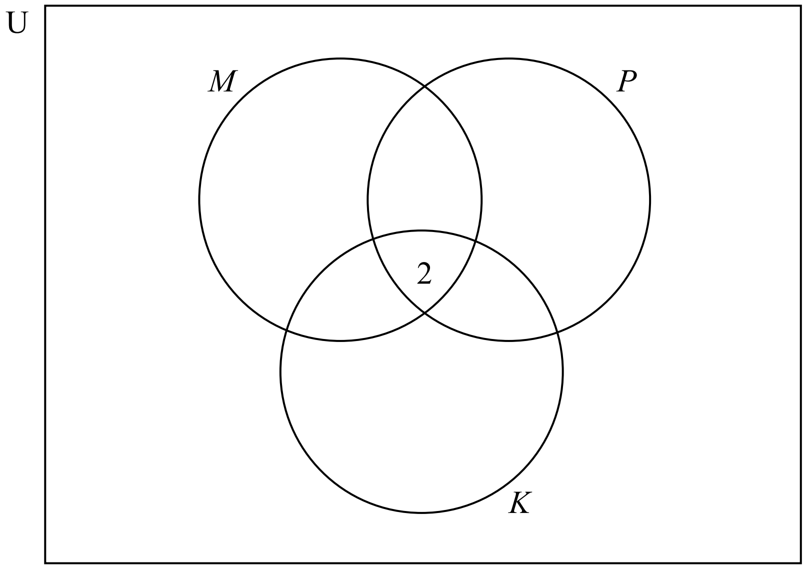
. . . .

. . . .

5. En la cafetería de un colegio se venden batidos de tres sabores distintos: mango (M), kiwi (K) y plátano (P). 85 alumnos participaron en una encuesta para saber cuáles de estos tres sabores les gustaban a cada uno.

A 35 alumnos les gusta el mango, a 37 el plátano y a 26 el kiwi
A 2 les gustan los tres sabores
A 20 les gustan el mango y el plátano
A 14 les gustan el mango y el kiwi
A 3 les gustan el plátano y el kiwi

(a) Utilizando esta información, complete el siguiente diagrama de Venn. [2]



(b) Halle a cuántos de los alumnos que participaron en la encuesta no les gusta ninguno de los tres sabores. [2]

(c) Se elige al azar a uno de los alumnos que participaron en la encuesta.

Halle la probabilidad de que a este alumno le gusten los batidos de kiwi, sabiendo que le gustan los batidos de mango. [2]

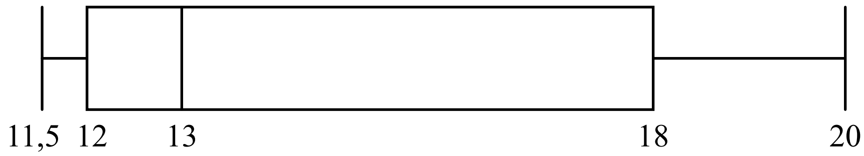
Operaciones:

Respuestas:

(b) . . . .

(c) . . . .

6. Una inspectora de sanidad analizó la cantidad de azúcar presente en 500 **aperitivos** distintos que se elaboran en las cafeterías de diversos colegios. Los datos recogidos se muestran en el siguiente diagrama de caja y bigotes.



Cantidad de azúcar (en gramos) que contiene cada aperitivo

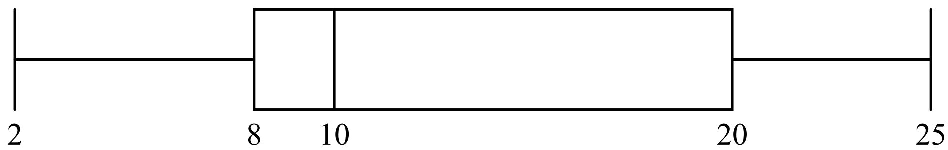
(a) Indique qué representa el 13 en este diagrama. [1]

(b) (i) Escriba el rango intercuartil para estos datos.

(ii) Escriba el número aproximado de aperitivos cuyo contenido de azúcar está comprendido entre 18 y 20 gramos. [3]

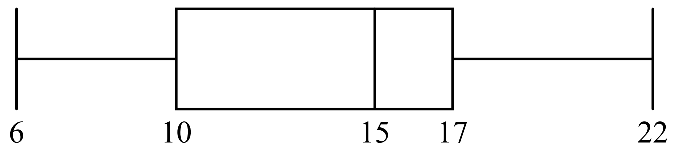
La inspectora de sanidad visita la cafetería de dos colegios distintos. En cada una de esas cafeterías inspecciona el mismo número de **comidas**. Los datos recogidos se muestran en los siguientes diagramas de caja y bigotes.

Cafetería del colegio 1



Cantidad de azúcar (en gramos) que contiene cada comida

Cafetería del colegio 2



Cantidad de azúcar (en gramos) que contiene cada comida

Las comidas que elaboran las cafeterías de los colegios tienen que tener menos de 10 gramos de azúcar.

(c) Indique —dando una razón— en cuál de las dos cafeterías hay más comidas que **no** cumplen este requisito. [2]

(Esta pregunta continúa en la página siguiente)

(Pregunta 6: continuación)

Operaciones:

Respuestas:

(a) . . . .

. . . .

(b) (i) . . . .

(ii) . . . .

(c) . . . .

. . . .

. . . .

7. La anchura de un jardín rectangular es $4{,}5$ metros menor que su longitud, que es igual a x metros.

(a) Escriba una expresión que dé la anchura del jardín en función de x. [1]

El perímetro del jardín es igual a $111\,\text{m}$.

(b) Escriba una ecuación que exprese el perímetro del jardín en función de x. [1]

(c) Halle el valor de x. [2]

Un jardinero mide la longitud del jardín y obtiene un valor de $25\,\text{m}$.

(d) Halle el porcentaje de error de su medición. [2]

Operaciones:

Respuestas:

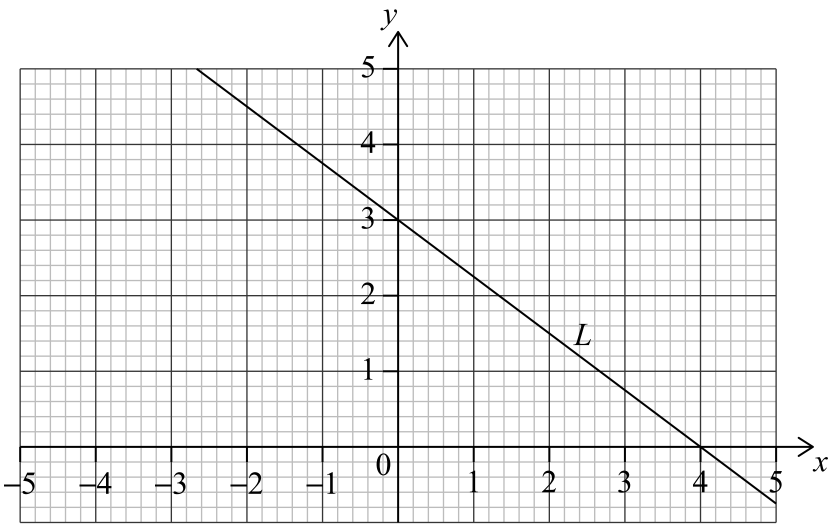
(a) . . . .

(b) . . . .

(c) . . . .

(d) . . . .

8. La recta L corta al eje y en $(0, 3)$ y al eje x en $(4, 0)$, tal y como se muestra en la siguiente figura.



(a) (i) Halle la pendiente de L.

(ii) Escriba la ecuación de L en la forma $y = mx + c$. [3]

La recta N es perpendicular a L y pasa por el punto $\mathrm{P}(2, 1)$.

(b) (i) Escriba la pendiente de N.

(ii) Halle la ecuación de N en la forma $y = mx + c$. [3]

Operaciones:

Respuestas:

(a) (i) . . . .

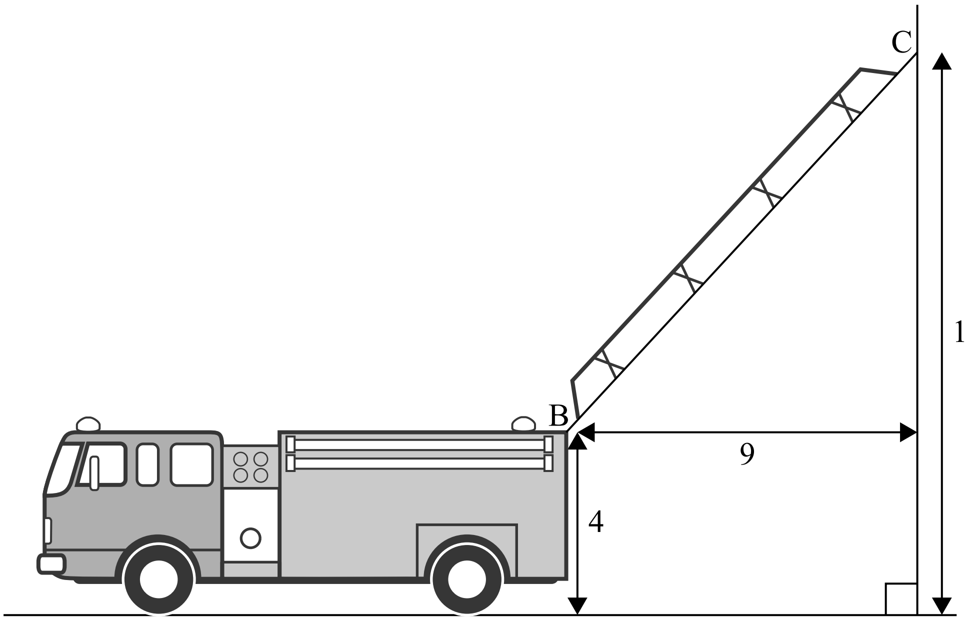
(ii) . . . .

(b) (i) . . . .

(ii) . . . .

9. La escalera de un camión de bomberos tiene su base en el punto B, que está situado a 4 metros del suelo. Cuando se despliega la escalera, el otro extremo se apoya en una pared vertical en el punto C, que está situado a 16 metros del suelo. La distancia horizontal que hay entre B y C es de 9 metros.

la figura no está dibujada a escala



(a) Halle el ángulo de elevación de C desde B. [3]

Llega un segundo camión y su escalera, cuando está totalmente desplegada, mide 30 metros de largo. La base de esta escalera también se encuentra a 4 metros del suelo. Por seguridad, el máximo ángulo de elevación que puede subtender la escalera es de 70°.

(b) Halle la máxima altura a la que se puede llegar en la pared con la escalera de este segundo camión. [3]

(Esta pregunta continúa en la página siguiente)

(Pregunta 9: continuación)

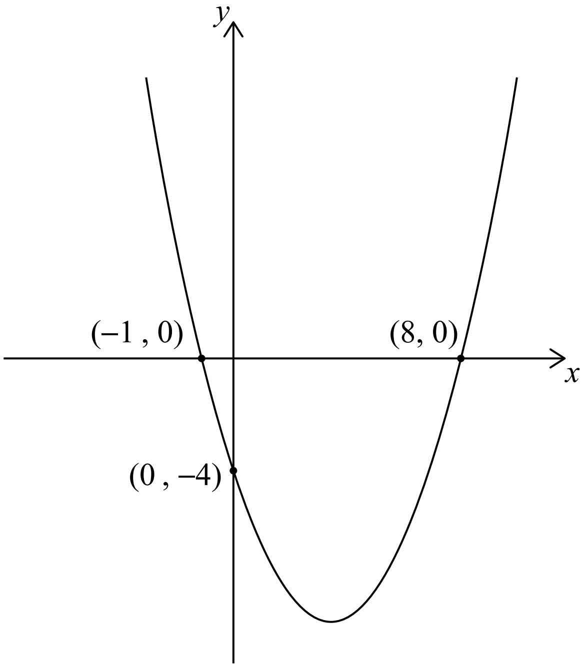
Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

10. Considere la función $f(x) = a(x-p)(x-q)$, que se muestra en el siguiente gráfico.



(a) Halle la ecuación del eje de simetría. [2]

(b) Halle el valor de p y el de q. [2]

(c) Halle el valor de a. [2]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

11. Considere los siguientes conjuntos:

El conjunto universal U consta de todos los números enteros positivos menores de 15;
A es el conjunto de todos los números que son múltiplos de 3;
B es el conjunto de todos los números pares.

(a) Escriba los elementos que pertenecen a $A \cap B$. [3]

(b) Escriba

(i) los elementos que pertenecen a $A \cap B'$;

(ii) $n(A \cap B')$. [3]

Operaciones:

Respuestas:

(a) . . . .

(b) (i) . . . .

(ii) . . . .

12. Un grupo de estudiantes universitarios participaron en una encuesta donde se les preguntó cuántas horas (h) trabajaban al mes. Los resultados se muestran en la siguiente tabla.

Horas al mes (h)	**Frecuencia**	**Frecuencia acumulada**
$0 < h \le 10$	3	3
$10 < h \le 20$	7	10
$20 < h \le 30$	10	20
$30 < h \le 40$	14	34
$40 < h \le 50$	p	44
$50 < h \le 60$	6	50
$60 < h \le 70$	4	54
$70 < h \le 80$	2	q

(a) Utilice la tabla para hallar los siguientes valores.

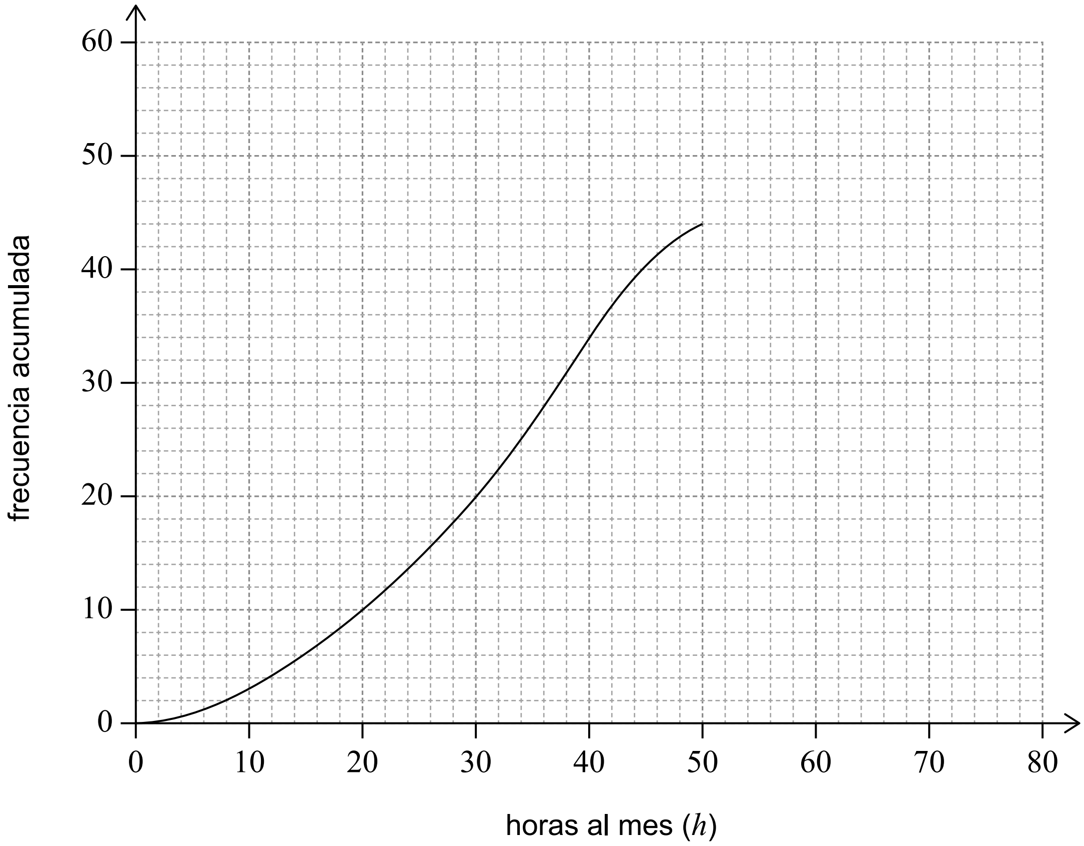
(i) p

(ii) q [2]

(Esta pregunta continúa en la página siguiente)

(Pregunta 12: continuación)

Se han usado los cinco primeros intervalos de clase (los que se indican en la tabla) para dibujar una parte de la curva de frecuencias acumuladas, tal y como se muestra en la figura.



(b) En esa misma cuadrícula, complete la curva de frecuencias acumuladas correspondiente a estos datos. [2]

(c) Utilice la curva de frecuencias acumuladas para hallar una estimación del número de alumnos que trabajan como mucho 35 horas al mes. [2]

Operaciones:

Respuestas:

(a) (i) . . . .

(ii) . . . .

(c) . . . .

13. En la isla Little Green no había tortugas, originariamente. Después de que se introdujeran 55 tortugas en la isla, la población de tortugas se puede ahora modelizar mediante la siguiente expresión:

$$N(t) = a \times 2^{-t} + 10,\ t \geq 0,$$

donde a es una constante y t es el tiempo transcurrido, en años, desde que se introdujeron las tortugas.

(a) Halle el valor de a. [2]

(b) Halle el tiempo (en años) que ha de transcurrir para que la población disminuya a 20 tortugas. [2]

Hay un número m a partir del cual la población de tortugas ya no disminuye.

(c) Halle el valor de m. Justifique su respuesta. [2]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

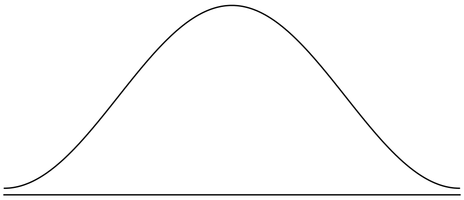
(c) . . . .

. . . .

. . . .

14. El precio (en euros) al que se vende un kilogramo de tomates en los diversos mercados de una ciudad dada sigue una distribución normal, de media $3{,}22$ y desviación típica igual a $0{,}84$.

(a) (i) En la siguiente figura, sombree la región que representa la probabilidad de que el precio de un kilogramo de tomates, elegido al azar, sea mayor que $3{,}22$ euros.



(ii) Halle el precio que está dos desviaciones típicas por encima de la media del precio. [2]

(b) Halle la probabilidad de que el precio de un kilogramo de tomates, elegido al azar, esté comprendido entre $2{,}00$ euros y $3{,}00$ euros. [2]

Para fomentar que se pongan unos precios razonables, la ciudad ofrece licencias de venta gratuitas a aquellos vendedores cuyo precio de venta de un kilogramo de tomates esté dentro del 20% más bajo.

(c) Halle el precio más alto que puede poner un vendedor si quiere tener una licencia gratuita. [2]

Operaciones:

Respuestas:

(a) (ii) . . . .

(b) . . . .

(c) . . . .

15. Un alfarero vende x jarrones al mes.

Los beneficios mensuales que obtiene, en dólares australianos (AUD), se pueden modelizar mediante la siguiente expresión:

$$B(x) = -\frac{1}{5}x^3 + 7x^2 - 120,\ x \geq 0 .$$

(a) Halle el valor de B cuando el alfarero no vende ningún jarrón. [1]

(b) Derive $B(x)$. [2]

(c) **A partir de lo anterior**, halle el número de jarrones para el cual se maximizan los beneficios. [3]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

Diploma Programme
Programme du diplôme
Programa del Diploma

Esquema de calificación

Mayo de 2019

Estudios matemáticos

Nivel medio

Prueba 1

Esquema de calificación de la Prueba 1
Instrucciones para los examinadores

Observaciones: **En caso de dudas sobre estas instrucciones o temas relacionados con la asignación de puntuaciones, póngase en contacto con su jefe de equipo.**

La puntuación máxima para cada pregunta es 6.

1 Siglas

En el esquema de calificación pueden aparecer las siguientes siglas:

M Puntos otorgados por el **método**

A Puntos otorgados por una **respuesta** o por la **precisión**

C Puntos otorgados por respuestas **correctas** (independientemente del trabajo mostrado)

R Puntos otorgados por un **razonamiento** claro

ft Se pueden otorgar puntos por la **coherencia** con resultados previos en la pregunta

2 Método para corregir

(a) Todas las correcciones deben realizarse en RM Assessor, utilizando las anotaciones de Estudios Matemáticos y de acuerdo con el documento vigente de orientaciones para la corrección electrónica de Estudios Matemáticos NM. Es fundamental leer el documento antes de empezar a corregir.

(b) Si el alumno obtiene la puntuación máxima en una pregunta utilice la anotación ***C6***, si lo ha intentado pero obtiene cero puntos utilice ***C0***. Si no lo ha intentado utilice la tecla de No respuesta. Si un alumno no ha obtenido ni la puntuación máxima ni cero puntos, entonces se **DEBEN** mostrar todas las anotaciones.

(c) En esta prueba, **si aparece la respuesta correcta en la línea de respuesta** se otorgará la puntuación máxima. **¡No es necesario comprobar el trabajo! Otorgue puntos C y siga adelante.**

(d) Si la respuesta no aparece en la línea de respuesta, pero la respuesta correcta se encuentra en el cuadro de operaciones sin trabajo posterior, otorgue la puntuación máxima.

(e) Si la **respuesta es incorrecta**, se deben otorgar puntos por el trabajo realizado, de acuerdo con el esquema de calificación.

(f) No se debe otorgar ningún punto al trabajo tachado por el alumno. Si el alumno ha dado dos respuestas a una pregunta, solo se debe corregir la primera respuesta.

(g) Una respuesta correcta en el cuadro de trabajo transcrita erróneamente a la línea de respuesta puede recibir la puntuación máxima.

(e) Si el procedimiento adecuado lleva a la respuesta correcta, pero esta va seguida de más desarrollos que indican una falta de comprensión matemática, entonces **no** se puede otorgar la puntuación máxima. En la mayoría de estos casos se perderá solo el último punto por respuesta. Una excepción podría ser las respuestas numéricas, donde una respuesta exacta que es correcta está seguida de una expresión decimal incorrecta.

Ejemplo:

	Se ve la respuesta correcta	**Hay más desarrollo**	**Acción**
1.	$8\sqrt{2}$	$5{,}65685...$ *(valor decimal incorrecto)*	Otorgue el ultimo ***(A1)*** *(ignore el desarrollo posterior)*
2.	$(x-6)\,(x+1)$	$x=6$ y -1	**No** otorgue el último ***(A1)*** *(vea el siguiente ejemplo)*

Ejemplo: Factorice $x^2 - 5x - 6$

Esquema de calificación		Examen del alumno		Corrección
$(x-6)(x+1)$	***(A1)(A1)***	(i)	Línea de respuesta: $(x+6)(x+1)$	***(A0)(A1)***
		(ii)	Cuadro de operaciones: $(x-6)(x+1)$ seguido de $x = 6$ y -1, o simplemente $6, -1$ bien en el cuadro de trabajo o en la línea de respuesta.	***(A1)*** ***(A0)***

3 **Puntos por la coherencia (ft)**

En cualquier paso de una resolución, un error puede afectar a todo el trabajo posterior. Para limitar la gravedad de la penalización, se pueden otorgar **puntos por la coherencia**. Los esquemas de calificación indicarán, con **"(ft)"**, dónde es adecuado aplicar el criterio de coherencia en una pregunta.

(a) La coherencia se aplica solo de un apartado de una pregunta a otro apartado posterior de la pregunta. No se aplica dentro del mismo apartado.

(b) Si una respuesta que es consecuencia de un procedimiento de coherencia es muy poco realista (por ejemplo, distancias negativas o de un orden de magnitud excesivo) entonces el punto ***A*** final no se debe otorgar.

(c) Si, debido a un error, una pregunta queda transformada en otra **distinta**, **mucho más sencilla**, entonces el criterio de coherencia podría no ser aplicado.

(d) Para otorgar puntos por coherencia en un apartado de una pregunta, **es necesario que haya un trabajo explícito relacionado con ese apartado**. Una respuesta aislada, obtenida por coherencia, pero sin un desarrollo explícito, se considera incorrecta, y no recibirá ningún punto **aunque se aproxime a la respuesta correcta**.

(e) Una excepción a lo anterior sería una pregunta donde lo que se evalúa es el manejo del alumno en el uso de la calculadora de pantalla gráfica, y donde no se pretende que se muestre ningún desarrollo escrito. **En el esquema de calificación vendrá indicado claramente cuando sea el caso.**

(f) El uso inadvertido de radianes será penalizado la primera vez que aparezca. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez un punto por el uso de radianes.

Ejemplo: Cálculo de ángulos y longitudes en trigonometría

Esquema de calificación		Examen del alumno	Corrección
(a) $\frac{\operatorname{sen} A}{3} = \frac{\operatorname{sen} 30}{4}$	***(M1)(A1)***	(a) $\frac{\operatorname{sen} A}{4} = \frac{\operatorname{sen} 30}{3}$	***(M1)(A0)*** *(uso del teorema del seno, pero con valores incorrectos)*
$A = 22{,}0^\circ$ $(22{,}0243\ldots)$	***(A1)***	$A = 41{,}8^\circ$	***(A0)*** ***(Nota:*** *Aquí, el segundo* ***(A1)*** *no ha sido corregido como* **(ft)** *y no se puede otorgar porque existe un error previo en el* ***mismo*** *apartado de la pregunta.)*
(b) $x = 7\operatorname{tg}(22{,}0243\ldots^\circ)$	***(M1)***	(b) caso (i) $x = 7\operatorname{tg} 41{,}8^\circ$	***(M1)***
$= 2{,}83$ $(2{,}831639\ldots)$	***(A1)*****(ft)**	$= 6{,}26$	***(A1)*****(ft)**
		pero caso (ii) $6{,}26$	***(C0)*** *pues no aparece un desarrollo explícito*

4 Uso del Esquema de calificación

(a) Los puntos ***A*** **dependen** de los puntos ***M*** obtenidos previamente, **no** es posible otorgar ***(M0)(A1)***. Una vez que se otorga un ***(M0)***, se pierden todos los puntos ***A*** que le siguen en ese apartado de la pregunta, incluso si los cálculos son correctos, hasta el siguiente punto ***M***.
La única excepción a lo anterior se hará en el caso de una respuesta donde la precisión se especifique en la pregunta (ver apartado 5).

(b) Los puntos ***A*** **dependen** de los puntos ***R*** obtenidos, **no** es posible otorgar ***(A1)(R0)***. Así pues, el ***(A1)*** no se otorga a una respuesta correcta si no aparece el razonamiento, o este es incorrecto.

(c) Los **métodos alternativos** no siempre se incluyen. Así, si una respuesta es errónea, entonces se debe analizar cuidadosamente el procedimiento, de modo que se asignen puntos por un método distinto de forma coherente con el esquema de calificación.
Cuando en el esquema de calificación se incluyen métodos alternativos para una pregunta, ello viene indicado mediante "**O**" *etc*.

(d) A menos que en la pregunta se especifique lo contrario, se aceptan **expresiones equivalentes**. Por ejemplo: $\frac{\operatorname{sen}\theta}{\cos\theta}$ por $\operatorname{tg}\theta$. En el esquema de calificación, estas expresiones algebraica o numéricamente equivalentes aparecerán a veces escritas entre paréntesis junto a la respuesta requerida.
Cuando se requieran respuestas numéricas como respuesta final de un apartado de una pregunta, en el esquema de calificación se mostrará en este orden;
la respuesta con 3 cifras significativas a partir de la solución completa de la calculadora;
el valor exacto (por ejemplo $\sqrt{3}$ si corresponde);
la solución completa de la calculadora en la forma $2{,}83163\ldots$ como en el ejemplo anterior.
Cuando se dan las respuestas con 3 cifras significativas y se utilizan después en apartados posteriores de la pregunta llevando a una solución con 3 cifras significativas **distinta**, también se darán estas soluciones.

(e) Dado que este es un examen internacional, se aceptarán todas las **formas de notación alternativas**. Algunos ejemplos:

Comas decimales: $1{,}7$**;** $1'7$; $1\cdot 7$; $1;7$.
Los números decimales menores que 1 podrían aparecer escritos sin un cero en el frente: 0.49 o $.49$.
Descripciones de un intervalo: $3 < x < 5$; $(3;\ 5)$; $]\ 3;\ 5\ [$.
Distintas formas de notación de las propiedades de los conjuntos (por ejemplo complementario): A'; $\overline{A}$; A^c; $U - A$;$(A$;$U \setminus A$.
Distintas formas de notación lógica: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p, -p$.
$p \Rightarrow q$; $p \to q$.
El nivel de significación podría escribirse como α .

(f) Puntos discrecionales: habrá ocasiones excepcionales en las que el esquema de calificación no cubra el trabajo que aparece. En estos casos se utilizará la nota DM para indicar que el examinador ha utilizado su criterio. La discrecionalidad debe utilizarse con moderación, y si existe duda se debe plantear una excepción a través de RM Assessor al jefe del equipo.

A partir de noviembre de 2011 ya no se aplicarán las penalizaciones AP, FP y UP. La precisión y las unidades serán evaluados en preguntas específicas y los puntos se otorgarán de acuerdo a las reglas dadas en los apartados 5, 6 y 7.

5 Precisión de las respuestas

Instrucciones: Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas correctamente a 3 cifras significativas.

1. Si el alumno muestra la solución a 4 o más cifras significativas, la cual redondeada a 3 cifras significativas daría la respuesta requerida entonces otorgue ***(A1)*** e ignore cualquier redondeo posterior. **Nota:** La solución sin redondear puede estar tanto en la línea de respuesta como en el cuadro de operaciones.

2. Si el alumno **no** muestra la solución sin redondear, entonces otorgue ***(A1)*** si la solución dada está aproximada correctamente a 2 o más cifras significativas, y ***(A0)*** en cualquier otro caso.

3. Si una solución aproximada a 2 cifras significativas se utiliza en apartados posteriores, entonces para otorgar más puntos debe aparecer el desarrollo. (Este criterio es el mismo que en el que se aplica coherencia a partir de una respuesta errónea.)

Estos 3 casos (vea los superíndices) se han resumido en la siguiente tabla e ilustrado después con ejemplos.

	Si la solución final de alumno se expresa…				
	Exacta o a 4 o más cifras significativas (y que redondeada **a 3 cifras significativas daría la respuesta correcta**)	**Aproximada incorrectamente a 3 cifras significativas**	Aproximada correctamente a 2 cifras significativas[3]	Aproximada incorrectamente a 2 cifras significativas	Aproximada correcta o incorrectamente a 1 cifra significativa
Aparece la solución sin redondear[1]	Otorgue ***(A1)*** independientemente del redondeo correcto o incorrecto				
No aparece la solución sin redondear[2]	***(A1)***	***(A0)***	***(A1)***	***(A0)***	***(A0)***
Criterio para los apartados posteriores	Como en MS	Como en el criterio de coherencia, solo si se muestra el desarrollo[3]			

Ejemplos:

Esquema de calificación	Examen del alumno	Corrección
9,43 (9,43398…) ***(A1)***	(i) 9,43398… se muestra en el cuadro de trabajo seguido de 9; 9,4; 9,43; 9,434 (redondeados correctamente)	***(A1)***
	(ii) 9,43398… se muestra en el cuadro de trabajo seguido de 9,433; 9,44, *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 9,4	***(A1)***
	(iv) 9	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 9,3	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 9,44	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Esquema de calificación	Examen del alumno	Corrección
7,44 (7,43798…) ***(A1)***	(i) 7,43798… se muestra en el cuadro de trabajo seguido de 7; 7,4; 7,44; 7,438 *etc.* (redondeados correctamente)	***(A1)***
	(ii) 7,43798… se muestra en el cuadro de trabajo seguido de 7,437 ó 7,43 *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 7,4	***(A1)***
	(iv) 7	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 7,5	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 7,43	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Ejemplo: ABC es un triángulo rectángulo con el ángulo $\text{ABC} = 90^\circ$, $\text{AC} = 32\text{ cm}$ y $\text{AB} = 30\text{ cm}$. Halle (a) la longitud de BC, (b) el área del triángulo ABC.

Esquema de calificación	Examen del alumno	Corrección
(a) $\text{BC} = \sqrt{32^2 - 30^2}$ ***(M1)***	(a) $\text{BC} = \sqrt{32^2 - 30^2}$	***(M1)***
*Otorgue **(M1)** por la sustitución correcta en el teorema de Pitágoras*	$11\,(\text{cm})$	***(A1)***
$= 11{,}1\ \left(\sqrt{124}\,11{,}1355\ldots\right)(\text{cm})$ ***(A1)***	*(solo se muestra la aproximación a 2 cifras significativas, pero correcta)*	
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11{,}1355\ldots$ ***(M1)***	(b) caso (i) $\text{Área} = \frac{1}{2} \times 30 \times 11$	***(M1)***
*Otorgue **(M1)** por la sustitución correcta en la fórmula del área de un triángulo*	*(se muestra el desarrollo)*	
$= 167\ \ (167{,}032\ldots)(\text{cm}^2)$ ***(A1)*(ft)**	$= 165\,(\text{cm}^2)$	***(A1)*(ft)**
	caso (ii) $= 165\,(\text{cm}^2)$	***(M0)(A0)*(ft)**
	(No se muestra el desarrollo, la solución 11 se trata como procedimiento de coherencia, por lo que no se deben otorgar puntos en este caso)	

La aproximación a 3 cifras significativas de una solución exacta **se debe aceptar si se realiza correctamente**. Las soluciones exactas del tipo $\frac{1}{4}$ se pueden escribir como decimales con menos de 3 cifras significativas si el resultado sigue siendo exacto. La simplificación de una fracción a su expresión irreducible **no** es imprescindible, sin embargo cuando una respuesta se simplifica a un entero, esto es lo esperado. Fracciones que incluyen un decimal en el numerador y/o en el denominador son aceptadas para demostrar una substitución correcta, sin embargo, no como respuesta final.

Razones de π y respuestas con expresiones de raíces cuadradas de enteros o cualquier potencia racional de un entero (por ejemplo, $\sqrt{13}, 2^{2/3}, \sqrt[4]{5}$), se pueden aceptar como respuestas exactas. Todas las otras potencias (por ejemplo de no enteros) y valores de funciones transcendentes tales como seno y coseno se deben evaluar.

Si el nivel de precisión viene especificado en la pregunta, se asignarán puntos por dar la respuesta con la precisión requerida. En **todos** estos casos no se obtiene el punto final si el redondeo no sigue las instrucciones dadas en la pregunta. El punto por la precisión especificada se puede considerar como punto por coherencia **(ft)** con independencia de un ***(M0)*** inmediatamente anterior.

6 Nivel de precisión en las preguntas sobre cuestiones financieras

El nivel de precisión de las respuestas se especificará en todas las preguntas que se relacionen con valores en dinero. Generalmente será a la unidad más cercana o de dos cifras decimales. La primera respuesta que no venga dada con el nivel de precisión especificado no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez la un punto por la precisión incorrecta en una pregunta sobre cuestiones financieras.

Ejemplo: En una pregunta sobre cuestiones financieras se pide una precisión de dos cifras decimales.

Esquema de calificación	Examen del alumno	Corrección
$\$231,62\ (231,6189)$ ***(A1)***	(i) $231,6$	***(A0)***
	(ii) 232 *(Aproximación correcta con un nivel de precisión distinto del requerido)*	***(A0)***
	(iii) $231,61$	***(A0)***
	(iv) $232,00$ *(Aproximación incorrecta al nivel de precisión requerido)*	***(A0)***

7 Unidades de medida en las respuestas

En determinadas preguntas será necesario expresar las unidades de medida, y esto vendrá claramente indicado en el esquema de calificación. La primera respuesta correcta pero sin unidades o con unidades incorrectas no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la falta de unidades o unidades incorrectas. Las unidades de medida se tendrán en cuenta solamente cuando se ha otorgado ***(A1)*** a la respuesta numérica siguiendo las indicaciones sobre la precisión dadas en el apartado 5.

Esquema de calificación	Examen del alumno	Corrección
(a) $37000\ \mathrm{m}^2$ ***(A1)***	(a) $36000\ \mathrm{m}^2$ *(respuesta incorrecta, por lo que no se consideran las unidades)*	***(A0)***
(b) $3200\ \mathrm{m}^3$ ***(A1)***	(b) $3200\ \mathrm{m}^2$ *(unidades incorrectas)*	***(A0)***

8 Calculadoras de pantalla gráfica

Con frecuencia los alumnos obtienen las soluciones directamente de la calculadora. Deben utilizar la notación matemática y no la notación de la calculadora. No se puede otorgar puntos por método a respuestas incorrectas basadas únicamente en notación de calculadora. No se puede otorgar puntos por método al comentario “usé la calculadora de pantalla gráfica”.

1. (a) $\dfrac{3474000\times\pi}{1000}$ ***(M1)(M1)***

> **Nota:** Conceda ***(M1)*** por el numerador correcto y ***(M1)*** por dividir por 1000
> **O** equivalente como $\dfrac{3474000\cdot 2\cdot\pi}{2000}$ es decir el diámetro.
> No acepte el uso de la fórmula del área, es decir πr^2.

$10\,913{,}89287\ldots$ (km) ***(A1)*** ***(C3)***

[3 puntos]

(b) $10\,900$ (km) ***(A1)*(ft)** ***(C1)***

> **Nota:** Se pueden otorgar puntos de arrastre de error desde el apartado (a).

[1 punto]

(c) $1{,}09\times10^4$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

> **Nota:** Se pueden otorgar puntos de arrastre de error únicamente desde el apartado (b). Conceda ***(A1)*(ft)** por el $1{,}09$ y ***(A1)*(ft)** $\times10^4$.
> Conceda ***(A0)(A0)*** por respuestas del tipo: $10{,}9\times10^3$.

[2 puntos]

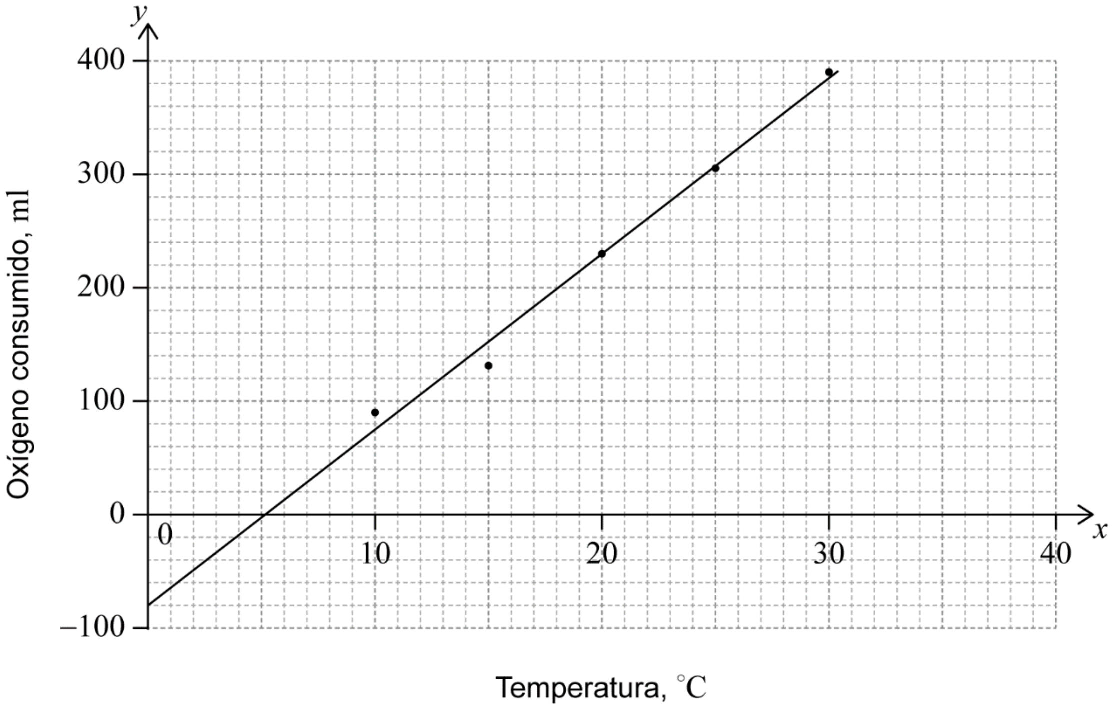
Total [6 puntos]

2. (a) $y = 15,5x - 80$ ***(A1)(A1)*** ***(C2)***

Nota: Conceda ***(A1)*** por el $15,5x$ y ***(A1)*** por el -80. Si la respuesta no es una ecuación, conceda como mucho ***(A1)(A0)***. Conceda ***(A0)(A1)*****(ft)** por responder $y = -80x + 15,5$.

[2 puntos]

(b)



(A1)(A1) ***(C2)***

Nota: Conceda ***(A1)*** por una recta, con regla, que pase por el $(20, 230)$; ***(A1)*** por un punto de corte con el eje Y que sea correcto. Si no se ha utilizado regla, conceda a lo más ***(A0)(A1)***.

[2 puntos]

(c) $a = 10$ **Y** $b = 30$ ***(A1)(A1)*** ***(C2)***

Nota: Acepte $[10, 30]$ o $10 \le x \le 30$.

[2 puntos]

Total [6 puntos]

3. (a) $1200 \times 7,0208$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por multiplicar por $7,0208$.

$8424,96$ (DKK) ***(A1)*** ***(C2)***

[2 puntos]

(b) (i) $0,95 \times 3450$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por multiplicar 3450 por $0,95$ (o equivalente).

$3277,50$ (DKK) ***(A1)*** ***(C2)***

> **Nota:** La respuesta tiene que haberla dado redondeando a dos lugares decimales, a no ser que ya se le haya penalizado por ello en el apartado (a).

(ii) $\dfrac{3277,50}{7,0208}$ ***(M1)***

> **Nota:** Se pueden otorgar puntos de arrastre de error desde el apartado (b)(i). Se pueden otorgar ***(M1)*** por dividir su respuesta del apartado (b)(i) entre $7,0208$.

$466,83$ (USD) ***(A1)*****(ft)** ***(C2)***

> **Nota:** La respuesta tiene que haberla dado redondeando a dos lugares decimales, a no ser que ya se le haya penalizado por ello en el apartado (a) o en el apartado (b)(i).

[4 puntos]

Total [6 puntos]

4. (a) El producto de a y b es divisible entre 9 si y solo si (es equivalente a) a es divisible entre 9 y b es divisible entre 9. ***(A1)(A1)(A1)*** ***(C3)***

O BIEN

El producto de a y b es divisible entre 9 si y solo si a y b son ambos divisibles entre 9. ***(A1)(A1)(A1)*** ***(C3)***

O BIEN

Si el producto de a y b es divisible entre 9 entonces a es divisible entre 9 y b es divisible entre 9 y si a es divisible entre 9 y b es divisible entre 9 entonces el producto de a y b es divisible entre 9. ***(A1)(A1)(A1)*** ***(C3)***

Nota: Conceda ***(A1)*** por el “es equivalente a” o el “si y solo si” o “ssi” que hay entre la proposición simple y la compuesta, ***(A1)*** por el “y” incluido en la proposición compuesta, ***(A1)*** por colocar la s en un lado y $p \wedge q$ en el otro lado de su “es equivalente a”.
Por ejemplo, conceda ***(A1)(A1)(A0)*** por expresar $p \Leftrightarrow (s \wedge q)$ con palabras.

[3 puntos]

(b) Conceda un ***(A1)*** por cada columna que sea correcta. ***(A1)(A1)*(ft)** ***(C2)***

p	q	s	$p \wedge q$	$s \Leftrightarrow (p \wedge q)$
V	V	V	V	V
V	V	F	V	F
V	F	V	F	F
V	F	F	F	V
F	V	V	F	F
F	V	F	F	V
F	F	V	F	F
F	F	F	F	V

Nota: Se pueden otorgar puntos de arrastre de error desde su columna $p \wedge q$ a su columna $s \Leftrightarrow (p \wedge q)$.

[2 puntos]

(c) La última columna contiene ambos tipos de valores: F y V. ***(R1)*** ***(C1)***

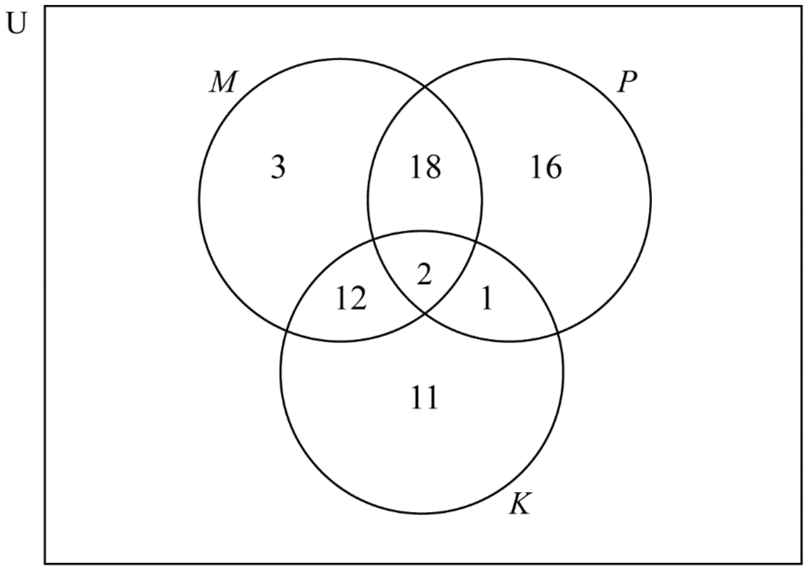
O BIEN

Los valores dados en la columna “s” son distintas de las dadas en la columna “ $p \wedge q$ ”. ***(R1)*** ***(C1)***

[1 punto]

Total [6 puntos]

5. (a)



(A1)(A1) ***(C2)***

> **Nota:** Conceda ***(A1)*** por colocar los números 18, 12 y 1 en el lugar correcto del diagrama de Venn; ***(A1)*** por colocar los números 3, 16 y 11 en el lugar correcto del diagrama de Venn.

[2 puntos]

(b) $85-(3+16+11+18+12+1+2)$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por restarle la suma de sus valores a 85.

22 ***(A1)*(ft)** ***(C2)***

> **Nota:** Se pueden otorgar puntos de arrastre de error desde su diagrama de Venn del apartado (a).
> Si cualquiera de los números que se restan son negativos conceda ***(M1)(A0).***

[2 puntos]

(c) $\frac{14}{35}\left(\frac{2}{5};\ 0,4;\ 40\%\right)$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

> **Nota:** Conceda ***(A1)*** por un numerador correcto; ***(A1)*** por un denominador correcto.
> Se pueden otorgar puntos de arrastre de error desde su diagrama de Venn.

[2 puntos]

Total [6 puntos]

6. (a) la mediana ***(A1)*** ***(C1)***

[1 punto]

(b) (i) $18-12$ ***(A1)***

Nota: Conceda ***(M1)*** si ha mostrado los cuartiles correctos.

6 (g) ***(A1)*** ***(C2)***

(ii) 125 ***(A1)*** ***(C1)***

[3 puntos]

(c) Cafetería 2 ***(A1)*** ***(C1)***

$75\% > 50\%$ (no cumplen el requisito) ***(R1)*** ***(C1)***

O BIEN

$25\% < 50\%$ (cumplen el requisito) ***(R1)*** ***(C1)***

Nota: No conceda ***(A1)(R0)***. Conceda el ***(R1)*** por una comparación correcta (que también puede hacerse con palabras) de los valores porcentuales de las dos cafeterías. El alumno tiene que haber incluido los valores porcentuales o las fracciones. Es posible conceder ***(A0)(R1)***.

[2 puntos]

Total [6 puntos]

7. (a) $x-4,5$ ***(A1)*** ***(C1)***

Nota: Acepte $a=x-4,5$ **O BIEN** ancho $=x-4,5$.

[1 punto]

(b) $2x+2(x-4,5)=111$ (o equivalente) ***(A1)*****(ft)** ***(C1)***

Nota: Se pueden otorgar puntos de arrastre de error desde su expresión del ancho que dieron en el apartado (a).

[1 punto]

(c) $4x-9=111$ (o equivalente) ***(M1)***

Nota: Conceda ***(M1)*** por eliminar correctamente los paréntesis y reunir los términos en x. También vale si esto lo incluyó en el apartado (b).

$(x=)\ 30$ ***(A1)*****(ft)** ***(C2)***

Nota: Se pueden otorgar puntos de arrastre de error desde su ecuación del apartado (b), siempre y cuando $x>4,5$.

[2 puntos]

(d) $\left|\frac{25-30}{30}\right|\times 100\%$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente en la fórmula del porcentaje de error. Acepte un método en dos pasos en el que el "$\times 100$" quede implícito en su respuesta.

$16,7\ (\%)\ (16,6666\ldots)$ ***(A1)*****(ft)** ***(C2)***

Nota: Se pueden otorgar puntos de arrastre de error desde su respuesta del apartado (c). Conceda como mucho ***(M1)(A0)*** si la respuesta final es un valor negativo.

[2 puntos]

Total [6 puntos]

8. (a) (i) $-\frac{3}{4}$ ***(A1)(A1)*** ***(C2)***

> **Nota**: Conceda ***(A1)*** por el $\frac{3}{4}$ $(0{,}75)$; ***(A1)*** si el signo es correcto.
>
> Conceda ***(A1)(A0)*** por $-\frac{3}{4}x$.

(ii) $y = -\frac{3}{4}x + 3$ ***(A1)*(ft)** ***(C1)***

> **Nota:** Se pueden otorgar puntos de arrastre de error desde su valor de la pendiente del apartado (a)(i). La ecuación tiene que ser de la forma $y = mx + c$.

[3 puntos]

(b) (i) $\frac{4}{3}$ ***(A1)*(ft)** ***(C1)***

> **Nota:** Se pueden otorgar puntos de arrastre de error desde su valor de la pendiente del apartado (a)(i).

(ii) $1 = \frac{4}{3} \times 2 + c$ ***(M1)***

O BIEN

$y - 1 = \frac{4}{3}(x - 2)$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por sustituir correctamente su pendiente y sus coordenadas del punto P en la ecuación de la recta.

$y = \frac{4}{3}x - \frac{5}{3}$ ***(A1)*(ft)** ***(C2)***

> **Nota:** Se pueden otorgar puntos de arrastre de error desde el apartado (b)(i). La ecuación tiene que ser de la forma $y = mx + c$, a menos que esto ya se haya penalizado en el apartado (a)(ii).

[3 puntos]

Total [6 puntos]

9. (a) $\tan B = \frac{12}{9}$ ***(A1)(M1)***

Nota:	Conceda ***(A1)*** por incluir el 12, ***(M1)*** por sustituir correctamente en $\tan$ (o equivalente). Acepte métodos equivalentes, como Pitágoras, para hallar BC y sustituir correctamente en otras razones trigonométricas. Si ha incluido $\tan^{-1}\left(\frac{16}{9}\right)$ conceda ***(A0)(M1)(A0)***.

$53{,}1^\circ \ \left(53{,}1301\ldots^\circ\right)$ ***(A1)*** ***(C3)***

Nota:	Si ha utilizado radianes la respuesta es $0{,}927295\ldots$; conceda como mucho ***(A1)(M1)(A0)***.

[3 puntos]

(b) $30\,\mathrm{sen}\,70^\circ + 4$ ***(M1)(M1)***

Nota:	Conceda ***(M1)*** por $\mathrm{sen}\,70^\circ = \frac{x}{30}$ (o equivalente) y ***(M1)*** por sumarle 4.

$32{,}2 \ (32{,}1907\ldots) \ (\mathrm{m})$ ***(A1)*** ***(C3)***

Nota:	Si ha utilizado radianes la respuesta es $27{,}2167\ldots$; conceda como mucho ***(M1)(M1)(A0)***.

[3 puntos]

Total [6 puntos]

10. (a) $x = 3,5$ ***(A1)(A1)*** ***(C2)***

Nota: Conceda ***(A1)*** por el "$x =$ una constante" y ***(A1)*** por incluir el $3,5$.

[2 puntos]

(b) -1 **Y** 8 ***(A1)(A1)*** ***(C2)***

Nota: Conceda ***(A0)(A1)*****(ft)** por responder "-8 y 1".

[2 puntos]

(c) $-4 = a(0+1)(0-8)$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente sus valores del apartado (b) y por sustituir $(0, -4)$ en $f(x)$.

$(a =)\ \dfrac{1}{2}$ ***(A1)*****(ft)** ***(C2)***

Nota: Se pueden otorgar puntos de arrastre de error desde el apartado (b).

[2 puntos]

Total [6 puntos]

11. (a) $A = \{3, 6, 9, 12\}$ **Y** $B = \{2, 4, 6, 8, 10, 12, 14\}$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por enumerar todos los elementos de los conjuntos A y B. También puede verse en el apartado (b). Condone la inclusión de 15 en el conjunto A al otorgar ***(M1)***.

$6, 12$ ***(A1)(A1)*** ***(C3)***

> **Nota:** Conceda un ***(A1)*** por cada elemento que sea correcto. Conceda ***(A1)(A0)*** si ha incluido un valor adicional. Conceda ***(A0)(A0)*** si ha incluido dos o más valores adicionales.

[3 puntos]

(b) (i) $3, 9$ ***(A1)*****(ft)*****(A1)*****(ft)** ***(C2)***

> **Nota:** Se pueden otorgar puntos de arrastre de error desde el apartado (a), pero solo si ha enumerado explícitamente su A y su B. Conceda un ***(A1)*****(ft)** por cada elemento que sea correcto. Conceda ***(A1)(A0)*** si ha incluido un valor adicional. Conceda ***(A0)(A0)*** si ha incluido dos o más valores adicionales.

(ii) 2 ***(A1)*****(ft)** ***(C1)***

> **Nota:** Se pueden otorgar puntos de arrastre de error desde el apartado (b)(i).

[3 puntos]

Total [6 puntos]

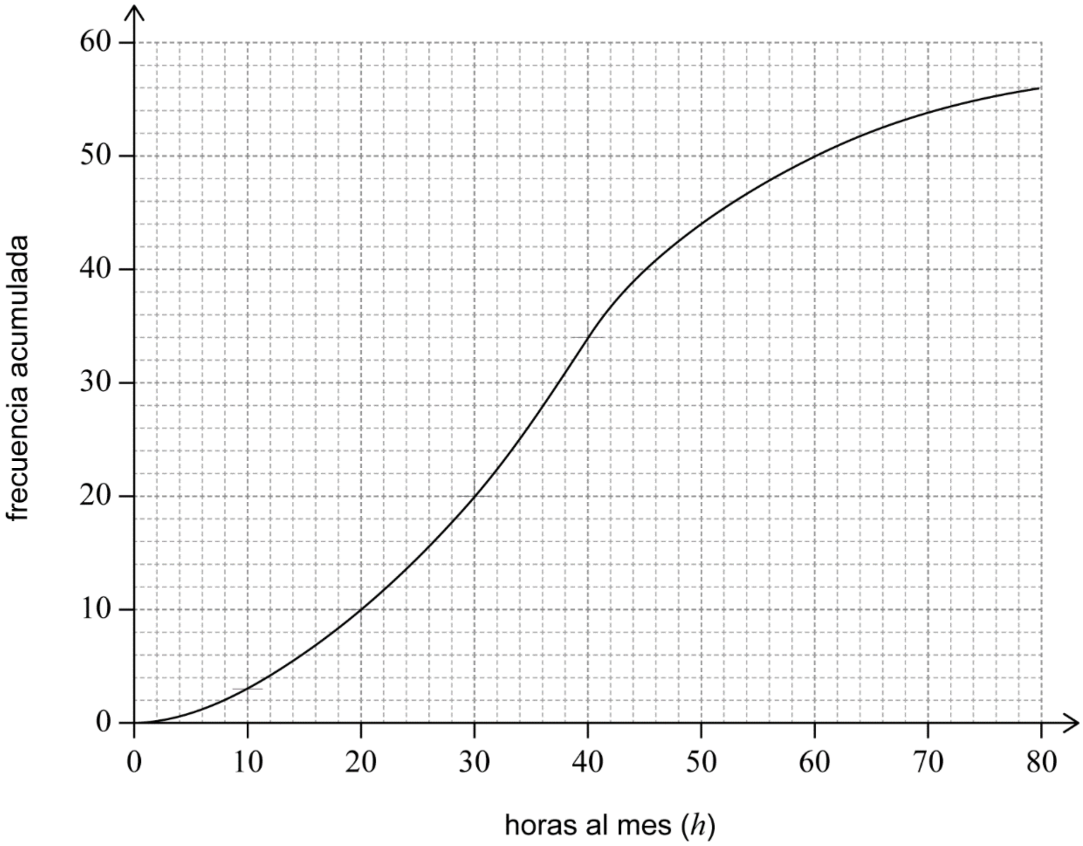
12. (a) (i) $p = 10$ ***(A1)*** ***(C1)***

(ii) $q = 56$ ***(A1)*** ***(C1)***

Nota: Conceda un ***(A1)*** por cada valor que sea correcto.

[2 puntos]

(b)



(A1)(A1) ***(C2)***

Nota: Conceda ***(A1)*****(ft)** si situó correctamente en el gráfico sus tres puntos; conceda ***(A1)*****(ft)** por completar la figura con una curva suave que pase por sus puntos. El segundo ***(A1)*****(ft)** puede ser un arrastre de error a partir de puntos incorrectos, siempre y cuando la pendiente de la curva no sea nunca negativa. Conceda ***(C2)*** por una curva suave totalmente correcta que pase por los puntos correctos.

[2 puntos]

(c) una recta vertical dibujada en 35 (acepte 35 ± 1) ***(M1)***

26 (estudiantes). ***(A1)*** ***(C2)***

Nota: Acepte valores entre 25 y 27 inclusive.

[2 puntos]

Total [6 puntos]

13. (a) $55 = a \times 2^0 + 10$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente en la función el cero **y** el 55.

45 ***(A1)*** ***(C2)***

[2 puntos]

(b) $45 \times 2^{-t} + 10 \le 20$ ***(M1)***

Nota: Conceda ***(M1)*** por comparar expresiones correctas donde aparece el 20 y su 45. Acepte una ecuación.

$t = 2{,}17 \ (2{,}16992\ldots)$ ***(A1)*(ft)** ***(C2)***

Nota: Se pueden otorgar puntos de arrastre de error desde su respuesta al apartado (a), pero solo si el valor es positivo. **La respuesta debe ser en años**, no acepte meses para el ***(A1)*** final.

[2 puntos]

(c) $(m =)\ 10$ ***(A1)***

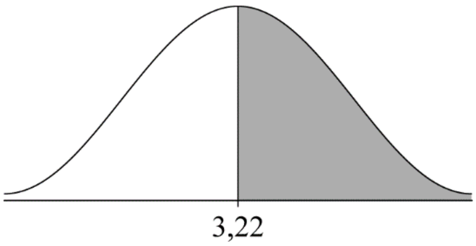
porque a medida que aumenta el número de años, el número de tortugas se aproxima a 10 ***(R1)*** ***(C2)***

Nota: Conceda ***(R1)*** por un bosquejo (dibujo aproximado) que tenga una asíntota aproximadamente en $y = 10$, **O BIEN** por una tabla con valores tales como $10{,}003$ y $10{,}001$ para $t = 14$ y $t = 15$ (por ejemplo), **O BIEN** cuando t se acerca a valores muy grandes, y se acerca a 10. No conceda ***(A1)(R0)***.

[2 puntos]

Total [6 puntos]

14. (a) (i)



(A1) *(C1)*

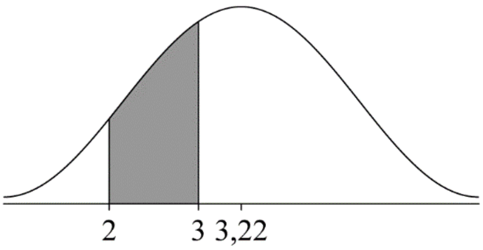
Nota:	Conceda ***(A1)*** por una recta vertical dibujada en la media (no es necesario que se haya incluido el valor $3{,}22$) y por una región sombreada correcta.

(ii) 4,90 *(A1)* *(C1)*

[2 puntos]

(b) 0,323 (0,323499…; 32,3 %) *(A2)* *(C2)*

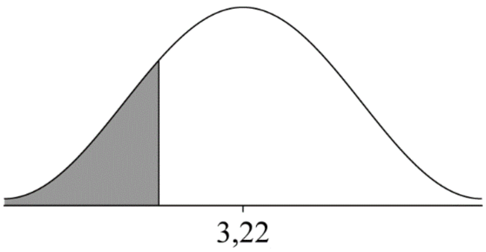
Nota: Si la respuesta final es incorrecta, se puede conceder ***(M1)(A0)*** por mostrar en un bosquejo (dibujado a continuación) una región sombreada que sea correcta, o por un enunciado de probabilidad correcto "$\mathrm{P}(2 \le X \le 3)$" (acepte otras variables para X o para el "precio" y también inecuaciones [desigualdades] estrictas).



[2 puntos]

(c) 2,51 (2,51303…) *(A2)* *(C2)*

Nota: Si la respuesta final es incorrecta, se puede conceder ***(M1)(A0)*** por mostrar en un bosquejo (dibujado a continuación) una región sombreada que sea correcta, o por un enunciado de probabilidad correcto "$\mathrm{P}(X \le a) = 0{,}2$" (acepte otras variables y también una inecuación [desigualdad] estricta).



[2 puntos]

Total [6 puntos]

15. (a) -120 (AUD) ***(A1)*** ***(C1)***

[1 punto]

(b) $-\frac{3}{5}x^2 + 14x$ ***(A1)(A1)*** ***(C2)***

Nota: Conceda un ***(A1)*** por cada término que sea correcto. Conceda como mucho ***(A1)(A0)*** si ha incluido términos adicionales.

[2 puntos]

(c) $-\frac{3}{5}x^2 + 14x = 0$ ***(M1)***

Nota: Conceda ***(M1)*** por igualar su derivada a cero.

O BIEN

por un bosquejo (dibujo aproximado) de su derivada (cuya forma sea aproximadamente correcta) que incluya el punto de corte con el eje X ***(M1)***

$23\frac{1}{3}$ $\left(23{,}3;\ 23{,}3333\ldots;\ \frac{70}{3}\right)$ ***(A1)*(ft)**

Nota: Conceda ***(C2)*** por $23\frac{1}{3}$ $\left(23{,}3;\ 23{,}3333\ldots;\ \frac{70}{3}\right)$ aunque no muestre trabajo.

23 ***(A1)*(ft)** ***(C3)***

Nota: Se pueden otorgar puntos de arrastre de error desde el apartado (b).

[3 puntos]

Total [6 puntos]

Diploma Programme
Programme du diplôme
Programa del Diploma

Estudios matemáticos
Nivel medio
Prueba 2

Martes 14 de mayo de 2019 (mañana)

1 hora 30 minutos

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Para esta prueba, se necesita una copia sin anotaciones del **cuadernillo de fórmulas de estudios matemáticos NM**.
- Conteste todas las preguntas en el cuadernillo de respuestas provisto.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán darse como valores exactos o con una aproximación de tres cifras significativas.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

2219 – 7410

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta. Se recomienda que muestre todos los cálculos, siempre que sea posible. Cuando la respuesta sea incorrecta se otorgarán algunos puntos siempre que aparezca el método empleado y éste sea correcto. Para los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el proceso seguido hasta su obtención. Por ejemplo, cuando deba utilizar un gráfico de una calculadora de pantalla gráfica para hallar soluciones, deberá dibujar esas gráficas en su respuesta.

1. [Puntuación máxima: 15]

En el Instituto Sila hay 110 alumnos. Cada uno de ellos estudia exactamente un idioma; pueden elegir entre inglés, español o chino. La siguiente tabla muestra el número de alumnas y de alumnos que han elegido cada uno de estos idiomas.

		Idioma elegido			
		Inglés	**Español**	**Chino**	**Total**
Sexo del alumno	**Mujer**	25	8	10	43
	Hombre	21	14	32	67
	Total	46	22	42	110

Se llevó a cabo una prueba de χ^2 a un nivel de significación del 5% para analizar la relación que existe entre el sexo del alumno y el idioma que decide estudiar.

(a) Escriba la hipótesis nula (H_0), para esta prueba. [1]

(b) Indique el número de grados de libertad. [1]

(c) Utilizando la calculadora de pantalla gráfica, escriba

(i) la frecuencia esperada de alumnas (mujeres) que deciden estudiar chino;

(ii) el estadístico χ^2. [3]

El valor crítico para esta prueba, a un nivel de significación del 5%, es igual a $5{,}99$.

(d) Indique si H_0 se debería (o no) rechazar. Justifique su respuesta. [2]

(e) Se elige al azar a un alumno (hombre o mujer) de este instituto.

(i) Halle la probabilidad de que el alumno o la alumna no esté estudiando español.

Se elige al azar a otro alumno (hombre o mujer) de este instituto.

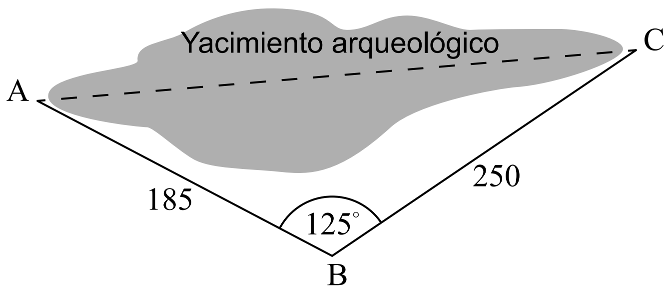
(ii) Halle la probabilidad de que ninguno de los dos alumnos (o alumnas) estén estudiando español.

(iii) Halle la probabilidad de que al menos uno de los dos alumnos sea mujer. [8]

2. [Puntuación máxima: 13]

Un yacimiento arqueológico se quiere hacer accesible para poder abrirlo al público. Para ello, los arqueólogos han hecho dos caminos rectos que van desde el punto A al punto B y desde el punto B al punto C, tal y como se muestra en la siguiente figura. El camino AB tiene una longitud de $185\ \mathrm{m}$, el camino BC tiene una longitud de $250\ \mathrm{m}$ y el ángulo $\mathrm{A\hat{B}C}$ mide 125°.

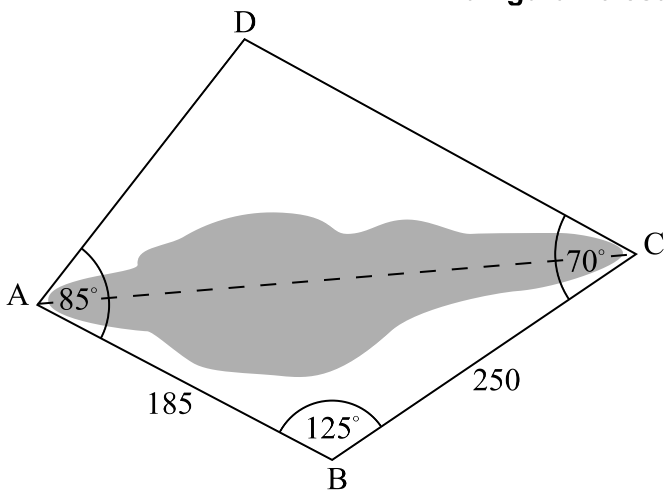
la figura no está dibujada a escala



(a) Halle la distancia que hay entre A y C. [3]

Los arqueólogos tienen previsto hacer otros dos caminos rectos: AD y DC. Para que los caminos rodeen al yacimiento, el ángulo $\mathrm{B\hat{A}D}$ se va a hacer igual a 85° y el ángulo $\mathrm{B\hat{C}D}$ se va a hacer igual a 70°, tal y como se muestra en la siguiente figura.

la figura no está dibujada a escala



(b) Halle el valor de

(i) el ángulo $\mathrm{B\hat{A}C}$;

(ii) el ángulo $\mathrm{C\hat{A}D}$. [4]

(c) Halle el valor del ángulo $\mathrm{A\hat{C}D}$. [2]

La longitud del camino AD es igual a $287\,\mathrm{m}$.

(d) Halle el área de la región ABCD. [4]

3. [Puntuación máxima: 15]

Una fábrica envasa agua de coco en recipientes con forma de cono, donde el radio de la base es de $5,2\,\text{cm}$ y la altura es igual a $13\,\text{cm}$.

(a) Halle el volumen de uno de estos recipientes con forma de cono. [2]

(b) Halle la generatriz de este recipiente con forma de cono. [2]

(c) Muestre que el área total de la superficie de este recipiente con forma de cono es igual a $314\,\text{cm}^2$, redondeando a tres cifras significativas. [3]

Los diseñadores de la fábrica están investigando si es posible sustituir los recipientes con forma de cono por recipientes con forma de cilindro que tengan el mismo radio y la misma superficie total.

(d) Halle la altura (h) de uno de estos recipientes con forma de cilindro. [4]

El director de la fábrica quiere aumentar el volumen de agua de coco que se vende con cada recipiente.

(e) Indique si debería (o no) sustituir los recipientes con forma de cono por recipientes con forma de cilindro. Justifique su respuesta. [4]

4. [Puntuación máxima: 16]

El primer día que está en el hospital, a Kiri le dan u_1 miligramos (mg) de un medicamento determinado. La cantidad de medicamento que le dan a Kiri va aumentando cada día, siempre en la misma cantidad (d). El séptimo día le dan $21\,\text{mg}$ de medicamento y el undécimo día le dan $29\,\text{mg}$.

(a) Escriba una ecuación, en función de u_1 y d, que represente la cantidad de medicamento que le dan

(i) el séptimo día;

(ii) el undécimo día. [2]

(b) Escriba el valor de d y el valor de u_1. [2]

Kiri toma este medicamento durante 30 días.

(c) Calcule la cantidad total del medicamento (en mg) que habrá tomado. [3]

Ted también está ingresado en el hospital y el primer día le ponen una inyección de $20\,\text{mg}$ de antibiótico. La cantidad de antibiótico que le dan a Ted va disminuyendo un $50\,\%$ cada día. Es decir, el segundo día a Ted le ponen una inyección de $10\,\text{mg}$ de antibiótico, el tercer día le ponen $5\,\text{mg}$, y así sucesivamente.

(d) (i) Halle la cantidad de antibiótico (en mg) que le ponen a Ted el quinto día.

(ii) La cantidad diaria de antibiótico que le ponen a Ted será por primera vez inferior a $0{,}06\,\text{mg}$ el $k.^{o}$ día. Halle el valor de k.

(iii) A partir de lo anterior, halle la cantidad total de antibiótico (en mg) que le habrán dado a Ted durante esos k primeros días. [9]

5. [Puntuación máxima: 20]

Considere la función $f(x)=\frac{1}{3}x^3+\frac{3}{4}x^2-x-1$.

(a) Halle $f(2)$. [2]

(b) Escriba el punto de corte con el eje y del gráfico de $y=f(x)$. [1]

(c) Dibuje aproximadamente el gráfico de $y=f(x)$ para $-3\le x\le 3$ y $-4\le y\le 12$. [4]

(d) Halle $f'(x)$. [3]

(e) Halle la pendiente del gráfico de $y=f(x)$ en $x=2$. [2]

(f) Halle la ecuación de la recta tangente al gráfico de $y=f(x)$ en $x=2$. Dé la ecuación de la forma $ax+by+d=0$ donde a, b y $d\in\mathbb{Z}$. [2]

La función tiene un máximo local en $x=p$ y un mínimo local en $x=q$.

(g) Utilice la derivada de $f(x)$ para hallar el valor de p y el valor de q. [3]

(h) Determine el recorrido (imagen) de $f(x)$ para $p\le x\le q$. [3]

6. [Puntuación máxima: 11]

Tommaso tiene previsto competir en una carrera ciclista regional cuando acabe el instituto, pero para eso necesita comprarse una bicicleta de carreras. Encuentra una bicicleta que cuesta 1100 euros (EUR). Tommaso tiene 950 EUR y decide invertir este dinero en una cuenta que paga al año un 5% de interés, **compuesto mensualmente**.

(a) Determine la cantidad que tendrá en esta cuenta al cabo de 3 años. Dé la respuesta redondeando a dos lugares decimales. [3]

El costo (C) de la bicicleta se puede modelizar mediante la expresión $C = 20x + 1100$, donde x es el número de años que han transcurrido desde que Tommaso decidió invertir su dinero.

(b) Halle la diferencia entre el costo de la bicicleta y la cantidad de dinero que tiene Tommaso en la cuenta al cabo de 3 años. Dé la respuesta redondeando a dos lugares decimales. [3]

Cuando hayan transcurrido m **meses** completos Tommaso tendrá en la cuenta, por primera vez, suficiente dinero para comprarse la bicicleta.

(c) Halle el valor de m. [5]

Esquema de calificación

Mayo de 2019

Estudios matemáticos

Nivel medio

Prueba 2

Instrucciones para los examinadores

Observaciones: En caso de dudas sobre estas instrucciones o temas relacionados con la asignación de puntuaciones, póngase en contacto con su jefe de equipo.

1 Siglas

M Puntos otorgados por el **método**
A Puntos otorgados por una **respuesta** o por la **precisión**
R Puntos otorgados por un **razonamiento** claro
G Puntos otorgados por soluciones correctas obtenidas mediante la **calculadora de pantalla gráfica**, independientemente del trabajo mostrado.
AG **Respuesta incluida** en la pregunta y, en consecuencia; no se otorgan puntos.
ft Se pueden otorgar puntos por la **coherencia** con resultados previos en la pregunta.

2 Método para corregir

(a) Todas las correcciones deben realizarse en RM Assessor, utilizando las anotaciones de Estudios Matemáticos y de acuerdo con el documento vigente de orientaciones para la corrección electrónica de Estudios Matemáticos NM. Es fundamental leer el documento antes de empezar a corregir.

(b) Si un apartado de una pregunta es del todo correcto use las anotaciones tic con números para otorgar la puntuación máxima. Si un apartado es completamente erróneo use la nota ***A0***, de lo contrario se deben mostrar todas las anotaciones.

(c) No se debe otorgar ningún punto al trabajo tachado por el alumno.

(d) Si el alumno ha dado dos respuestas a una pregunta, solo se debe corregir la primera respuesta.

(e) Si el procedimiento adecuado lleva a la respuesta correcta, pero esta va seguida de más desarrollos que indican una falta de comprensión matemática, entonces **no** se puede otorgar la puntuación máxima. En la mayoría de estos casos se perderá solo el último punto por respuesta. Una excepción podría ser las respuestas numéricas, donde una respuesta exacta que es correcta está seguida de una expresión decimal incorrecta.

Ejemplo:

	Se ve la respuesta correcta	**Hay más desarrollo**	**Acción**
1.	$8\sqrt{2}$	$5{,}65685...$ *(valor decimal incorrecto)*	Otorgue el ultimo ***(A1)*** *(ignore el desarrollo posterior)*
2.	$(x-6)\,(x+1)$	$x=6$ and -1	**No** otorgue el último ***(A1)***

Ejemplo: Calcule la pendiente de la recta que pasa por los puntos $(5;3)$ y $(0;9)$.

Esquema de calificación	**Examen del alumno**	**Corrección**
$\frac{9-3}{0-5}$ ***(M1)*** *Otorgue (M1) por la sustitución correcta en la fórmula de la pendiente* $=-\frac{6}{5}$ ***(A1)***	(i) $\frac{9-3}{0-5}=-\frac{6}{5}$ La pendiente es $=-\frac{6}{5}$ *(Existe una comprensión clara de la pendiente.)* $y=-\frac{6}{5}x+9$	***(M1)*** ***(A1)***
	(ii) $\frac{9-3}{0-5}=-\frac{6}{5}$ $y=-\frac{6}{5}x+9$ *(Existe confusión sobre lo requerido.)*	***(M1)*** ***(A0)***

3 Puntos por la coherencia (ft)

En cualquier paso de una resolución, un error puede afectar a todo el trabajo posterior. Para limitar la gravedad de la penalización, se pueden otorgar puntos por la **coherencia (ft)**. Los esquemas de calificación indicarán, con **"(ft)"**, dónde es adecuado aplicar el criterio de coherencia en una pregunta.

(a) La coherencia se aplica solo de un apartado de una pregunta a otro apartado posterior de la pregunta. No se aplica dentro del mismo apartado.

(b) Si una respuesta que es consecuencia de un procedimiento de coherencia es muy poco realista (por ejemplo, distancias negativas o de un orden de magnitud excesivo) entonces el punto ***A*** final no se debe otorgar.

(c) Si, debido a un error, una pregunta queda transformada en otra **distinta**, **mucho más sencilla**, entonces el criterio de coherencia podría no ser aplicado.

(d) Para otorgar puntos por coherencia en un apartado de una pregunta, **es necesario que haya un trabajo explícito relacionado con ese apartado**. Una respuesta aislada, obtenida por coherencia, pero sin un desarrollo explícito, se considera incorrecta, y no recibirá ningún punto **aunque se aproxime a la respuesta correcta**.

(e) Una excepción a lo anterior sería una pregunta donde lo que se evalúa es el manejo del alumno en el uso de la calculadora de pantalla gráfica, y donde no se pretende que se muestre ningún desarrollo escrito. **En el esquema de calificación vendrá indicado claramente cuando sea el caso.**

(f) El uso inadvertido de radianes será penalizado la primera vez que aparezca. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo un punto por el uso de radianes.

Ejemplo: Cálculo de ángulos y longitudes en trigonometría

Esquema de calificación	Examen del alumno	Corrección
(a) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ ***(M1)(A1)*** *Otorgue **(M1)** por la sustitución en el teorema del seno, **(A1)** por las sustituciones correctas.*	(a) $\frac{\text{sen } A}{4} = \frac{\text{sen } 30}{3}$	***(M1)(A0)*** *(uso del teorema del seno, pero con valores incorrectos)*
$A = 22{,}0^\circ\ (22{,}0243\ldots)$ ***(A1)(G2)***	$A = 41{,}8^\circ$	***(A0)*** (***Observación****: Aquí, el segundo **(A1)** no ha sido corregido como* **(ft)** *y no se puede otorgar porque existe un error previo en el **mismo** apartado de la pregunta.*)
(b) $x = 7\text{tg}\,(22{,}0243\ldots^\circ)$ ***(M1)*** $= 2{,}83\ (2{,}83163\ldots)$ ***(A1)*(ft)**	(b) caso (i) $x = 7\text{tg}\,41{,}8^\circ$ $= 6{,}26$ ***pero*** caso (ii) $6{,}26$	***(M1)*** ***(A1)*(ft)** ***(G0)*** *pues no aparece un desarrollo explícito*

4 Uso del Esquema de calificación

(a) Los puntos ***A* dependen** de los puntos ***M*** obtenidos previamente, **no** es posible otorgar ***(M0)(A1)***. Una vez que se otorga un ***(M0)***, se pierden todos los puntos ***A*** que le siguen en ese apartado de la pregunta, incluso si los cálculos son correctos, hasta el siguiente punto ***M***.
La única excepción a lo anterior se hará en el caso de una respuesta donde la precisión se especifique en la pregunta (ver apartado 5).

(b) Los puntos ***A* dependen** de los puntos ***R*** obtenidos, **no** es posible otorgar ***(A1)(R0)***. Así pues, el ***(A1)*** no se puede otorgar a una respuesta que sea correcta cuando no aparece el razonamiento, o este es incorrecto.

(c) En la prueba 2 se espera que los alumnos demuestren su destreza en la comunicación matemática mediante el uso de desarrollos adecuados. Las respuestas que sean correctas, pero no se basen en un desarrollo adecuado **no siempre recibirán la puntuación máxima**. Estas respuestas sin desarrollo que las sustente vienen designadas por ***G*** en el esquema de calificación, como una alternativa a la puntuación máxima. Ejemplo ***(M1)(A1)(A1)(G2).***

Ejemplo: Uso de la trigonometría para el cálculo de un ángulo de un triángulo.

Esquema de calificación	Examen del alumno	Corrección
(a) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ ***(M1)(A1)*** *Otorgue **(M1)** por la sustitución en el teorema del seno, **(A1)** por las sustituciones correctas.* $A = 22{,}0^\circ$ $(22{,}0243\ldots)$ ***(A1)(G2)***	(i) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ $A = 22{,}0^\circ$ (ii) $A = 22{,}0^\circ$ ***Observación:*** *Los puntos **G** se utilizan solo si no se muestra ningún desarrollo, pero la respuesta es correcta.*	***(M1)(A1)*** ***(A1)*** ***(G2)***

(d) Los **métodos alternativos** no siempre se incluyen. Así, si una respuesta es errónea, entonces se debe analizar cuidadosamente el procedimiento, de modo que se asignen puntos por un método distinto de forma coherente con el esquema de calificación.
Cuando en el esquema de calificación se incluyen métodos alternativos para una pregunta, ello viene indicado mediante "**O**" *etc*.

(e) A menos que en la pregunta se especifique lo contrario, se aceptan **expresiones equivalentes**.
Por ejemplo: $\frac{\text{sen } \theta}{\cos \theta}$ por $\text{tg } \theta$.

En el esquema de calificación, estas expresiones algebraica o numéricamente equivalentes aparecerán a veces escritas entre paréntesis junto a la respuesta requerida.
Cuando se requieran respuestas numéricas como respuesta final de un apartado de una pregunta, en el esquema de calificación se mostrará en este orden:
la respuesta con 3 cifras significativas a partir de la solución completa de la calculadora;
el valor exacto (por ejemplo $\frac{2}{3}$ si corresponde);
la solución completa de la calculadora en la forma $2{,}83163\ldots$ como en el ejemplo anterior.
Cuando se dan las respuestas con 3 cifras significativas y se utilizan después en apartados posteriores de la pregunta llevando a una solución con 3 cifras significativas **distinta**, también se darán estas soluciones.

(f) Dado que este es un examen internacional, se aceptarán todas las **formas de notación alternativas**. Algunos ejemplos:

Comas decimales: 1,7; 1’7; $1 \cdot 7$; 1,7.

Los números decimales menores que 1 podrían aparecer escritos sin un cero en el frente: 0.49 o .49.

Distintas descripciones de un intervalo: $3 < x < 5$; (3; 5);] 3; 5 [.

Distintas formas de notación de las propiedades de los conjuntos (por ejemplo complementario): A'; $\overline{A}$; A^c; $U - A$;$(A$; $U \setminus A$.

Distintas formas de notación lógica: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p$.
$p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

El nivel de significación podría escribirse como α .

(g) Puntos discrecionales: habrá ocasiones excepcionales en las que el esquema de calificación no cubra el trabajo que aparece. En estos casos se utilizará la nota DM para indicar que el examinador ha utilizado su criterio. La discrecionalidad debe utilizarse con moderación, y si existe duda se debe plantear una excepción a través de RM Assessor al jefe del equipo.

A partir de noviembre de 2011 no habrá una única penalización por prueba por precisión AP, precisión financiera FP y unidades UP. En lugar de ello, estas destrezas serán evaluadas en preguntas específicas y los puntos se otorgarán de acuerdo a lo especificado en los apartados 5, 6 y 7.

5 Precisión de las respuestas

Una precisión incorrecta debe ser penalizada una sola vez en cada pregunta de acuerdo a las siguientes reglas.

Instrucciones: Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas correctamente a 3 cifras significativas.

1. Si el alumno muestra la solución a 4 o más cifras significativas, la cual redondeada a 3 cifras significativas daría la respuesta requerida entonces otorgue ***(A1)*** e ignore cualquier redondeo posterior.
Observación: La solución sin redondear puede estar tanto en la línea de respuesta como en el cuadro de trabajo.

2. Si el alumno **no** muestra la solución sin redondear, entonces otorgue ***(A1)*** si la solución dada está aproximada **correctamente** a 2 o más cifras significativas, y ***(A0)*** en cualquier otro caso.

3. Si una solución aproximada a 2 cifras significativas se utiliza en apartados posteriores, entonces para otorgar más puntos debe aparecer el desarrollo. (Este criterio es el mismo que en el que se aplica coherencia a partir de una respuesta errónea.)

Estos 3 casos (vea los supra índices) se han resumido en la siguiente tabla e ilustrado después con ejemplos.

	Si la solución final de alumno se expresa…				
	Exacta o a 4 o más cifras significativas (y que redondeada **a 3 cifras significativas daría la respuesta correcta**)	**Aproximada incorrectamente a 3 cifras significativas**	Aproximada correctamente a 2 cifras significativas[3]	Aproximada incorrectamente a 2 cifras significativas	Aproximada correcta o incorrectamente a 1 cifra significativa
Aparece la solución sin redondear[1]	Otorgue ***(A1)*** independientemente del redondeo correcto o incorrecto				
No aparece la solución sin redondear[2]	***(A1)***	***(A0)***	***(A1)***	***(A0)***	***(A0)***
Criterio para los apartados posteriores	Como en MS	Como en el criterio de coherencia, solo si se muestra el desarrollo[3]			

Ejemplos:

Esquema de calificación	Examen del alumno	Corrección
$9{,}43\ (9{,}43398\ldots)$ ***(A1)***	(i) $9{,}43398\ldots$ se muestra en el cuadro de trabajo seguido de 9; 9,4; 9,43; 9,434 (redondeados correctamente) ***(A1)***	
	(ii) $9{,}43398\ldots$ se muestra en el cuadro de trabajo seguido de 9,433; 9,44, *etc.* (redondeados incorrectamente) ***(A1)***	
	(iii) 9,4 ***(A1)***	
	(iv) 9 ***(A0)***	(aproximado correctamente a 1 cifra significativa)
	(v) 9,3 ***(A0)***	(aproximado incorrectamente a 2 cifras significativas)
	(vi) 9,44 ***(A0)***	(aproximado incorrectamente a 3 cifras significativas)

Esquema de calificación	Examen del alumno	Corrección
$7{,}44\ (7{,}43798\ldots)$ ***(A1)***	(i) $7{,}43798\ldots$ se muestra en el cuadro de trabajo seguido de 7; 7,4; 7,44; 7,438 *etc.* (redondeados correctamente) ***(A1)***	
	(ii) $7{,}43798\ldots$ se muestra seguido de 7,437; 7,43 *etc.* (redondeados incorrectamente) ***(A1)***	
	(iii) 7,4 ***(A1)***	
	(iv) 7 ***(A0)***	(aproximado correctamente a 1 cifra significativa)
	(v) 7,5 ***(A0)***	(aproximado incorrectamente a 2 cifras significativas)
	(vi) 7,43 ***(A0)***	(aproximado incorrectamente a 3 cifras significativas)

Ejemplo: ABC es un triángulo rectángulo con el ángulo $ABC = 90^\circ$, $AC = 32$ cm y $AB = 30$ cm. Halle (a) la longitud de BC, (b) el área del triángulo ABC.

Esquema de calificación	Examen del alumno	Corrección
(a) $BC = \sqrt{32^2 - 30^2}$ ***(M1)*** *Otorgue **(M1)** por la sustitución correcta en el teorema de Pitágoras* $= 11{,}1\left(\sqrt{124}; 11{,}1355...\right)(\text{cm})$ ***(A1)***	(a) $BC = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$ *(solo se muestra la aproximación a 2 cifras significativas, pero correcta)*	***(M1)*** ***(A1)***
(b) $\text{Área} = \frac{1}{2} \times 30 \times 11{,}1355...$ ***(M1)*** *Otorgue **(M1)** por la sustitución correcta en la fórmula del área de un triángulo* $= 167(167{,}032...)(\text{cm}^2)$ ***(A1)*(ft)**	(b) caso (i) $\text{Área} = \frac{1}{2} \times 30 \times 11$ *(se muestra el desarrollo)* $= 165\,(\text{cm}^2)$ caso (ii) $= 165\,(\text{cm}^2)$ *(No se muestra el desarrollo, la solución* 11 *se trata como procedimiento de coherencia, por lo que no se deben otorgar puntos en este caso)*	***(M1)*** ***(A1)*(ft)** ***(M0)(A0)*(ft)**

La aproximación a 3 cifras significativas de una solución exacta **se debe aceptar si se realiza correctamente**. Las soluciones exactas del tipo $\frac{1}{4}$ se pueden escribir como decimales con menos de 3 cifras significativas si el resultado sigue siendo exacto. La simplificación de una fracción a su expresión irreducible **no** es imprescindible. Las fracciones que contienen un numerador y/o un denominador decimal se aceptan para mostrar sustituciones pero no como respuesta final.

Razones de π y respuestas con expresiones de raíces cuadradas de enteros o cualquier potencia racional de un entero (por ejemplo, $\sqrt{13}, 2^{2/3}, \sqrt[4]{5}$), se pueden aceptar como respuestas exactas. Todas las otras potencias (por ejemplo de no enteros) y valores de funciones transcendentes tales como seno y coseno se deben evaluar.

Si el nivel de precisión viene especificado en la pregunta, se asignarán puntos por dar la respuesta con la precisión requerida. En **todos** estos casos no se obtiene el punto final si el redondeo no sigue las instrucciones dadas en la pregunta. El punto por la precisión especificada se puede considerar como punto por coherencia **(ft)** con independencia de un ***(M0)*** inmediatamente anterior.

6 Nivel de precisión en las preguntas sobre cuestiones financieras

El nivel de precisión de las respuestas se especificará en todas las preguntas que se relacionen con valores en dinero. Generalmente será a la unidad más cercana o a dos cifras decimales. La primera respuesta que no venga dada con el nivel de precisión especificado no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la precisión incorrecta en una pregunta sobre cuestiones financieras.

Ejemplo: En una pregunta sobre cuestiones financieras se pide una precisión de 2 cifras decimales.

Esquema de calificación	Examen del alumno	Corrección
$\$231{,}62\,(231{,}6189)$ ***(A1)***	(i) $231{,}6$	***(A0)***
	(ii) 232	***(A0)***
	(Aproximación correcta con un nivel de precisión distinto del requerido)	
	(iii) $231{,}61$	***(A0)***
	(iv) $232{,}00$	***(A0)***
	(Aproximación incorrecta al nivel de precisión requerido)	

7 Unidades de medida en las respuestas

En determinadas preguntas será necesario expresar las unidades de medida, y esto vendrá claramente indicado en el esquema de calificación. La primera respuesta correcta pero sin unidades o con unidades incorrectas no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la falta de unidades o unidades incorrectas.

Las unidades de medida se tendrán en cuenta solamente cuando se ha otorgado ***(A1)*** a la respuesta numérica siguiendo las indicaciones sobre la precisión dadas en el apartado 5.

Ejemplo:

Esquema de calificación	Examen del alumno	Corrección
(a) $37000\ \text{m}^2$ ***(A1)***	(a) $36000\ \text{m}^2$	***(A0)***
	(respuesta incorrecta, por lo que no se consideran las unidades)	
(b) $3200\ \text{m}^3$ ***(A1)***	(b) $3200\ \text{m}^2$	***(A0)***
	(unidades incorrectas)	

8 Calculadoras de pantalla gráfica

Con frecuencia los alumnos van a obtener las soluciones directamente de la calculadora. Deben utilizar la notación matemática y no la notación de la calculadora. No se puede otorgar puntos por método a respuestas incorrectas basadas únicamente en notación de calculadora. No se puede otorgar puntos por método al comentario “usé la calculadora de pantalla gráfica”.

1. (a) (H_0:) el idioma (elegido/que decide estudiar) es independiente del sexo (del alumno)

(A1)

Nota:	Acepte “no existe ninguna relación entre el idioma (elegido/que decide estudiar) y el sexo (del alumno)”. Acepte “el idioma (elegido/que decide estudiar) no depende del sexo (del alumno)”. No acepte “no está relacionado” o “no está correlacionado” o “no influye”.

[1 punto]

(b) 2 ***(A1)***

[1 punto]

(c) (i) $16{,}4\ (16{,}4181\ldots)$ ***(G1)***

(ii) $\chi^2_{\text{calc}} = 8{,}69\ (8{,}68507\ldots)$ ***(G2)***

[3 puntos]

(d) (Nosotros) rechazamos la hipótesis nula ***(A1)*(ft)**

$8{,}68507\ldots > 5{,}99$ ***(R1)*(ft)**

Nota:	Puede otorgar puntos de arrastre de error desde el subapartado (c)(ii). Acepte “no aceptamos/no se debería aceptar” en lugar de “rechazamos”. No conceda ***(A1)*(ft)*(R0)***.

O BIEN

(Nosotros) rechazamos la hipótesis nula ***(A1)***

$0{,}0130034 < 0{,}05$ ***(R1)***

Nota:	Acepte “no aceptamos/no se debería aceptar” en lugar de “rechazamos”. No conceda ***(A1)*(ft)*(R0)***.

[2 puntos]

continúa en la página siguiente…

Continuación de la Pregunta 1

(e) (i) $\frac{88}{110}\left(\frac{4}{5};\ 0{,}8;\ 80\%\right)$ ***(A1)(A1)(G2)***

Nota: Conceda ***(A1)*** por un numerador correcto; ***(A1)*** por un denominador correcto.

(ii) $\frac{88}{110}\times\frac{87}{109}$ ***(M1)(M1)***

Nota: Conceda ***(M1)*** por haber multiplicado dos fracciones. Conceda ***(M1)*** por haber multiplicado sus fracciones y que éstas fueran correctas.

O BIEN

$\left(\frac{46}{110}\right)\left(\frac{45}{109}\right)+2\left(\frac{46}{110}\right)\left(\frac{42}{109}\right)+\left(\frac{42}{110}\right)\left(\frac{41}{109}\right)$ ***(M1)(M1)***

Nota: Conceda ***(M1)*** por los productos correctos; ***(M1)*** por la suma de cuatro productos.

$0{,}639\ \left(0{,}638532\ldots;\ \frac{348}{545};\ 63{,}9\%\right)$ ***(A1)*****(ft)*****(G2)***

Nota: Puede otorgar puntos de arrastre de error desde la respuesta dada en el subapartado (e)(i).

(iii) $1-\frac{67}{110}\times\frac{66}{109}$ ***(M1)(M1)***

Nota: Conceda ***(M1)*** por multiplicar dos fracciones correctas. Conceda ***(M1)*** por restarle a 1 su producto de esas dos fracciones.

O BIEN

$\frac{43}{110}\times\frac{42}{109}+\frac{43}{110}\times\frac{67}{109}+\frac{67}{110}\times\frac{43}{109}$ ***(M1)(M1)***

Nota: Conceda ***(M1)*** si todos los productos son correctos y ***(M1)*** por sumar tres productos.

$0{,}631\ \left(0{,}631192\ldots;\ 63{,}1\%;\ \frac{344}{545}\right)$ ***(A1)(G2)***

[8 puntos]
Total [15 puntos]

2. (a) $\text{AC}^2 = 185^2 + 250^2 - 2 \times 185 \times 250 \times \cos(125^\circ)$ ***(M1)(A1)***

> **Nota:** Conceda ***(M1)*** por sustituir los valores en la fórmula del coseno; ***(A1)*** por haberlos sustituido correctamente.

$387\ (\text{m})\ (387{,}015\ldots)(\text{m})$ ***(A1)(G2)***

> **Nota:** Si ha utilizado radianes la respuesta es $154\ (154{,}471\ldots)$; en ese caso conceda como mucho ***(M1)(A1)(A0)***.

[3 puntos]

(b) (i) $\dfrac{250}{\text{sen B}\hat{\text{A}}\text{C}} = \dfrac{387{,}015\ldots}{\text{sen}(125^\circ)}$ ***(M1)(A1)*****(ft)**

> **Nota:** Conceda ***(M1)*** por sustituir los valores en la fórmula del seno; ***(A1)*****(ft)** por haberlos sustituido correctamente.

O BIEN

$\cos^{-1}\left(\dfrac{185^2 + (387{,}015...)^2 - 250^2}{2 \times 185 \times 387{,}015...}\right)$ ***(M1)(A1)(ft)***

> **Nota:** Conceda ***(M1)*** por sustituir los valores en la fórmula del seno; ***(A1)*****(ft)** por haberlos sustituido correctamente.

$\text{B}\hat{\text{A}}\text{C} = 31{,}9^\circ\ (31{,}9478\ldots^\circ)$ ***(A1)*****(ft)*****(G2)***

> **Nota:** Puede otorgar puntos de arrastre de error desde el apartado (a).

(ii) $(\text{C}\hat{\text{A}}\text{D} =)\ 53{,}1^\circ\ (53{,}0521\ldots^\circ)$ ***(A1)*****(ft)**

> **Nota:** Puede otorgar puntos de arrastre de error desde el apartado (b)(i) solo si ha incluido el desarrollo para obtener la respuesta.

[4 puntos]

continúa en la página siguiente…

Continuación de la Pregunta 2

(c) $(\hat{ACD} =)\ 70^\circ - (180^\circ - 125^\circ - 31,9478^\circ \ldots)$ ***(M1)***

Nota:	Conceda ***(M1)*** por restarle a 70° su ángulo $\hat{ACB}$.

O BIEN

$$(\hat{ADC} =)\ 360 - (85 + 70 + 125) = 80$$

$$(\hat{ACD} =)\ 180 - (80 + 53,0521...)$$

$46,9^\circ\ (46,9478\ldots^\circ)$ ***(A1)*(ft)*(G2)***

Nota:	Puede otorgar puntos de arrastre de error desde el apartado (b)(i).

[2 puntos]

(d) $\frac{185 \times 250 \times \text{sen}\,(125^\circ)}{2} + \frac{287 \times 387,015\ldots \times \text{sen}\,(53,0521\ldots^\circ)}{2}$ ***(M1)(M1)(M1)***

Nota:	Conceda ***(M1)*** por sustituir los valores de uno de los dos triángulos en la fórmula del área; ***(M1)*** por sustituir correctamente los valores de ambos triángulos; ***(M1)*** por sumar las dos áreas resultantes.

$18942,8\ldots + 44383,97\ldots$

$63300\ (\text{m}^2)\ (63326,8...\ (\text{m}^2))$ ***(A1)*(ft)*(G3)***

Nota:	Puede otorgar puntos de arrastre de error desde el apartado (a) y el subapartado (b)(ii).

O BIEN

$$\text{DC} = \frac{287 \times \text{sen}\,(53,0521...)}{\text{sen}\,(46,9478...)} = 313,884...$$

$0,5 \times 287 \times 185 \times \text{sen}\,85^\circ + 0,5 \times 250 \times 313,884... \times \text{sen}\,70^\circ$ ***M1M1M1***

Nota:	Conceda ***(M1)*** por sustituir los valores de uno de los dos triángulos en la fórmula del área; ***(M1)*** por sustituir correctamente los valores de ambos triángulos; ***(M1)*** por sumar las dos áreas resultantes.

$26446,4... + 36869,3...$

$63300\ \ (63315,8\ldots)\ \ (\text{m}^2)$ ***(A1)*(ft)*(G3)***

[4 puntos]

Total [13 puntos]

3. (a) $\frac{\pi(5,2)^2 \times 13}{3}$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente los valores en la fórmula del volumen de un cono.

$368\ (368{,}110\ldots)\ \text{cm}^3$ ***(A1)(G2)***

Nota: Acepte $117{,}173...\pi\ \text{cm}^3$ o $\frac{8788}{75}\pi\ \text{cm}^3$.

[2 puntos]

(b) $(\text{generatriz}^2) = (5,2)^2 + 13^2$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente los valores en la fórmula.

$14{,}0\ (14{,}0014\ldots)\ (\text{cm})$ ***(A1)(G2)***

[2 puntos]

(c) $14{,}0014... \times (5,2) \times \pi + (5,2)^2 \times \pi$ ***(M1)(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente sus valores en la fórmula del área de la superficie curva de un cono; ***(M1)*** por sumarle el área de la base y que ésta sea correcta. Para poder conceder el segundo ***(M1)*** tiene que haber mostrado explícitamente la suma mencionada. No acepte aquí valores redondeados puesto que es posible que el alumno haya hecho los cálculos "hacia atrás" (es decir, partiendo de la respuesta dada en el enunciado).

$313{,}679\ldots\ (\text{cm}^2)$ ***(A1)***

Nota: Si se utiliza el valor de 3 cifras significativas 14,0 se obtiene una respuesta sin redondear de 313,656….

$314\ (\text{cm}^2)$ ***(AG)***

Nota: Para que se pueda conceder el ***(A1)*** final es necesario que haya escrito la respuesta sin redondear y también la respuesta tras haber aplicado el redondeo.

[3 puntos]

continúa en la página siguiente…

Continuación de la Pregunta 3

(d) $2\times\pi\times(5,2)\times h+2\times\pi\times(5,2)^2=314$ ***(M1)(M1)(M1)***

> **Nota:** Conceda ***(M1)*** por sustituir correctamente los valores en la fórmula del área de la superficie curva de un cilindro; ***(M1)*** por sumarle el área de las dos bases del cilindro y que ésta sea correcta; ***(M1)*** por igualar el valor resultante a 314 $(313,679\ldots)$. Para poder conceder este punto, tiene que haber sumado las áreas de las dos bases a la superficie curva e igualado a 314. Conceda como máximo ***(M1)(M0)(A0)*** por igualar la superficie curva del cilindro a 314.

$(h=)\ 4,41\ (4,41051\ldots)\ (\text{cm})$ ***(A1)(G3)***

[4 puntos]

(e) $\pi\times(5,2)^2\times 4,41051\ldots$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por sustituir correctamente los valores en la fórmula del volumen de un cilindro.

$375\ (374,666\ldots)\ (\text{cm}^3)$ ***(A1)*****(ft)*****(G2)***

> **Nota:** Puede otorgar puntos de arrastre de error desde el apartado (d).

$375\ (\text{cm}^3) > 368\ (\text{cm}^3)$ ***(R1)*****(ft)**

O BIEN

el volumen del cilindro es mayor que el volumen del cono o similar ***(R1)*****(ft)**

> **Nota:** Puede otorgar puntos de arrastre de error desde el apartado (a). Para conceder ***R1***, el enunciado verbal debe ser coherente con las respuestas dadas en los apartados (a) y (e).

sustituir (los conos) por los recipientes cilíndricos. ***(A1)*****(ft)**

> **Nota:** No conceda ***(A1)*****(ft)*****(R0)***. En este apartado de la pregunta se pueden conceder puntos por arrastre de error a partir de su volumen incorrecto del cilindro, pero solo si ha mostrado la sustitución en la fórmula del volumen.

[4 puntos]

Total [15 puntos]

4. (a) (i) (cantidad que le dan el 7° día): $u_1 + 6d = 21$ ***(A1)***

(ii) (cantidad que le dan el 11° día): $u_1 + 10d = 29$ ***(A1)***

> **Nota:** Acepte $u_1 + (7-1)d = 21$ y $u_1 + (11-1)d = 29$. No es necesario que las ecuaciones estén simplificadas. Para conceder los puntos las ecuaciones han de ser escritas en función de u_1 y d.

[2 puntos]

(b) $(u_1 =)\ 9$ ***(A1)*****(ft)**

$(d =)\ 2$ ***(A1)*****(ft)**

> **Nota:** Puede otorgar puntos de arrastre de error desde el apartado (a), pero solo si los valores son positivos y $u_1 < 21$.

[2 puntos]

(c) $(S_{30} =)\ \dfrac{30}{2}(2\times 9 + (30-1)\times 2)$ ***(M1)(A1)*****(ft)**

> **Nota:** Conceda ***(M1)*** por sustituir los valores en la fórmula de la progresión aritmética; ***(A1)*****(ft)** por haberlos sustituido correctamente.

1140 (mg) ***(A1)*****(ft)*****(G3)***

> **Nota:** Puede otorgar puntos por arrastre de error desde su valor de u_1 y d que dio en el apartado (b).

[3 puntos]

(d) (i) $20\times(0{,}5)^4$ ***(M1)(A1)***

> **Nota:** Conceda ***(M1)*** por sustituir los valores en la fórmula de la progresión geométrica; ***(A1)*** por haberlos sustituido correctamente.

$1{,}25$ (mg) ***(A1)(G3)***

continúa en la página siguiente…

Continuación de la Pregunta 4

(ii) $20 \times (0,5)^{k-1} < 0,06$ ***(M1)(M1)***

Nota:	Conceda ***(M1)*** por sustituir correctamente los valores en la fórmula de la progresión geométrica; ***(M1)*** por comparar su expresión con $0{,}06$. Acepte una ecuación en lugar de una inecuación (desigualdad).

$(k =)10\ (10°$ día$)$ ***(A1)*(ft)*(G3)***

Nota:	Puede otorgar puntos de arrastre de error desde el subapartado (d)(i), si $0 < r < 1$. Las respuestas con arrastre de error tienen que darse redondeadas para poder optar al punto final.

(iii) $\dfrac{20\left(1-0,5^{10}\right)}{1-0,5}$ ***(M1)(A1)*(ft)**

Nota:	Conceda ***(M1)*** por sustituir los valores en la fórmula de la suma de los términos de una progresión geométrica; ***(A1)*(ft)** por haberlos sustituido correctamente. Puede otorgar puntos de arrastre de error desde el valor de u_1 y r que dieron en el subapartado (d)(i), pero solo si la progresión es geométrica y $0 < r < 1$. Puede otorgar puntos de arrastre de error desde el valor de k que dieron en el subapartado (d)(ii), pero solo si k es un número entero positivo.

$40,0\ (39,9609\ldots)$ (mg) ***(A1)*(ft)*(G2)***

[9 puntos]
Total [16 puntos]

5. (a) $\frac{1}{3}\times 2^3+\frac{3}{4}\times 2^2-2-1$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente los valores en la función.

$2{,}67\left(\frac{8}{3};\ 2{,}66666\ldots\right)$ ***(A1)(G2)***

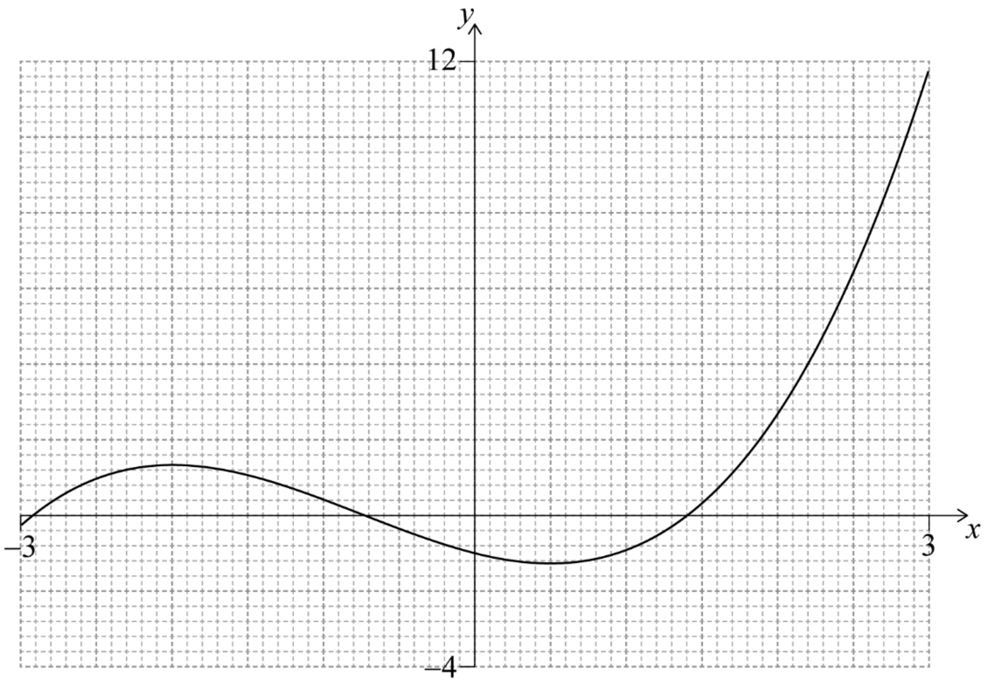
[2 puntos]

(b) -1 ***(A1)***

Nota: Acepte $(0,-1)$.

[1 punto]

(c)



(A1)(A1)(A1)(A1)

Nota: Conceda ***(A1)*** por ventana correcta y ejes rotulados, -3 y 3 deben estar escritos en el eje x, y -4 y 12 en el eje y.
(A1) por una curva suave con una forma cúbica y dibujada en la ventana;
(A1) por cortes con el eje x: uno cerca de -3, el segundo entre -1 y 0 y el tercero entre 1 y 2. El corte con el eje y aproximadamente en -1;
(A1) por el mínimo local en el cuarto cuadrante y el máximo local en el segundo cuadrante, ambos en posiciones aproximadamente correctas.
No es necesario papel milimetrado. Si no se muestra una ventana conceda como máximo ***(A0)(A1)(A0)(A1)***.

[4 puntos]

continúa en la página siguiente…

Continuación de la Pregunta 5

(d) $x^2 + \frac{3}{2}x - 1$ ***(A1)(A1)(A1)***

Nota:	Conceda un ***(A1)*** por cada término que sea correcto. Conceda como mucho ***(A1)(A1)(A0)*** si hay otros términos adicionales.

[3 puntos]

(e) $2^2 + \frac{3}{2} \times 2 - 1$ ***(M1)***

Nota:	Conceda ***(M1)*** por sustituir correctamente el 2 en su derivada de la función.

6 ***(A1)*(ft)*(G2)***

Nota:	Puede otorgar puntos de arrastre de error desde el apartado (d).

[2 puntos]

(f) $\frac{8}{3} = 6(2) + c$ ***(M1)***

Nota:	Conceda ***(M1)*** por sustituir en la ecuación de la recta el 2, su respuesta al apartado (a) y su respuesta al apartado (e).

$c = -\frac{28}{3}$

O BIEN

$\left(y - \frac{8}{3}\right) = 6(x - 2)$ ***(M1)***

Nota:	Conceda ***(M1)*** por sustituir en la ecuación de la recta el 2, su respuesta al apartado (a) y su respuesta al apartado (e).

O BIEN

$y = 6x - \frac{28}{3}$ $(y = 6x - 9.33333...)$ ***(M1)***

Nota:	Conceda ***(M1)*** por sustituir en la ecuación de la recta su respuesta al apartado (e) y $-\frac{28}{3}$.

$-18x + 3y + 28 = 0$ (Acepte múltiplos enteros) ***(A1)*(ft)*(G2)***

Nota:	Puede otorgar puntos de arrastre de error desde los apartados (a) y (e).

[2 puntos]

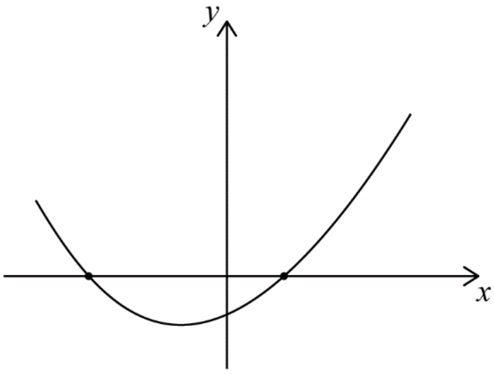
continúa en la página siguiente…

Continuación de la Pregunta 5

(g) $x^2 + \frac{3}{2}x - 1 = 0$ ***(M1)***

Nota:	Conceda ***(M1)*** por plantear que la derivada $= 0$.

O BIEN



(M1)

Nota:	Conceda ***(M1)*** por un bosquejo (dibujo aproximado) de la función derivada cuyas raíces estén en el lugar aproximadamente correcto.

$p = -2$ ***(A1)*****(ft)**
$q = 0{,}5$ ***(A1)*****(ft)**

Nota:	Acepte "La función tiene un máximo local en $x = -2$ y un mínimo local en $x = 0.5$" en lugar de $p = -2$ y $q = 0{,}5$. Puede otorgar puntos de arrastre de error desde su derivada del apartado (d), solo si sus valores de p y q están entre -3 y 3. Conceda como mucho ***(M1)(A0)(A1)*****(ft)** si incluye el desarrollo del ejercicio que lleva a una respuesta de $p = 0{,}5$, $q = -2$. Se debe usar la derivada para conceder ***(M1)***. Conceda como mucho ***(G1)*** si ha escrito los valores correctos de p y q, pero no ha incluido el desarrollo del ejercicio **O BIEN** si ha mostrado un bosquejo (dibujo aproximado) de la función con los puntos máximo y mínimo identificados, y los valores correctos de p y q. Conceda ***(G1)*** si ha escrito 0.5 y -2 (sin especificar si son p o q). Conceda ***(G1)*** si ha escrito las coordenadas del máximo y del mínimo en lugar de los valores de p y de q.

[3 puntos]

continúa en la página siguiente…

Continuación de la Pregunta 5

(h) $-1,27 \le f(x) \le 1,33$

$\left(-1,27083... \le f(x) \le 1,33333..., -\frac{61}{48} \le f(x) \le \frac{4}{3}\right)$ ***(A1)*(ft)*(A1)*(ft)*(A1)***

Nota: Conceda ***(A1)*** por incluir en su respuesta $-1,27$, ***(A1)*** por incluir $1,33$ y ***(A1)*** por desigualdades débiles (o sea no estrictas) correctas con **sus** extremos en el orden correcto. Por ejemplo, conceda ***(A0)(A0)(A0)*** por respuestas del tipo de $5 \le f(x) \le 2$.
Acepte *y* en lugar de $f(x)$. Acepte otras formas de notación correctas; p. ej., $[-1,27; 1,33]$.
Puede otorgar puntos de arrastre de error a partir de los valores de p y de q del apartado (g) solo si sus valores de $f(p)$ y $f(q)$ están entre -4 y 12.
Conceda ***(A0)(A0)(A0)*** si utiliza como extremos del intervalo los valores obtenidos en (g).

[3 puntos]

Total [20 puntos]

6. (a) $950\times\left(1+\frac{5}{12\times 100}\right)^{12\times 3}$ ***(M1)(A1)***

Nota: Conceda ***(M1)*** por sustituir los valores en la fórmula del interés compuesto; ***(A1)*** si la sustitución se hizo correctamente.

O BIEN

$N = 3$
$I\% = 5$
$PV = 950$
$P/Y = 1$
$C/Y = 12$ ***(A1)(M1)***

Nota: Conceda ***(A1)*** si ha incluido $C/Y = 12$, ***(M1)*** por otros elementos correctos.

O BIEN

$N = 36$
$I\% = 5$
$PV = 950$
$P/Y = 12$
$C/Y = 12$ ***(A1)(M1)***

Nota: Conceda ***(A1)*** si ha incluido $C/Y = 12$, ***(M1)*** por otros elementos correctos.

$1103{,}40$ (EUR) ***(A1)(G3)***

Nota: La respuesta se ha de dar redondeando a 2 lugares decimales.

[3 puntos]

continúa en la página siguiente…

Continuación de la Pregunta 6

(b) $(20\times 3+1100)-1103,40$ ***(M1)(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente los valores en la fórmula del costo de la bicicleta; ***(M1)*** por restar su respuesta dada en el apartado (a). Esta resta puede haber quedado implícita en su respuesta final (puede otorgar puntos de arrastre de error desde su respuesta dada en el apartado (a) para esta resta implícita).

$56,60$ (EUR) ***(A1)*(ft)*(G3)***

Nota: Puede otorgar puntos de arrastre de error desde el apartado (a). La respuesta tiene que haberla dado redondeando a dos lugares decimales.

[3 puntos]

(c) **MÉTODO 1**

$$950\times\left(1+\frac{5}{12\times 100}\right)^{12x}=20x+1100$$ ***(M1)(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente los valores en la fórmula del interés compuesto con una variable en el exponente, ***(M1)*** por comparar sus expresiones, siempre y cuando haya utilizado la misma variable (no una expresión con x representando años y otra con x representando meses). Conceda como máximo ***(M0)(M1)(A0)(M1)(A0)*** por sustituir un entero en ambas expresiones y comparar ambos resultados. Acepte una desigualdad.

$(x=)\ 4,52157\ldots$ (años) ***(A1)*(ft)**

$4,52157\ldots\times 12\ (=54,2588\ldots)$ ***(M1)***

Nota: Conceda ***(M1)*** por multiplicar **su** valor de x por 12. Vale también si esta operación ha quedado implícita.

$m=55$ (meses) ***(A1)*(ft)*(G4)***

MÉTODO 2

$$950\times\left(1+\frac{5}{12\times 100}\right)^{m}=20\times\frac{m}{12}+1100$$ ***(M1)(M1)(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente los valores en la fórmula del interés compuesto con una variable en el exponente; ***(M1)*** por comparar sus expresiones, siempre y cuando haya utilizado la misma variable; ***(M1)*** por convertir los años en meses en estas expresiones. Conceda como máximo ***(M0)(M1)(A0)(M1)(A0)*** por sustituir un entero en ambas expresiones y comparar ambos resultados. Acepte una desigualdad.

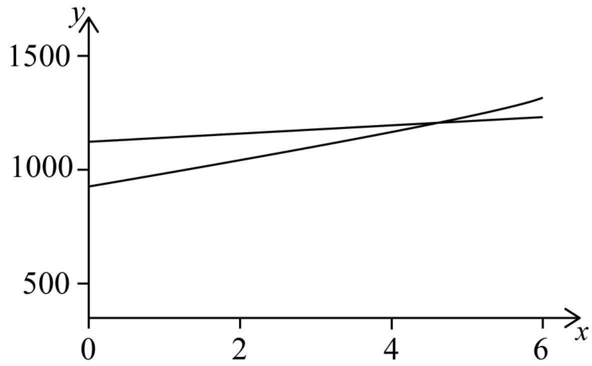
$m=54,2588\ldots$ (meses) ***(A1)*(ft)**

$m=55$ (meses) ***(A1)*(ft)*(G4)***

continúa en la página siguiente…

Continuación de la Pregunta 6

MÉTODO 3



(M1)(M1)

Nota:	Conceda ***(M1)*** por cada gráfico dibujado.

$(x =)\ 4,52157\ldots$ (años) ***(A1)*(ft)**

$4,52157\ldots \times 12\ (= 54,2588\ldots)$ ***(M1)***

Nota:	Conceda ***(M1)*** por multiplicar **su** valor de x por 12. Vale también si esta operación ha quedado implícita. Si los gráficos dibujados están en función de meses, y llevan al valor $54{,}2588\ldots$, conceda ***(M1)(M1)(M1)(A1)***, coherente con el MÉTODO 2.

$m = 55$ (meses) ***(A1)*(ft)*(G4)***

Nota:	Puede otorgar puntos de arrastre de error por una fórmula del interés compuesto que sea coherente con lo que respondió en el apartado (a). El ***(A1)*(ft)** final solo se puede conceder si da la respuesta correcta, o si da **su** respuesta correcta tras arrastrar un error desde algún apartado anterior, pero solo si el valor que dé está redondeado. Por ejemplo, no conceda ***(M0)(M0)(A0)(M1)(A1)*(ft)** por una respuesta tipo "5 años$\times 12 = 60$" (o similar) no avalada por los cálculos/razonamientos pertinentes.

[5 puntos]

Total [11 puntos]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel Superior
Prueba 1

Lunes 18 de noviembre de 2019 (tarde)

Número de convocatoria del alumno

2 horas

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba no se permite el uso de ninguna calculadora.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de Matemáticas NS y de Ampliación de Matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[100 puntos]**.

8819 – 7225

No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 5]

La distribución de probabilidad de la variable aleatoria discreta X se muestra en la siguiente tabla, donde N y p son constantes.

x	1	5	10	N
$\mathrm{P}(X=x)$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{5}$	p

(a) Halle el valor de p. [2]

(b) Sabiendo que $\mathrm{E}(X)=10$, halle el valor de N. [3]

2. [Puntuación máxima: 6]

Sabiendo que $\int_0^{\ln k} \mathrm{e}^{2x}\,\mathrm{d}x = 12$, halle el valor de k.

3. [Puntuación máxima: 5]

Sean tres planos cuyas ecuaciones son:

$$\begin{aligned} 2x - y + z &= 5 \\ x + 3y - z &= 4 \\ 3x - 5y + az &= b \end{aligned} \quad \text{, donde } a, b \in \mathbb{R}.$$

Halle el conjunto de valores de a y b para los cuales estos tres planos no tienen ningún punto de intersección.

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4. [Puntuación máxima: 7]

A y B son ángulos agudos tales que $\cos A = \frac{2}{3}$ y $\operatorname{sen} B = \frac{1}{3}$.

Muestre que $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$.

5. [Puntuación máxima: 7]

Considere la ecuación $z^4 = -4$, donde $z \in \mathbb{C}$.

(a) Resuelva la ecuación, y dé las soluciones en la forma $a + \mathrm{i}b$, donde $a, b \in \mathbb{R}$. [5]

(b) Las soluciones constituyen los vértices de un polígono en el plano complejo. Halle el área de dicho polígono. [2]

6. [Puntuación máxima: 7]

Considere la función $f(x) = x\,\mathrm{e}^{2x}$, donde $x \in \mathbb{R}$. Sea $f^{(n)}(x)$ la derivada n-ésima de $f(x)$.

Demuestre por inducción matemática que $f^{(n)}(x) = (2^n x + n2^{n-1})\mathrm{e}^{2x},\ n \in \mathbb{Z}^+$.

7. [Puntuación máxima: 7]

(a) Escriba $2x - x^2$ en la forma $a(x-h)^2 + k$, donde a, h, $k \in \mathbb{R}$. [2]

(b) A partir de lo anterior, halle el valor de $\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{\sqrt{2x-x^2}}\,\mathrm{d}x$. [5]

8. [Puntuación máxima: 6]

La recta L_θ tiene por ecuación vectorial $\boldsymbol{r} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ \operatorname{sen}\theta \\ \cos\theta \end{pmatrix}$, $\lambda, \theta \in \mathbb{R}$.

El plano, Π_p, tiene por ecuación $x = p$, $p \in \mathbb{R}$.

Muestre que el ángulo que forman L_θ y Π_p es independiente tanto de θ como de p.

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No escriba soluciones en esta página.

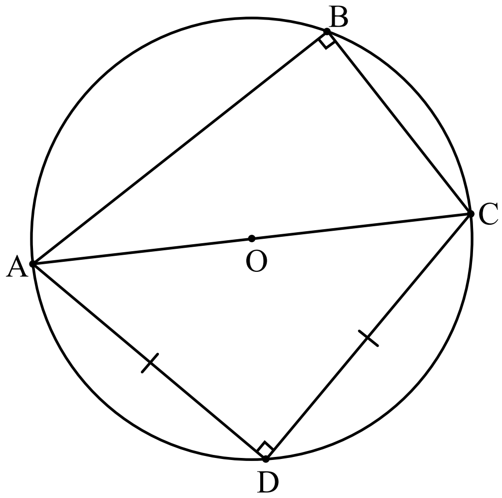
Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

9. [Puntuación máxima: 14]

(a) Sabiendo que $\cos 75° = q$, muestre que $\cos 105° = -q$. [1]

En la siguiente figura, los puntos A, B, C y D pertenecen a la circunferencia de un círculo de centro O y radio r. $[\mathrm{AC}]$ es un diámetro del círculo. $\mathrm{BC} = r$, $\mathrm{AD} = \mathrm{CD}$ y $\mathrm{A\hat{B}C} = \mathrm{A\hat{D}C} = 90^\circ$.



(b) Muestre que $\mathrm{B\hat{A}D} = 75^\circ$. [3]

(c) (i) Partiendo del triángulo ABD, muestre que $\mathrm{BD}^2 = 5r^2 - 2r^2 q\sqrt{6}$.

(ii) Partiendo del triángulo CBD, halle otra expresión que dé BD^2 en función de r y q. [7]

(d) Utilice las respuestas que dio en el apartado (c) para mostrar que $\cos 75^\circ = \dfrac{1}{\sqrt{6}+\sqrt{2}}$. [3]

No escriba soluciones en esta página.

10. [Puntuación máxima: 19]

Considere $f\left(x\right)=\frac{2x-4}{x^2-1}$, $-1<x<1$.

(a) (i) Halle $f'(x)$.

(ii) Muestre que si $f'(x)=0$, entonces $x=2-\sqrt{3}$. [5]

(b) Para el gráfico de $y=f(x)$,

(i) halle las coordenadas del punto de corte con el eje y;

(ii) muestre que no hay ningún punto de corte con el eje x;

(iii) dibuje aproximadamente el gráfico, mostrando claramente cualquier comportamiento asintótico. [5]

(c) Muestre que $\frac{3}{x+1}-\frac{1}{x-1}=\frac{2x-4}{x^2-1}$. [2]

(d) El área de la región encerrada por el gráfico de $y=f(x)$ y la recta $y=4$ puede expresarse como $\ln v$. Halle el valor de v. [7]

No escriba soluciones en esta página.

11. [Puntuación máxima: 17]

Los puntos $\mathrm{A}(0\,,0\,,10)$, $\mathrm{B}(0\,,10\,,0)$, $\mathrm{C}(10\,,0\,,0)$, $V(p\,,p\,,p)$ son los vértices de un tetraedro.

(a) (i) Muestre que $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AV}} = -10\begin{pmatrix} 10-2p \\ p \\ p \end{pmatrix}$ y halle una expresión similar para $\overrightarrow{\mathrm{AC}} \times \overrightarrow{\mathrm{AV}}$.

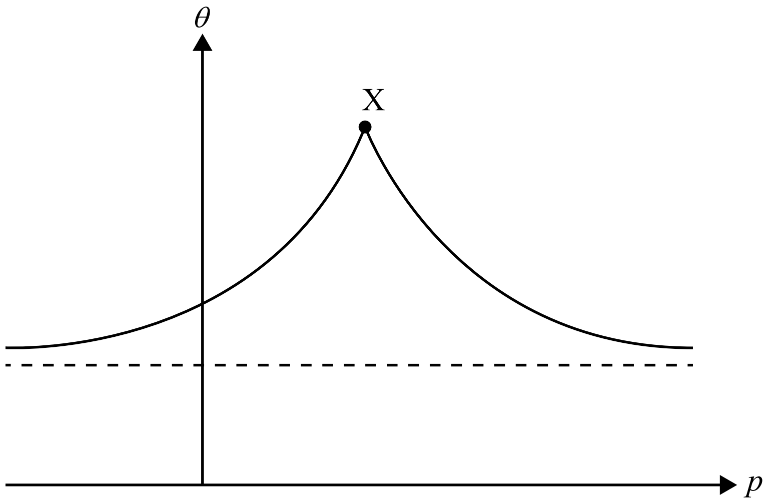
(ii) A partir de lo anterior, muestre que si el ángulo que forman las caras ABV y ACV es θ, entonces $\cos\theta = \dfrac{p(3p-20)}{6p^2-40p+100}$. [8]

(b) Considere el caso en que las caras ABV y ACV son perpendiculares.

(i) Halle las dos posibles coordenadas de V.

(ii) Comente las posiciones de V respecto al plano ABC. [4]

(c) La siguiente figura muestra el gráfico de θ en función de p. El máximo se encuentra en el punto X.



(i) Halle el valor de p y el valor de θ en X.

(ii) Halle la ecuación de la asíntota horizontal que tiene el gráfico. [5]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2019

Mathematics

Higher level

Paper 1

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives

$$f'(x) = \left(2\cos(5x-3)\right)5 \; \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT***.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. (a) $p = 1 - \frac{1}{2} - \frac{1}{5} - \frac{1}{5}$ ***(M1)***

$= \frac{1}{10}$ ***A1***

[2 marks]

(b) attempt to find $\mathrm{E}(X)$ ***(M1)***

$\frac{1}{2} + 1 + 2 + \frac{N}{10} = 10$ ***A1***

$\Rightarrow N = 65$ ***A1***

Note: Do not allow FT in part (b) if their p is outside the range $0 < p < 1$.

[3 marks]

Total [5 marks]

2. $\frac{1}{2}\mathrm{e}^{2x}$ seen ***(A1)***

attempt at using limits in an integrated expression $\left(\left[\frac{1}{2}\mathrm{e}^{2x}\right]_0^{\ln k} = \frac{1}{2}\mathrm{e}^{2\ln k} - \frac{1}{2}\mathrm{e}^{0}\right)$ ***(M1)***

$= \frac{1}{2}\mathrm{e}^{\ln k^2} - \frac{1}{2}\mathrm{e}^{0}$ ***(A1)***

Setting their equation $= 12$ ***M1***

Note: their equation must be an integrated expression with limits substituted.

$\frac{1}{2}k^2 - \frac{1}{2} = 12$ ***A1***

$\left(k^2 = 25 \Rightarrow\right) k = 5$ ***A1***

Note: Do not award final ***A1*** for $k = \pm 5$.

[6 marks]

3. attempt to eliminate a variable (or attempt to find det A) ***M1***

$$\left(\begin{array}{ccc|c} 2 & -1 & 1 & 5 \\ 1 & 3 & -1 & 4 \\ 3 & -5 & a & b \end{array}\right) \to \left(\begin{array}{ccc|c} 2 & -1 & 1 & 5 \\ 0 & 7 & -3 & 3 \\ 0 & -14 & a+3 & b-12 \end{array}\right)$$ (or $\det A = 14(a-3)$)

(or two correct equations in two variables) ***A1***

$$\to \left(\begin{array}{ccc|c} 2 & -1 & 1 & 5 \\ 0 & 7 & -3 & 3 \\ 0 & 0 & a-3 & b-6 \end{array}\right)$$ (or solving $\det A = 0$)

(or attempting to reduce to one variable, e.g. $(a-3)z = b-6$) ***M1***

$a = 3,\ b \neq 6$ ***A1A1***

[5 marks]

4. attempt to use $\cos(2A+B) = \cos 2A \cos B - \sin 2A \sin B$ (may be seen later) ***M1***

attempt to use any double angle formulae (seen anywhere) ***M1***

attempt to find either $\sin A$ or $\cos B$ (seen anywhere) ***M1***

$\cos A = \frac{2}{3} \Rightarrow \sin A \left(= \sqrt{1 - \frac{4}{9}} \right) = \frac{\sqrt{5}}{3}$ ***(A1)***

$\sin B = \frac{1}{3} \Rightarrow \cos B \left(= \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3} \right) = \frac{2\sqrt{2}}{3}$ ***A1***

$\cos 2A \left(= 2\cos^2 A - 1 \right) = -\frac{1}{9}$ ***A1***

$\sin 2A \left(= 2 \sin A \cos A \right) = \frac{4\sqrt{5}}{9}$ ***A1***

So $\cos(2A+B) = \left(-\frac{1}{9}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{4\sqrt{5}}{9}\right)\left(\frac{1}{3}\right)$

$= -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$ ***AG***

[7 marks]

5. (a) **METHOD 1**

$|z| = \sqrt[4]{4}\left(= \sqrt{2}\right)$ ***(A1)***

$\arg(z_1) = \dfrac{\pi}{4}$ ***(A1)***

first solution is $1+\mathrm{i}$ ***A1***

valid attempt to find all roots (De Moivre or +/– their components) ***(M1)***

other solutions are $-1+\mathrm{i}$, $-1-\mathrm{i}$, $1-\mathrm{i}$ ***A1***

[5 marks]

METHOD 2

$z^4 = -4$

$(a+\mathrm{i}b)^4 = -4$

attempt to expand and equate **both** reals and imaginaries. ***(M1)***

$a^4 + 4a^3b\mathrm{i} - 6a^2b^2 - 4ab^3\mathrm{i} + b^4 = -4$

$\left(a^4 - 6a^4 + a^4 = -4 \Rightarrow\right) a = \pm 1$ ***and*** $\left(4a^3b - 4ab^3 = 0 \Rightarrow\right) a = \pm b$ ***(A1)***

first solution is $1+\mathrm{i}$ ***A1***

valid attempt to find all roots (De Moivre or +/- their components) ***(M1)***

other solutions are $-1+\mathrm{i}$, $-1-\mathrm{i}$, $1-\mathrm{i}$ ***A1***

[5 marks]

(b) complete method to find area of ‘rectangle’ ***(M1)***

$= 4$ ***A1***

[2 marks]

Total [7 marks]

6. $f'(x) = e^{2x} + 2x e^{2x}$ ***A1***

Note: This must be obtained from the candidate differentiating $f(x)$.

$= \left(2^1 x + 1 \times 2^{1-1}\right) e^{2x}$ ***A1***

(hence true for $n = 1$)

assume true for $n = k$: ***M1***

$f^{(k)}(x) = \left(2^k x + k2^{k-1}\right) e^{2x}$

Note: Award ***M1*** if truth is assumed. Do not allow "let $n = k$".

consider $n = k + 1$:

$f^{(k+1)}(x) = \frac{d}{dx}\left(\left(2^k x + k2^{k-1}\right)e^{2x}\right)$

attempt to differentiate $f^{(k)}(x)$ ***M1***

$f^{(k+1)}(x) = 2^k e^{2x} + 2\left(2^k x + k2^{k-1}\right) e^{2x}$ ***A1***

$f^{(k+1)}(x) = \left(2^k + 2^{k+1} x + k2^k\right) e^{2x}$

$f^{(k+1)}(x) = \left(2^{k+1} x + (k+1) 2^k\right) e^{2x}$ ***A1***

$= \left(2^{k+1} x + (k+1) 2^{(k+1)-1}\right) e^{2x}$

True for $n = 1$ and $n = k$ true implies true for $n = k + 1$.

Therefore the statement is true for all $n \left(\in \mathbb{Z}^+\right)$ ***R1***

Note: Do not award final ***R1*** if the two previous ***M1s*** are not awarded. Allow full marks for candidates who use the base case $n = 0$.

[7 marks]

7. (a) attempt to complete the square or multiplication and equating coefficients ***(M1)***

$2x - x^2 = -(x-1)^2 + 1$ ***A1***

$a = -1,\ h = 1,\ k = 1$

[2 marks]

(b) use of their identity from part (a) $\left(\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{\sqrt{1-(x-1)^2}}\, \mathrm{d}x \right)$ ***(M1)***

$= \left[\arcsin(x-1) \right]_{\frac{1}{2}}^{\frac{3}{2}}$ or $\left[\arcsin(u) \right]_{-\frac{1}{2}}^{\frac{1}{2}}$ ***A1***

Note: Condone lack of, or incorrect limits up to this point.

$= \arcsin\left(\frac{1}{2}\right) - \arcsin\left(-\frac{1}{2}\right)$ ***(M1)***

$= \frac{\pi}{6} - \left(-\frac{\pi}{6}\right)$ ***(A1)***

$= \frac{\pi}{3}$ ***A1***

[5 marks]

Total [7 marks]

8. a vector normal to Π_p is $\begin{pmatrix}1\\0\\0\end{pmatrix}$ ***(A1)***

Note: Allow any scalar multiple of $\begin{pmatrix}1\\0\\0\end{pmatrix}$, including $\begin{pmatrix}p\\0\\0\end{pmatrix}$

attempt to find scalar product (or vector product) of direction vector of line with any scalar multiple of $\begin{pmatrix}1\\0\\0\end{pmatrix}$ ***M1***

$\begin{pmatrix}1\\0\\0\end{pmatrix}\bullet\begin{pmatrix}5\\ \sin\theta\\ \cos\theta\end{pmatrix}=5$ (or $\begin{pmatrix}1\\0\\0\end{pmatrix}\times\begin{pmatrix}5\\ \sin\theta\\ \cos\theta\end{pmatrix}=\begin{pmatrix}0\\ -\cos\theta\\ \sin\theta\end{pmatrix}$) ***A1***

(if α is the angle between the line and the normal to the plane)

$\cos\alpha=\dfrac{5}{1\times\sqrt{25+\sin^2\theta+\cos^2\theta}}$ (or $\sin\alpha=\dfrac{1}{1\times\sqrt{25+\sin^2\theta+\cos^2\theta}}$) ***A1***

$\Rightarrow\cos\alpha=\dfrac{5}{\sqrt{26}}$ or $\sin\alpha=\dfrac{1}{\sqrt{26}}$ ***A1***

this is independent of p and θ, hence the angle between the line and the plane, $(90-\alpha)$, is also independent of p and θ ***R1***

Note: The final ***R*** mark is independent, but is conditional on the candidate obtaining a value independent of p and θ.

[6 marks]

Section B

9. (a) $\cos 105^\circ = \cos\left(180^\circ - 75^\circ\right) = -\cos 75^\circ$ ***R1***

$= -q$ ***AG***

Note: Accept arguments using the unit circle or graphical/diagrammatical considerations.

[1 mark]

(b) $\mathrm{AD} = \mathrm{CD} \Rightarrow \mathrm{C\hat{A}D} = 45^\circ$ ***A1***

valid method to find $\mathrm{B\hat{A}C}$ ***(M1)***

for example: $\mathrm{BC} = r \Rightarrow \mathrm{B\hat{C}A} = 60^\circ$

$\Rightarrow \mathrm{B\hat{A}C} = 30^\circ$ ***A1***

hence $\mathrm{B\hat{A}D} = 45^\circ + 30^\circ = 75^\circ$ ***AG***

[3 marks]

(c) (i) $\mathrm{AB} = r\sqrt{3}$, $\mathrm{AD}\left(= \mathrm{CD}\right) = r\sqrt{2}$ ***A1A1***

applying cosine rule ***(M1)***

$\mathrm{BD}^2 = \left(r\sqrt{3}\right)^2 + \left(r\sqrt{2}\right)^2 - 2\left(r\sqrt{3}\right)\left(r\sqrt{2}\right)\cos 75^\circ$ ***A1***

$= 3r^2 + 2r^2 - 2r^2\sqrt{6}\cos 75^\circ$

$= 5r^2 - 2r^2 q\sqrt{6}$ ***AG***

(ii) $\mathrm{B\hat{C}D} = 105^\circ$ ***(A1)***

attempt to use cosine rule on $\Delta\mathrm{BCD}$ ***(M1)***

$\mathrm{BD}^2 = r^2 + \left(r\sqrt{2}\right)^2 - 2r\left(r\sqrt{2}\right)\cos 105^\circ$

$= 3r^2 + 2r^2 q\sqrt{2}$ ***A1***

[7 marks]

(d) $5r^2 - 2r^2 q\sqrt{6} = 3r^2 + 2r^2 q\sqrt{2}$ ***(M1)(A1)***

$2r^2 = 2r^2 q\left(\sqrt{6} + \sqrt{2}\right)$ ***A1***

Note: Award ***A1*** for any correct intermediate step seen using only two terms.

$$q = \frac{1}{\sqrt{6} + \sqrt{2}}$$ ***AG***

Note: Do not award the final ***A1*** if follow through is being applied.

[3 marks]

Total [14 marks]

10. (a) (i) attempt to use quotient rule (or equivalent) ***(M1)***

$$f'(x) = \frac{(x^2-1)(2)-(2x-4)(2x)}{(x^2-1)^2}$$ ***A1***

$$= \frac{-2x^2+8x-2}{(x^2-1)^2}$$

(ii) $f'(x) = 0$

simplifying numerator (may be seen in part (i)) ***(M1)***

$\Rightarrow x^2 - 4x + 1 = 0$ or equivalent quadratic equation ***A1***

EITHER

use of quadratic formula

$$\Rightarrow x = \frac{4 \pm \sqrt{12}}{2}$$ ***A1***

OR

use of completing the square

$(x-2)^2 = 3$ ***A1***

THEN

$x = 2 - \sqrt{3}$ (since $2+\sqrt{3}$ is outside the domain) ***AG***

Note: Do not condone verification that $x = 2-\sqrt{3} \Rightarrow f'(x) = 0$. Do not award the final ***A1*** as follow through from part (i).

[5 marks]

(b) (i) $(0, 4)$ ***A1***

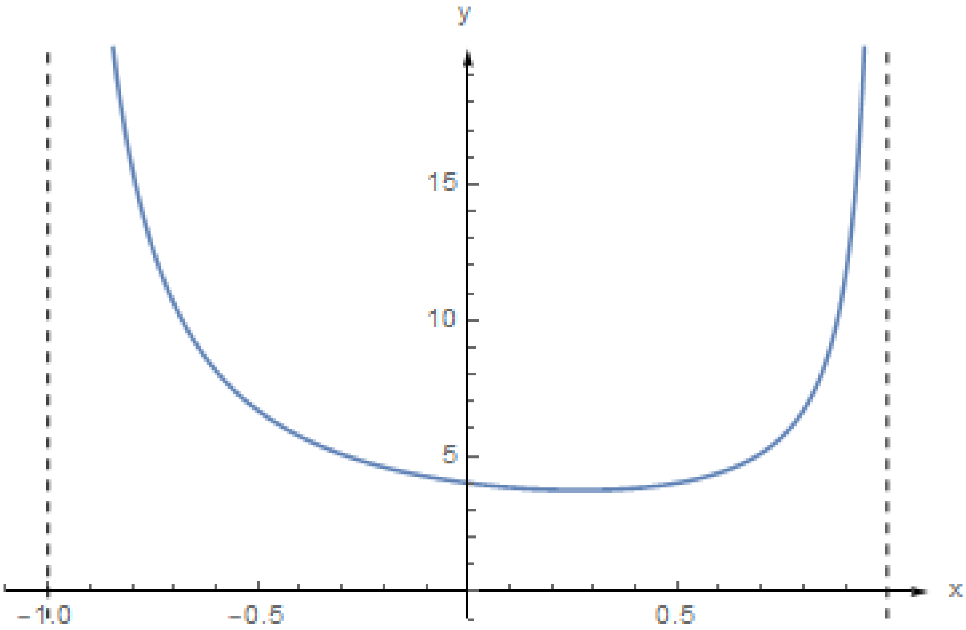
(ii) $2x - 4 = 0 \Rightarrow x = 2$ ***A1***

outside the domain ***R1***

continued…

Question 10 continued

(iii)



A1A1

award ***A1*** for concave up curve over correct domain with one minimum point in the first quadrant
award ***A1*** for approaching $x = \pm 1$ asymptotically

[5 marks]

(c) valid attempt to combine fractions (using common denominator) ***M1***

$$\frac{3(x-1)-(x+1)}{(x+1)(x-1)}$$ ***A1***

$$= \frac{3x-3-x-1}{x^2-1}$$

$$= \frac{2x-4}{x^2-1}$$ ***AG***

[2 marks]

continued…

Question 10 continued

(d)

$f(x)=4 \Rightarrow 2x-4=4x^2-4$ **M1**

($x=0$ or) $x=\dfrac{1}{2}$ **A1**

area under the curve is $\int_0^{\frac{1}{2}} f(x)\,\mathrm{d}x$ **M1**

$=\int_0^{\frac{1}{2}} \dfrac{3}{x+1}-\dfrac{1}{x-1}\,\mathrm{d}x$

Note: Ignore absence of, or incorrect limits up to this point.

$=\left[3\ln|x+1|-\ln|x-1|\right]_0^{\frac{1}{2}}$ **A1**

$=3\ln\dfrac{3}{2}-\ln\dfrac{1}{2}(-0)$

$=\ln\dfrac{27}{4}$ **A1**

area is $2-\int_0^{\frac{1}{2}} f(x)\,\mathrm{d}x$ or $\int_0^{\frac{1}{2}} 4\,\mathrm{d}x-\int_0^{\frac{1}{2}} f(x)\,\mathrm{d}x$ **M1**

$=2-\ln\dfrac{27}{4}$

$=\ln\dfrac{4\mathrm{e}^2}{27}$ **A1**

$\left(\Rightarrow v=\dfrac{4\mathrm{e}^2}{27}\right)$

[7 marks]

Total [19 marks]

11. (a) (i) $\overrightarrow{AV} = \begin{pmatrix} p \\ p \\ p-10 \end{pmatrix}$ ***A1***

$$\overrightarrow{AB} \times \overrightarrow{AV} = \begin{pmatrix} 0 \\ 10 \\ -10 \end{pmatrix} \times \begin{pmatrix} p \\ p \\ p-10 \end{pmatrix} = \begin{pmatrix} 10(p-10)+10p \\ -10p \\ -10p \end{pmatrix}$$ ***A1***

$$= \begin{pmatrix} 20p-100 \\ -10p \\ -10p \end{pmatrix} = -10\begin{pmatrix} 10-2p \\ p \\ p \end{pmatrix}$$ ***AG***

$$\overrightarrow{AC} \times \overrightarrow{AV} = \begin{pmatrix} 10 \\ 0 \\ -10 \end{pmatrix} \times \begin{pmatrix} p \\ p \\ p-10 \end{pmatrix} = \begin{pmatrix} 10p \\ 100-20p \\ 10p \end{pmatrix} \left(= 10\begin{pmatrix} p \\ 10-2p \\ p \end{pmatrix} \right)$$ ***A1***

(ii) attempt to find a scalar product ***M1***

$$-10\begin{pmatrix} 10-2p \\ p \\ p \end{pmatrix} \bullet 10\begin{pmatrix} p \\ 10-2p \\ p \end{pmatrix} = 100\left(3p^2-20p\right)$$

OR $-\begin{pmatrix} 10-2p \\ p \\ p \end{pmatrix} \bullet \begin{pmatrix} p \\ 10-2p \\ p \end{pmatrix} = 3p^2-20p$ ***A1***

attempt to find magnitude of either $\overrightarrow{AB} \times \overrightarrow{AV}$ or $\overrightarrow{AC} \times \overrightarrow{AV}$ ***M1***

$$\left|-10\begin{pmatrix} 10-2p \\ p \\ p \end{pmatrix}\right| = \left|10\begin{pmatrix} p \\ 10-2p \\ p \end{pmatrix}\right| = 10\sqrt{(10-2p)^2+2p^2}$$ ***A1***

$$100\left(3p^2-20p\right) = 100\left(\sqrt{(10-2p)^2+2p^2}\right)^2 \cos\theta$$

$$\cos\theta = \frac{3p^2-20p}{(10-2p)^2+2p^2}$$ ***A1***

Note: Award ***A1*** for any intermediate step leading to the correct answer.

$$= \frac{p(3p-20)}{6p^2-40p+100}$$ ***AG***

Note: Do not allow FT marks from part (a)(i).

[8 marks]

continued…

Question 11 continued

(b) (i) $p(3p-20)=0 \Rightarrow p=0$ or $p=\frac{20}{3}$ ***M1A1***

coordinates are $(0,0,0)$ and $\left(\frac{20}{3},\frac{20}{3},\frac{20}{3}\right)$ ***A1***

Note: Do not allow column vectors for the final ***A*** mark.

(ii) two points are mirror images in the plane
or opposite sides of the plane
or equidistant from the plane
or the line connecting the two Vs is perpendicular to the plane ***R1***

[4 marks]

(c) (i) geometrical consideration or attempt to solve $-1=\frac{p(3p-20)}{6p^2-40p+100}$ ***(M1)***

$p=\frac{10}{3}$, $\theta=\pi$ or $\theta=180^\circ$ ***A1A1***

(ii) $p\to\infty \Rightarrow \cos\theta \to \frac{1}{2}$ ***M1***

hence the asymptote has equation $\theta=\frac{\pi}{3}$ ***A1***

[5 marks]

Total [17 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel Superior
Prueba 2

Martes 19 de noviembre de 2019 (mañana)

Número de convocatoria del alumno

2 horas

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de Matemáticas NS y de Ampliación de Matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[100 puntos]**.

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No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 5]

Sea una progresión geométrica donde $u_4 = -70$ y $u_7 = 8,75$. Halle el segundo término de la progresión.

2. [Puntuación máxima: 6]

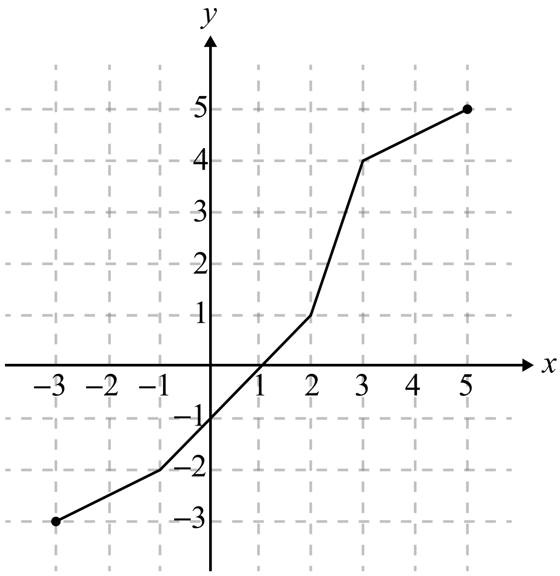
El número de maratones que Audrey corre cada año se puede modelizar mediante una distribución de Poisson de media $1{,}3$.

(a) Calcule la probabilidad de que Audrey corra al menos dos maratones en un determinado año. [2]

(b) Halle la probabilidad de que corra al menos dos maratones al año en exactamente cuatro de los próximos cinco años. [4]

3. [Puntuación máxima: 6]

La siguiente figura muestra el gráfico de $y = f(x)$, $-3 \le x \le 5$.



(a) Halle el valor de $(f \circ f)(1)$. [2]

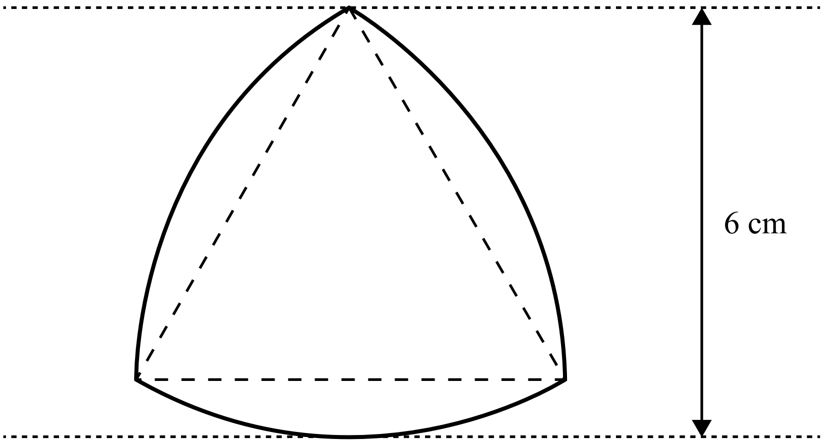
(b) Sabiendo que $f^{-1}(a) = 3$, determine el valor de a. [2]

(c) Sabiendo que $g(x) = 2f(x-1)$, halle el dominio y el recorrido de g. [2]

4. [Puntuación máxima: 7]

La siguiente forma geométrica consta de tres arcos de circunferencia, cada uno con centro en el vértice opuesto de un triángulo equilátero, tal y como se muestra en la figura.

la figura no está dibujada a escala



Para esta forma geométrica, calcule

(a) el perímetro; [2]

(b) el área. [5]

5. [Puntuación máxima: 6]

Considere el desarrollo de $(2+x)^n$, donde $n \geq 3$ y $n \in \mathbb{Z}$.

El coeficiente de x^3 es cuatro veces mayor que el coeficiente de x^2. Halle el valor de n.

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6. [Puntuación máxima: 6]

Sea $P(z) = az^3 - 37z^2 + 66z - 10$, donde $z \in \mathbb{C}$ y $a \in \mathbb{Z}$.

Una de las raíces de $P(z) = 0$ es $3 + \mathrm{i}$. Halle el valor de a.

7. [Puntuación máxima: 6]

Las mejores marcas de la temporada logradas por los corredores de un equipo de atletismo en los $100\,\mathrm{m}$ lisos siguen una distribución normal de media $11,6$ segundos y desviación típica $0,8$ segundos. Para clasificarse para una competición dada, los corredores necesitan que su mejor marca de la temporada sea inferior a 11 segundos. Se elige al azar a uno de los corredores de este equipo que se han clasificado para la competición. Halle la probabilidad de que su mejor marca de la temporada sea inferior a $10,7$ segundos.

8. [Puntuación máxima: 8]

Ocho chicos y dos chicas están sentados en un banco. Determine el número de disposiciones posibles, sabiendo que

(a) las chicas no se sientan juntas; [3]

(b) las chicas no se sientan en ninguno de los dos extremos; [2]

(c) las chicas no se sientan en ninguno de los dos extremos y tampoco se sientan juntas. [3]

No escriba soluciones en esta página.

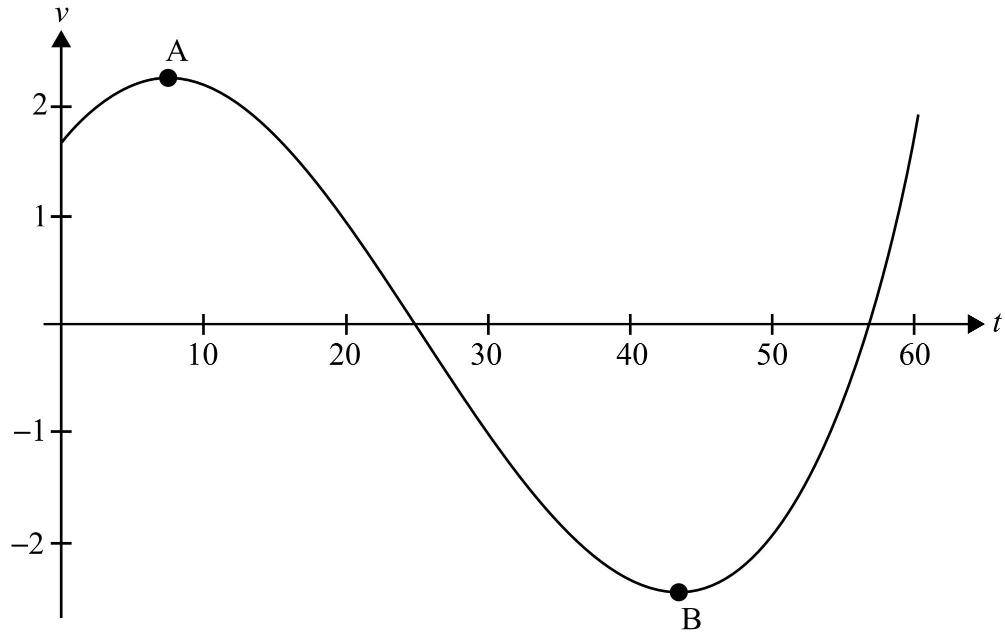
Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

9. [Puntuación máxima: 14]

Un cuerpo se mueve en línea recta, de modo tal que su velocidad v $\mathrm{m\,s^{-1}}$ después de t segundos viene dada por $v = 2\,\mathrm{sen}\left(\frac{t}{10}+\frac{\pi}{5}\right)\csc\left(\frac{t}{30}+\frac{\pi}{4}\right)$ para $0 \le t \le 60$.

La siguiente figura muestra el gráfico de v en función de t. El punto A es un máximo local y el punto B es un mínimo local.



(a) (i) Determine las coordenadas del punto A y las coordenadas del punto B.

(ii) A partir de lo anterior, escriba la velocidad máxima que alcanza el cuerpo. [5]

(b) El cuerpo se detiene por primera vez en el instante $t = t_1$. Halle

(i) el valor de t_1;

(ii) la distancia que ha recorrido entre $t = 0$ y $t = t_1$;

(iii) la aceleración en el instante $t = t_1$. [6]

(c) Halle la distancia que ha recorrido en los primeros 30 segundos. [3]

No escriba soluciones en esta página.

10. [Puntuación máxima: 19]

La variable aleatoria X tiene la siguiente función de densidad de probabilidad:

$$f(x)=\begin{cases} 3a & , \quad 0 \le x < 2 \\ a(x-5)(1-x) & , \quad 2 \le x \le b \\ 0 & , \quad \text{resto de valores} \end{cases} \qquad a,\, b \in \mathbb{R}^{+},\, 3 < b \le 5\,.$$

(a) Halle, en función de a, la probabilidad de que X esté comprendida entre 1 y 3. [4]

Considere el caso en que $b = 5$.

(b) Dibuje aproximadamente el gráfico de f. Indique las coordenadas de los extremos de cada tramo y de cualquier máximo o mínimo local; dé las respuestas en función de a. [4]

(c) Halle el valor de

(i) a;

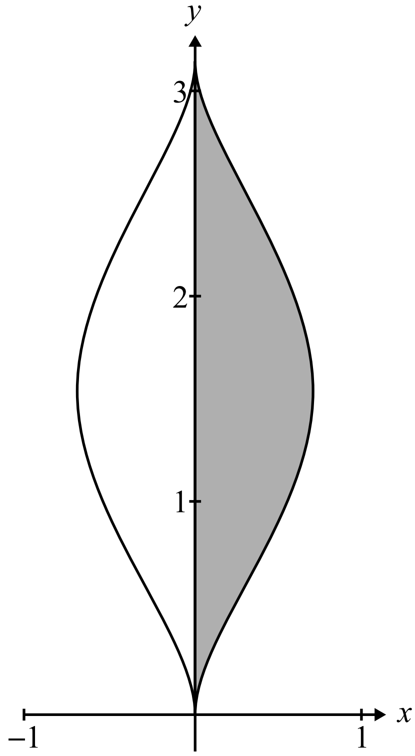
(ii) $\mathrm{E}(X)$;

(iii) la mediana de X. [11]

No escriba soluciones en esta página.

11. [Puntuación máxima: 17]

La siguiente figura muestra una parte del gráfico de $2x^2 = \operatorname{sen}^3 y$ para $0 \leq y \leq \pi$.



(a) (i) Utilizando la derivación implícita, halle una expresión para $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(ii) Halle la ecuación de la tangente a la curva en el punto $\left(\frac{1}{4}, \frac{5\pi}{6}\right)$. [8]

La región sombreada R está delimitada por la curva, el eje y y las rectas $y = 0$ e $y = \pi$.

(b) Halle el área de R. [3]

Ahora se rota la región R 2π radianes alrededor del eje y para generar un sólido.

(c) Escribiendo $\operatorname{sen}^3 y$ como $(1 - \cos^2 y) \operatorname{sen} y$, muestre que el volumen del sólido así generado es igual a $\frac{2\pi}{3}$. [6]

Markscheme

November 2019

Mathematics

Higher level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

Award ***N*** *marks for* ***correct*** *answers where there is* ***no*** *working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets eg (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives

$$f'(x) = \left(2\cos(5x-3)\right)5 \ \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

Section A

1. $u_1r^3 = -70$, $u_1r^6 = 8.75$ ***(M1)***

$r^3 = \frac{8.75}{-70} = -0.125$ ***(A1)***

$\Rightarrow r = -0.5$ ***(A1)***

valid attempt to find u_2 ***(M1)***

for example: $u_1 = \frac{-70}{-0.125} = 560$

$u_2 = 560 \times -0.5$

$= -280$ ***A1***

[5 marks]

2. (a) $X \sim \text{Po}(1.3)$

$\text{P}(X \geq 2) = 0.373$ ***(M1)A1***

[2 marks]

(b) $V \sim \text{B}(5, 0.373)$ ***(M1)A1***

Note: Award ***(M1)*** for recognition of binomial or equivalent, ***A1*** for correct parameters.

$\text{P}(V = 4) = 0.0608$ ***(M1)A1***

[4 marks]

Total [6 marks]

3. (a) $f(1) = 0$ ***(A1)***

$f(0) = -1$ ***A1***

[2 marks]

(b) $a = f(3)$ ***(M1)***

$\Rightarrow a = 4$ ***A1***

[2 marks]

(c) domain is $-2 \leq x \leq 6$ ***A1***

range is $-6 \leq y \leq 10$ ***A1***

[2 marks]

Total [6 marks]

4. (a) each arc has length $r\theta = 6 \times \frac{\pi}{3} = 2\pi \left(= 6.283...\right)$ ***(M1)***

perimeter is therefore $6\pi \left(= 18.8\right) (\text{cm})$ ***A1***

[2 marks]

(b) area of sector, s, is $\frac{1}{2}r^2\theta = 18 \times \frac{\pi}{3} = 6\pi \left(= 18.84...\right)$ ***(A1)***

area of triangle, t, is $\frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3} \left(= 15.58...\right)$ ***(M1)(A1)***

Note: area of segment, k, is 3.261… implies area of triangle

finding $3s - 2t$ or $3k + t$ or similar
area $= 3s - 2t = 18\pi - 18\sqrt{3} \left(= 25.4\right) (\text{cm}^2)$ ***(M1)A1***

[5 marks]

Total [7 marks]

5. attempt to find coefficients in binomial expansion ***(M1)***

coefficient of x^2 : $\binom{n}{2} \times 2^{n-2}$; coefficient of x^3 : $\binom{n}{3} \times 2^{n-3}$ ***A1A1***

Note: Condone terms given rather than coefficients. Terms may be seen in an equation such as that below.

$\binom{n}{3} \times 2^{n-3} = 4\binom{n}{2} \times 2^{n-2}$ ***(A1)***

attempt to solve equation using GDC or algebraically ***(M1)***

$\binom{n}{3} = 8\binom{n}{2}$

$\frac{n!}{3!(n-3)!} = \frac{8n!}{2!(n-2)!}$

$\frac{1}{3} = \frac{8}{n-2}$

$n = 26$ ***A1***

[6 marks]

6. **METHOD 1**

one other root is $3-\mathrm{i}$ ***A1***
let third root be α ***(M1)***
considering sum or product of roots ***(M1)***
sum of roots $=6+\alpha=\frac{37}{a}$ ***A1***
product of roots $=10\alpha=\frac{10}{a}$ ***A1***
hence $a=6$ ***A1***

[6 marks]

METHOD 2

one other root is $3-\mathrm{i}$ ***A1***
quadratic factor will be $z^2-6z+10$ ***(M1)A1***
$P(z)=az^3-37z^2+66z-10=\left(z^2-6z+10\right)(az-1)$ ***M1***
comparing coefficients ***(M1)***
hence $a=6$ ***A1***

[6 marks]

METHOD 3

substitute $3+\mathrm{i}$ into $P(z)$ ***(M1)***
$a(18+26\mathrm{i})-37(8+6\mathrm{i})+66(3+\mathrm{i})-10=0$ ***(M1)A1***
equating real or imaginary parts or dividing ***M1***
$18a-296+198-10=0$ or $26a-222+66=0$ or $\frac{10-66(3+\mathrm{i})+37(8+6\mathrm{i})}{18+26\mathrm{i}}$ ***A1***
hence $a=6$ ***A1***

[6 marks]

7. $T \sim \mathrm{N}\left(11.6, 0.8^2\right)$

$\mathrm{P}(T<10.7 \mid T<11)$ ***(M1)***

$=\frac{\mathrm{P}(T<10.7 \cap T<11)}{\mathrm{P}(T<11)}$ ***(M1)***

$=\frac{\mathrm{P}(T<10.7)}{\mathrm{P}(T<11)}$ ***(M1)***

$\mathrm{P}(T<10.7)=0.1302\ldots$ ***(A1)***

$\mathrm{P}(T<11)=0.2266\ldots$ ***(A1)***

$\mathrm{P}(T<10.7 \mid T<11)=0.575$ ***A1***

Note: Accept only 0.575.

[6 marks]

8. (a) **METHOD 1**

$10!-2\times 9!\,(=2903040)$ ***(A1)(A1)A1***

Note: Award ***A1*** for $10!$, ***A1*** for $2\times 9!$, ***A1*** for final answer.

METHOD 2

$9\times 8\times 8!$ ***(A1)(A1)A1***

Note: Award ***A1*** for 9×8 or equivalent, ***A1*** for $8!$ and ***A1*** for answer.

[3 marks]

(b) **METHOD 1**

$8\times 7\times 8!\,(=2257920)$ ***(A1)A1***

Note: Award ***(A1)*** for 8×7, ***A1*** for final answer.

METHOD 2

$10!-2\times 8!-2\times 2\times 7\times 8!$

Note: Award ***A1*** for $10!$ minus EITHER subtracted terms and ***A1*** for final correct answer.

[2 marks]

(c) **METHOD 1**

$8\times 7\times (8!-2\times 7!)\,(=1693440)$ ***(A1)(A1)A1***

Note: Award ***(A1)*** for 8×7, ***(A1)*** for $2\times 7!$, ***A1*** for final answer.
$(8!-2\times 7!)$ can be replaced by $6\times 7!$ *or* ${}^{7}P_{2}\times 6!$ which may be awarded the second ***A1***.

METHOD 2

their answer to (a) $-2\times 8!-2\times 2\times 7\times 8!$ ***(A1)(A1)A1***

Note: Award ***A1*** for subtracting each of the terms and ***A1*** for final answer.

METHOD 3

their answer to (b) $-2\times 7\times 8!$ or equivalent ***(A1)A2***

Note: Award ***A1*** for the subtraction and ***A2*** for final answer.

[3 marks]

Total [8 marks]

Section B

9. (a) (i) $\mathrm{A}(7.47, 2.28)$ and $\mathrm{B}(43.4, -2.45)$ ***A1A1A1A1***

(ii) maximum speed is $2.45\ (\mathrm{m\,s^{-1}})$ ***A1***

[5 marks]

(b) (i) $v = 0 \Rightarrow t_1 = 25.1\,(\mathrm{s})$ ***(M1)A1***

(ii) $\int_0^{t_1} v\,\mathrm{d}t$ ***(M1)***

$= 41.0\,(\mathrm{m})$ ***A1***

(iii) $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ at $t = t_1 = 25.1$ ***(M1)***

$a = -0.200\ (\mathrm{m\,s^{-2}})$ ***A1***

Note: Accept $a = -0.2$.

[6 marks]

(c) attempt to integrate between 0 and 30 ***(M1)***

Note: An unsupported answer of 38.6 can imply integrating from 0 to 30.

EITHER

$\int_0^{30} |v|\,\mathrm{d}t$ ***(A1)***

OR

$41.0 - \int_{t_1}^{30} v\,\mathrm{d}t$ ***(A1)***

THEN

$= 43.3\,(\mathrm{m})$ ***A1***

[3 marks]

Total [14 marks]

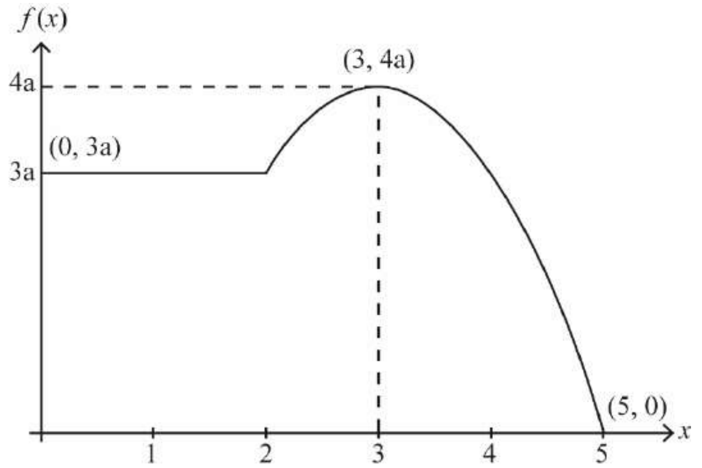
10. (a) $\left(\mathrm{P}(1 < X < 3) =\right) \int_1^2 3a \, \mathrm{d}x + a\int_2^3 -x^2 + 6x - 5 \, \mathrm{d}x$ ***(M1)(A1)(A1)***

$= 3a + \frac{11}{3}a$

$= \frac{20}{3}a \left(= 6.67a\right)$ ***A1***

[4 marks]

(b)



A4

award ***A1*** for $(0,3a)$, ***A1*** for continuity at $(2,3a)$, ***A1*** for maximum at $(3,4a)$, ***A1*** for $(5,0)$

Note: Award ***A3*** if correct four points are not joined by a straight line and a quadratic curve.

[4 marks]

(c) (i) $\mathrm{P}(0 \le X \le 5) = 6a + a\int_2^5 -x^2 + 6x - 5 \, \mathrm{d}x$ ***(M1)***

$= 15a$ ***(A1)***

$15a = 1$ ***(M1)***

$\Rightarrow a = \frac{1}{15} \left(= 0.0667\right)$ ***A1***

(ii) $\mathrm{E}(X) = \frac{1}{5}\int_0^2 x \, \mathrm{d}x + \frac{1}{15}\int_2^5 -x^3 + 6x^2 - 5x \, \mathrm{d}x$ ***(M1)(A1)***

$= 2.35$ ***A1***

continued…

Question 10 continued

(iii) attempt to use $\int_0^m f(x)\mathrm{d}x = 0.5$ ***(M1)***

$0.4 + a\int_2^m -x^2 + 6x - 5\,\mathrm{d}x = 0.5$ ***(A1)***

$a\int_2^m -x^2 + 6x - 5\,\mathrm{d}x = 0.1$

attempt to solve integral using GDC and/or analytically ***(M1)***

$\frac{1}{15}\left[-\frac{1}{3}x^3 + 3x^2 - 5x\right]_2^m = 0.1$

$m = 2.44$ ***A1***

[11 marks]

Total [19 marks]

11. (a) (i) valid attempt to differentiate implicitly ***(M1)***

$4x = 3\sin^2 y \cos y \frac{\mathrm{d}y}{\mathrm{d}x}$ ***A1A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x}{3\sin^2 y \cos y}$ ***A1***

(ii) at $\left(\frac{1}{4}, \frac{5\pi}{6}\right)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x}{3\sin^2 y \cos y} = \frac{1}{3\left(\frac{1}{2}\right)^2\left(-\frac{\sqrt{3}}{2}\right)}$ ***(M1)***

$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{8}{3\sqrt{3}}\left(= -1.54\right)$ ***A1***

hence equation of tangent is

$y - \frac{5\pi}{6} = -1.54\left(x - \frac{1}{4}\right)$ **OR** $y = -1.54x + 3.00$ ***(M1)A1***

Note: Accept $y = -1.54x + 3$.

[8 marks]

(b) $x = \sqrt{\frac{1}{2}\sin^3 y}$ ***(M1)***

$\int_0^\pi \sqrt{\frac{1}{2}\sin^3 y}\,\mathrm{d}y$ ***(A1)***

$= 1.24$ ***A1***

[3 marks]

continued…

Question 11 continued

(c) use of volume $= \int \pi x^2 \mathrm{d}y$ ***(M1)***

$= \int_0^{\pi} \frac{1}{2}\pi \sin^3 y \, \mathrm{d}y$ ***A1***

$= \frac{1}{2}\pi \int_0^{\pi} \left(\sin y - \sin y \cos^2 y\right) \mathrm{d}y$

Note: Condone absence of limits up to this point.

reasonable attempt to integrate ***(M1)***

$= \frac{1}{2}\pi \left[-\cos y + \frac{1}{3}\cos^3 y\right]_0^{\pi}$ ***A1A1***

Note: Award ***A1*** for correct limits (not to be awarded if previous ***M1*** has not been awarded) and ***A1*** for correct integrand.

$= \frac{1}{2}\pi\left(1 - \frac{1}{3}\right) - \frac{1}{2}\pi\left(-1 + \frac{1}{3}\right)$ ***A1***

$= \frac{2\pi}{3}$ ***AG***

Note: Do not accept decimal answer equivalent to $\frac{2\pi}{3}$.

[6 marks]

Total [17 marks]

Matemáticas
Nivel Superior
Prueba 3 – Análisis

Jueves 21 de noviembre de 2019 (tarde)

1 hora

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de Matemáticas NS y de Ampliación de Matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

8819 – 7228

Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 7]

La función f viene dada por $f(x)=\begin{cases} \dfrac{x-3}{x-5}, & x<3 \\ \ln(x-2), & x\geq 3 \end{cases}$.

(a) Muestre que f es continua en $x=3$. [3]

(b) Muestre que f no es derivable en $x=3$. [4]

2. [Puntuación máxima: 10]

Determine si cada una de las siguientes series infinitas converge o diverge.

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{3n}{2n^2+5}$ [4]

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{(2n)!}{3^n(n!)^2}$ [6]

3. [Puntuación máxima: 11]

La función f viene dada por $f(x)=\operatorname{arcsen}(2x)$, donde $-\dfrac{1}{2}\leq x\leq\dfrac{1}{2}$.

(a) Hallando un número apropiado de derivadas de f, halle los dos primeros términos no nulos del desarrollo en serie de Maclaurin de f. [8]

(b) A partir de lo anterior o de cualquier otro modo, halle $\displaystyle\lim_{x\to 0}\frac{\operatorname{arcsen}(2x)-2x}{(2x)^3}$. [3]

4. [Puntuación máxima: 22]

Considere la ecuación diferencial $\frac{dy}{dx} = \frac{4x^2 + y^2 - xy}{x^2}$, con $y = 2$ cuando $x = 1$.

(a) Utilice el método de Euler con un paso $h = 0{,}1$ para hallar el valor aproximado de y cuando $x = 1{,}4$. [5]

(b) Dibuje aproximadamente las isoclinas para $\frac{dy}{dx} = 4$. [3]

(c) (i) Exprese $m^2 - 2m + 4$ en la forma $(m - a)^2 + b$, donde $a, b \in \mathbb{Z}$.

(ii) Resuelva la ecuación diferencial, para $x > 0$, y dé la respuesta en la forma $y = f(x)$.

(iii) Dibuje aproximadamente el gráfico de $y = f(x)$ para $1 \le x \le 1{,}4$.

(iv) Haciendo referencia a la curvatura del dibujo aproximado que realizó en el apartado (c)(iii), y sin hacer ningún cálculo adicional, explique si conjetura que $f(1{,}4)$ será menor, igual o mayor que la respuesta que dio en el apartado (a). [14]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2019

Calculus

Higher level

Paper 3

14 pages

Instructions to Examiners

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- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685\ldots$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets** eg **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

7 Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) attempt to substitute $x = 3$ in both parts of f ***(M1)***

$\lim_{x \to 3^-} f(x) \left(= \frac{3-3}{3-5} \right) = 0$ ***A1***

$\lim_{x \to 3^+} f(x) \left(= \ln(3-2) \right) = 0$ ***A1***

(since $\lim_{x \to 3^-} f(x) = 0 = \lim_{x \to 3^+} f(x)$), f is continuous at $x = 3$ ***AG***

[3 marks]

(b) **METHOD 1**

for $x < 3$, $f'(x) = \frac{-2}{(x-5)^2}$ ***M1***

$\Rightarrow \lim_{x \to 3^-} f'(x) = -\frac{1}{2}$ (or equivalent) ***A1***

Note: Award ***A0*** for $f'(3) = -\frac{1}{2}$.

for $x > 3$, $f'(x) = \frac{1}{x-2}$ ***M1***

Note: Condone $x \geq 3$.

$\Rightarrow \lim_{x \to 3^+} f'(x) = 1$ (or equivalent) ***A1***

Note: Award ***A0*** for $f'(3) = 1$.

(since $-\frac{1}{2} \neq 1$), f is not differentiable at $x = 3$ ***AG***

METHOD 2

$\lim_{h \to 0^-} \frac{f(3+h) - f(3)}{h}$ ***M1***

$= \lim_{h \to 0^-} \frac{\frac{h}{h-2} - 0}{h} = \lim_{h \to 0^-} \frac{1}{h-2}$

$= -\frac{1}{2}$ ***A1***

$\lim_{h \to 0^+} \frac{f(3+h) - f(3)}{h}$ ***M1***

$= \lim_{h \to 0^+} \frac{\ln(1+h)}{h} = \lim_{h \to 0^+} \frac{1}{1+h}$ (by L'Hôpital)

$= 1$ ***A1***

(since $-\frac{1}{2} \neq 1$), f is not differentiable at $x = 3$ ***AG***

[4 marks]

Total [7 marks]

2. (a) **METHOD 1**

attempt to use limit comparison test and choosing an appropriate b_n ***M1***

let $a_n = \dfrac{3n}{2n^2+5}$ and $b_n = \dfrac{1}{n}$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\frac{3n}{2n^2+5}}{\frac{1}{n}}$$ ***(A1)***

$$= \lim_{n\to\infty}\frac{3}{2+\frac{5}{n^2}}$$

$$= \frac{3}{2}\ (>0)$$ ***A1***

since $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, $\sum_{n=1}^{\infty}\frac{3n}{2n^2+5}$ also diverges (by the limit comparison test) ***R1***

NOTE: Do not award ***R1*** if candidates omit sigma.

METHOD 2

attempt to find $\int_1^{\infty}\frac{3x}{2x^2+5}\mathrm{d}x$ ***M1***

$$= \lim_{R\to\infty}\int_1^{R}\frac{3x}{2x^2+5}\mathrm{d}x$$

$$= \lim_{R\to\infty}\left[\frac{3}{4}\ln(2x^2+5)\right]_1^R$$ ***A1***

NOTE: Condone use of ∞ as the upper limit.

$$= \lim_{R\to\infty}\frac{3}{4}\ln(2R^2+5) - \frac{3}{4}\ln 7$$

$= \infty$ (accept limit DNE) ***A1***

since $\int_1^{\infty}\frac{3x}{2x^2+5}\mathrm{d}x$ diverges, $\sum_{n=1}^{\infty}\frac{3n}{2n^2+5}$ also diverges (by the integral test.) ***R1***

continued…

Question 2 continued

METHOD 3

attempt to use comparison test and choosing an appropriate b_n ***M1***

EITHER

for $n > 2$ ***A1***

$\frac{3n}{2n^2+5} > \frac{1}{n}$ ***A1***

OR

for $n \geq 1$ ***(A1)***

$\frac{3n}{2n^2+5} > \frac{1}{3n}$ ***A1***

THEN

since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, $\sum_{n=1}^{\infty} \frac{3n}{2n^2+5}$ also diverges (by the comparison test.) ***R1***

Note: In both cases accept valid alternative inequalities. Do not award ***R1*** if candidates omit sigma.

[4 marks]

(b) attempt to use ratio test

$\frac{a_{n+1}}{a_n} = \frac{(2n+2)!}{3^{n+1}\left((n+1)!\right)^2} \times \frac{3^n (n!)^2}{(2n)!}$ (***M1***)

attempt to simplify factorials (***M1***)

$\frac{a_{n+1}}{a_n} = \frac{(2n+2)(2n+1)}{3(n+1)^2} \quad \left(= \frac{4n^2+6n+2}{3n^2+6n+3}\right)$ ***A1***

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\left(2+\frac{2}{n}\right)\left(2+\frac{1}{n}\right)}{3\left(1+\frac{1}{n}\right)^2}$ ***(M1)***

$= \frac{4}{3}$ ***A1***

since $\frac{4}{3} > 1$, $\sum_{n=1}^{\infty} \frac{(2n)!}{3^n (n!)^2}$ diverges (by the ratio test) ***R1***

Note: Award ***R1*** for correct reasoning consistent with their limit.

[6 marks]

Total [10 marks]

3. (a) $f(x) = \arcsin(2x)$

$f'(x) = \dfrac{2}{\sqrt{1-4x^2}}$ ***M1A1***

> **Note:** Award ***M1A0*** for $f'(x) = \dfrac{1}{\sqrt{1-4x^2}}$.

$f''(x) = \dfrac{8x}{\left(1-4x^2\right)^{\frac{3}{2}}}$ ***A1***

EITHER

$$f'''(x) = \frac{8\left(1-4x^2\right)^{\frac{3}{2}} - 8x\left(\frac{3}{2}(-8x)\left(1-4x^2\right)^{\frac{1}{2}}\right)}{\left(1-4x^2\right)^3} \quad \left(= \frac{8\left(1-4x^2\right)^{\frac{3}{2}} + 96x^2\left(1-4x^2\right)^{\frac{1}{2}}}{\left(1-4x^2\right)^3}\right)$$ ***A1***

OR

$$f'''(x) = 8\left(1-4x^2\right)^{-\frac{3}{2}} + 8x\left(-\frac{3}{2}\left(1-4x^2\right)^{-\frac{5}{2}}\right)(-8x) \quad \left(= 8\left(1-4x^2\right)^{-\frac{3}{2}} + 96x^2\left(1-4x^2\right)^{-\frac{5}{2}}\right)$$ ***A1***

THEN

substitute $x = 0$ into f or any of its derivatives ***(M1)***

$f(0) = 0$, $f'(0) = 2$ and $f''(0) = 0$ ***A1***

$f'''(0) = 8$

the Maclaurin series is

$f(x) = 2x + \dfrac{8x^3}{6} + \ldots \left(= 2x + \dfrac{4x^3}{3} + \ldots\right)$ ***(M1)A1***

[8 marks]

continued…

Question 3 continued

(b) **METHOD 1**

$$\lim_{x\to 0}\frac{\arcsin(2x)-2x}{(2x)^3}=\lim_{x\to 0}\frac{2x+\dfrac{4x^3}{3}+\ldots-2x}{8x^3}$$ ***M1***

$$=\lim_{x\to 0}\frac{\dfrac{4}{3}+\ldots\text{ terms with }x}{8}$$ ***(M1)***

$$=\frac{1}{6}$$ ***A1***

Note: Condone the omission of $+\ldots$ in their working.

METHOD 2

$$\lim_{x\to 0}\frac{\arcsin(2x)-2x}{(2x)^3}=\frac{0}{0}$$ indeterminate form, using L'Hôpital's rule

$$=\lim_{x\to 0}\frac{\dfrac{2}{\sqrt{1-4x^2}}-2}{24x^2}$$ ***M1***

$$=\frac{0}{0}$$ indeterminate form, using L'Hôpital's rule again

$$=\lim_{x\to 0}\frac{\dfrac{8x}{\left(1-4x^2\right)^{\frac{3}{2}}}}{48x}\left(=\lim_{x\to 0}\frac{1}{6\left(1-4x^2\right)^{\frac{3}{2}}}\right)$$ ***M1***

Note: Award ***M1*** only if their previous expression is in indeterminate form.

$$=\frac{1}{6}$$ ***A1***

Note: Award ***FT*** for use of their derivatives from part (a).

[3 marks]

Total [11 marks]

4. (a)

x	y	$\frac{dy}{dx}$	
1	2	6	***(M1)***
1.1	2.6	7.22	***(A1)***
1.2	3.32	8.89652	***(A1)***
1.3	4.21	11.26	***(A1)***
1.4	5.34		***A1***

$y(1.4) \approx 5.34$

Note: Award ***A1*** for each correct y value.
For the intermediate y values, accept answers that are accurate to 2 significant figures.
The final y value must be accurate to 3 significant figures or better.

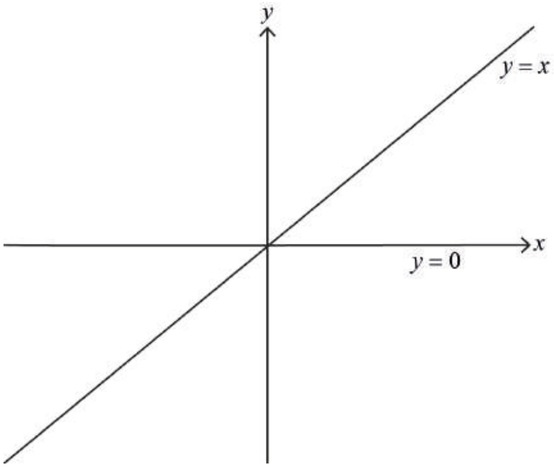
[5 marks]

(b) attempt to solve $\frac{4x^2 + y^2 - xy}{x^2} = 4$ ***(M1)***

$\Rightarrow y^2 - xy = 0$

$y(y-x) = 0$

$y = 0$ or $y = x$



A1A1

[3 marks]

(c) (i) $m^2 - 2m + 4 = (m-1)^2 + 3 \quad (a = 1, b = 3)$ ***A1***

continued…

Question 4 continued

(ii) recognition of homogeneous equation,
let $y = vx$ ***M1***

the equation can be written as

$v + x\frac{\mathrm{d}v}{\mathrm{d}x} = 4 + v^2 - v$ ***(A1)***

$x\frac{\mathrm{d}v}{\mathrm{d}x} = v^2 - 2v + 4$

$\int \frac{1}{v^2 - 2v + 4}\mathrm{d}v = \int \frac{1}{x}\mathrm{d}x$ ***M1***

Note: Award ***M1*** for attempt to separate the variables.

$\int \frac{1}{(v-1)^2 + 3}\mathrm{d}v = \int \frac{1}{x}\mathrm{d}x$ from part (c)(i) ***M1***

$\frac{1}{\sqrt{3}}\arctan\left(\frac{v-1}{\sqrt{3}}\right) = \ln x \ (+c)$ ***A1A1***

$x = 1, y = 2 \Rightarrow v = 2$

$\frac{1}{\sqrt{3}}\arctan\left(\frac{1}{\sqrt{3}}\right) = \ln 1 + c$ ***M1***

Note: Award ***M1*** for using initial conditions to find c.

$\Rightarrow c = \frac{\pi}{6\sqrt{3}} \quad (= 0.302)$ ***A1***

$\arctan\left(\frac{v-1}{\sqrt{3}}\right) = \sqrt{3}\ln x + \frac{\pi}{6}$

substituting $v = \frac{y}{x}$ ***M1***

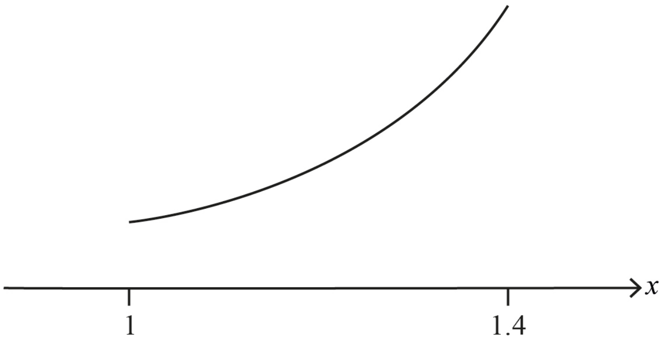
Note: This ***M1*** may be awarded earlier.

$y = x\left(\sqrt{3}\tan\left(\sqrt{3}\ln x + \frac{\pi}{6}\right) + 1\right)$ ***A1***

continued…

Question 4 continued

(iii)



curve drawn over correct domain ***A1***

(iv) the sketch shows that f is concave up ***A1***

Note: Accept f' is increasing.

this means the tangent drawn using Euler's method will give an underestimate of the real value, so $f(1.4) >$ estimate in part (a) ***R1***

Note: The ***R1*** is dependent on the ***A1***.

[14 marks]

Total [22 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel Superior
Prueba 3 – Matemáticas discretas

Jueves 21 de noviembre de 2019 (tarde)

1 hora

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de Matemáticas NS y de Ampliación de Matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

8819 – 7227

Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 10]

Un conductor tiene que repartir mercancías a cinco tiendas A, B, C, D y E.
El conductor inicia y acaba su recorrido en el almacén W. El conductor quiere hallar la ruta más corta que pase por todas las tiendas y le lleve de vuelta al almacén. En la siguiente tabla se muestran las distancias, en kilómetros, que hay entre estos lugares.

	A	***B***	***C***	***D***	***E***	***W***
A	-	11	28	15	20	40
B	11	-	25	20	32	36
C	28	25	-	16	22	39
D	15	20	16	-	12	42
E	20	32	22	12	-	41
W	40	36	39	42	41	-

(a) Borrando W, utilice el algoritmo de vértice borrado para hallar un límite inferior para la longitud de una ruta que pase por todas las tiendas y que empiece y acabe en W. [6]

(b) Empezando en W, utilice el algoritmo del vecino más próximo para hallar una ruta que represente un límite superior para este problema y calcule su longitud. [4]

2. [Puntuación máxima: 15]

(a) (i) Indique el pequeño teorema de Fermat.

(ii) Halle el resto que se obtiene al dividir 15^{1207} entre 13. [7]

En los apartados (b) y (c), $(abc\ldots)_n$ denota el número $abc\ldots$ escrito en base n, donde $n \in \mathbb{Z}^+$. Por ejemplo, $(359)_n = 3n^2 + 5n + 9$.

(b) Convierta $(7A2)_{16}$ a base 5, donde $(A)_{16} = (10)_{10}$. [4]

(c) Considere la ecuación $(1251)_n + (30)_n = (504)_n + (504)_n$.

Halle el valor de n. [4]

3. [Puntuación máxima: 6]

Sea una relación de recurrencia lineal definida por

$$u_n = au_{n-1} + b\,,\text{ donde } a\,, b \in \mathbb{R}\,.$$

Los dos primeros términos son $u_1 = 25$ y $u_2 = 16$.

(a) Sabiendo que $\lim_{n\to\infty} u_n = 10$, muestre que $a = \frac{2}{5}$. [4]

(b) A partir de lo anterior, halle el valor de u_3. [2]

4. [Puntuación máxima: 14]

(a) G es un grafo conexo y simple que tiene ocho vértices.

(i) Escriba el número mínimo de aristas que puede tener G.

(ii) Halle el número máximo de aristas que puede tener G.

(iii) Halle el número máximo de aristas que puede tener G, sabiendo que G contiene un circuito euleriano. [5]

(b) H es un grafo planario y conexo que tiene v vértices, e aristas y f caras. Cada una de las caras de H está delimitada por exactamente k aristas.

(i) Explique por qué $2e = kf$.

(ii) Halle el valor de f cuando $v = 9$ y $k = 3$.

(iii) Halle los posibles valores de f cuando $v = 13$. [9]

5. [Puntuación máxima: 5]

Se colocan diez puntos en cualquier lugar del interior o del perímetro de un cuadrado de lado igual a 1.

Utilice el principio del palomar para demostrar que al menos dos de estos puntos están separados por una distancia menor o igual que $\frac{\sqrt{2}}{3}$.

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2019

Discrete mathematics

Higher level

Paper 3

11 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

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  Examples

	Correct answer seen	**Further working seen**	**Action**
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Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

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*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

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13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) deleting W and its adjacent edges, the minimal spanning tree is

Edge	Weight	
AB	11	***A1***
DE	12	***A1***
AD	15	***A1***
CD	16	***A1***

Note: Award the ***A1's*** for either the edges or their weights.

the minimum spanning tree has weight $= 54$

Note: Accept a correct drawing of the minimal spanning tree.

adding in the weights of 2 deleted edges of least weight WB and WC ***(M1)***
lower bound $= 54 + 36 + 39$
$= 129$ ***A1***

[6 marks]

(b) attempt at the nearest-neighbour algorithm ***M1***
WB
BA
AD
DE
EC
CW ***A1***

Note: Award ***M1*** for a route that begins with WB and then BA.

upper bound $= 36 + 11 + 15 + 12 + 22 + 39 = 135$ ***(M1)A1***

[4 marks]
Total [10 marks]

2. (a) (i) **EITHER**

$a^p \equiv a \pmod p$ ***A1***
where p is prime ***A1***

OR

$a^{p-1} \equiv 1 \pmod p$ ***A1***
where p is prime and p does not divide a (or equivalent statement) ***A1***

continued…

Question 2 continued

(ii) $15^{1207} \equiv 2^{1207} \left(\bmod 13\right)$

$2^{12} \equiv 1(\bmod 13)$ ***(M1)(A1)***

$2^{1207} = \left(2^{12}\right)^{100} 2^{7}$ ***(M1)***

$2^{1207} \left(\equiv 2^{7}\right) \equiv 11 \left(\bmod 13\right)$ ***(M1)A1***

the remainder is 11

Note: Award as above for using 15 instead of 2.

[7 marks]

(b) $\left(7A2\right)_{16} = 7 \times 16^{2} + 10 \times 16 + 2$ ***M1***

$= 1954$ ***A1***

EITHER

$$\begin{array}{r|l} 5 & \underline{1954} \\ & 390\ r\ 4 \\ & 78\ r\ 0 \\ & 15\ r\ 3 \\ & 3\ r\ 0 \\ & 0\ r\ 3 \end{array}$$

M1

OR

$1954 = 3 \times 5^{4} + 0 \times 5^{3} + 3 \times 5^{2} + 0 \times 5^{1} + 4$ ***M1***

THEN

$\left(7A2\right)_{16} = \left(30304\right)_{5}$ ***A1***

[4 marks]

(c) the equation can be written as

$n^{3} + 2n^{2} + 5n + 1 + 3n = 2\left(5n^{2} + 4\right)$ ***M1A1***

$\Rightarrow n^{3} - 8n^{2} + 8n - 7 = 0$ ***(M1)***

Note: The ***(M1)*** is for an attempt to solve the original equation.

$n = 7$ ***A1***

[4 marks]

Total [15 marks]

3. METHOD 1

(a) solution is of the form $u_n = Aa^n + C$ ***M1***

(since the limit is 10) $C = 10$ ***A1***

substituting $n = 1$ and $n = 2$ into u_n with $C = 10$ ***(M1)***

$15 = Aa$ and $6 = Aa^2$ ***A1***

$\Rightarrow a = \dfrac{2}{5}$ ***AG***

[4 marks]

(b) $A = \dfrac{75}{2}$ (may be seen in part (a)) ***A1***

$u_n = \dfrac{75}{2}\left(\dfrac{2}{5}\right)^n + 10$

$\Rightarrow u_3 = \dfrac{62}{5}(=12.4)$ ***A1***

[2 marks]

METHOD 2

(a) $16 = 25a + b$ ***A1***

$\lim_{n\to\infty} u_n = \lim_{n\to\infty} u_{n-1} = 10$ ***(M1)***

$10 = 10a + b$ ***A1***

solving their two equations simultaneously ***M1***

$6 = 15a$

$\Rightarrow a = \dfrac{2}{5}$ ***AG***

[4 marks]

(b) $b = 6$ (may be seen in part (a)) ***A1***

$u_3 = \dfrac{2}{5}(16) + 6$

$= \dfrac{62}{5}(=12.4)$ ***A1***

[2 marks]

Total [6 marks]

4. (a) (i) 7 (a tree) ***A1***

(ii) $\frac{8\times 7}{2}$ $(7+6+5+4+3+2+1)$ (a complete graph) ***(M1)***

$=28$ ***A1***

(iii) $\frac{8\times 6}{2}$ (since every vertex must be of degree 6) ***(M1)***

$=24$ ***A1***

[5 marks]

(b) (i) counting the edges around every face gives kf edges ***A1***

but as every edge is counted in 2 faces ***R1***

$\Rightarrow kf = 2e$ ***AG***

(ii) using $v-e+f=2$ with $v=9$ ***M1***

EITHER

substituting $2e=3f$ into $2(9)-2e+2f=4$ ***(M1)***

OR

substituting $e=\frac{3f}{2}$ into $9-e+f=2$ ***(M1)***

THEN

$18-f=4$

$f=14$ ***A1***

(iii) $2v-kf+2f=4$ (or equivalent) ***M1***

when $v=13$

$(k-2)f=22$ or $(2-k)f=-22$ ***A1***

EITHER

$(k-2)f=1\times 2\times 11$ ***M1***

OR

substituting at least two of $k=13,4,3$ into $f=\frac{22}{k-2}$ (or equivalent) ***M1***

THEN

$f=2,11,22$ (since $f>1$) ***A1***

[9 marks]

Total [14 marks]

5. divide the square into 9 equal squares (of side length $\frac{1}{3}$) ***M1A1***

Note: Award ***M1*** for an attempt to divide the square into 9 parts.

by the pigeon-hole principle, at least 2 points must be inside or on the perimeter of one of the smaller squares ***R1***

the maximum distance between two points of a small square is

$$\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$$ ***M1A1***

$$= \frac{\sqrt{2}}{3}$$ ***AG***

Total [5 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel Superior
Prueba 3 – Conjuntos, relaciones y grupos

Jueves 21 de noviembre de 2019 (tarde)

1 hora

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- Conteste todas las preguntas.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de Matemáticas NS y de Ampliación de Matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

8819 – 7229

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 12]

Sean $A = \{1, 3, 4, 5, 8, 9\}$, $B = \{1, 5, 6, 7, 9\}$ y $C = \{1, 2, 7, 8, 9\}$.

(a) (i) Halle $(A \setminus B) \setminus C$, donde \ representa la diferencia de conjuntos.

(ii) Halle $A \setminus (B \setminus C)$.

(iii) A partir de lo anterior, determine si la diferencia de conjuntos es asociativa. [5]

(b) Halle $(A \Delta B) \Delta C$, donde Δ representa la diferencia simétrica. [2]

(c) Partiendo de los conjuntos A, B y C, determine si la diferencia simétrica es distributiva respecto a la intersección. [5]

2. [Puntuación máxima: 14]

El conjunto $\{-4, -3, -2, -1, 0, 1, 2, 3\}$ junto con la operación binaria $*$ constituye un grupo, definido en la siguiente tabla de Cayley.

$*$	-4	-3	-2	-1	0	1	2	3
-4	0	1	2	3	-4	-3	-2	-1
-3	1	a	3	-4	-3	-2	-1	0
-2	2	3	-4	-3	-2	-1	0	1
-1	3	-4	-3	-2	-1	0	1	2
0	-4	-3	-2	-1	0	1	2	3
1	-3	b	-1	0	1	2	c	-4
2	-2	-1	0	1	2	3	-4	-3
3	-1	0	1	2	3	-4	-3	-2

(a) (i) Explique lo que significa el término cuadrado latino.

(ii) A partir de lo anterior, escriba los valores de a, b y c. [4]

(b) (i) Escriba el elemento neutro de este grupo.

(ii) A partir de lo anterior, indique cuál es el simétrico del elemento -4. [2]

(c) Hallando el orden de los elementos, determine si este grupo es cíclico. [3]

(Esta pregunta continúa en la página siguiente)

(Pregunta 2: continuación)

(d) Halle un subgrupo de orden 4. [2]

Existe un isomorfismo f entre el grupo $\{\{-4, -3, -2, -1, 0, 1, 2, 3\}, *\}$ y el grupo $\{\{0, 1, 2, 3, 4, 5, 6, 7\}, +_8\}$, donde $+_8$ es la operación adición módulo 8.

(e) Sabiendo que $f(1) = 1$, halle el valor de $f(-3)$. [3]

3. [Puntuación máxima: 13]

(a) Sea V el conjunto de todos los vectores tridimensionales. Se define una relación R sobre V de modo que $\boldsymbol{a}R\boldsymbol{b}$ si y solo si $\boldsymbol{a} \bullet \boldsymbol{b} = 0$. Determine, de manera razonada, si R es

(i) reflexiva;

(ii) simétrica;

(iii) transitiva. [3]

(b) Sea W el conjunto de todos los vectores tridimensionales **no nulos**. Se define una relación S sobre W de modo que $\boldsymbol{a}S\boldsymbol{b}$ si y solo si $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0}$. Determine, de manera razonada, si S es

(i) reflexiva;

(ii) simétrica;

(iii) transitiva. [5]

(c) (i) Exactamente una de las relaciones R y S es una relación de equivalencia. Indique cuál.

(ii) Para esa relación de equivalencia, $\begin{pmatrix} -2 \\ y \\ -4 \end{pmatrix}$ pertenece a la clase de equivalencia que contiene a $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$. Halle el valor de y. [3]

(d) La relación S del apartado (b) se define ahora sobre el conjunto V del apartado (a). Determine, de manera razonada, si S es transitiva sobre V. [2]

4. [Puntuación máxima: 11]

(a) Sea el grupo $\{G, *\}$.
Demuestre que $\{G, *\}$ tiene **exactamente** un elemento neutro. [3]

(b) La operación binaria $\otimes$ se define sobre el conjunto de los números reales mediante $a \otimes b = a|b|$.

(i) Determine si $\otimes$ es asociativa. Justifique su respuesta.

(ii) Determine si existe un elemento neutro para $\otimes$. Justifique su respuesta. [8]

Markscheme

November 2019

Sets, relations and groups

Higher level

Paper 3

10 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685\ldots$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 ***N*** marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets** eg **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

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1. (a) (i) $(A \setminus B) = \{3, 4, 8\}$ *(A1)*

$(A \setminus B) \setminus C = \{3, 4\}$ ***A1***

(ii) $B \setminus C = \{5, 6\}$ *(A1)*

$A \setminus (B \setminus C) = \{1, 3, 4, 8, 9\}$ ***A1***

(iii) not associative as $(A \setminus B) \setminus C \neq A \setminus (B \setminus C)$ ***R1***

[5 marks]

(b) $A \Delta B = \{3, 4, 6, 7, 8\}$ *(A1)*

$(A \Delta B) \Delta C = \{1, 2, 3, 4, 6, 9\}$ ***A1***

[2 marks]

(c) $A \Delta (B \cap C) = \{3, 4, 5, 7, 8\}$ *(M1)A1*

$(A \Delta B) \cap (A \Delta C) = \{3, 4, 7\}$ *(M1)A1*

these two sets are not equal hence symmetric difference is not distributive over intersection ***R1***

Note: Allow investigating distributive from the right instead of from the left.

[5 marks]

Total [12 marks]

2. (a) (i) each element appears (once and only once) in each row and each column ***A1***

(ii) since table is a Latin square
$a = 2$, $b = -2$, $c = 3$ ***A1A1A1***

[4 marks]

(b) (i) identity is 0 ***A1***

(ii) inverse is -4 ***A1***

[2 marks]

(c) attempt to find the order of an element (other than 0 or -4) ***(M1)***
1 (or -1or 3 or -3) is a generator ***A1***
hence the group is cyclic ***A1***

[3 marks]

(d) 2 (or -2) is of order 4 ***(M1)***
subgroup is $\{2, -4, -2, 0\}$ ***A1***

[2 marks]

(e) $f(-3) = f(1^5)$ ***M1***
$= \left(f(1)\right)^5 = 1^5$ ***R1***
$\Rightarrow f(-3) = 5$ ***A1***

Note:	Allow ***A1*** for correct final answer regardless of the earlier work and ***M*** and ***R*** marks. There are other valid methods.

[3 marks]
Total [14 marks]

3. (a) (i) $\boldsymbol{a} \bullet \boldsymbol{a} = |\boldsymbol{a}|^2$ hence R is not reflexive ***R1***

(ii) $\boldsymbol{a}R\boldsymbol{b} \Rightarrow \boldsymbol{a} \bullet \boldsymbol{b} = 0 \Rightarrow \boldsymbol{b} \bullet \boldsymbol{a} = 0 \Rightarrow \boldsymbol{b}R\boldsymbol{a}$ hence R is symmetric ***R1***

(iii) for example, $\boldsymbol{i}R\boldsymbol{j}$ and $\boldsymbol{j}R\boldsymbol{i}$ but $\boldsymbol{i}R\boldsymbol{i}$ is false hence R is not transitive ***R1***

Note: Accept geometrical reasoning.

[3 marks]

(b) (i) $\boldsymbol{a} \times \boldsymbol{a} = \boldsymbol{0}\ \forall \boldsymbol{a}$ so S is reflexive ***R1***

(ii) $\boldsymbol{a}S\boldsymbol{b} \Rightarrow \boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0} \Rightarrow \boldsymbol{b} \times \boldsymbol{a} = -\boldsymbol{0} = \boldsymbol{0} \Rightarrow \boldsymbol{b}S\boldsymbol{a}$ hence S is symmetric ***R1***

(iii) $\boldsymbol{a}S\boldsymbol{b}$ and $\boldsymbol{b}S\boldsymbol{c} \Rightarrow \boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0}$ and $\boldsymbol{b} \times \boldsymbol{c} = \boldsymbol{0}$ ***M1***
$\Rightarrow \boldsymbol{a} = \mu \boldsymbol{b}$ and $\boldsymbol{b} = \lambda \boldsymbol{c}$ (or $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel and $\boldsymbol{b}$ and $\boldsymbol{c}$ are parallel) $\Rightarrow \boldsymbol{a} = \mu\lambda \boldsymbol{c}$ (or $\boldsymbol{a}$ and $\boldsymbol{c}$ are parallel) $\Rightarrow \boldsymbol{a} \times \boldsymbol{c} = \boldsymbol{0} \Rightarrow \boldsymbol{a}S\boldsymbol{c}$ ***R1***
hence S is transitive ***A1***

Note: The ***A*** mark depends on the ***R*** mark.

[5 marks]

(c) (i) S is an equivalence relation ***A1***

(ii) as cross product is zero, (the two vectors are parallel) ***(M1)***
$\Rightarrow y = -6$ ***A1***

[3 marks]

(d) for example $\boldsymbol{i}S\boldsymbol{0}$ and $\boldsymbol{0}S\boldsymbol{j}$ since $\boldsymbol{i} \times \boldsymbol{0} = \boldsymbol{0}$ and $\boldsymbol{0} \times \boldsymbol{j} = \boldsymbol{0}$ but $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$
hence $\boldsymbol{i}S\boldsymbol{j}$ is false ***R1***
S is not transitive ***A1***

Note: The ***A*** mark depends on the ***R*** mark.

[2 marks]

Total [13 marks]

4. (a) suppose that there are 2 identity elements e and $E,\ e \neq E$ ***M1***

EITHER

consider $e * E$ ***M1***

as e is an identity $e * E = E$. As E is an identity $e * E = e$ ***R1***

OR

$a * e = a * E \Rightarrow a^{-1} * (a * e) = a^{-1} * (a * E)$ ***M1***

$\Rightarrow (a^{-1} * a) * e = (a^{-1} * a) * E \Rightarrow e * e = e * E$ ***R1***

THEN

so $e = E \Rightarrow\Leftarrow$ proving that $\{G, *\}$ has exactly one identity element ***AG***

[3 marks]

(b) (i) $(a \otimes b) \otimes c = a|b| \otimes c = a|b||c|$ ***A1***

$a \otimes (b \otimes c) = a \otimes b|c| = a\big|b|c|\big| = a|b||c|$ ***A1***

hence it is associative ***A1***

(ii) $e \otimes a = a \otimes e = a \Rightarrow e|a| = a|e| = a$ ***M1***

$\Rightarrow e = \pm 1$ ***A1***

for $e = 1$ example $1 \otimes -2 = |-2| = 2 \neq -2$ so $e \neq 1$ ***R1***

for $e = -1$ example $-1 \otimes 2 = -|2| = -2 \neq 2$ so $e \neq -1$ ***R1***

hence there is no identity element ***A1***

[8 marks]

Total [11 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel Superior
Prueba 3 – Estadística y probabilidad

Jueves 21 de noviembre de 2019 (tarde)

1 hora

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- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de Matemáticas NS y de Ampliación de Matemáticas NS** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[50 puntos]**.

8819 – 7230

International Baccalaureate
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Por favor comience cada pregunta en una página nueva. No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

1. [Puntuación máxima: 7]

Peter, el director de un centro educativo, cree que existe una relación entre la nota de sus alumnos en un examen de matemáticas, X, y el tiempo que tardan en correr $500\,\text{m}$, Y segundos. Se recogen los siguientes datos pareados.

Nota en el examen de matemáticas X	70	75	76	66	60	61
Tiempo que tarda en correr $500\,\text{m}$ Y	100	105	95	109	89	101

Se puede suponer que (X, Y) sigue una distribución normal bidimensional, cuyo coeficiente de correlación momento-producto es ρ.

(a) (i) Indique hipótesis H_0 y H_1 apropiadas para poner a prueba la afirmación de Peter, utilizando un contraste de dos colas.

(ii) Realice un contraste apropiado a un nivel de significación del 5%. Haciendo referencia al valor del parámetro p, indique cuál es su conclusión respecto a la afirmación de Peter. [5]

(b) Peter utiliza una recta de regresión de y sobre x dada por $y = 0{,}248x + 83{,}0$ y calcula que un alumno que saque una nota de 73 en el examen de matemáticas tendrá una marca de 101 segundos en la carrera. Comente la validez de este cálculo. [2]

2. [Puntuación máxima: 15]

(a) Se toman tres variables aleatorias independientes X_1, X_2, X_3 de una distribución de media μ y varianza σ^2. Se proponen tres estimadores para μ.

$$T_1 = \frac{X_1 + X_2 + X_3}{3},\ T_2 = \frac{X_1 + 2X_2 + 3X_3}{3},\ T_3 = \frac{X_1 + 2X_2}{3}$$

(i) Muestre que uno de estos tres estimadores para μ es sesgado; muestre que los otros dos son estimadores sin sesgo.

(ii) De entre los dos estimadores sin sesgo, determine, de manera razonada, cuál es el más eficiente. [9]

(Esta pregunta continúa en la página siguiente)

(Pregunta 2: continuación)

(b) Considere la variable aleatoria Y, que sigue una distribución binomial negativa $Y \sim \mathrm{NB}(4, p)$. Tomamos una muestra aleatoria de esta distribución, y a la media la denominamos $\bar{Y}$.

(i) Halle $\mathrm{E}(\bar{Y})$.

(ii) A partir de lo anterior, sugiera un estimador sin sesgo de $\frac{1}{p}$ en función de $\bar{Y}$. [2]

(c) La variable aleatoria discreta W tiene la distribución de probabilidad que se muestra en la siguiente tabla.

w	1	2
$\mathrm{P}(W = w)$	$0{,}5$	$0{,}5$

(i) Calcule $\mathrm{E}(W)$.

(ii) Calcule $\mathrm{E}\left(\frac{1}{W}\right)$.

(iii) A partir de lo anterior, explique por qué el estimador de $\frac{1}{p}$ que dio en (b)(ii) no sugiere directamente un estimador sin sesgo de p. [4]

3. [Puntuación máxima: 14]

(a) Indique el teorema central del límite, aplicado a una muestra aleatoria de tamaño n que se toma de una distribución de media μ y varianza σ^2. [2]

La variable aleatoria X tiene una distribución de media μ y varianza 4. Tomamos una muestra aleatoria de tamaño 100 de esta distribución de X.

(b) Jack toma una muestra aleatoria de tamaño 100 y calcula que $\bar{x} = 60{,}2$. Halle un intervalo de confianza aproximado del $90\,\%$ para μ. [2]

(c) Josie toma una muestra aleatoria de tamaño 100 distinta para contrastar la hipótesis nula de que $\mu = 60$, frente a la hipótesis alternativa de que $\mu > 60$, a un nivel del $5\,\%$.

(i) Halle la región crítica para el contraste de Josie. Dé la respuesta redondeando a dos lugares decimales.

(ii) Escriba la probabilidad de que Josie cometa un error de tipo I.

(iii) Sabiendo que la probabilidad de que Josie cometa un error de tipo II es igual a $0{,}25$, halle el valor de μ. Dé la respuesta redondeando a tres cifras significativas. [10]

4. [Puntuación máxima: 14]

Considere la variable aleatoria X, que sigue una distribución binomial negativa $X \sim \mathrm{NB}(r, p)$. La función generatriz de probabilidad para X es

$$G_X(t) = \frac{p^r t^r}{(1-qt)^r}\text{, donde } q = 1 - p\,.$$

(a) Utilice esta función generatriz de probabilidad para hallar y simplificar $\mathrm{E}(X)$. [5]

Considere otra variable aleatoria independiente Y, donde $Y \sim \mathrm{NB}(s, p)$.
Sea $W = X + Y$.

(b) (i) Halle la función generatriz de probabilidad para W.

(ii) A partir de lo anterior, identifique qué distribución sigue W e indique sus parámetros.

(iii) Sabiendo que $r = 2$ y $s = 3$, calcule $\mathrm{P}(X = 3 \mid W = 7)$. [9]

Markscheme

November 2019

Statistics and probability

Higher level

Paper 3

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

  Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685\ldots$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets** eg **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(= 10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) (i) $H_0: \rho = 0 \quad H_1: \rho \neq 0$ ***A1***

Note: It must be ρ.

(ii) $p = 0.649$ ***A2***

Note: Accept anything that rounds to 0.65.

$0.649 > 0.05$ ***R1***

hence, we accept H_0 and conclude that Peter's claim is wrong ***A1***

Note: The ***A*** mark depends on the ***R*** mark and the answer must be given in context. Follow through the p-value in part (b).

[5 marks]

(b) a statement along along the lines of '(we have accepted that) the two variables are independent' or 'the two variables are weakly correlated' ***R1***

a statement along the lines of 'the use of the regression line is invalid' or 'it would give an inaccurate result' ***R1***

Note: Award the second ***R1*** only if the first ***R1*** is awarded.

Note: FT the conclusion in(a)(ii). If a candidate concludes that the claim is correct, mark as follows: (as we have accepted H_1) the 2 variables are dependent and 73 lies in the range of x values ***R1***, hence the use of the regression line is valid ***R1***.

[2 marks]

Total [7 marks]

2. (a) (i) attempt to find expected values *eg* $\mathrm{E}(T_1)$ ***(M1)***

$\mathrm{E}(T_1) = \frac{1}{3}\mathrm{E}(X_1 + X_2 + X_3) = \frac{1}{3}(\mathrm{E}(X_1) + \mathrm{E}(X_2) + \mathrm{E}(X_3))$

$= \mu$ ***A1***

$\mathrm{E}(T_2) = \frac{1}{3}\mathrm{E}(X_1 + 2X_2 + 3X_3) = \frac{1}{3}(\mathrm{E}(X_1) + 2\mathrm{E}(X_2) + 3\mathrm{E}(X_3))$

$= 2\mu$ ***A1***

$\mathrm{E}(T_3) = \frac{1}{3}\mathrm{E}(X_1 + 2X_2) = \frac{1}{3}(\mathrm{E}(X_1) + 2\mathrm{E}(X_2))$

$= \mu$ ***A1***

Note: Order does not matter.

$(2\mu \neq \mu)$ hence T_2 is biased, T_1 and T_3 are unbiased

(ii) use of variance of linear combinations ***(M1)***

$\mathrm{Var}(T_1) = \frac{1}{9}\left(\mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \mathrm{Var}(X_3)\right)$

$= \frac{3}{9}\sigma^2 \left(= \frac{\sigma^2}{3}\right)$ ***A1***

$\mathrm{Var}(T_3) = \frac{1}{9}\left(\mathrm{Var}(X_1) + 4\mathrm{Var}(X_2)\right)$

$= \frac{5}{9}\sigma^2$ ***A1***

$\frac{3}{9}\sigma^2 < \frac{5}{9}\sigma^2$ so T_1 is the more efficient estimator ***R1A1***

Note: Award ***A1*** only if the ***R1*** is awarded.

Note: Follow through their variances and award ***R1*** for a comparison and ***A1*** if the ***M1*** was awarded.

[9 marks]

(b) (i) $\mathrm{E}(\bar{Y}) = \mathrm{E}(Y) = \frac{4}{p}$ ***A1***

(ii) $\frac{\bar{Y}}{4}$ ***A1***

[2 marks]

(c) (i) $\mathrm{E}(W)\left(= 1 \times \frac{1}{2} + 2 \times \frac{1}{2}\right) = \frac{3}{2}$ ***A1***

(ii) $\mathrm{E}\left(\frac{1}{W}\right)\left(= 1 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}\right) = \frac{3}{4}$ ***(M1)A1***

(iii) the above example shows that in general $\mathrm{E}\left(\frac{1}{T}\right) \neq \frac{1}{\mathrm{E}(T)}$ (so that $E\left(\frac{4}{\bar{Y}}\right)$ may not equal $\frac{1}{E(\bar{Y}/4)} = p$). ***R1***

Note: Do not award ***R1*** if the statement is given only in terms of W.

[4 marks]
Total [15 marks]

3. (a) for n (sufficiently) large the sample mean $\overline{X}$ approximately ***A1***

$\sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ ***A1***

Note: Award the first ***A1*** for n large and reference to the sample mean $\left(\overline{X}\right)$, the second ***A1*** is for normal and the two parameters.

Note: Award the second ***A1*** only if the first ***A1*** is awarded.

Note: Allow 'n tends to infinity' or '$n \geq 30$' in place of 'large'.

[2 marks]

(b) $[59.9, 60.5]$ ***A1A1***

Note: Accept answers which round to the correct 3sf answers.

[2 marks]

(c) (i) under H_0, $\overline{X} \sim \mathrm{N}\left(60, \frac{4}{100}\right)$ ***(A1)***

required to find k such that $P(\overline{X} > k) = 0.05$ ***(M1)***

use of any valid method, *eg* GDC Inv(Normal) or $k = 60 + z\frac{\sigma}{\sqrt{n}}$ ***(M1)***

hence critical region is $\overline{x} > 60.33$ ***A1***

(ii) 0.05 ***A1***

(iii) P(Type II error) = P(H_0 is accepted / H_0 is false) ***(R1)***

Note: Accept Type II error means H_0 is accepted given H_0 is false.

$\Rightarrow \mathrm{P}(\overline{X} < 60.33) = 0.25$ when $\overline{X} \sim \mathrm{N}\left(\mu, \frac{4}{100}\right)$ ***(M1)***

$$\Rightarrow \mathrm{P}\left(\frac{\overline{X}-\mu}{\frac{2}{10}} < \frac{60.33-\mu}{\frac{2}{10}}\right) = 0.25$$ ***(M1)***

$$\Rightarrow \mathrm{P}\left(Z < \frac{60.33-\mu}{\frac{2}{10}}\right) = 0.25 \text{ where } Z \sim \mathrm{N}\left(0, 1^2\right)$$

$$\frac{60.33-\mu}{\frac{2}{10}} = -0.6744\ldots$$ ***(A1)***

$$\mu = 60.33 + \frac{2}{10} \times 0.6744\ldots$$

$$\mu = 60.5$$ ***A1***

[10 marks]

Total [14 marks]

4. (a) $G_x'(t) = \dfrac{p^r r t^{r-1}(1-qt)^r - p^r t^r r(1-qt)^{r-1}(-q)}{(1-qt)^{2r}}$ ***M1A1***

use of $\mathrm{E}(X) = G_x'(1)$ ***M1***

$G_x'(1) = \dfrac{rp^r(1-q)^r - rp^r(1-q)^{r-1}(-q)}{(1-q)^{2r}}$ ***A1***

Note: Accept correct substitution of $t=1$ in any correct form of $G_x'(t)$.

$= \dfrac{rp^{2r} + rp^{2r-1} - rp^{2r}}{p^{2r}}$ ***A1***

Note: Accept any equivalent simplified expression which leads immediately to the final result $\dfrac{r}{p}$, for example $r\left(1+\dfrac{q}{p}\right)$.

$= \dfrac{r}{p}$

[5 marks]

(b) (i) $G_w(t) = G_x(t)G_y(t)$ ***(M1)***

$= \dfrac{p^r t^r}{(1-qt)^r} \times \dfrac{p^s t^s}{(1-qt)^s} = \dfrac{p^{r+s}t^{r+s}}{(1-qt)^{r+s}}$ ***A1***

(ii) $W \sim \mathrm{NB}(r+s,\ p)$ ***A1A1***

(iii) $\mathrm{P}(X=3 \mid W=7) = \dfrac{\mathrm{P}(X=3 \cap W=7)}{\mathrm{P}(W=7)}$ ***(M1)***

$= \dfrac{P(X=3)\times P(Y=4)}{P(W=7)}$ ***(A1)***

$= \dfrac{\binom{2}{1}p^2q\binom{3}{2}p^3q}{\binom{6}{4}p^5q^2}$ ***M1A1***

$= \dfrac{2}{5}$ ***A1***

[9 marks]

Total [14 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel Medio
Prueba 1

Lunes 18 de noviembre de 2019 (tarde)

Número de convocatoria del alumno

1 hora 30 minutos

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba no se permite el uso de ninguna calculadora.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de Matemáticas NM** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

8819 – 7309

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No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 6]

Sea una progresión aritmética donde $u_2 = 5$ y $u_3 = 11$.

(a) Halle la diferencia de la progresión. [2]

(b) Halle el primer término. [2]

(c) Halle la suma de los 20 primeros términos. [2]

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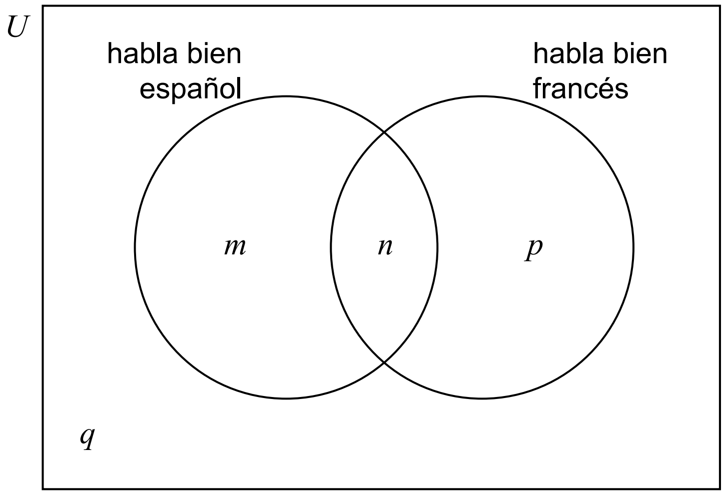
2. [Puntuación máxima: 6]

En una clase de 30 alumnos hay 18 que hablan bien español, 10 que hablan bien francés y 5 que no hablan bien ninguno de estos dos idiomas. El siguiente diagrama de Venn muestra los sucesos “habla bien español” y “habla bien francés”.

Los valores m, n, p y q representan números de alumnos.



(a) Escriba el valor de q. [1]

(b) Halle el valor de n. [2]

(c) Escriba el valor de m y el de p. [3]

3. [Puntuación máxima: 7]

Sea $g(x) = x^2 + bx + 11$. El punto $(-1, 8)$ pertenece al gráfico de g.

(a) Halle el valor de b. [3]

(b) Se transforma el gráfico de $f(x) = x^2$ hasta obtenerse el gráfico de g.

Describa esta transformación. [4]

4. [Puntuación máxima: 6]

Considere $\begin{pmatrix} 11 \\ a \end{pmatrix} = \dfrac{11!}{a!\,9!}$.

(a) Halle el valor de a. [2]

(b) A partir de lo anterior o de cualquier otro modo alternativo, halle el coeficiente del término en x^9 del desarrollo de $(x+3)^{11}$. [4]

5. [Puntuación máxima: 6]

Considere la función f cuya derivada es $f'(x) = 2x^2 + 5kx + 3k^2 + 2$, donde $x,\ k \in \mathbb{R}$.

(a) Muestre que el discriminante de $f'(x)$ es $k^2 - 16$. [2]

(b) Sabiendo que f es una función creciente, halle todos los posibles valores de k. [4]

6. [Puntuación máxima: 8]

Sea $f(x) = 4\cos\left(\frac{x}{2}\right) + 1$, para $0 \le x \le 6\pi$. Halle los valores de x para los que se cumple que $f(x) > 2\sqrt{2} + 1$.

7. [Puntuación máxima: 6]

Las variables X e Y siguen una distribución normal, con $X \sim \mathrm{N}(14\,,\, a^2)$ e $Y \sim \mathrm{N}(22\,,\, a^2)$, $a > 0$.

(a) Halle b de modo que se cumpla que $\mathrm{P}(X > b) = \mathrm{P}(Y < b)$. [2]

Se sabe que $\mathrm{P}(X > 20) = 0{,}112$.

(b) Halle $\mathrm{P}(16 < Y < 28)$. [4]

No escriba soluciones en esta página.

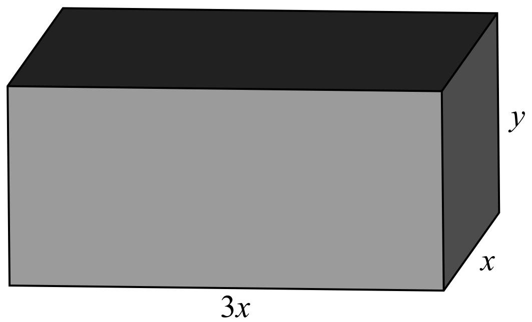
Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

8. [Puntuación máxima: 14]

Sea una pequeña caja con forma de ortoedro cuya base rectangular tiene una longitud de $3x$ cm y una anchura de x cm, donde $x > 0$. De altura mide y cm, donde $y > 0$.

la figura no está dibujada a escala



La suma de la longitud, la anchura y la altura es igual a $12\,\text{cm}$.

(a) Escriba una expresión para y en función de x. [1]

El volumen de la caja es igual a $V\,\text{cm}^3$.

(b) Halle una expresión para V en función de x. [2]

(c) Halle $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

(d) (i) Halle el valor de x para el cual V es máximo.

(ii) Justifique su respuesta. [7]

(e) Halle el volumen máximo. [2]

No escriba soluciones en esta página.

9. [Puntuación máxima: 17]

Los puntos A y B tienen por vectores de posición $\begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix}$ y $\begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}$ respectivamente.

El punto C tiene por vector de posición $\begin{pmatrix} -1 \\ k \\ 0 \end{pmatrix}$. Sea O el origen.

(a) Halle, en función de k,

(i) $\overrightarrow{\mathrm{OA}} \bullet \overrightarrow{\mathrm{OC}}$;

(ii) $\overrightarrow{\mathrm{OB}} \bullet \overrightarrow{\mathrm{OC}}$. [3]

(b) Sabiendo que $\mathrm{A\hat{O}C} = \mathrm{B\hat{O}C}$, muestre que $k = 7$. [8]

(c) Calcule el área del triángulo AOC. [6]

No escriba soluciones en esta página.

10. [Puntuación máxima: 14]

Sea $g(x) = p^x + q$, donde x, p, $q \in \mathbb{R}$, $p > 1$. El punto $\mathrm{A}(0, a)$ pertenece al gráfico de g.

Sea $f(x) = g^{-1}(x)$. El punto B pertenece al gráfico de f y es el simétrico del punto A respecto de la recta $y = x$.

(a) Escriba las coordenadas de B. [2]

La recta L_1 es tangente al gráfico de f en B.

(b) Sabiendo que $f'(a) = \dfrac{1}{\ln p}$, halle la ecuación de L_1 **en función de** x, p y q. [5]

La recta L_2 es tangente al gráfico de g en A y tiene por ecuación $y = (\ln p)x + q + 1$.

La recta L_2 pasa por el punto $(-2, -2)$.

La pendiente de la normal a g en A es igual a $\dfrac{1}{\ln\left(\dfrac{1}{3}\right)}$.

(c) Halle la ecuación de L_1 en función de x. [7]

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

Diploma Programme
Programme du diplôme
Programa del Diploma

Esquema de calificación

Noviembre de 2019

Matemáticas

Nivel Medio

Prueba 1

Instrucciones para los Examinadores

Abreviaturas

M Puntos concedidos por tratar de utilizar un Método correcto; el procedimiento (es decir, el razonamiento que se ha seguido y los cálculos realizados) tiene que estar incluido.

(M) Puntos concedidos por el **Método** utilizado; dicho método puede inferirse de un procedimiento posterior **correcto**.

A Puntos concedidos por una Respuesta (en inglés, Answer) o por Precisión (en inglés, Accuracy); a menudo dependen de los puntos M precedentes.

(A) Puntos concedidos por una **Respuesta** o por **Precisión**; dicha respuesta/precisión puede inferirse de un procedimiento posterior **correcto**.

R Puntos concedidos por un **Razonamiento** claro.

N Puntos concedidos por respuestas **correctas** cuando no se muestra **ningún** procedimiento.

AG Respuesta dada (del inglés answer given) en la propia pregunta, por lo que no se concede ningún punto.

Uso del esquema de calificación

1 General

Se deberá calificar siguiendo las instrucciones que aparecen en RM Assessor

2 Puntuación por Método y por Respuesta/Precisión

- **No** conceda automáticamente la puntuación máxima cuando la respuesta sea correcta; es **obligatorio** comprobar todo el procedimiento y puntuar la pregunta conforme al esquema de calificación.
- No se puede conceder ***M0*** seguido de ***A1***, puesto que los puntos ***A*** dependen de los puntos ***M*** precedentes, de haber alguno. Una excepción a esta regla es el caso en el que no se haya incluido ningún desarrollo que permita conceder un ***M1***, a diferencia de aquellos casos en los que el método utilizado haya sido incorrecto (véase el punto 4).
- Cuando se indica en la misma línea una puntuación ***M*** y otra ***A*** (p. ej., ***M1A1***), esto normalmente significa que se conceda ***M1*** por **intentar** utilizar un método adecuado (p. ej., sustitución en una fórmula) y ***A1*** por utilizar los valores **correctos**.
- Cuando existen dos o más puntuaciones ***A*** en la misma línea, se pueden otorgar de forma independiente; de manera que si el primer valor es incorrecto, pero los dos siguientes son correctos, se otorga ***A0A1A1***.
- Cuando en el esquema de calificación se especifica ***(M2)***, ***N3***, etc., **no** separe las notas, a menos que
- exista una observación
- La mayoría de las puntuaciones ***M*** se han de conceder por el empleo de un método **válido**; es decir, de un método capaz de conducir a la respuesta que pide el enunciado. Por ello, dicho método ha de propiciar algún tipo de avance en pos de la respuesta.
- Una vez que aparezca en la hoja la respuesta correcta a una pregunta o a un apartado de una pregunta, ignore cualquier desarrollo adicional correcto. Sin embargo, si el desarrollo adicional revela falta de comprensión matemática, no conceda el ***A1*** final.

3 Puntos *N*

*Si **no** se muestra ningún desarrollo, otorgue puntos **N** a las respuestas **correctas**. En ese caso, ignore la distribución de notas (**M, A, R**).*

- **No** otorgue una mezcla de notas ***N*** y otras notas
- Pueden existir menos notas ***N*** disponibles que el total de notas ***M***, ***A*** y ***R***; esto se hace de forma deliberada para penalizar a los alumnos por no seguir las instrucciones respecto a que muestren el trabajo.
- Es posible que no exista una relación directa entre las puntuaciones ***N*** y las puntuaciones implícitas. Hay veces en las que todos los puntos que hay en juego son implícitos, pero la puntuación ***N*** no es la máxima puntuación: esto indica que queremos que el alumno plasme parte del desarrollo del ejercicio, sin especificar cuál.
- Para ser coherentes con el esquema de calificación, la puntuación ***N*** se indica para cada apartado, incluso en aquellos casos en los que coincida con el desglose de puntos ***M, A*** y ***R***.
- Si un alumno muestra un trabajo incorrecto que, de algún modo, le lleva a la respuesta correcta, **no** se deben otorgar puntos ***N*** a esa respuesta correcta. Sin embargo, si el alumno ha indicado (generalmente tachándolo) que el trabajo debe ser ignorado, otorgue los puntos ***N*** a la respuesta correcta.

4 Puntuaciones implícitas

*Las puntuaciones implícitas se muestran entre **paréntesis; p. ej., (M1)**.*

- Las puntuaciones implícitas solo se pueden conceder si el alumno ha incluido el desarrollo del ejercicio o si dicho desarrollo queda implícito en apartados subsiguientes de la pregunta (el que el alumno haya dado una respuesta final correcta no implica necesariamente que se le tengan que conceder todos los puntos implícitos que hay en juego). Hay preguntas en las que es necesario plasmar algo de desarrollo, pero puesto que se acepta que no todo el mundo vaya a escribir los mismos pasos, todos los puntos que hay en juego son implícitos, pero la puntuación ***N*** no es la máxima puntuación asignada a la pregunta
- Normalmente el desarrollo correcto del ejercicio aparece escrito en la línea siguiente.
- Allí donde se haya asignado un ***(M1)*** seguido de un ***A1*** para cada respuesta correcta, si no se incluye el desarrollo del ejercicio una respuesta correcta será prueba suficiente para poder conceder el ***(M1)***.

*Las puntuaciones 'se ha de ver' aparecen en el esquema **sin paréntesis;*** p. ej., ***M1***.

- Las puntuaciones 'se ha de ver' solo se pueden conceder si se ha incluido el desarrollo del ejercicio (cálculos realizados/razonamiento seguido).
- Si una puntuación 'se ha de ver' dada no se ha concedido porque no se hubiera incluido el desarrollo del ejercicio (a diferencia de ***M0*** o ***A0,*** que se conceden cuando el desarrollo mostrado sea incorrecto), en ese caso sí que se pueden conceder todas las puntuaciones subsiguientes si resulta pertinente.

5 Puntuación de arrastre de error (FT)

*Las puntuaciones de arrastre de error (**FT**, del inglés follow-through) se conceden cuando tras dar una respuesta —final o intermedia— incorrecta en uno de los **apartados** de una pregunta, dicha respuesta se utiliza correctamente en apartados o subapartados **posteriores** de esa pregunta. Por lo general, para poder conceder puntos de arrastre de error (**FT**), **el alumno tiene que haber incluido el desarrollo del ejercicio** (es decir, los cálculos/razonamientos que ha seguido); no puede haberse limitado a dar una respuesta final basada en esa respuesta incorrecta que dio en el apartado anterior. Sin embargo, si en un subapartado dado los únicos puntos que tiene asignados son por la respuesta **final** que se dé, en ese caso sí se deberían conceder puntos **FT** si resulta pertinente. Se espera que los examinadores comprueben el desarrollo del ejercicio que ha incluido el alumno antes de concederle puntos **FT** allí donde resulte pertinente.*

- Dentro de un apartado dado, una vez que se ha cometido un **error** ya no se pueden conceder más puntos ***A*** a otras partes del desarrollo que hagan uso de ese error. Sin embargo, sí se pueden conceder puntos ***M*** y ***R*** si resulta pertinente. (No obstante, tal y como se indicó anteriormente, si no se concedió una puntuación ***A*** determinada porque no se había incluido el desarrollo del ejercicio, en ese caso sí que se pueden conceder las puntuaciones subsiguientes si resulta pertinente).
- Las excepciones a esta regla se indicarán explícitamente en el esquema de calificación.
- Si la pregunta resulta mucho más sencilla debido a un error, entonces se ha de utilizar el propio criterio para otorgar menos puntos ***FT***.
- Si el error lleva a un valor inadecuado (por ejemplo, probabilidad mayor que 1, uso de $r > 1$ para la suma infinita de una progresión geométrica, sen $\theta = 1,5$, un valor no entero cuando se requiere un entero), entonces no se debe otorgar la puntuación de la respuesta final.
- En el esquema de calificación puede aparecer la expresión "del alumno" en una descripción, para indicar que el alumno podría estar utilizando un valor incorrecto.
- Un alumno comete un error en un apartado, pero obtiene la respuesta o respuestas correctas en los apartados posteriores, se otorgan puntos cuando proceda, a menos que la pregunta diga a partir de lo anterior. Con frecuencia, es posible utilizar en apartados posteriores un enfoque distinto, que no depende de la solución obtenida en los apartados previos.
- En una pregunta de tipo «mostrar que», si un error cometido en un subapartado anterior hace que el alumno no pueda mostrar la respuesta requerida (la dada en el enunciado), en ese caso no conceda el ***A1*** final. Tenga presente que si dicho error se comete dentro del mismo subapartado, las reglas del arrastre de errores (***FT***) quizá conduzcan a la pérdida de puntos adicionales

6 Error de lectura

*Si un alumno copia de forma incorrecta la información de la pregunta, se considera un error de lectura (MR). A un alumno sólo se le puede penalizar una vez por un error de lectura dado. Utilice el sello **MR** para indicar que se trata de un error de lectura. No conceda el primer punto que haya en juego en dicha pregunta, incluso aunque se trate de una puntuación **M**, pero sí que conceda todas las restantes (si resulta pertinente), para que así el alumno solo pierda un punto por el error de lectura cometido.*

- Si la pregunta resulta mucho más sencilla debido al error de lectura (MR), entonces se ha de utilizer el propio criterio para otorgar menos puntos.
- Si el error de lectura (***MR***) lleva a un valor inadecuado (por ejemplo, probabilidad mayor que 1, uso de $r > 1$ para la suma infinita de una progresión geométrica, $\operatorname{sen}\theta = 1{,}5$, un valor no entero cuando se requiere un entero), entonces no se debe otorgar la puntuación de la respuesta final.
- El error que el alumno pueda cometer al copiar su propio trabajo **no** constituye un error de lectura, sino un error.

7 Puntuación discrecional (d)

*En las contadas ocasiones en las que el esquema de calificación no cubra el procedimiento incluido por el alumno, el examinador utilizará su propio criterio para conceder una puntuación apropiada. En esos casos se ha de utilizar la anotación DM y, al lado de la puntuación, se ha de escribir una **nota** breve en la que se explique el porqué de esta decisión.*

8 Métodos alternativos

En ocasiones, los alumnos utilizan métodos distintos de aquellos que aparecen en el esquema de calificación. A menos que en la pregunta se especifique qué método se ha de utilizar, el uso de métodos alternativos correctos no se ha de penalizar, sino que se han de puntuar en sintonía con lo que indica el esquema de calificación. Si tiene alguna duda al respecto, póngase en contacto con el jefe de equipo (su team leader) y pídale consejo.

- Cuando para toda una pregunta se incluyen varios métodos alternativos, estos aparecen señalados mediante los encabezamientos **MÉTODO 1**, **MÉTODO 2**, etc.
- Las soluciones alternativas para un apartado de una pregunta se indican mediante el encabezamiento **O BIEN… O BIEN**. Siempre que sea posible, también se empleará la alineación del texto (sangría del párrafo) como recurso para que el examinador pueda identificar más fácilmente dónde comienzan y dónde terminan las distintas alternativas.

9 Formas alternativas

*A menos que en la pregunta se especifique lo contrario, **acepte** formas equivalentes.*
- Dado que se trata de un examen internacional, acepte todas las formas alternativas de **notación**.
- En el esquema de calificación, las formas **numéricas** y **algebraicas** equivalentes aparecen generalmente escritas entre paréntesis, justo a continuación de la respuesta.
- En el esquema de calificación, las respuestas **simplificadas** (que los alumnos suelen no incluir en los exámenes) normalmente aparecen escritas entre paréntesis. La puntuación se ha de conceder si el alumno da la respuesta bien en la forma que precede al paréntesis o bien en la forma que aparece entre paréntesis (de habérsela incluido).

10 Calculadoras

No se permite el uso de calculadoras. El uso de cualquier tipo de calculadora durante la prueba 1 se considera conducta improcedente y, como consecuencia de ello, no se concederá ninguna calificación final. Si se topa con un ejercicio que sugiera que el alumno ha utilizado algún tipo de calculadora, siga por favor los procedimientos establecidos para abordar la conducta improcedente. Ejemplos: hallar un ángulo a partir de una razón trigonométrica de 0,4235.

11 Estilo

El objetivo del esquema de calificación es presentar las respuestas mediante una expresión clara, por ejemplo, si la pregunta pide hallar el valor de k, en el esquema de calificación aparecerá k = 3, pero los puntos se otorgarán al valor 3 (normalmente no hay necesidad del " k = ”). En estos casos, también es usualmente aceptable que el nombre de la variable sea distinto, siempre que no exista ambigüedad en la pregunta, por ejemplo, si la pregunta pide hallar el valor de p y de q, entonces la respuesta del alumno ha de ser clara. En general, la única situación donde se requiere la respuesta completa es en una pregunta donde lo que se pide es una ecuación (en este caso, en el esquema de calificación aparecerá “debe ser una ecuación”).

En el esquema de calificación aparece con frecuencia un texto que describe a qué se deben otorgar los puntos, seguido de ejemplos. Estos ejemplos no son exhaustivos, y los examinadores deben comprobar lo que han escrito los alumnos para ver si satisface las descripciones. Cuando se trata de puntos M, algunos de los ejemplos que se incluyen pueden presentar una notación deficiente, para indicar lo que es aceptable.

12 Respuestas del alumno

Si el alumno, en las hojas que contienen sus respuestas, ha trazado una línea cubriendo parte del desarrollo de alguna pregunta o si de algún otro modo ha tachado parte del desarrollo de algún ejercicio, no conceda ningún punto por esa parte del desarrollo.

Se supone que los alumnos han de escribir las respuestas a las preguntas de la Sección A en el cuestionario de examen (CE), y las de la Sección B en los cuadernillos de respuestas. En ocasiones los alumnos necesitan más espacio para la Sección A y utilizan el cuadernillo (y con frecuencia así lo indican en el CE) o escriben fuera de las casillas. Este desarrollo hay que calificarlo.

En las instrucciones se les dice a los alumnos que no han de escribir en la Sección B del CE. Así, es posible que hayan utilizado este espacio como hoja borrador, para hacer cálculos que dan por hecho se van a ignorar. Si han escrito soluciones en el cuadernillo de respuestas, no hay necesidad de mirar el CE. Sin embargo, si hay preguntas enteras o apartados enteros que no aparezcan resueltos en el cuadernillo de respuestas, por favor eche un vistazo al CE y compruebe que no estén resueltos ahí. En caso de que lo estén, puntúe esas preguntas enteras o esos apartados enteros que el alumno no escribió en el cuadernillo de respuesta.

13. Diagramas

Las indicaciones de cómo puntuar bosquejos (dibujos aproximados) suelen mencionar que el dibujo ha de pasar por determinados puntos o que tiene que tener una serie de características concretas. Estos puntos solo se pueden conceder si el bosquejo tiene (aproximadamente) la forma correcta. Todos los valores que se den en el esquema de calificación constituyen una guía aproximada que indica dónde se encuentran esos puntos/características relevantes. En algunas preguntas el primer ***A1*** se concede por la forma del bosquejo; en otras, los puntos solo se conceden si aparecen plasmados esos puntos/características relevantes. En ambos casos, a no ser que la forma del bosquejo sea aproximadamente correcta, no se podrá conceder ningún punto (a menos que se indique explícitamente lo contrario). No obstante, si el gráfico está basado en cálculos previos, se deberán conceder puntos ***FT*** si resulta pertinente.

14. Precisión de las respuestas

Cuando el grado de precisión se especifique en el enunciado de la pregunta, el esquema asignará un punto a la respuesta dada con la precisión requerida. Cuando esto no se especifique en el enunciado de la pregunta, todas las respuestas numéricas se tendrán que dar exactas o con una aproximación de tres cifras significativas.

No acepte una respuesta final numérica que esté a medias o sin terminar (p. ej., 3/0,1), a no ser que se haya indicado explícitamente lo contrario. Como regla general, las respuestas numéricas que consten de más de una parte (como es el caso de las fracciones) se deberían dar utilizando números enteros (p. ej., 6/8). Aquellos cálculos que conducen a un número entero se han de terminar, a excepción de aquellas fracciones que no sean números enteros. No es necesario dar los valores intermedios redondeándolos a tres cifras significativas. Más aún, hay que tener presente que si el alumno trabaja con valores redondeados puede acabar obteniendo una respuesta incorrecta; en casos así se ha de conceder un ***A0*** por la respuesta final.

12 Trabajo del alumno

Si el alumno, en las hojas que contienen sus respuestas, ha trazado una línea cubriendo parte del desarrollo de alguna pregunta o si de algún otro modo ha tachado parte del desarrollo de algún ejercicio, no conceda ningún punto por esa parte del desarrollo.

Se supone que los alumnos han de escribir las respuestas correspondientes a la Sección A en el cuestionario de examen (CE) y las de la Sección B en los cuadernillos de respuestas. En ocasiones los alumnos necesitan más espacio para la Sección A y deciden utilizar el cuadernillo (y con frecuencia comentan este hecho en el CE) o escriben fuera del cuadro de texto. Esto resulta totalmente aceptable, y esa parte del ejercicio escrita fuera del CE se ha de puntuar también.

En las instrucciones se les dice a los alumnos que no han de escribir en la Sección B del CE. Por este motivo, es posible que hayan utilizado este espacio como hoja «de sucio», para hacer cálculos, pues dan por hecho que estos cálculos se van a ignorar. Si ya han escrito soluciones en el cuadernillo de respuestas no hay necesidad de mirar el CE. Sin embargo, si hay preguntas enteras o apartados enteros que no aparezcan resueltos en el cuadernillo de respuestas, por favor eche un vistazo al CE y compruebe que no estén resueltos ahí. En caso de que lo estén, puntúe esas preguntas enteras o esos apartados enteros que el alumno no escribió en el cuadernillo de respuesta.

13. Diagramas

Las indicaciones de cómo puntuar bosquejos (dibujos aproximados) suelen mencionar que el dibujo ha de pasar por determinados puntos o que tiene que tener una serie de características concretas. Estos puntos solo se pueden conceder si el bosquejo tiene (aproximadamente) la forma correcta. Todos los valores que se den en el esquema de calificación constituyen una guía aproximada que indica dónde se encuentran esos puntos/características relevantes. En algunas preguntas el primer ***A1*** se concede por la forma del bosquejo; en otras, los puntos solo se conceden si aparecen plasmados esos puntos/características relevantes. En ambos casos, a no ser que la forma del bosquejo sea aproximadamente correcta, no se podrá conceder ningún punto (a menos que se indique explícitamente lo contrario). No obstante, si el gráfico está basado en cálculos previos, se deberán conceder puntos ***FT*** si resulta pertinente.

14. Precisión de las respuestas

*Cuando el grado de precisión se especifique en el enunciado de la pregunta, habrá en juego un punto por dar la respuesta final con la precisión requerida. Cuando esto no aparezca especificado en el enunciado de la pregunta, todas las respuestas numéricas se tendrán que dar exactas o habrán de darse redondeando a tres cifras significativas. A partir de ahí, a los alumnos YA NO se les debería penalizar por los errores de precisión (**AP**, del inglés Accuracy Penalty) cometidos. Los examinadores han puntuar conforme a las reglas dadas en estas instrucciones y a lo que se indica en el esquema de calificación. Precisión no es lo mismo que exactitud: con un valor incorrecto no se logran puntos **A** relevantes. Solo con las respuestas finales (y no con los valores intermedios) se pueden quitar puntos por haberse cometido errores de precisión. Por favor, revise cuidadosamente el desarrollo del ejercicio por si se pudieran conceder puntos **FT**.*

No acepte una respuesta final numérica que esté a medias o sin terminar (p. ej., 3/0,1), a no ser que se haya indicado explícitamente lo contrario. Como regla general, las respuestas numéricas que consten de más de una parte (como es el caso de las fracciones) se deberían dar utilizando números enteros (p. ej., 6/8). Aquellos cálculos que conducen a un número entero se han de terminar, a excepción de aquellas fracciones que no sean números enteros.

Sección A

1. (a) Por utilizar un enfoque válido *(M1)*

P. ej., $11-5$, $11=5+d$

$d=6$ ***A1*** ***N2***

[2 puntos]

(b) Por utilizar un enfoque válido *(M1)*

P. ej., u_2-d, $5-6$, $u_1+(3-1)(6)=11$

$u_1=-1$ ***A1*** ***N2***

[2 puntos]

(c) Por sustituir correctamente en la fórmula de la suma

P. ej., $\frac{20}{2}\left(2(-1)+19(6)\right)$, $\frac{20}{2}(-1+113)$ *(A1)*

$S_{20}=1120$ ***A1*** ***N2***

[2 puntos]

Total [6 puntos]

2. (a) $q=5$ ***A1*** ***N1***

[1 punto]

(b) Por utilizar un enfoque válido *(M1)*

P. ej., $(18+10+5)-30$, $28-25$, $18+10-n=25$

$n=3$ ***A1*** ***N2***

[2 puntos]

(c) Por utilizar un enfoque válido para hallar m o p
(puede verse en el apartado (b)) *(M1)*

P. ej., $18-3$, $3+p=10$

$m=15$, $p=7$ ***A1A1*** ***N3***

[3 puntos]

[Total: 6 puntos]

3. (a) Por un intento válido de sustituir las coordenadas ***(M1)***
P. ej., $g(-1)=8$

Por sustituir correctamente los valores ***(A1)***
P. ej., $(-1)^2+b(-1)+11=8$, $1-b+11=8$

$b=4$ ***A1*** ***N2***
[3 puntos]

(b) Por un intento válido de resolverlo ***(M1)***
P. ej., $\left(x^2+4x+4\right)+7$, $h=\dfrac{-4}{2}$, $k=g(-2)$

Por un desarrollo correcto ***A1***
P. ej., $(x+2)^2+7$, $h=-2,\ k=7$

Traslación o desplazamiento (no acepte otras descripciones) mediante el vector $\begin{pmatrix}-2\\7\end{pmatrix}$

(acepte 2 a la izquierda y 7 hacia arriba) ***A1A1*** ***N2***

[4 puntos]

Total [7 puntos]

4. (a) Por utilizar un enfoque válido ***(M1)***

P. ej., $11-a=9$, $\dfrac{11!}{9!(11-9)!}$

$a=2$ ***A1*** ***N2***

[2 puntos]

(b) Por utilizar un enfoque válido para el desarrollo usando $n=11$ ***(M1)***

P. ej., $\binom{11}{r}x^{11-r}3^r$, $a^{11}b^0+\binom{11}{1}a^{10}b^1+\binom{11}{2}a^9b^2+\ldots$

Si hay pruebas de elegir el término correcto ***A1***

P. ej., $\binom{11}{2}3^2$, $\binom{11}{2}x^9 3^2$, $\binom{11}{9}3^2$

Por un desarrollo correcto para el coeficiente binomial

(visto aquí o en algún otro lugar de la pregunta, no acepte factoriales) ***A1***

P. ej., 55, $\binom{11}{2}=55$, 55×3^2, $(55\times9)x^9$, $\dfrac{11\times10}{2}\times9$

495 ***A1*** ***N2***

Nota: Si hay evidencia clara de sumar en lugar de multiplicar, conceda ***A1*** por el trabajo correcto para el coeficiente binomial, pero no otros puntos. Por ejemplo, $55x^9+3^2$, obtendría ***M0A0A1A0***.

No conceda el ***A1*** final si la respuesta final dada es $495x^9$, incluso aunque el 495 aparezca en pasos anteriores del ejercicio. Si no ha incluido ningún desarrollo del ejercicio, conceda ***N1*** por $495x^9$.

[4 puntos]

Total [6 puntos]

5. (a) Por sustituir correctamente los valores en $b^2 - 4ac$ ***(A1)***

P. ej., $(5k)^2 - 4(2)(3k^2 + 2)$, $(5k)^2 - 8(3k^2 + 2)$

Por haber desarrollado correctamente todos los términos ***A1***

P. ej., $25k^2 - 24k^2 - 16$, $25k^2 - (24k^2 + 16)$

$k^2 - 16$ ***AG*** ***N0***

[2 puntos]

(b) Por utilizar un enfoque válido ***M1***

P. ej., $f'(x) > 0$, $f'(x) \geq 0$

Por darse cuenta de que el disciminante <0 o ≤ 0 ***M1***

P. ej., $D < 0$, $k^2 - 16 \leq 0$, $k^2 < 16$

Por dos valores correctos para k/extremos
(incluso si las desigualidades son incorrectas) ***(A1)***

P. ej., $k = \pm 4$, $k < -4$ y $k > 4$, $|k| < 4$

Intervalo correcto ***A1*** ***N2***

P. ej., $-4 < k < 4$, $-4 \leq k \leq 4$

Nota: Los candidatos pueden trabajar con una ecuación, luego escribir los intervalos con desigualidades al final. Si no se observan desigualidades hasta la respuesta correcta final del candidato, se puede concedar ***M0M0A1A1***.

Si el candidato está trabajando con desigualidades incorrectas al principio, y luego obtiene la respuesta final correcta, se puede conceder ***M0M0A1A0*** o ***M1M0A1A0*** o ***M0M1A1A0***.

[4 puntos]

Total [6 puntos]

6. MÉTODO 1 – HALLAR INTERVALOS PARA x

$4\cos\left(\frac{x}{2}\right)+1>2\sqrt{2}+1$

Por un desarrollo correcto ***(A1)***

P. ej., $4\cos\left(\frac{x}{2}\right)=2\sqrt{2},\ \cos\left(\frac{x}{2}\right)>\frac{\sqrt{2}}{2}$

Por darse cuenta de que $\cos^{-1}\frac{\sqrt{2}}{2}=\frac{\pi}{4}$ ***(A1)***

Por dar un valor correcto adicional para $\frac{x}{2}$

(ignorando el dominio y ecuación/desigualdades) ***(A1)***

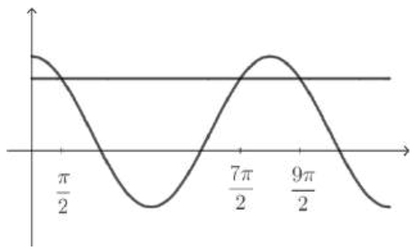
P. ej., $-\frac{\pi}{4},\ \frac{7\pi}{4},\ 315°,\ \frac{9\pi}{4},\ -45°,\ \frac{15\pi}{4}$

Por dar tres valores de x correctos ***A1A1***

P. ej., $\frac{\pi}{2},\ \frac{7\pi}{2},\ \frac{9\pi}{2}$

Por utilizar un enfoque válido para hallar los intervalos ***(M1)***

P. ej.,



Por dar los intervalos correctos (debe estar en radianes) ***A1A1*** ***N2***

$0\le x<\frac{\pi}{2},\ \frac{7\pi}{2}<x<\frac{9\pi}{2}$

Nota:	Si aparece el desarrollo del ejercicio, conceda ***A1A0*** si la inclusión/exclusión de los extremos es incorrecta. Si no ha incluido ningún desarrollo del ejercicio, conceda ***N1***. Si aparece el desarrollo del ejercicio, conceda ***A1A0*** si ha dado ambos intervalos correctos, **y además** ha dado intervalos adicionales. Si no ha incluido ningún desarrollo del ejercicio, conceda ***N1***. Conceda ***A0A0*** si la inclusión/exclusión de los extremos es incorrecta **y además** ha dado intervalos adicionales.

Continúa en la pág. siguiente...

Continuación de la pregunta 6

MÉTODO 2 – HALLAR INTERVALOS PARA $\frac{x}{2}$

$4\cos\left(\frac{x}{2}\right)+1 > 2\sqrt{2}+1$

Por un desarrollo correcto **(A1)**

P. ej., $4\cos\left(\frac{x}{2}\right)=2\sqrt{2},\ \cos\left(\frac{x}{2}\right)>\frac{\sqrt{2}}{2}$

Por darse cuenta de que $\cos^{-1}\frac{\sqrt{2}}{2}=\frac{\pi}{4}$ **(A1)**

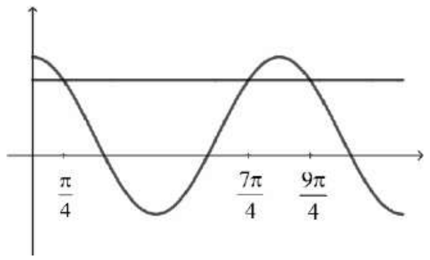
Por dar un valor correcto adicional para $\frac{x}{2}$
(ignorando el dominio y ecuación/desigualidades) **(A1)**

P. ej., $-\frac{\pi}{4},\ \frac{7\pi}{4},\ 315°,\ \frac{9\pi}{4},\ -45°,\ \frac{15\pi}{4}$

Por dar tres valores de $\frac{x}{2}$ correctos **A1**

P. ej., $\frac{\pi}{4},\ \frac{7\pi}{4},\ \frac{9\pi}{4}$

Por utilizar un enfoque válido para hallar los intervalos **(M1)**
P. ej.,



Por dar un intervalo para $\frac{x}{2}$ correcto **A1**

P. ej., $0\le\frac{x}{2}<\frac{\pi}{4},\ \frac{7\pi}{4}<\frac{x}{2}<\frac{9\pi}{4}$

Por dar los intervalos correctos (debe estar en radianes) **A1A1** **N2**

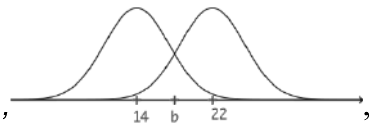
$0\le x<\frac{\pi}{2},\ \frac{7\pi}{2}<x<\frac{9\pi}{2}$

> **Nota**: Si aparece el desarrollo del ejercicio, conceda ***A1A0*** si la inclusión/exclusión de los extremos es incorrecta. Si no ha incluido ningún desarrollo del ejercicio, conceda ***N1***.
> Si aparece el desarrollo del ejercicio, conceda ***A1A0*** si ha dado ambos intervalos correctos, **y además** ha dado intervalos adicionales. Si no ha incluido ningún desarrollo del ejercicio, conceda ***N1***.
> Conceda ***A0A0*** si la inclusión/exclusión de los extremos es incorrecta **y además** ha dado intervalos adicionales.

Total [8 puntos]

7. (a) **MÉTODO 1**

Por darse cuenta de que b está a medio camino entre las medias 14 y 22 ***(M1)***

P. ej., 

, $b = \frac{14+22}{2}$

$b = 18$ ***A1*** ***N2***

MÉTODO 2

Por un intento válido de comparar las distribuciones ***(M1)***

P. ej., $\frac{b-14}{a} = -\frac{b-22}{a}$, $b-14 = 22-b$

$b = 18$ ***A1*** ***N2***

[2 puntos]

(b) Por un intento válido de comparar las distribuciones
(visto aquí o en algún otro lugar de la pregunta) ***(M1)***
P. ej., Y es una traslación horizontal de X de 8 unidades a la derecha,
$\mathrm{P}(16 < Y < 28) = \mathrm{P}(8 < X < 20)$, $\mathrm{P}(Y > 22+6) = \mathrm{P}(X > 14+6)$

Por un intento válido usando simetría ***(M1)***
P. ej., $1-2\mathrm{P}(X > 20)$, $1-2\mathrm{P}(Y < 16)$, $2\times\mathrm{P}(14 < x < 20)$, $\mathrm{P}(X < 8) = \mathrm{P}(X > 20)$

Por un desarrollo correcto ***(A1)***
P. ej., $1-2(0{,}112)$, $2\times(0{,}5-0{,}112)$, $2\times 0{,}388$, $0{,}888-0{,}112$

$\mathrm{P}(16 < Y < 28) = 0{,}776$ ***A1*** ***N3***

[4 puntos]

Total [6 puntos]

Sección B

8. (a) $y = 12 - 4x$ ***A1*** ***N1***

[1 punto]

(b) Por sustituir correctamente los valores en la fórmula del volumen ***(A1)***
P. ej., $3x \times x \times y$, $x \times 3x \times (12 - x - 3x)$, $(12 - 4x)(x)(3x)$

$V = 3x^2(12 - 4x)\ \left(= 36x^2 - 12x^3\right)$ ***A1*** ***N2***

Nota: Conceda ***A0*** por respuestas incompletas como $3x^2(12 - x - 3x)$.

[2 puntos]

(c) $\frac{\mathrm{d}V}{\mathrm{d}x} = 72x - 36x^2$ ***A1A1*** ***N2***

Nota: Conceda ***A1*** por $72x$ y ***A1*** por $-36x^2$.

[2 puntos]

(d) (i) Por un enfoque válido para hallar el máximo ***(M1)***
P. ej., $V' = 0$, $72x - 36x^2 = 0$

Por un desarrollo correcto ***(A1)***

P. ej., $x(72 - 36x)$, $\frac{-72 \pm \sqrt{72^2 - 4 \cdot (-36) \cdot 0}}{2(-36)}$, $36x = 72$, $36x(2 - x) = 0$

$x = 2$ ***A2*** ***N2***

Nota: Conceda ***A1*** por $x = 2$ y $x = 0$.

(ii) Por un enfoque válido para explicar que V es máximo cuando $x = 2$ ***(M1)***
*P. ej.,*por tratar de hallar V'', tabla de signos (ha de estar rotulada V')

Por dar valor(es) correcto(s) ***A1***
P. ej., $V''(2) = 72 - 72 \times 2$, $V'(a)$ para $a < 2$ **y** $V'(b)$ para $b > 2$

Por hacer un razonamiento correcto ***R1***
P. ej., $V''(2) < 0$, V' es positiva para $x < 2$ **y** es negativa para $x > 2$

Nota: No conceda ***R1*** a menos que ***A1*** haya sido otorgado.

V es máximo cuando $x = 2$ ***AG*** ***N0***

[7 puntos]

(e) Por sustituir correctamente los valores en **su** expresión del volumen ***A1***
P. ej., $3 \times 2^2(12 - 4 \times 2)$, $36\left(2^2\right) - 12\left(2^3\right)$

$V = 48\ (\mathrm{cm}^3)$ ***A1*** ***N1***

[2 puntos]

Total [14 puntos]

9. (a) (i) Por sustituir correctamente los valores en $\overrightarrow{\mathrm{OA}} \bullet \overrightarrow{\mathrm{OC}}$ o en $\overrightarrow{\mathrm{OB}} \bullet \overrightarrow{\mathrm{OC}}$ (en (ii)) ***(A1)***

P. ej., $-2\times(-1)+4\times k$, $6\times(-1)+8\times k$

Por expresión correcta ***A1*** ***N1***

P. ej., $2+4k$, $4k+2$

(ii) Por expresión correcta ***A1*** ***N1***

P. ej., $8k-6$, $-6+8k$

[3 puntos]

(b) Por hallar los módulos (visto aquí o en algún otro lugar de la pregunta) ***A1A1***

P. ej., $\sqrt{(-2)^2+(4)^2+(-4)^2}$ $(=6)$, $\sqrt{(6)^2+(8)^2+0^2}$ $(=10)$

Por sustituir correctamente sus valores en la fórmula correspondiente al ángulo AOC ***(A1)***

P. ej., $\cos\theta = \dfrac{2+4k}{\sqrt{(-2)^2+(4)^2+(-4)^2}\left|\overrightarrow{\mathrm{OC}}\right|}$

Por sustituir correctamente sus valores en la fórmula correspondiente al ángulo BOC ***(A1)***

P. ej., $\cos\theta = \dfrac{8k-6}{\sqrt{(6)^2+(8)^2+0^2}\left|\overrightarrow{\mathrm{OC}}\right|}$

Por darse cuenta de que $\cos \mathrm{A\hat{O}C} = \cos \mathrm{B\hat{O}C}$ (visto aquí o en algún otro lugar de la pregunta) ***(M1)***

P. ej., $\dfrac{2+4k}{\left|\overrightarrow{\mathrm{OC}}\right|\sqrt{(-2)^2+(4)^2+(-4)^2}} = \dfrac{8k-6}{\left|\overrightarrow{\mathrm{OC}}\right|\sqrt{6^2+(8)^2+0^2}}$, $\dfrac{2+4k}{6\sqrt{1+k^2}} = \dfrac{8k-6}{10\sqrt{1+k^2}}$

Por un desarrollo correcto (sin radicales) ***(A2)***

P. ej., $10(2+4k)=6(8k-6)$, $11k^2-79k+14=0$

Por un desarrollo correcto que conduzca claramente a la respuesta requerida ***A1***

P. ej., $20+36=48k-40k$, $56=8k$, $k=7$ y $k=\dfrac{2}{11}$, $(k-7)(11k-2)=0$

$k=7$ ***AG*** ***N0***

[8 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 9

(c) Por hallar el módulo de $\overrightarrow{\mathrm{OC}}$ (visto aquí o en algún otro lugar de la pregunta) ***A1***

P. ej., $\sqrt{(-1)^2+7^2+0^2}$, $\sqrt{50}$

Por un intento válido de hallar $\cos\theta$ ***(M1)***

P. ej., $\cos\theta = \dfrac{2+28}{6\sqrt{(-1)^2+7^2+0^2}}$, $\cos\theta = \dfrac{56-6}{10\sqrt{(-1)^2+7^2+0^2}}$,

$$\left(\sqrt{26}\right)^2 = 6^2 + \left(\sqrt{50}\right)^2 - 2(6)\sqrt{50}\cos\theta$$

Por hallar $\cos\theta$ ***A1***

P. ej., $\cos\theta = \dfrac{5}{\sqrt{50}}$ $\left(=\dfrac{1}{\sqrt{2}}\right)$

Por un intento válido de hallar $\operatorname{sen}\theta$
(visto aquí o en algún otro lugar de la pregunta) ***(M1)***

P. ej., $\theta = \dfrac{\pi}{4}$, $\operatorname{sen}\theta = \cos\theta$, $\operatorname{sen}\theta = \sqrt{1-\dfrac{25}{50}}$, $\operatorname{sen}\theta = \sqrt{1-\cos^2\theta}$, $\operatorname{sen}\theta = \dfrac{\sqrt{2}}{2}$

Por sustituir correctamente **sus** valores en $\dfrac{1}{2}ab\operatorname{sen}\mathrm{C}$ ***(A1)***

P. ej., $\dfrac{1}{2}\times 6\times\sqrt{50}\times\sqrt{1-\dfrac{25}{50}}$, $\dfrac{1}{2}\times 6\times\sqrt{50}\times\dfrac{5}{\sqrt{50}}$

El área es igual a 15 ***A1*** ***N3***

[6 puntos]

Total [17 puntos]

10. (a) $\mathrm{B}(a,\, 0)$ (acepte $\mathrm{B}(q+1,\, 0)$) ***A2*** ***N2***

[2 puntos]

(b)

Nota:	Hay muchos enfoques posibles para este apartado, y los pasos se pueden acometer en cualquier orden. Por favor, revise el desarrollo del ejercicio y puntúelo conforme a lo que aparece en el esquema de calificación; tenga presente que es posible que los alumnos trabajen con la ecuación de la recta antes de hallar a.

POR HALLAR *a*

Por un intento válido de hallar una expresión para a en función de q ***(M1)***

$g(0)=a$, $p^0+q=a$

$a=q+1$ ***(A1)***

POR HALLAR LA ECUACIÓN DE L_1

O BIEN

Por tratar de sustituir la pendiente de la tangente y las coordenadas en la ecuación de la recta ***(M1)***

P. ej., $y-0=f'(a)(x-a)$, $y=f'(a)\left(x-(q+1)\right)$

Por dar una ecuación correcta en función de a y de p ***(A1)***

P. ej., $y-0=\dfrac{1}{\ln(p)}(x-a)$

O BIEN

Por tratar de sustituir la pendiente de la tangente y las coordenadas para hallar b ***(M1)***

P. ej., $0=\dfrac{1}{\ln(p)}(a)+b$

$b=\dfrac{-a}{\ln(p)}$ ***(A1)***

ENTONCES (ha de estar en función **tanto** de p **como** de q)

$y=\dfrac{1}{\ln p}(x-q-1)$, $y=\dfrac{1}{\ln p}x-\dfrac{q+1}{\ln p}$ ***A1*** ***N3***

Nota:	Conceda ***A0*** por respuestas finales en la forma $L_1=\dfrac{1}{\ln p}(x-q-1)$.

[5 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 10

(c)

Nota: Hay muchos enfoques posibles para este apartado, y los pasos se pueden acometer en cualquier orden. Por favor, revise el desarrollo del ejercicio y puntúelo conforme a lo que aparece en el esquema de calificación; tenga presente que es posible que los alumnos hallen q en función de p antes de hallar el valor de p.

POR HALLAR *p*

Por utilizar un enfoque válido para hallar la pendiente de la tangente ***(M1)***

P. ej., $m_1 m_2 = -1$, $-\dfrac{1}{\dfrac{1}{\ln\left(\frac{1}{3}\right)}}$, $-\ln\left(\dfrac{1}{3}\right)$, $-\dfrac{1}{\ln p} = \dfrac{1}{\ln\left(\frac{1}{3}\right)}$

Por aplicar correctamente las propiedades de los logaritmos
(visto aquí o en algún otro lugar de la pregunta) ***(A1)***

P. ej., $\ln\left(\dfrac{1}{3}\right)^{-1}$, $-(\ln(1) - \ln(3))$

Por dar una ecuación correcta (visto aquí o en otro lugar de la pregunta) ***A1***
P. ej., $\ln p = \ln 3$, $p = 3$

POR HALLAR *q*

Por sustitución correcta de $(-2, -2)$ en la ecuación de L_2 ***(A1)***
P. ej., $-2 = (\ln p)(-2) + q + 1$

$q = 2\ln p - 3$, $q = 2\ln 3 - 3$ (visto aquí o en algún otro lugar de la pregunta) ***A1***

POR HALLAR L_1

Por sustituir correctamente **su** p y **su** q en **su** L_1 ***(A1)***

P. ej., $y = \dfrac{1}{\ln 3}\left(x - (2\ln 3 - 3) - 1\right)$

$y = \dfrac{1}{\ln 3}(x - 2\ln 3 + 2)$, $y = \dfrac{1}{\ln 3}x - \dfrac{2\ln 3 - 2}{\ln 3}$ ***A1*** ***N2***

Nota: Conceda ***A0*** por respuestas finales en la forma $L_1 = \dfrac{1}{\ln 3}(x - 2\ln 3 + 2)$.

[7 puntos]

Total [14 puntos]

Diploma Programme
Programme du diplôme
Programa del Diploma

Matemáticas
Nivel Medio
Prueba 2

Martes 19 de noviembre de 2019 (mañana)

Número de convocatoria del alumno

1 hora 30 minutos

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Sección A: conteste todas las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto.
- Sección B: conteste todas las preguntas en el cuadernillo de respuestas provisto. Escriba su número de convocatoria en la parte delantera del cuadernillo de respuestas, y adjúntelo a este cuestionario de examen y a su portada utilizando los cordeles provistos.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas con tres cifras significativas.
- Se necesita una copia sin anotaciones del **cuadernillo de fórmulas de Matemáticas NM** para esta prueba.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

8819 – 7310

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No se otorgará necesariamente la máxima puntuación a una respuesta correcta que no esté acompañada de un procedimiento. Las respuestas deben estar sustentadas en un procedimiento o en explicaciones. En particular, junto a los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido para su obtención; por ejemplo, si se utiliza un gráfico para hallar una solución, se deberá dibujar aproximadamente el mismo como parte de la respuesta. Aun cuando una respuesta sea errónea, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Por lo tanto, se aconseja mostrar todo el procedimiento seguido.

Sección A

Conteste **todas** las preguntas. Escriba sus respuestas en las casillas provistas a tal efecto. De ser necesario, se puede continuar desarrollando la respuesta en el espacio que queda debajo de las líneas.

1. [Puntuación máxima: 6]

La siguiente tabla muestra el número de mensajes, M, que seis adolescentes elegidos al azar enviaron durante el mes de octubre. La tabla también muestra el tiempo, T, que hablaron por teléfono a lo largo de ese mismo mes.

Tiempo que hablaron por teléfono (T minutos)	50	55	105	128	155	200
Número de mensajes (M)	358	340	740	731	800	992

La relación entre estas variables se puede modelizar mediante la ecuación de regresión $M = aT + b$.

(a) Escriba el valor de a y el de b. [3]

(b) Utilice su ecuación de regresión para predecir el número de mensajes enviados por un adolescente que habló por teléfono 154 minutos en octubre. [3]

2. [Puntuación máxima: 5]

Considere las rectas L_1 y L_2 cuyas ecuaciones son, respectivamente,

$$L_1 : y = -\frac{2}{3}x + 9 \text{ y } L_2 : y = \frac{2}{5}x - \frac{19}{5}.$$

(a) Halle el punto de intersección de L_1 y L_2. [2]

Sea una tercera recta L_3 cuya pendiente es $-\frac{3}{4}$.

(b) Escriba un vector director de L_3. [1]

L_3 pasa por el punto de intersección de L_1 y L_2.

(c) Escriba una ecuación vectorial para L_3. [2]

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3. [Puntuación máxima: 7]

Sea $f(x) = x - 8$, $g(x) = x^4 - 3$ y $h(x) = f(g(x))$.

(a) Halle $h(x)$. [2]

Sea C un punto del gráfico de h. La tangente al gráfico de h en C es paralela al gráfico de f.

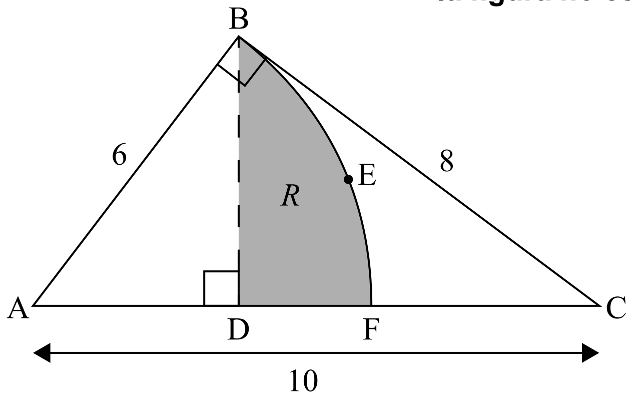
(b) Halle la coordenada x de C. [5]

4. [Puntuación máxima: 7]

En la siguiente figura se muestra un triángulo rectángulo ABC, siendo $AC = 10\,cm$, $AB = 6\,cm$ y $BC = 8\,cm$.

Los puntos D y F pertenecen a [AC].
[BD] es perpendicular a [AC].
BEF es el arco de una circunferencia con centro en A.
La región R está delimitada por [BD], [DF] y el arco BEF.

la figura no está dibujada a escala



(a) Halle $\text{B}\hat{\text{A}}\text{C}$. [2]

(b) Halle el área de R. [5]

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5. [Puntuación máxima: 7]

Los dos primeros términos de una progresión geométrica son $u_1 = 2{,}1$ y $u_2 = 2{,}226$.

(a) Halle el valor de r. [2]

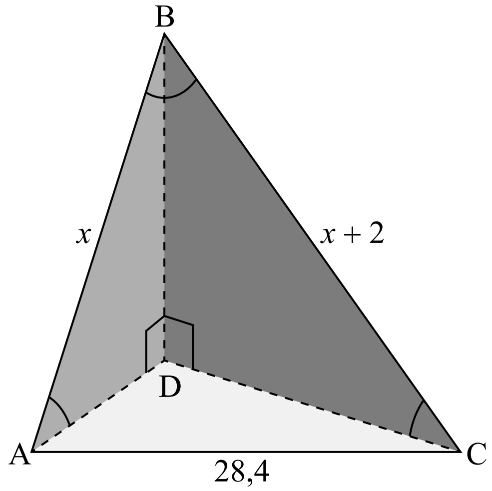
(b) Halle el valor de u_{10}. [2]

(c) Halle el valor más pequeño de n para el que se cumple que $S_n > 5543$. [3]

6. [Puntuación máxima: 6]

En la siguiente figura se muestra una pirámide de base triangular; dicha base es ADC.
La arista BD es perpendicular a las aristas AD y CD.

la figura no está dibujada a escala



$$\mathrm{AC} = 28{,}4\,\mathrm{cm},\ \mathrm{AB} = x\,\mathrm{cm},\ \mathrm{BC} = x + 2\,\mathrm{cm},\ \mathrm{A\hat{B}C} = 0{,}667,\ \mathrm{B\hat{A}D} = 0{,}611$$

Calcule AD.

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7. [Puntuación máxima: 7]

La siguiente tabla muestra la distribución de probabilidad de una variable aleatoria discreta X, con $a \geq 0$ y $b \geq 0$.

x	1	4	a	$a + b - 0{,}5$
$\mathrm{P}(X = x)$	0,2	0,5	b	a

(a) Muestre que $b = 0{,}3 - a$. [1]

(b) Halle la diferencia entre el mayor valor posible de la esperanza matemática y el menor valor posible de la esperanza matemática. [6]

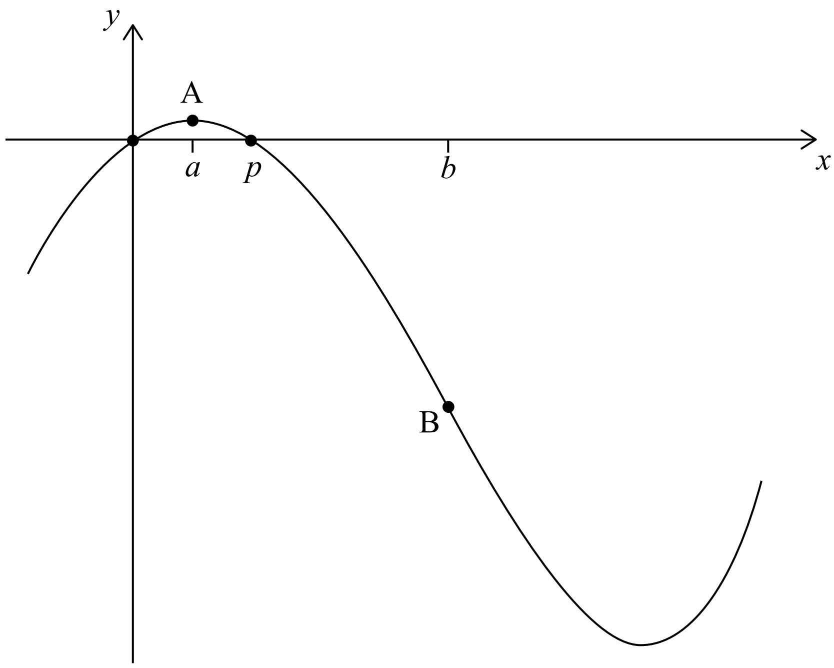
No escriba soluciones en esta página.

Sección B

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta.

8. [Puntuación máxima: 16]

Sea $f(x) = x^4 - 54x^2 + 60x$, para $-1 \le x \le 6$. La siguiente figura muestra el gráfico de f.



Los puntos de corte con el eje x están en $x = 0$ y en $x = p$. Hay un máximo en el punto A, donde $x = a$, y un punto de inflexión en el punto B, donde $x = b$.

(a) Halle el valor de p. [2]

(b) (i) Escriba las coordenadas de A.

(ii) Halle la ecuación de la tangente al gráfico de f en A. [4]

(c) (i) Halle las coordenadas de B.

(ii) Halle la razón de cambio de f en B. [7]

(d) Sea R la región delimitada por el gráfico de f, el eje x y las rectas $x = p$ y $x = b$. La región R se rota 360° alrededor del eje x. Halle el volumen del sólido así formado. [3]

No escriba soluciones en esta página.

9. [Puntuación máxima: 15]

La aerolínea SpeedWay vuela de la ciudad A a la ciudad B. El tiempo de vuelo sigue una distribución normal de media 260 minutos y desviación típica 15 minutos.

Se considera que un vuelo llega con retraso si dura más de 275 minutos.

(a) Calcule la probabilidad de que un vuelo **no** llegue con retraso. [2]

Se considera que el vuelo llega **puntual** si dura entre m y 275 minutos. La probabilidad de que un vuelo llegue puntual es igual a $0{,}830$.

(b) Halle el valor de m. [3]

Cada semana hay 12 vuelos de SpeedWay que van de la ciudad A a la ciudad B. La duración de cada vuelo es independiente de la duración de cualquier otro vuelo.

(c) (i) Calcule la probabilidad de que al menos 7 de estos vuelos lleguen **puntuales**.

(ii) Sabiendo que al menos 7 de estos vuelos han llegado puntuales, calcule la probabilidad de que haya exactamente 10 vuelos que han llegado puntuales. [7]

SpeedWay aumenta el número de vuelos que van de la ciudad A a la ciudad B hasta los 20 vuelos por semana; además, mejora su eficiencia para que haya más vuelos que lleguen puntuales. La probabilidad de que al menos 19 vuelos lleguen puntuales es igual a $0{,}788$.

(d) Se escoge un vuelo al azar. Calcule la probabilidad de que llegue puntual. [3]

No escriba soluciones en esta página.

10. [Puntuación máxima: 14]

Un cohete se mueve en línea recta, siendo su velocidad inicial de $140\,\mathrm{m\,s^{-1}}$. Va acelerando en dos fases hasta alcanzar una velocidad de $500\,\mathrm{m\,s^{-1}}$.

Durante la primera fase, la aceleración del cohete, $a\,\mathrm{m\,s^{-2}}$, después de t segundos viene dada por $a(t) = 240\,\mathrm{sen}(2t)$, donde $0 \le t \le k$.

(a) Halle una expresión para la velocidad del cohete, $v\,\mathrm{m\,s^{-1}}$, durante esta primera fase. [4]

La primera fase dura k segundos, hasta que la velocidad del cohete alcanza los $375\,\mathrm{m\,s^{-1}}$.

(b) Halle la distancia que recorre el cohete durante esta primera fase. [4]

Durante la segunda fase, el cohete presenta una aceleración constante. La distancia que recorre el cohete durante esta segunda fase es la misma que la que recorre durante la primera fase.

(c) Halle el tiempo total que duran las dos fases. [6]

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

N19/5/MATME/SP2/SPA/TZ0/XX/M

Esquema de calificación

Noviembre de 2019

Matemáticas

Nivel Medio

Prueba 2

17 páginas

Instrucciones para los Examinadores

Abreviaturas

M Puntos concedidos por tratar de utilizar un Método correcto; el procedimiento (es decir, el razonamiento que se ha seguido y los cálculos realizados) tiene que estar incluido.

(M) Puntos concedidos por el **Método** utilizado; dicho método puede inferirse de un procedimiento posterior **correcto**.

A Puntos concedidos por una Respuesta (en inglés, Answer) o por Precisión (en inglés, Accuracy); a menudo dependen de los puntos M precedentes.

(A) Puntos concedidos por una **Respuesta** o por **Precisión**; dicha respuesta/precisión puede inferirse de un procedimiento posterior **correcto**.

R Puntos concedidos por un **Razonamiento** claro.

N Puntos concedidos por respuestas **correctas** cuando no se muestra **ningún** procedimiento.

AG Respuesta dada (del inglés answer given) en la propia pregunta, por lo que no se concede ningún punto.

Uso del esquema de calificación

1 General

Se deberá calificar siguiendo las instrucciones que aparecen en RM Assessor

2 Puntuación por Método y por Respuesta/Precisión

- **No** conceda automáticamente la puntuación máxima cuando la respuesta sea correcta; es **obligatorio** comprobar todo el procedimiento y puntuar la pregunta conforme al esquema de calificación.
- No se puede conceder ***M0*** seguido de ***A1***, puesto que los puntos ***A*** dependen de los puntos ***M*** precedentes, de haber alguno. Una excepción a esta regla es el caso en el que no se haya incluido ningún desarrollo que permita conceder un ***M1***, a diferencia de aquellos casos en los que el método utilizado haya sido incorrecto (véase el punto 4).
- Cuando se indica en la misma línea una puntuación ***M*** y otra ***A*** (p. ej., ***M1A1***), esto normalmente significa que se conceda ***M1*** por **intentar** utilizar un método adecuado (p. ej., sustitución en una fórmula) y ***A1*** por utilizar los valores **correctos**.
- Cuando existen dos o más puntuaciones ***A*** en la misma línea, se pueden otorgar de forma independiente; de manera que si el primer valor es incorrecto, pero los dos siguientes son correctos, se otorga ***A0A1A1***.
- Cuando en el esquema de calificación se especifica ***(M2)***, ***N3***, etc., **no** separe las notas, a menos que
- exista una observación
- La mayoría de las puntuaciones ***M*** se han de conceder por el empleo de un método **válido**; es decir, de un método capaz de conducir a la respuesta que pide el enunciado. Por ello, dicho método ha de propiciar algún tipo de avance en pos de la respuesta.
- Una vez que aparezca en la hoja la respuesta correcta a una pregunta o a un apartado de una pregunta, ignore cualquier desarrollo adicional correcto. Sin embargo, si el desarrollo adicional revela falta de comprensión matemática, no conceda el ***A1*** final.

3 Puntos *N*

*Si **no** se muestra ningún desarrollo, otorgue puntos **N** a las respuestas **correctas**. En ese caso, ignore la distribución de notas (**M**, **A**, **R**).*

- **No** otorgue una mezcla de notas ***N*** y otras notas
- Pueden existir menos notas ***N*** disponibles que el total de notas ***M***, ***A*** y ***R***; esto se hace de forma deliberada para penalizar a los alumnos por no seguir las instrucciones respecto a que muestren el trabajo.
- Es posible que no exista una relación directa entre las puntuaciones ***N*** y las puntuaciones implícitas. Hay veces en las que todos los puntos que hay en juego son implícitos, pero la puntuación ***N*** no es la máxima puntuación: esto indica que queremos que el alumno plasme parte del desarrollo del ejercicio, sin especificar cuál.
- Para ser coherentes con el esquema de calificación, la puntuación ***N*** se indica para cada apartado, incluso en aquellos casos en los que coincida con el desglose de puntos ***M, A*** y ***R***.
- Si un alumno muestra un trabajo incorrecto que, de algún modo, le lleva a la respuesta correcta, **no** se deben otorgar puntos ***N*** a esa respuesta correcta. Sin embargo, si el alumno ha indicado (generalmente tachándolo) que el trabajo debe ser ignorado, otorgue los puntos ***N*** a la respuesta correcta.

4 Puntuaciones implícitas

*Las puntuaciones implícitas se muestran entre **paréntesis; p. ej., (M1)**.*

- Las puntuaciones implícitas solo se pueden conceder si el alumno ha incluido el desarrollo del ejercicio o si dicho desarrollo queda implícito en apartados subsiguientes de la pregunta (el que el alumno haya dado una respuesta final correcta no implica necesariamente que se le tengan que conceder todos los puntos implícitos que hay en juego). Hay preguntas en las que es necesario plasmar algo de desarrollo, pero puesto que se acepta que no todo el mundo vaya a escribir los mismos pasos, todos los puntos que hay en juego son implícitos, pero la puntuación ***N*** no es la máxima puntuación asignada a la pregunta
- Normalmente el desarrollo correcto del ejercicio aparece escrito en la línea siguiente.
- Allí donde se haya asignado un ***(M1)*** seguido de un ***A1*** para cada respuesta correcta, si no se incluye el desarrollo del ejercicio una respuesta correcta será prueba suficiente para poder conceder el ***(M1)***.

*Las puntuaciones ‘se ha de ver’ aparecen en el esquema **sin paréntesis;*** p. ej., ***M1***.

- Las puntuaciones ‘se ha de ver’ solo se pueden conceder si se ha incluido el desarrollo del ejercicio (cálculos realizados/razonamiento seguido).
- Si una puntuación ‘se ha de ver’ dada no se ha concedido porque no se hubiera incluido el desarrollo del ejercicio (a diferencia de ***M0*** o ***A0,*** que se conceden cuando el desarrollo mostrado sea incorrecto), en ese caso sí que se pueden conceder todas las puntuaciones subsiguientes si resulta pertinente.

5 Puntuación de arrastre de error (FT)

*Las puntuaciones de arrastre de error (**FT**, del inglés follow-through) se conceden cuando tras dar una respuesta —final o intermedia— incorrecta en uno de los **apartados** de una pregunta, dicha respuesta se utiliza correctamente en apartados o subapartados **posteriores** de esa pregunta. Por lo general, para poder conceder puntos de arrastre de error (**FT**), **el alumno tiene que haber incluido el desarrollo del ejercicio** (es decir, los cálculos/razonamientos que ha seguido); no puede haberse limitado a dar una respuesta final basada en esa respuesta incorrecta que dio en el apartado anterior. Sin embargo, si en un subapartado dado los únicos puntos que tiene asignados son por la respuesta **final** que se dé, en ese caso sí se deberían conceder puntos **FT** si resulta pertinente. Se espera que los examinadores comprueben el desarrollo del ejercicio que ha incluido el alumno antes de concederle puntos **FT** allí donde resulte pertinente.*

- Dentro de un apartado dado, una vez que se ha cometido un **error** ya no se pueden conceder más puntos ***A*** a otras partes del desarrollo que hagan uso de ese error. Sin embargo, sí se pueden conceder puntos ***M*** y ***R*** si resulta pertinente. (No obstante, tal y como se indicó anteriormente, si no se concedió una puntuación ***A*** determinada porque no se había incluido el desarrollo del ejercicio, en ese caso sí que se pueden conceder las puntuaciones subsiguientes si resulta pertinente).
- Las excepciones a esta regla se indicarán explícitamente en el esquema de calificación.
- Si la pregunta resulta mucho más sencilla debido a un error, entonces se ha de utilizar el propio criterio para otorgar menos puntos ***FT***.
- Si el error lleva a un valor inadecuado (por ejemplo, probabilidad mayor que 1, uso de $r > 1$ para la suma infinita de una progresión geométrica, $\text{sen}\,\theta = 1{,}5$, un valor no entero cuando se requiere un entero), entonces no se debe otorgar la puntuación de la respuesta final.
- En el esquema de calificación puede aparecer la expresión “del alumno” en una descripción, para indicar que el alumno podría estar utilizando un valor incorrecto.
- Un alumno comete un error en un apartado, pero obtiene la respuesta o respuestas correctas en los apartados posteriores, se otorgan puntos cuando proceda, a menos que la pregunta diga a partir de lo anterior. Con frecuencia, es posible utilizar en apartados posteriores un enfoque distinto, que no depende de la solución obtenida en los apartados previos.
- En una pregunta de tipo «mostrar que», si un error cometido en un subapartado anterior hace que el alumno no pueda mostrar la respuesta requerida (la dada en el enunciado), en ese caso no conceda el ***A1*** final. Tenga presente que si dicho error se comete dentro del mismo subapartado, las reglas del arrastre de errores (***FT***) quizá conduzcan a la pérdida de puntos adicionales

6 Error de lectura

*Si un alumno copia de forma incorrecta la información de la pregunta, se considera un error de lectura (MR). A un alumno sólo se le puede penalizar una vez por un error de lectura dado. Utilice el sello **MR** para indicar que se trata de un error de lectura. No conceda el primer punto que haya en juego en dicha pregunta, incluso aunque se trate de una puntuación **M**, pero sí que conceda todas las restantes (si resulta pertinente), para que así el alumno solo pierda un punto por el error de lectura cometido.*

- Si la pregunta resulta mucho más sencilla debido al error de lectura (MR), entonces se ha de utilizer el propio criterio para otorgar menos puntos.
- Si el error de lectura (***MR***) lleva a un valor inadecuado (por ejemplo, probabilidad mayor que 1, uso de r > 1 para la suma infinita de una progresión geométrica, senθ = 1,5 , un valor no entero cuando se requiere un entero), entonces no se debe otorgar la puntuación de la respuesta final.
- El error que el alumno pueda cometer al copiar su propio trabajo **no** constituye un error de lectura, sino un error.

7 Puntuación discrecional (d)

*En las contadas ocasiones en las que el esquema de calificación no cubra el procedimiento incluido por el alumno, el examinador utilizará su propio criterio para conceder una puntuación apropiada. En esos casos se ha de utilizar la anotación DM y, al lado de la puntuación, se ha de escribir una **nota** breve en la que se explique el porqué de esta decisión.*

8 Métodos alternativos

En ocasiones, los alumnos utilizan métodos distintos de aquellos que aparecen en el esquema de calificación. A menos que en la pregunta se especifique qué método se ha de utilizar, el uso de métodos alternativos correctos no se ha de penalizar, sino que se han de puntuar en sintonía con lo que indica el esquema de calificación. Si tiene alguna duda al respecto, póngase en contacto con el jefe de equipo (su team leader) y pídale consejo.

- Cuando para toda una pregunta se incluyen varios métodos alternativos, estos aparecen señalados mediante los encabezamientos **MÉTODO 1**, **MÉTODO 2**, etc.
- Las soluciones alternativas para un apartado de una pregunta se indican mediante el encabezamiento **O BIEN… O BIEN**. Siempre que sea posible, también se empleará la alineación del texto (sangría del párrafo) como recurso para que el examinador pueda identificar más fácilmente dónde comienzan y dónde terminan las distintas alternativas.

9 Formas alternativas

*A menos que en la pregunta se especifique lo contrario, **acepte** formas equivalentes.*

- Dado que se trata de un examen internacional, acepte todas las formas alternativas de **notación**.
- En el esquema de calificación, las formas **numéricas** y **algebraicas** equivalentes aparecen generalmente escritas entre paréntesis, justo a continuación de la respuesta.
- En el esquema de calificación, las respuestas **simplificadas** (que los alumnos suelen no incluir en los exámenes) normalmente aparecen escritas entre paréntesis. La puntuación se ha de conceder si el alumno da la respuesta bien en la forma que precede al paréntesis o bien en la forma que aparece entre paréntesis (de habérsela incluido).

10 Calculadoras

Notación de calculadora – La guía de Matemáticas NM dice lo siguiente:
Los alumnos deben utilizar siempre la notación matemática correcta y no la propia de las calculadoras.
No acepte ninguna respuesta final que se haya escrito utilizando notación de calculadora. Sin embargo, no penalice el uso de notación de calculadora durante el desarrollo del ejercicio.

11 Estilo

El objetivo del esquema de calificación es presentar las respuestas mediante una expresión clara, por ejemplo, si la pregunta pide hallar el valor de k, en el esquema de calificación aparecerá k = 3, pero los puntos se otorgarán al valor 3 (normalmente no hay necesidad del " k = "). En estos casos, también es usualmente aceptable que el nombre de la variable sea distinto, siempre que no exista ambigüedad en la pregunta, por ejemplo, si la pregunta pide hallar el valor de p y de q, entonces la respuesta del alumno ha de ser clara. En general, la única situación donde se requiere la respuesta completa es en una pregunta donde lo que se pide es una ecuación (en este caso, en el esquema de calificación aparecerá "debe ser una ecuación").

En el esquema de calificación aparece con frecuencia un texto que describe a qué se deben otorgar los puntos, seguido de ejemplos. Estos ejemplos no son exhaustivos, y los examinadores deben comprobar lo que han escrito los alumnos para ver si satisface las descripciones. Cuando se trata de puntos M, algunos de los ejemplos que se incluyen pueden presentar una notación deficiente, para indicar lo que es aceptable.

12 Respuestas del alumno

Si el alumno, en las hojas que contienen sus respuestas, ha trazado una línea cubriendo parte del desarrollo de alguna pregunta o si de algún otro modo ha tachado parte del desarrollo de algún ejercicio, no conceda ningún punto por esa parte del desarrollo.

Se supone que los alumnos han de escribir las respuestas a las preguntas de la Sección A en el cuestionario de examen (CE), y las de la Sección B en los cuadernillos de respuestas. En ocasiones los alumnos necesitan más espacio para la Sección A y utilizan el cuadernillo (y con frecuencia así lo indican en el CE) o escriben fuera de las casillas. Este desarrollo hay que calificarlo.

En las instrucciones se les dice a los alumnos que no han de escribir en la Sección B del CE. Así, es posible que hayan utilizado este espacio como hoja borrador, para hacer cálculos que dan por hecho se van a ignorar. Si han escrito soluciones en el cuadernillo de respuestas, no hay necesidad de mirar el CE. Sin embargo, si hay preguntas enteras o apartados enteros que no aparezcan resueltos en el cuadernillo de respuestas, por favor eche un vistazo al CE y compruebe que no estén resueltos ahí. En caso de que lo estén, puntúe esas preguntas enteras o esos apartados enteros que el alumno no escribió en el cuadernillo de respuesta.

13. Diagramas

Las indicaciones de cómo puntuar bosquejos (dibujos aproximados) suelen mencionar que el dibujo ha de pasar por determinados puntos o que tiene que tener una serie de características concretas. Estos puntos solo se pueden conceder si el bosquejo tiene (aproximadamente) la forma correcta. Todos los valores que se den en el esquema de calificación constituyen una guía aproximada que indica dónde se encuentran esos puntos/características relevantes. En algunas preguntas el primer ***A1*** se concede por la forma del bosquejo; en otras, los puntos solo se conceden si aparecen plasmados esos puntos/características relevantes. En ambos casos, a no ser que la forma del bosquejo sea aproximadamente correcta, no se podrá conceder ningún punto (a menos que se indique explícitamente lo contrario). No obstante, si el gráfico está basado en cálculos previos, se deberán conceder puntos ***FT*** si resulta pertinente.

14. Precisión de las respuestas

Cuando el grado de precisión se especifique en el enunciado de la pregunta, el esquema asignará un punto a la respuesta dada con la precisión requerida. Cuando esto no se especifique en el enunciado de la pregunta, todas las respuestas numéricas se tendrán que dar exactas o con una aproximación de tres cifras significativas.

No acepte una respuesta final numérica que esté a medias o sin terminar (p. ej., 3/0,1), a no ser que se haya indicado explícitamente lo contrario. Como regla general, las respuestas numéricas que consten de más de una parte (como es el caso de las fracciones) se deberían dar utilizando números enteros (p. ej., 6/8). Aquellos cálculos que conducen a un número entero se han de terminar, a excepción de aquellas fracciones que no sean números enteros. No es necesario dar los valores intermedios redondeándolos a tres cifras significativas. Más aún, hay que tener presente que si el alumno trabaja con valores redondeados puede acabar obteniendo una respuesta incorrecta; en casos así se ha de conceder un ***A0*** por la respuesta final.
Cuando en un apartado de una pregunta haya que dar una respuesta numérica como respuesta **final**, en el correspondiente esquema de calificación se incluirá:

Un valor truncado con 6 cs
El valor exacto (si resulta pertinente), la respuesta correcta tras redondear a 3 cs

Sección A

1. (a) Si hay pruebas de que ha planteado el ejercicio *(M1)*
P. ej., por dar un valor correcto de a o de b (acepte $r = 0{,}966856$)

$4{,}30161,\ 163{,}330$
$a = 4{,}30$, $b = 163$ (acepte $y = 4{,}30x + 163$) ***A1A1*** ***N3***
[3 puntos]

(b) Por utilizar un enfoque válido *(M1)*
P. ej., $4{,}30(154) + 163$

$825{,}778$ ($825{,}2$ si partió de valores dados con 3 c. s.) *(A1)*

número de mensajes $= 826$ (ha de ser un n.º entero) ***A1*** ***N3***
[3 puntos]

Total [6 puntos]

2. (a) Por utilizar un enfoque válido *(M1)*
P. ej., $L_1 = L_2$, $x = 12$, $y = 1$

$(12, 1)$ (exacto) ***A1*** ***N2***
[2 puntos]

(b) $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ (o algún múltiplo de $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$) ***A1*** ***N1***

[1 punto]

(c) Por cualquier ecuación correcta de la forma $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$
(acepte cualquier parámetro para t), donde a es el vector de posición de un punto perteneciente a L_1, y b es un múltiplo escalar de $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ ***A2*** ***N2***

P. ej., $\boldsymbol{r} = \begin{pmatrix} 12 \\ 1 \end{pmatrix} + t\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

Nota: Conceda ***A1*** por la forma $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** por la forma $L = \boldsymbol{a} + t\boldsymbol{b}$ y ***A0*** por la forma $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 puntos]

Total [5 puntos]

3. (a) Por tratar de formar la composición de funciones (sea en el orden que sea) ***(M1)***
P. ej., $f(x^4-3)$, $(x-8)^4-3$

$h(x)=x^4-11$ ***A1*** ***N2***

[2 puntos]

(b) Por darse cuenta de que la pendiente de la tangente es igual a la derivada ***(M1)***
P. ej., h'

Por dar correctamente la derivada (visto aquí o en algún otro lugar de la pregunta) ***(A1)***
$h'(x)=4x^3$

Por dar correctamente la pendiente de f (visto aquí o en algún otro lugar de la pregunta) ***(A1)***
$f'(x)=1$, $m=1$

Por igualar **su** derivada a 1 ***(M1)***
$4x^3=1$

$0{,}629960$

$x=\sqrt[3]{\frac{1}{4}}$ (exacto) $0{,}630$ ***A1*** ***N3***

[5 puntos]

Total [7 puntos]

4. (a) Por un desarrollo correcto ***(A1)***

P. ej., $\operatorname{sen}\alpha = \dfrac{8}{10}$, $\cos\theta = \dfrac{6}{10}$, $\cos \mathrm{B\hat{A}C} = \dfrac{6^2+10^2-8^2}{2\times 6\times 10}$

$0{,}927295$

$\mathrm{B\hat{A}C} = 0{,}927 \quad (=53{,}1^\circ)$ ***(A1)*** ***N2***

[2 puntos]

(b)

> **Nota:** Puede haber pequeñas diferencias en la respuesta final, dependiendo del enfoque utilizado en el apartado (b). Acepte una respuesta final que esté de acuerdo con el desarrollo utilizado.

Por dar correctamente el área del sector circular ABF
(visto aquí o en algún otro lugar de la pregunta) ***(A1)***

P. ej., $\dfrac{1}{2}\times 6^2 \times 0{,}927$, $\dfrac{53{,}1301^\circ}{360^\circ}\times\pi\times 6^2$, $16{,}6913$

Por dar una expresión correcta (o un valor correcto) para $[\mathrm{AD}]$ o para $[\mathrm{BD}]$
(visto aquí o en algún otro lugar de la pregunta) ***(A1)***

P. ej., $\mathrm{AD} = 6\cos(\mathrm{B\hat{A}C})$ $(=3{,}6)$

$\mathrm{BD} = 6\operatorname{sen}(53{,}1^\circ)$ $(=4{,}8)$

Por dar correctamente el área del triángulo ABD
(visto aquí o en algún otro lugar de la pregunta) ***(A1)***

P. ej., $\dfrac{1}{2}\times 6\cos \mathrm{B\hat{A}D}\times 6\operatorname{sen}\mathrm{B\hat{A}D}$, $9\operatorname{sen}(2\mathrm{B\hat{A}C})$, $8{,}64$ (exacto)

Por utilizar un enfoque válido
(visto aquí o en algún otro lugar de la pregunta) ***(M1)***

P. ej., $\mathrm{A}_{\text{triángulo ABD}} - \mathrm{A}_{\text{sector}}$, **su** sector – **su** triángulo ABD

$8{,}05131$

área de la región sombreada = $8{,}05\ (\mathrm{cm}^2)$ ***A1*** ***N2***

[5 puntos]

Total [7 puntos]

5. (a) Por utilizar un enfoque válido *(M1)*

P. ej., $\frac{u_1}{u_2}$; $\frac{2,226}{2,1}$; $2,226 = 2,1r$

$r = 1,06$ (exacto) ***A1*** ***N2***

[2 puntos]

(b) Por sustituir correctamente los valores *(A1)*

P. ej., $2,1 \times 1,06^9$

3,54790 ***A1*** ***N2***

$u_{10} = 3,55$

[2 puntos]

(c) Por sustituir correctamente los valores en la fórmula de S_n *(A1)*

P. ej., $\frac{2,1\left(1,06^n - 1\right)}{1,06 - 1}$, $\frac{2,1\left(1,06^n - 1\right)}{1,06 - 1} > 5543$, $2,1\left(1,06^n - 1\right) = 332,58$,

por hacer un dibujo aproximado de S_n e $y = 5543$

Por dar la inecuación correcta **O BIEN** dando "los valores de cruce" ***A1***

P. ej., $n > 87,0316$, $S_{87} = 5532,73$ **Y** $S_{88} = 5866,79$

$n = 88$ ***A1*** ***N2***

[3 puntos]

Total [7 puntos]

6. Si hay pruebas de que escogió el teorema del coseno *(M1)*

P. ej., $a^2 = b^2 + c^2 - 2bc\cos A$

Por sustituir correctamente los valores para hallar AB *(A1)*

P. ej., $28,4^2 = x^2 + (x+2)^2 - 2x(x+2)\cos(0,667)$

$x = 42,2822$ ***A2***

Por utilizar un enfoque apropiado para hallar AD *(M1)*

P. ej., $\mathrm{AD} = x\cos(0,611)$, $\cos(0,611) = \frac{\mathrm{AD}}{42,2822}$

34,6322

$\mathrm{AD} = 34,6$ ***A1*** ***N3***

Total [6 puntos]

7. (a) Por utilizar un enfoque correcto ***A1***
P. ej., $0{,}2+0{,}5+b+a=1$, $0{,}7+a+b=1$

$b=0{,}3-a$ ***AG*** ***N0***

[1 punto]

(b) Por sustituir correctamente los valores en $\mathrm{E}(X)$ ***(A1)***
P. ej., $0{,}2+4\times 0{,}5+a\times b+(a+b-0{,}5)\times a$, $0{,}2+2+a\times b-0{,}2a$

Por un intento válido de expresar $\mathrm{E}(X)$ en función de una variable ***M1***
P. ej., $0{,}2+4\times 0{,}5+a\times(0{,}3-a)+(-0{,}2)\times a$, $2{,}2+0{,}1a-a^2$,
$0{,}2+4\times 0{,}5+(0{,}3-b)\times b+(-0{,}2)\times(0{,}3-b)$, $2{,}14+0{,}5b-b^2$

Por dar correctamente el mayor valor posible de $\mathrm{E}(X)$ ***(A1)***
$2{,}2025$ (exacto)

Por un intento válido de hallar el menor valor posible ***(M1)***
P. ej., gráfico con el punto mínimo indicado, $\mathrm{E}(0)$ y $\mathrm{E}(0{,}3)$
$(0;\ 2{,}2)$ y $(0{,}3;\ 2{,}14)$ si $\mathrm{E}(X)$ está en función de a
$(0;\ 2{,}14)$ y $(0{,}3;\ 2{,}2)$ si $\mathrm{E}(X)$ está en función de b

Por dar correctamente el menor valor posible de $\mathrm{E}(X)$ ***(A1)***
P. ej., $2{,}14$ (exacto)

diferencia = $0{,}0625$ (exacto) ***A1*** ***N2***

[6 puntos]

Total [7 puntos]

Sección B

8. (a) Si hay pruebas de que ha utilizado un enfoque válido ***(M1)***
P. ej., $f(x)=0$, $y=0$

$1{,}13843$
$p=1{,}14$ ***A1*** ***N2***
[2 puntos]

(b) (i) $0{,}562134,\ 16{,}7641$
$(0{,}562\ ;\ 16{,}8)$ ***A2*** ***N2***

(ii) Por utilizar un enfoque válido ***(M1)***
*P. ej.,*en el máximo la tangente es horizontal, $f'=0$

$y=16{,}8$ (ha de ser una ecuación) ***A1*** ***N2***
[4 puntos]

(c) (i) **MÉTODO 1 (empleando la calculadora de pantalla gráfica)**

Por utilizar un enfoque válido ***M1***
P. ej., $f''=0$, máx./mín. de f', $x=-3$

Por un bosquejo de f' o f'', con el máx./mín. o la raíz (respectiv.) ***(A1)***

$x=3$ ***A1*** ***N1***

Por sustituir **su** valor de x en f ***(M1)***
P. ej., $f(3)$

$y=-225$ (exacto) (acepte $(3,-225)$) ***A1*** ***N1***

MÉTODO 2 (analítico)

$f''=12x^2-108$ ***A1***

Por un enfoque válido ***(M1)***

P. ej., $f''=0$, $x=\pm 3$

$x=3$ ***A1*** ***N1***

Por sustituir **su** valor de x en f ***(M1)***
P. ej., $f(3)$

$y=-225$ (exacto) (acepte $(3,-225)$) ***A1*** ***N1***

(ii) Por darse cuenta de que la razón de cambio es f' ***(M1)***
P. ej., y', $f'(3)$
La razón de cambio es igual a -156 (exacto) ***A1*** ***N2***
[7 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 8

(d) Por tratar de sustituir, **o bien sus** límites, **o bien** la función en la fórmula del volumen ***(M1)***

P. ej., $\int_{1,14}^{3} f^2$; $\pi\int (x^4 - 54x^2 + 60x)^2\,\mathrm{d}x$; 25 752,0

80 902,3

volumen = 80 900 ***A2*** ***N3***

[3 puntos]

Total [16 puntos]

9. (a) Por utilizar un enfoque válido ***(M1)***

P. ej., $\mathrm{P}(X < 275)$; $1 - 0{,}158655$

0,841344

0,841 ***A1*** ***N2***

[2 puntos]

(b) Por utilizar un enfoque válido ***(M1)***

P. ej., $\mathrm{P}(X < 275) - \mathrm{P}(X < m) = 0{,}830$

Por un desarrollo correcto (cálculos/razonamiento) ***(A1)***

P. ej., $\mathrm{P}(X < m) = 0{,}0113447$

225,820

226 (minutos) ***A1*** ***N3***

[3 puntos]

(c) (i) Si hay pruebas de que se ha percatado de que se trata de una distribución binomial (visto aquí o en algún otro lugar de la pregunta) ***(M1)***

P. ej., ${}_nC_a \times p^a \times q^{n-a}$, $\mathrm{B}(n,\ p)$

Si hay evidencia de sumar las probabilidades desde 7 hasta 12 ***(M1)***

P. ej., $\mathrm{P}(X = 7) + \mathrm{P}(X = 8) + \ldots + \mathrm{P}(X = 12)$, $1 - \mathrm{P}(X \le 6)$

0,991248

0,991 ***A1*** ***N2***

Continúa en la pág. siguiente...

Continuación de la pregunta 9

(ii) Por hallar $\mathrm{P}(X=10)$ (visto aquí o en algún otro lugar de la pregunta) ***A1***

P. ej., $\binom{12}{10} \times 0,83^{10} \times 0,17^{2} \ (=0,295952)$

Por darse cuenta de que se trata de probabilidad condicionada ***(M1)***

P. ej., $\mathrm{P}(A \mid B)$, $\mathrm{P}(X=10 \mid X \geq 7)$, $\dfrac{\mathrm{P}(X=10 \cap X \geq 7)}{\mathrm{P}(X \geq 7)}$

Por un desarrollo correcto (cálculos/razonamiento) ***(A1)***

P. ej., $\dfrac{0,295952}{0,991248}$

0,298565

0,299 ***A1*** ***N1***

Nota:	**Excepción a los puntos *FT*:** si el candidato usa un valor incorrecto para la probabilidad de que un vuelo llegue puntual en (i) y ha mostrado desarrollo, conceda todos los puntos ***FT*** en (ii) según sea apropiado.

[7 puntos]

(d) Por dar una ecuación correcta ***(A1)***

P. ej., $\binom{20}{19} p^{19}(1-p)+p^{20}=0,788$

Por un intento válido de resolverla ***(M1)***

P. ej., mediante un gráfico

0,956961

0,957 ***A1*** ***N1***

[3 puntos]

Total [15 puntos]

10. (a) Por darse cuenta de que $v = \int a$ ***(M1)***

Por integrar correctamente ***A1***
P. ej., $-120\cos(2t) + c$

Por tratar de hallar el valor de c utilizando **su** valor de $v(t)$ ***(M1)***
P. ej., $-120\cos(0) + c = 140$

$v(t) = -120\cos(2t) + 260$ ***A1*** ***N3***
[4 puntos]

(b) Si hay pruebas de que ha utilizado un enfoque válido para hallar el tiempo que dura la primera fase ***(M1)***
P. ej., mediante un gráfico, $-120\cos(2t) + 260 = 375$

$k = 1,42595$ ***A1***

Por tratar de sustituir **su** v y/o **sus** límites en la fórmula de la distancia ***(M1)***
P. ej., $\int_0^{1,42595} |v|$, $\int 260 - 120\cos(2t)$, $\int_0^k \left(260 - 120\cos(2t)\right)\mathrm{d}t$

$353,608$
La distancia es igual a 354 (m) ***A1*** ***N3***
[4 puntos]

(c) Por darse cuenta de que, en la segunda fase, la velocidad es lineal (visto aquí o en algún otro lugar de la pregunta) ***R1***
P. ej., mediante un gráfico, $s = \frac{1}{2}h(a+b)$, $v = mt + c$

Por utilizar un enfoque válido ***(M1)***
P. ej., $\int v = 353,608$

Por dar una ecuación correcta ***(A1)***
P. ej., $\frac{1}{2}h(375 + 500) = 353,608$

tiempo para la fase dos = $0,808248$
($0,809142$ si partió de valores dados con 3 c. s.) ***A2***

$2,23420$ ($2,23914$ si partió de valores dados con 3 c. s.)
$2,23$ (segundos) ($2,24$ si partió de valores dados con 3 c. s.) ***A1*** ***N3***
[6 puntos]

Total [14 puntos]

Diploma Programme
Programme du diplôme
Programa del Diploma

Estudios Matemáticos
Nivel Medio
Prueba 1

Lunes 18 de noviembre de 2019 (tarde)

Número de convocatoria del alumno

1 hora 30 minutos

Instrucciones para los alumnos

- Escriba su número de convocatoria en las casillas de arriba.
- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Para esta prueba, se necesita una copia sin anotaciones del **cuadernillo de fórmulas de Estudios Matemáticos NM**.
- Conteste todas las preguntas.
- Escriba sus respuestas en las casillas provistas a tal efecto.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán darse como valores exactos o con una aproximación de tres cifras significativas.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

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Se otorgará la máxima puntuación a las respuestas correctas. Aun cuando una respuesta sea incorrecta, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Escriba sus respuestas en las casillas provistas a tal efecto. Para los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el procedimiento seguido hasta su obtención. Por ejemplo, cuando deba utilizar un gráfico de una calculadora de pantalla gráfica para hallar soluciones, deberá dibujarlo aproximadamente en su respuesta.

1. Complete la siguiente tabla, colocando los tics (✓) necesarios para indicar a cuáles de los conjuntos de números $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ y $\mathbb{R}$ pertenece cada uno de estos números. Hemos completado la primera fila a modo de ejemplo. [6]

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
$-\frac{3}{11}$			✓	✓
42				
$3{,}14$				
$\sqrt{23}$				
-113				
$\frac{24}{6}$				
$-5\frac{1}{5}$				

Operaciones:

2. M-Line es una empresa que imprime y vende camisetas con diseños personalizados. En cada pedido cobran una cantidad inicial por el diseño y luego una cantidad adicional por cada camiseta que imprimen.

M-Line cobra M euros por pedido. Este precio está modelizado por la función lineal $M(x) = 5x + 40$, donde x es el número de camisetas del pedido.

(a) Escriba la cantidad inicial que cobran por el diseño en cada pedido. [1]

(b) Halle la cantidad total que cobran por un pedido de 94 camisetas. [2]

EnYear es otra empresa que imprime y vende camisetas. El precio, N euros, que cobran por un pedido se puede modelizar mediante la función lineal $N(x) = 9x$, donde x es el número de camisetas del pedido.

(c) Escriba el número de camisetas de un pedido por el que EnYear ha cobrado 63 euros. [1]

Hay un pedido de p camisetas por el que M-Line y EnYear cobrarán la misma cantidad.

(d) Halle el valor de p. [2]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

(d) . . . .

3. En Internet se han popularizado una serie de nombres graciosos para referirse a distintos tipos de perros. Tres de estos nombres aparecen mencionados en las siguientes proposiciones.

p: no es un chucho
q: es un peludo
r: es un perrazo

(a) Escriba con palabras la proposición compuesta $p \Rightarrow \neg r$. [2]

(b) Escriba en forma simbólica la recíproca de $p \Rightarrow (\neg q \wedge r)$. [1]

(c) Complete la siguiente tabla de verdad. [2]

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$((p \Rightarrow q) \wedge (q \Rightarrow r)) \veebar (p \Rightarrow r)$
V	V	V	V	V		V	
V	V	F	V	F		F	
V	F	V	F	V		V	
V	F	F	F	V		F	
F	V	V	V	V		V	
F	V	F	V	F		V	
F	F	V	V	V		V	
F	F	F	V	V		V	

(d) **A partir de lo anterior**, justifique por qué $((p \Rightarrow q) \wedge (q \Rightarrow r)) \veebar (p \Rightarrow r)$ no es una contradicción lógica. [1]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(d) . . . .

. . . .

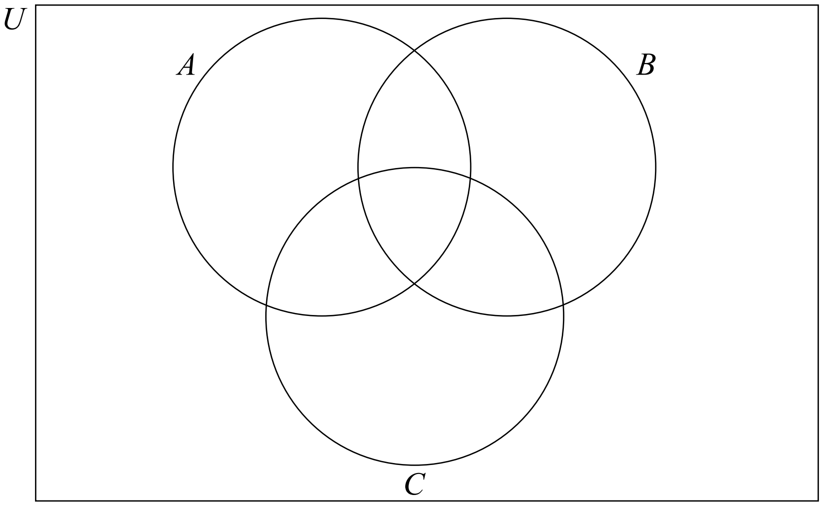
. . . .

4. Sea U (conjunto universal) el conjunto de todos los números enteros x tales que $1 \le x < 11$. A, B y C son subconjuntos de U.

$$A = \{1, 2, 3, 4, 6, 8\}$$
$$B = \{2, 3, 5, 7\}$$
$$C = \{1, 3, 5, 7, 9\}$$

(a) Escriba $n(B)$. [1]

(b) Complete el siguiente diagrama de Venn utilizando **todos** los elementos de U. [4]



(c) Escriba un elemento que pertenezca a $(A \cup B)' \cap C$. [1]

Operaciones:

Respuestas:

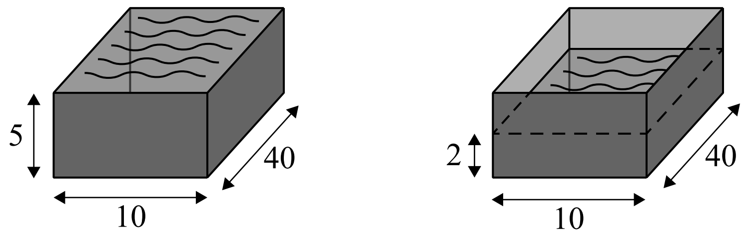
(a) . . . .

(c) . . . .

5. Yao extrae todo el aceite de su motocicleta y lo vierte en dos ortoedros idénticos cuya base rectangular mide $10\,\text{cm}$ de ancho por $40\,\text{cm}$ de largo. La altura de cada ortoedro es igual a $5\,\text{cm}$.

El aceite de la motocicleta llena el primer ortoedro completamente y el segundo ortoedro hasta una altura de $2\,\text{cm}$. Toda esta información se muestra en la siguiente figura.

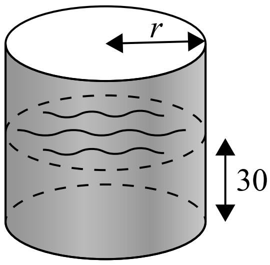
la figura no está dibujada a escala



(a) Calcule el volumen de aceite que se ha extraído de la motocicleta de Yao. [3]

A continuación Yao vierte todo el aceite de los ortoedros en un contenedor cilíndrico vacío. En dicho contenedor, el aceite alcanza una altura de $30\,\text{cm}$.

la figura no está dibujada a escala



(b) Halle el radio interno, r, de este contenedor. [3]

(Esta pregunta continúa en la página siguiente)

(Pregunta 5: continuación)

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

6. La aerolínea Galois Airways tiene vuelos desde el Aeropuerto Internacional de Hong Kong a diversos destinos. La siguiente tabla muestra la distancia, x kilómetros, que hay entre Hong Kong y los diversos destinos, junto con el correspondiente precio del vuelo, y, en dólares de Hong Kong (HKD).

Destino	Bali, Indonesia	Sídney, Australia	Bengaluru, India	Singapur	Auckland, Nueva Zelanda	Bangkok, Tailandia
Distancia x (km)	3400	7400	4000	2600	9200	1700
Precio del vuelo y (HKD)	1550	3600	2800	1300	4000	1400

Para estos datos, el coeficiente de correlación momento-producto de Pearson es igual a $0{,}948$, redondeando a tres cifras significativas.

(a) Utilice la calculadora de pantalla gráfica para hallar la ecuación de la recta de regresión de y sobre x. [2]

La distancia que hay entre Hong Kong y Tokio es de $2900\,\text{km}$.

(b) Utilice su ecuación de regresión para estimar el precio de un vuelo de Galois Airways que vaya de Hong Kong a Tokio. [2]

(c) Explique por qué es válido utilizar la ecuación de regresión para estimar el precio del vuelo entre Hong Kong y Tokio. [2]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

. . . .

. . . .

7. Sea una progresión geométrica donde el primer término es $\frac{8}{3}$ y el cuarto término es 9.

(a) Halle la razón de la progresión. [2]

(b) Escriba el segundo término de esta progresión. [1]

La suma de los k primeros términos es mayor que 2500.

(c) Halle el valor más pequeño posible de k. [3]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

8. Siân invierte $50\,000$ dólares australianos (AUD) en una cuenta de ahorro que ofrece un tipo de interés nominal anual del $5{,}6\,\%$ **compuesto mensualmente**.

(a) Calcule cuál será el valor de la inversión de Siân al cabo de cuatro años. Dé la respuesta redondeando a dos lugares decimales. [3]

Transcurrido ese período de cuatro años, Siân saca $40\,000$ AUD de su cuenta de ahorro y utiliza este dinero para comprarse un coche. Sabemos que el coche se deprecia un $18\,\%$ cada año.

Después de t años, el coche tendrá un valor de 2500 AUD.

(b) Halle el valor de t. [3]

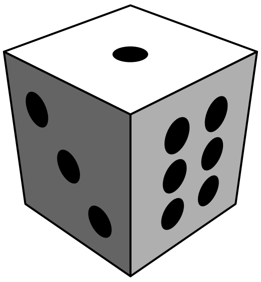
Operaciones:

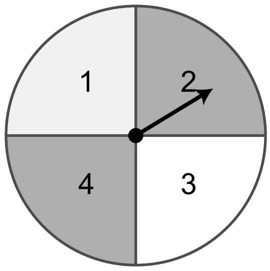
Respuestas:

(a) . . . .

(b) . . . .

9. Sungwon juega a un juego en el que tira un dado equilibrado de 6 caras y hace girar una ruleta equilibrada que tiene 4 sectores iguales. En cada ronda del juego se tira el dado una vez y se hace girar la ruleta una vez. La **puntuación** de cada ronda es la suma de los dos valores obtenidos. Por ejemplo, si saca un 1 con el dado y un 2 con la ruleta, la puntuación será igual a 3.





La siguiente figura representa el espacio muestral.

		Dado					
		1	**2**	**3**	**4**	**5**	**6**
Ruleta	**1**	●	●	●	●	●	●
	2	●	●	●	●	●	●
	3	●	●	●	●	●	●
	4	●	●	●	●	●	●

(a) Halle la probabilidad de que la puntuación de Sungwon en la primera ronda sea mayor que 4. [2]

A continuación Sungwon juega una segunda ronda.

(b) Halle la probabilidad de que la puntuación de Sungwon sea mayor que 4 en cada una de esas dos primeras rondas. [2]

Sungwon juega 11 rondas de este juego.

(c) Halle el número esperado de veces que la puntuación de la ronda será mayor que 4. [2]

(Esta pregunta continúa en la página siguiente)

(Pregunta 9: continuación)

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

10. En esta pregunta, dé todas las respuestas redondeando al número entero más próximo.

Criselda viajó a Kota Kinabalu, en Malasia. En el aeropuerto vio la siguiente información en la oficina de cambio de divisas.

Cambio de divisas – ringgit malasio (MYR)		
Divisa	**Venta**	**Compra**
1 dólar de Singapur (SGD)	3,22	3,07
1 dólar estadounidense (USD)	4,45	4,25

Esto significa que en esta oficina de cambio de divisas **compran** USD a los viajeros y, a cambio, les entregan MYR según el tipo de cambio 1 USD = 4,25 MYR. No se cobra ninguna comisión.

Criselda cambió 460 SGD a MYR.

(a) Calcule cuántos MYR recibió Criselda. [3]

Durante el tiempo que pasó en Kota Kinabalu, Criselda gastó 440 MYR. Volvió a la oficina de cambio de divisas y cambió el resto de sus MYR por USD.

(b) Calcule cuántos USD recibió Criselda. [3]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

11. Elvis Presley fue un cantante muy famoso. A pesar de que murió en 1977, sigue habiendo muchos aficionados a su música que le rinden homenaje vistiéndose de Elvis y cantando sus canciones.

El número de imitadores de Elvis, $N(t)$, se puede modelizar mediante la siguiente función:

$$N(t) = 170 \times 1{,}31^{t},$$

donde t es el número de años que han transcurrido desde 1977.

(a) Escriba el número de imitadores de Elvis que había en 1977. [1]

(b) Calcule el tiempo que pasó hasta que el número de imitadores de Elvis llegó a los $130\,000$. [2]

(c) Calcule el número de imitadores de Elvis que habrá cuando $t = 70$. [2]

En el año 2047 se espera que la población mundial llegue a los $9\,500\,000\,000$ de personas.

(d) Utilice esta información para explicar por qué este modelo del número de imitadores de Elvis no es realista. [1]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

(d) . . . .

. . . .

. . . .

12. La carrera solidaria Malthouse se celebró sobre una distancia de 5 kilómetros. Se registró el tiempo que tardó cada corredor en acabar la carrera y se comprobó que los datos seguían una distribución normal de media 28 minutos y desviación típica 5 minutos.

Se elige al azar a uno de los corredores que completaron la carrera.

(a) Escriba la probabilidad de que este corredor tardara más de 28 minutos en completar la carrera. [1]

(b) Calcule la probabilidad de que este corredor tardara menos de 26 minutos en completar la carrera. [2]

Se sabe que el 20% de los corredores tardaron más de 28 minutos y menos de k minutos en completar la carrera.

(c) Halle el valor de k. [3]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

13. Los huevos de gallina se clasifican en categorías (4, 5, 6, 7 u 8) en función de su peso. En una caja mixta vienen 12 huevos y puede haber huevos de cualquier categoría. Como parte de su proyecto de ciencias, Rocky compra 9 cajas mixtas y ordena los huevos por peso.

Categoría	**Peso, w (gramos)**	**Frecuencia**
4	$40 \le w < 50$	3
5	$50 \le w < 60$	30
6	$60 \le w < 70$	45
7	$70 \le w < 80$	25
8	$80 \le w < 90$	5

(a) Indique si el peso de los huevos es una variable discreta o continua. [1]

(b) Escriba la categoría modal de los huevos. [1]

(c) Utilice la calculadora de pantalla gráfica para hallar una estimación de la desviación típica del peso de los huevos. [2]

El peso medio de estos huevos es igual a $64{,}9$ gramos, si redondeamos a tres cifras significativas.

(d) Utilice la tabla y la respuesta que dio en el apartado (c) para hallar el número **más pequeño posible** de huevos que podrían encontrarse a menos de una desviación típica de la media. [2]

Operaciones:

Respuestas:

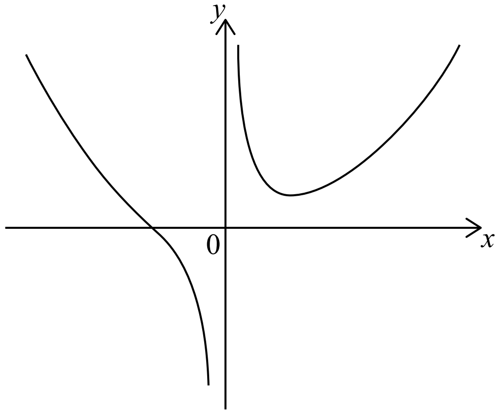
(a) . . . .

(b) . . . .

(c) . . . .

(d) . . . .

14. La siguiente figura muestra la curva $y = \frac{x^2}{2} + \frac{2a}{x},\ x \neq 0$.



La ecuación de la asíntota vertical de esta curva es $x = k$.

(a) Escriba el valor de k. [1]

(b) Halle $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

En el punto donde $x = 2$, la pendiente de la tangente a la curva es igual a $0{,}5$.

(c) Halle el valor de a. [2]

Operaciones:

Respuestas:

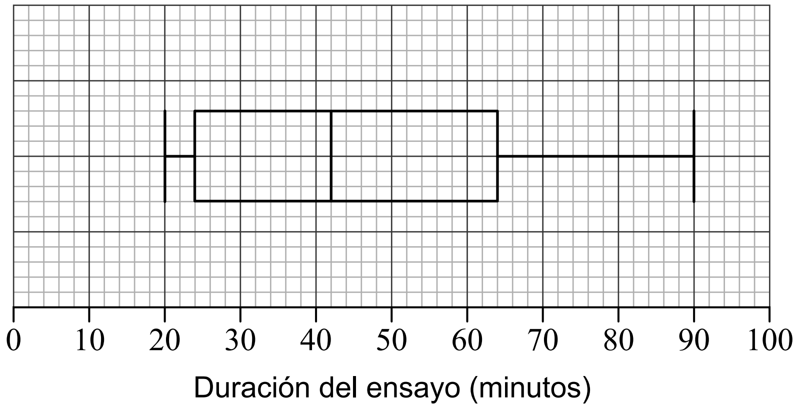
(a) . . . .

(b) . . . .

(c) . . . .

15. A Stephen lo invitaron a tocar un recital de piano. A modo de preparación para el concierto, Stephen fue anotando el tiempo, en minutos, que ensayaba cada día para dicho recital de piano.

Stephen estuvo ensayando 32 días; en el siguiente diagrama de caja y bigotes se muestran los datos correspondientes a todos estos días.



(a) Escriba la mediana de la duración del ensayo. [1]

Stephen afirma que ensayó todos y cada uno de esos 32 días.

(b) Indique si lo que dice Stephen es cierto. Dé una razón que justifique su respuesta. [2]

Hay k días en los que Stephen estuvo practicando exactamente 24 minutos.

(c) Halle los posibles valores de k. [3]

Operaciones:

Respuestas:

(a) . . . .

(b) . . . .

(c) . . . .

No escriba en esta página.

Las respuestas que se escriban en esta página no serán corregidas.

Diploma Programme
Programme du diplôme
Programa del Diploma

Esquema de calificación

Noviembre de 2019

Estudios Matemáticos

Nivel Medio

Prueba 1

23 páginas

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Esquema de calificación de la Prueba 1
Instrucciones para los examinadores

Observaciones: **En caso de dudas sobre estas instrucciones o temas relacionados con la asignación de puntuaciones, póngase en contacto con su jefe de equipo.**

La puntuación máxima para cada pregunta es 6.

1 Siglas

En el esquema de calificación pueden aparecer las siguientes siglas:

M Puntos otorgados por el **método**

A Puntos otorgados por una **respuesta** o por la **precisión**

C Puntos otorgados por respuestas **correctas** (independientemente del trabajo mostrado)

R Puntos otorgados por un **razonamiento** claro

ft Se pueden otorgar puntos por la **coherencia** con resultados previos en la pregunta

2 Método para corregir

(a) Todas las correcciones deben realizarse en RM Assessor, utilizando las anotaciones de Estudios Matemáticos y de acuerdo con el documento vigente de orientaciones para la corrección electrónica de Estudios Matemáticos NM. Es fundamental leer el documento antes de empezar a corregir.

(b) Si el alumno obtiene la puntuación máxima en una pregunta utilice la anotación ***C6***, si lo ha intentado pero obtiene cero puntos utilice ***C0***. Si no lo ha intentado utilice la tecla de No respuesta. Si un alumno no ha obtenido ni la puntuación máxima ni cero puntos, entonces se **DEBEN** mostrar todas las anotaciones.

(c) En esta prueba, **si aparece la respuesta correcta en la línea de respuesta** se otorgará la puntuación máxima. **¡No es necesario comprobar el trabajo! Otorgue puntos C y siga adelante.**

(d) Si la respuesta no aparece en la línea de respuesta, pero la respuesta correcta se encuentra en el cuadro de operaciones sin trabajo posterior, otorgue la puntuación máxima.

(e) Si la **respuesta es incorrecta**, se deben otorgar puntos por el trabajo realizado, de acuerdo con el esquema de calificación.

(f) No se debe otorgar ningún punto al trabajo tachado por el alumno. Si el alumno ha dado dos respuestas a una pregunta, solo se debe corregir la primera respuesta.

(g) Una respuesta correcta en el cuadro de trabajo transcrita erróneamente a la línea de respuesta puede recibir la puntuación máxima.

(e) Si el procedimiento adecuado lleva a la respuesta correcta, pero esta va seguida de más desarrollos que indican una falta de comprensión matemática, entonces **no** se puede otorgar la puntuación máxima. En la mayoría de estos casos se perderá solo el último punto por respuesta. Una excepción podría ser las respuestas numéricas, donde una respuesta exacta que es correcta está seguida de una expresión decimal incorrecta.

Ejemplo:

	Se ve la respuesta correcta	**Hay más desarrollo**	**Acción**
1.	$8\sqrt{2}$	$5,65685...$ *(valor decimal incorrecto)*	Otorgue el ultimo ***(A1)*** *(ignore el desarrollo posterior)*
2.	$(x-6)(x+1)$	$x=6$ y -1	**No** otorgue el último ***(A1)*** *(vea el siguiente ejemplo)*

Ejemplo: Factorice $x^2 - 5x - 6$

Esquema de calificación	Examen del alumno	Corrección
$(x-6)(x+1)$ ***(A1)(A1)***	(i) Línea de respuesta: $(x+6)(x+1)$	***(A0)(A1)***
	(ii) Cuadro de operaciones: $(x-6)(x+1)$	***(A1)***
	seguido de $x=6$ y -1, o simplemente $6, -1$ bien en el cuadro de trabajo o en la línea de respuesta.	***(A0)***

3 Puntos por la coherencia (ft)

En cualquier paso de una resolución, un error puede afectar a todo el trabajo posterior. Para limitar la gravedad de la penalización, se pueden otorgar **puntos por la coherencia**. Los esquemas de calificación indicarán, con **"(ft)"**, dónde es adecuado aplicar el criterio de coherencia en una pregunta.

(a) La coherencia se aplica solo de un apartado de una pregunta a otro apartado posterior de la pregunta. No se aplica dentro del mismo apartado.

(b) Si una respuesta que es consecuencia de un procedimiento de coherencia es muy poco realista (por ejemplo, distancias negativas o de un orden de magnitud excesivo) entonces el punto ***A*** final no se debe otorgar.

(c) Si, debido a un error, una pregunta queda transformada en otra **distinta**, **mucho más sencilla**, entonces el criterio de coherencia podría no ser aplicado.

(d) Para otorgar puntos por coherencia en un apartado de una pregunta, **es necesario que haya un trabajo explícito relacionado con ese apartado**. Una respuesta aislada, obtenida por coherencia, pero sin un desarrollo explícito, se considera incorrecta, y no recibirá ningún punto **aunque se aproxime a la respuesta correcta**.

(e) Una excepción a lo anterior sería una pregunta donde lo que se evalúa es el manejo del alumno en el uso de la calculadora de pantalla gráfica, y donde no se pretende que se muestre ningún desarrollo escrito. **En el esquema de calificación vendrá indicado claramente cuando sea el caso.**

(f) El uso inadvertido de radianes será penalizado la primera vez que aparezca. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez un punto por el uso de radianes.

Ejemplo: Cálculo de ángulos y longitudes en trigonometría

Esquema de calificación			Examen del alumno		Corrección
(a)	$\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$	***(M1)(A1)***	(a)	$\frac{\text{sen } A}{4} = \frac{\text{sen } 30}{3}$	***(M1)(A0)*** *(uso del teorema del seno, pero con valores incorrectos)*
	$A = 22{,}0^\circ \ (22{,}0243\ldots)$	***(A1)***		$A = 41{,}8^\circ$	***(A0)***
				(Nota: *Aquí, el segundo* ***(A1)*** *no ha sido corregido como* **(ft)** *y no se puede otorgar porque existe un error previo en el* ***mismo*** *apartado de la pregunta.)*	
(b)	$x = 7\text{tg}\,(22{,}0243\ldots^\circ)$	***(M1)***	(b)	caso (i) $x = 7\text{tg}\,41{,}8^\circ$	***(M1)***
	$= 2{,}83\ (2{,}831639\ldots)$	***(A1)*(ft)**		$= 6{,}26$	***(A1)*(ft)**
			pero	caso (ii) $6{,}26$	***(C0)*** *pues no aparece un desarrollo explícito*

4 Uso del Esquema de calificación

(a) Los puntos ***A*** **dependen** de los puntos ***M*** obtenidos previamente, **no** es posible otorgar ***(M0)(A1)***. Una vez que se otorga un ***(M0)***, se pierden todos los puntos ***A*** que le siguen en ese apartado de la pregunta, incluso si los cálculos son correctos, hasta el siguiente punto ***M***.
La única excepción a lo anterior se hará en el caso de una respuesta donde la precisión se especifique en la pregunta (ver apartado 5).

(b) Los puntos ***A*** **dependen** de los puntos ***R*** obtenidos, **no** es posible otorgar ***(A1)(R0)***. Así pues, el ***(A1)*** no se otorga a una respuesta correcta si no aparece el razonamiento, o este es incorrecto.

(c) Los **métodos alternativos** no siempre se incluyen. Así, si una respuesta es errónea, entonces se debe analizar cuidadosamente el procedimiento, de modo que se asignen puntos por un método distinto de forma coherente con el esquema de calificación.
Cuando en el esquema de calificación se incluyen métodos alternativos para una pregunta, ello viene indicado mediante "**O**" *etc*.

(d) A menos que en la pregunta se especifique lo contrario, se aceptan **expresiones equivalentes**. Por ejemplo: $\frac{\operatorname{sen}\theta}{\cos\theta}$ por $\operatorname{tg}\theta$. En el esquema de calificación, estas expresiones algebraica o numéricamente equivalentes aparecerán a veces escritas entre paréntesis junto a la respuesta requerida.
Cuando se requieran respuestas numéricas como respuesta final de un apartado de una pregunta, en el esquema de calificación se mostrará en este orden;
la respuesta con 3 cifras significativas a partir de la solución completa de la calculadora;
el valor exacto (por ejemplo $\sqrt{3}$ si corresponde);
la solución completa de la calculadora en la forma $2{,}83163\ldots$ como en el ejemplo anterior.
Cuando se dan las respuestas con 3 cifras significativas y se utilizan después en apartados posteriores de la pregunta llevando a una solución con 3 cifras significativas **distinta**, también se darán estas soluciones.

(e) Dado que este es un examen internacional, se aceptarán todas las **formas de notación alternativas**. Algunos ejemplos:

Comas decimales: $1{,}7$**;** $1’7$; $1\cdot 7$; $1;7$.
Los números decimales menores que 1 podrían aparecer escritos sin un cero en el frente: 0.49 o $.49$.
Descripciones de un intervalo: $3 < x < 5$; $(3;\ 5)$; $]\ 3;\ 5\ [$.
Distintas formas de notación de las propiedades de los conjuntos (por ejemplo complementario): A'; $\overline{A}$; A^c; $U - A$;$(A$;$U \setminus A$.
Distintas formas de notación lógica: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p, -p$.
$p \Rightarrow q$; $p \rightarrow q$.
El nivel de significación podría escribirse como α .

(f) Puntos discrecionales: habrá ocasiones excepcionales en las que el esquema de calificación no cubra el trabajo que aparece. En estos casos se utilizará la nota DM para indicar que el examinador ha utilizado su criterio. La discrecionalidad debe utilizarse con moderación, y si existe duda se debe plantear una excepción a través de RM Assessor al jefe del equipo.

A partir de noviembre de 2011 ya no se aplicarán las penalizaciones AP, FP y UP. La precisión y las unidades serán evaluados en preguntas específicas y los puntos se otorgarán de acuerdo a las reglas dadas en los apartados 5, 6 y 7.

5 Precisión de las respuestas

Instrucciones: Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas correctamente a 3 cifras significativas.

1. Si el alumno muestra la solución a 4 o más cifras significativas, la cual redondeada a 3 cifras significativas daría la respuesta requerida entonces otorgue ***(A1)*** e ignore cualquier redondeo posterior. **Nota:** La solución sin redondear puede estar tanto en la línea de respuesta como en el cuadro de operaciones.

2. Si el alumno **no** muestra la solución sin redondear, entonces otorgue ***(A1)*** si la solución dada está aproximada correctamente a 2 o más cifras significativas, y ***(A0)*** en cualquier otro caso.

3. Si una solución aproximada a 2 cifras significativas se utiliza en apartados posteriores, entonces para otorgar más puntos debe aparecer el desarrollo. (Este criterio es el mismo que en el que se aplica coherencia a partir de una respuesta errónea.)

Estos 3 casos (vea los superíndices) se han resumido en la siguiente tabla e ilustrado después con ejemplos.

	Si la solución final de alumno se expresa…				
	Exacta o a 4 o más cifras significativas (y que redondeada **a 3 cifras significativas daría la respuesta correcta**)	**Aproximada incorrectamente a 3 cifras significativas**	Aproximada correctamente a 2 cifras significativas[3]	Aproximada incorrectamente a 2 cifras significativas	Aproximada correcta o incorrectamente a 1 cifra significativa
Aparece la solución sin redondear[1]	Otorgue ***(A1)*** independientemente del redondeo correcto o incorrecto				
No aparece la solución sin redondear[2]	***(A1)***	***(A0)***	***(A1)***	***(A0)***	***(A0)***
Criterio para los apartados posteriores	Como en MS	Como en el criterio de coherencia, solo si se muestra el desarrollo[3]			

Ejemplos:

Esquema de calificación	Examen del alumno	Corrección
9,43 (9,43398…) ***(A1)***	(i) 9,43398… se muestra en el cuadro de trabajo seguido de 9; 9,4; 9,43; 9,434 (redondeados correctamente)	***(A1)***
	(ii) 9,43398… se muestra en el cuadro de trabajo seguido de 9,433; 9,44, *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 9,4	***(A1)***
	(iv) 9	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 9,3	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 9,44	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Esquema de calificación	Examen del alumno	Corrección
7,44 (7,43798…) ***(A1)***	(i) 7,43798… se muestra en el cuadro de trabajo seguido de 7; 7,4; 7,44; 7,438 *etc.* (redondeados correctamente)	***(A1)***
	(ii) 7,43798… se muestra en el cuadro de trabajo seguido de 7,437 ó 7,43 *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 7,4	***(A1)***
	(iv) 7	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 7,5	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 7,43	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Ejemplo: ABC es un triángulo rectángulo con el ángulo $\text{ABC} = 90^\circ$, $\text{AC} = 32\text{ cm}$ y $\text{AB} = 30\text{ cm}$. Halle (a) la longitud de BC, (b) el área del triángulo ABC.

Esquema de calificación	Examen del alumno	Corrección
(a) $\text{BC} = \sqrt{32^2 - 30^2}$ ***(M1)*** *Otorgue **(M1)** por la sustitución correcta en el teorema de Pitágoras* $= 11{,}1\ \left(\sqrt{124}\,11{,}1355...\right)(\text{cm})$ ***(A1)***	(a) $\text{BC} = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$ *(solo se muestra la aproximación a 2 cifras significativas, pero correcta)*	***(M1)*** ***(A1)***
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11{,}1355...$ ***(M1)*** *Otorgue **(M1)** por la sustitución correcta en la fórmula del área de un triángulo* $= 167\ \ (167{,}032...)(\text{cm}^2)$ ***(A1)*(ft)**	(b) caso (i) $\text{Área} = \frac{1}{2} \times 30 \times 11$ *(se muestra el desarrollo)* $= 165\,(\text{cm}^2)$ caso (ii) $= 165\,(\text{cm}^2)$ *(No se muestra el desarrollo, la solución 11 se trata como procedimiento de coherencia, por lo que no se deben otorgar puntos en este caso)*	***(M1)*** ***(A1)*(ft)** ***(M0)(A0)*(ft)**

La aproximación a 3 cifras significativas de una solución exacta **se debe aceptar si se realiza correctamente**.

Las soluciones exactas del tipo $\frac{1}{4}$ se pueden escribir como decimales con menos de 3 cifras significativas si el resultado sigue siendo exacto. La simplificación de una fracción a su expresión irreducible **no** es imprescindible, sin embargo cuando una respuesta se simplifica a un entero, esto es lo esperado. Fracciones que incluyen un decimal en el numerador y/o en el denominador son aceptadas para demostrar una substitución correcta, sin embargo, no como respuesta final.

Razones de π y respuestas con expresiones de raíces cuadradas de enteros o cualquier potencia racional de un entero (por ejemplo, $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$), se pueden aceptar como respuestas exactas. Todas las otras potencias (por ejemplo de no enteros) y valores de funciones transcendentes tales como seno y coseno se deben evaluar.

Si el nivel de precisión viene especificado en la pregunta, se asignarán puntos por dar la respuesta con la precisión requerida. En **todos** estos casos no se obtiene el punto final si el redondeo no sigue las instrucciones dadas en la pregunta. El punto por la precisión especificada se puede considerar como punto por coherencia **(ft)** con independencia de un ***(M0)*** inmediatamente anterior.

6 Nivel de precisión en las preguntas sobre cuestiones financieras

El nivel de precisión de las respuestas se especificará en todas las preguntas que se relacionen con valores en dinero. Generalmente será a la unidad más cercana o de dos cifras decimales. La primera respuesta que no venga dada con el nivel de precisión especificado no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez la un punto por la precisión incorrecta en una pregunta sobre cuestiones financieras.

Ejemplo: En una pregunta sobre cuestiones financieras se pide una precisión de dos cifras decimales.

Esquema de calificación	Examen del alumno	Corrección
$\$231,62$ $(231,6189)$ ***(A1)***	(i) $231,6$	***(A0)***
	(ii) 232 *(Aproximación correcta con un nivel de precisión distinto del requerido)*	***(A0)***
	(iii) $231,61$	***(A0)***
	(iv) $232,00$ *(Aproximación incorrecta al nivel de precisión requerido)*	***(A0)***

7 Unidades de medida en las respuestas

En determinadas preguntas será necesario expresar las unidades de medida, y esto vendrá claramente indicado en el esquema de calificación. La primera respuesta correcta pero sin unidades o con unidades incorrectas no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la falta de unidades o unidades incorrectas. Las unidades de medida se tendrán en cuenta solamente cuando se ha otorgado ***(A1)*** a la respuesta numérica siguiendo las indicaciones sobre la precisión dadas en el apartado 5.

Esquema de calificación	Examen del alumno	Corrección
(a) $37000\ \mathrm{m}^2$ ***(A1)***	(a) $36000\ \mathrm{m}^2$ *(respuesta incorrecta, por lo que no se consideran las unidades)*	***(A0)***
(b) $3200\ \mathrm{m}^3$ ***(A1)***	(b) $3200\ \mathrm{m}^2$ *(unidades incorrectas)*	***(A0)***

8 Calculadoras de pantalla gráfica

Con frecuencia los alumnos obtienen las soluciones directamente de la calculadora. Deben utilizar la notación matemática y no la notación de la calculadora. No se puede otorgar puntos por método a respuestas incorrectas basadas únicamente en notación de calculadora. No se puede otorgar puntos por método al comentario “usé la calculadora de pantalla gráfica”.

1.

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
$-\frac{3}{11}$			✓	✓
42	✓	✓	✓	✓
$3{,}14$			✓	✓
$\sqrt{23}$				✓
-113		✓	✓	✓
$\frac{24}{6}$	✓	✓	✓	✓
$-5\frac{1}{5}$			✓	✓

Nota: Conceda ***(A1)*** por cada fila completamente correcta; en caso contrario conceda ***(A0)***.

Total [6 puntos]

2. (a) 40 (euros) ***(A1)*** ***(C1)***

[1 punto]

(b) $\left(M(94) =\right)\ 5(94)+40$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente el 94 en la función dada.

510 (euros) ***(A1)*** ***(C2)***

[2 puntos]

(c) 7 (camisetas) ***(A1)*** ***(C1)***

[1 punto]

(d) $9p = 5p + 40$ ***(M1)***

Nota: Conceda ***(M1)*** por igualar las funciones dadas. Acepte un bosquejo donde se muestren ambas funciones.

$(p =)\ 10$ (camisetas) ***(A1)*** ***(C2)***

[2 puntos]

Total [6 puntos]

3. (a) **Si** no es un chucho **entonces** no es un perrazo ***(A1)(A1)*** ***(C2)***

> **Nota:** Conceda ***(A1)*** por el "Si… entonces". Conceda ***(A1)*** si las dos proposiciones correctas están en el orden correcto.

[2 puntos]

(b) $(\neg q \wedge r) \Rightarrow p$ ***(A1)*** ***(C1)***

> **Nota:** Conceda ***(A0)*** por $\neg q \wedge r \Rightarrow p$.

[1 punto]

(c)

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$((p \Rightarrow q) \wedge (q \Rightarrow r)) \veebar (p \Rightarrow r)$
V	V	V	V	V	**V**	V	**F**
V	V	F	V	F	**F**	F	**F**
V	F	V	F	V	**F**	V	**V**
V	F	F	F	V	**F**	F	**F**
F	V	V	V	V	**V**	V	**F**
F	V	F	V	F	**F**	V	**V**
F	F	V	V	V	**V**	V	**F**
F	F	F	V	V	**V**	V	**F**

(A1)(A1)*(ft)** ***(C2)

> **Nota:** Conceda ***(A1)*** por cada columna correcta.
> En la última columna corrija con error de arrastre de las columnas anteriores.

[2 puntos]

(d) No todos / solo algunos de los elementos de la **columna final (o última)** son falsos. ***(R1)*** ***(C1)***

O BIEN

Algunos de los elementos de la **columna final (o última)** son verdaderos. ***(R1)*** ***(C1)***

> **Nota:** Para conceder el ***(R1)*** es necesario que se haya referido explícitamente a la columna final (o última).

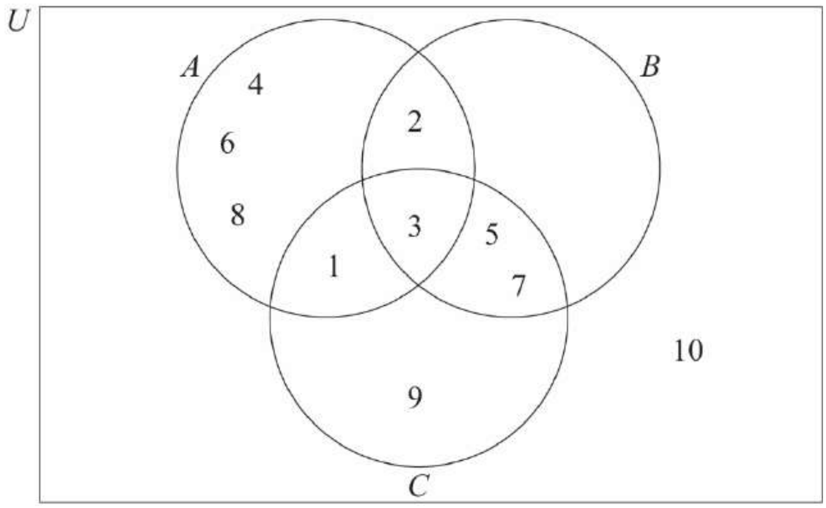
[1 punto]

Total [6 puntos]

4. (a) 4 ***(A1)*** ***(C1)***

[1 punto]

(b)



(A1)(A1)(A1)(A1) ***(C4)***

> **Nota:** Conceda ***(A1)*** si el 3 está en el lugar correcto. Conceda ***(A1)*** si 1, 2, 5 y 7 están en el lugar correcto. Conceda ***(A1)*** si 4, 6, 8 y 9 están en el lugar correcto. Conceda ***(A1)*** por un 10 fuera de los tres círculos **y** si 11 no aparece en el diagrama.
>
> Si se duplica algún elemento dentro de su región conceda como máximo ***(A3)***.

[4 puntos]

(c) 9 ***(A1)*(ft)** ***(C1)***

> **Nota:** Conceda ***(A1)*** por dar el elemento correcto. Se puede otorgar error de arrastre (ft) a partir de su diagrama de Venn del apartado (b). Conceda ***(A0)*** si ha incluido en su respuesta elementos adicionales incorrectos.

[1 punto]

Total [6 puntos]

5. Las unidades se requieren en ambos apartados.

(a) $(V =)\ 5\times 10\times 40 + 2\times 10\times 40$ ***(M1)(M1)***

> **Nota:** Conceda ***(M1)*** por sustituir correctamente los valores en la fórmula del volumen para los dos ortoedros. Conceda ***(M1)*** por sumar los volúmenes de los dos ortoedros.

$2800\,\text{cm}^3$ ***(A1)*** ***(C3)***

[3 puntos]

(b) $2800 = \pi\times r^2 \times 30$ ***(M1)(M1)***

> **Nota:** Conceda ***(M1)*** por sustituir correctamente los valores en la fórmula del volumen del cilindro. Conceda ***(M1)*** por igualar *su* expresión (ha de incluir π y r) a *su* 2800.

$(r =)\ 5{,}45\,\text{cm}\ (5{,}45058\ldots\text{cm})$ ***(A1)*(ft)** ***(C3)***

> **Nota:** Corrija con error de arrastre desde el apartado (a).

[3 puntos]

Total [6 puntos]

6. (a) $y = 0{,}384x + 629$

$y = (0{,}384221\ldots)\,x + (629{,}421\ldots)$ ***(A1)(A1)*** ***(C2)***

> **Nota:** Conceda ***(A1)*** por $0{,}384x$ y ***(A1)*** por 629. Si no da la respuesta como una ecuación conceda como máximo ***(A1)(A0)***.

[2 puntos]

(b) $y = 0{,}384221\ldots\times 2900 + 629{,}421\ldots$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido en *su* ecuación de regresión.

$1740\,(1744;\ 1743{,}66\ldots)\,(\text{HKD})$ ***(A1)*(ft)** ***(C2)***

> **Nota:** Corrija con error de arrastre (ft) desde el apartado (a).

[2 puntos]

(c) la correlación es (muy) fuerte ***(R1)***

2900 (km) está dentro del intervalo de datos dado (interpolación) ***(R1)*** ***(C2)***

> **Nota:** Para conceder ***(C2)*** es necesario que el alumno haya dado dos razones correctas.

[2 puntos]

Total [6 puntos]

7. (a) $9 = \left(\frac{8}{3}\right)r^3$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula de la progresión geométrica y haberla igualado a 9.

$r = 1{,}5 \ \left(\frac{3}{2}\right)$ ***(A1)*** ***(C2)***

[2 puntos]

(b) 4 ***(A1)*(ft)** ***(C1)***

Nota: Otorgue el punto de error de arrastre (ft) desde el apartado (a).

[1 punto]

(c) $2500 < \frac{\left(\frac{8}{3}\right)\left((1{,}5)^k - 1\right)}{1{,}5 - 1}$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente *sus* valores en la fórmula de la serie geométrica y haberla comparado con 2500.

$k = 15{,}2 \ (15{,}17319\ldots)$ ***(A1)*(ft)**

$(k =)\ 16$ ***(A1)*(ft)** ***(C3)***

Nota: Para poder conceder el ***(A1)*(ft)** final es necesario que la respuesta sea un número entero. Corrija con error de arrastre del apartado (a).

[3 puntos]

Total [6 puntos]

8. (a) $FV = 50\,000\left(1+\frac{5{,}6}{100\times 12}\right)^{12\times 4}$ ***(M1)(A1)***

Nota: Conceda ***(M1)*** por haber sustituido los valores en la fórmula del interés compuesto y ***(A1)*** por haberlos sustituido correctamente.

O BIEN

$N = 4$
$I\,\% = 5{,}6$
$PV = (\mp)50\,000$
$P/Y = 1$
$C/Y = 12$ ***(A1)(M1)***

Nota: Conceda ***(A1)*** si ha incluido $C/Y = 12$ en su desarrollo; conceda ***(M1)*** por el resto de elementos correctos.

O BIEN

$N = 48$
$I\,\% = 5{,}6$
$PV = (\mp)50\,000$
$P/Y = 12$
$C/Y = 12$ ***(A1)(M1)***

Nota: Conceda ***(A1)*** si ha incluido $C/Y = 12$ en su desarrollo; conceda ***(M1)*** por el resto de elementos correctos.

$(FV =)\ 62\,520{,}97$ (AUD) ***(A1)*** ***(C3)***

Nota: No conceda el ***(A1)*** final si la respuesta no se ha redondeado correctamente a 2 lugares decimales.

[3 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 8

(b) $2500 = 40\,000\left(1 - \frac{18}{100}\right)^{t}$ **O BIEN** $2500 = 40\,000\,(0{,}82)^{t-1}$ ***(M1)(A1)***

Nota:	Conceda ***(M1)*** por sustituir valores en la fórmula del interés compuesto o término de una sucesión geométrica e igualarlas a 2500; conceda ***(A1)*** por la sustitución correcta.

O BIEN

$I\% = -18$
$PV = (\pm)\,40\,000$
$FV = (\mp)\,2500$
$P/Y = 1$
$C/Y = 1$ ***(A1)(M1)***

Nota:	Conceda ***(A1)*** si ha incluido $FV = (\mp)\,2500$ en su desarrollo; conceda ***(M1)*** por el resto de elementos correctos. PV y FV deben tener signos opuestos.

O BIEN

$32800;\ 26896;\ 22054{,}72;\ 18084{,}87;\ 14829{,}59;\ldots$ ***(M1)***

$t_{13} = 3031{,}38$ y $t_{14} = 2485{,}73$ ***(A1)***

Nota:	Conceda ***(M1)*** por una lista de al menos 5 términos correctos comenzando con 32800, ***(A1)*** por identificar t_{13} y t_{14}.

$(t =)\ 14{,}0\ (13{,}9711\ldots)$ ***(A1)*** ***(C3)***

[3 puntos]

Total [6 puntos]

9. (a) $\frac{18}{24}\left(\frac{3}{4}, 0{,}75, 75\%\right)$ ***(A1)(A1)*** ***(C2)***

Nota: Conceda ***(A1)*** por un numerador correcto y ***(A1)*** por un denominador correcto.

[2 puntos]

(b) $\frac{18}{24}\times\frac{18}{24}$ ***(M1)***

Nota: Conceda ***(M1)*** por elevar al cuadrado *su* probabilidad, dada en el apartado (a).

$= 0{,}563\left(\frac{9}{16}, \frac{324}{576}, 0{,}5625\ , 56{,}3\%\right)$ ***(A1)*(ft)** ***(C2)***

Nota: Se puede corregir con error de arrastre (ft) desde el apartado (a), siempre y cuando *su* respuesta sea menor o igual a 1.

[2 puntos]

(c) $11\times\frac{18}{24}$ ***(M1)***

Nota: Conceda ***(M1)*** por haber multiplicado *su* respuesta de (a) por 11.

$8{,}25\left(\frac{33}{4}\right)$ ***(A1)*(ft)** ***(C2)***

Nota: Corrija con arrastre de error (ft) desde el apartado (a), siempre y cuando la respuesta de (a) sea menor o igual a 1.

[2 puntos]

Total [6 puntos]

10. (a) $460 \times 3{,}07$ ***(A1)(M1)***

> **Nota:** Conceda ***(A1)*** por haber seleccionado $3{,}07$ como tasa de cambio. Conceda ***(M1)*** por haber multiplicado 460 por una tasa de cambio de la tabla.

1412 (MYR) ***(A1)*** ***(C3)***

> **Nota:** No conceda el ***(A1)*** final si ha dado la respuesta con un grado de precisión incorrecto.

[3 puntos]

(b) $\frac{1412 - 440}{4{,}45}$ ***(M1)(M1)***

> **Nota:** Conceda ***(M1)*** por *su* sustracción correcta o por 972 $(972{,}2)$ o *su* correcta diferencia vista. Conceda ***(M1)*** por haber dividido entre $4{,}45$. Corrija con error de arrastre de (a).

218 (USD) ***(A1)*(ft)** ***(C3)***

> **Nota:** No conceda el ***(A1)*** final si ha dado la respuesta con un grado de precisión incorrecto.

[3 puntos]

Total [6 puntos]

11. (a) 170 ***(A1)*** ***(C1)***

[1 punto]

(b) $130\,000 = 170 \times 1{,}31^{t}$ ***(M1)***

Nota: Conceda ***(M1)*** por igualar $130\,000$ a la función exponencial.

$(t =)$ $24{,}6$ $(24{,}5882\ldots$ (años)) ***(A1)*** ***(C2)***

[2 puntos]

(c) $170 \times 1{,}31^{70}$ ***(M1)***

Nota: Conceda ***(M1)*** por sustituir correctamente los valores en la función $N(t)$.

$2{,}75 \times 10^{10}$ $(2{,}75067\ldots \times 10^{10}$, $27\,500\,000\,000$, $27\,506\,771\,343)$ ***(A1)*** ***(C2)***

[2 puntos]

(d) En 2047, el número de imitadores de Elvis sería mayor que la población mundial. ***(R1)*** ***(C1)***

O BIEN

$2{,}75 \times 10^{10} > 9\,500\,000\,000$ ***(R1)*** ***(C1)***

Nota: Conceda ***(R1)*** por comparar correctamente *su* número de imitadores con la población mundial. Corrija con error de arrastre (ft) desde el apartado (c) si el alumno ofrece un argumento razonable de que el modelo no resulta realista. Conceda ***(R0)*** si no ha mencionado explícitamente el número de imitadores en el apartado (c) o en el apartado (d).

[1 punto]

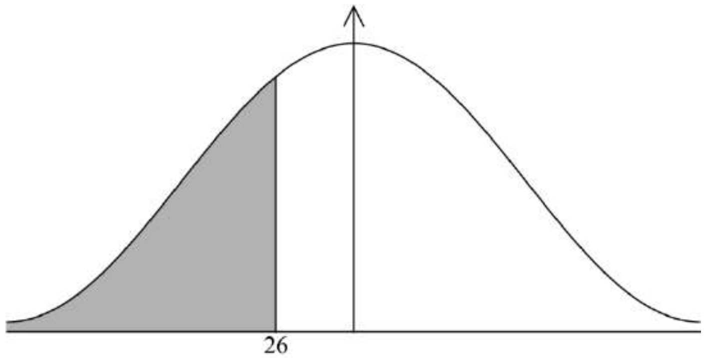
Total [6 puntos]

12. (a) $0,5 \left(\frac{1}{2}, 50\%\right)$ ***(A1)*** ***(C1)***

[1 punto]

(b) $\mathrm{P}(X \le 26)$ ***(M1)***

Nota: Conceda ***(M1)*** por dar un enunciado matemático correcto.
O BIEN
Conceda ***(M1)*** por un diagrama que muestre el valor 26 etiquetado a la izquierda de la media y la región correctamente sombreada.



$0,345$ $(0,344578\ldots, 34,5\%)$ ***(A1)*** ***(C2)***

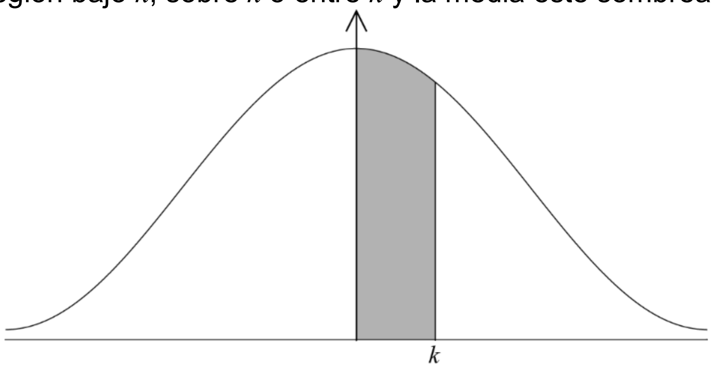
[2 puntos]

(c) $0,7$ **O BIEN** $0,3$ (visto) ***(A1)***

Nota: Conceda ***(A1)*** por el $0,7$ o el $0,3$.

$\mathrm{P}(\text{tiempo} < k) = 0,7$ **O BIEN** $\mathrm{P}(\text{tiempo} > k) = 0,3$ ***(M1)***

Nota: Conceda ***(M1)*** por dar un enunciado matemático correcto.
O BIEN
Conceda ***(M1)*** por un diagrama que muestre k mayor que la media y la región bajo k, sobre k o entre k y la media esté sombreada.



$k = 30,6$ $(30,6220\ldots)$ (minutos) ***(A1)*** ***(C3)***

Nota: Acepte "30 minutos y 37 segundos" o (de un valor de k que tuviera 3 c. s.) "30 minutos y 36 segundos".

[3 puntos]

Total [6 puntos]

13. (a) continua *(A1)* *(C1)*
[1 punto]

(b) 6 *(A1)* *(C1)*

> **Nota:** Conceda *(A0)* si la respuesta es $60 \le w < 70$.

[1 punto]

(c) $8{,}97\ (8{,}97479\ldots)\,(\mathrm{g})$ *(A2)* *(C2)*
[2 puntos]

(d) $[55{,}9,\ 73{,}9]$ **O BIEN** $55{,}9252\ldots \le w \le 73{,}8747\ldots$ *(M1)*

> **Nota:** Conceda *(M1)* por los extremos correctos observados. Si la respuesta a la parte (c) es $14{,}1421\ldots$, conceda *(M1)* por los extremos $[50{,}7578\ldots;\ 79{,}0421\ldots]$.

45 *(A1)*(ft) *(C2)*

> **Nota:** Se pueden otorgar puntos error de arrastre (ft) desde *su* apartado (c). Para una desviación estándar entre 0 y 5 inclusive, la respuesta ft es 0.

[2 puntos]

Total [6 puntos]

14. (a) $(k =)\ 0$ *(A1)* *(C1)*

> **Nota:** Conceda *(A1)* por la respuesta "$x = 0$".

[1 punto]

(b) $x - \dfrac{2a}{x^2}$ *(A1)(A1)(A1)* *(C3)*

> **Nota:** Conceda *(A1)* por x, *(A1)* por $-2a$ y *(A1)* por x^{-2} o $\dfrac{1}{x^2}$.
> Conceda a lo más *(A1)(A1)(A0)* si hay términos adicionales.

[3 puntos]

(c) $0{,}5 = 2 - \dfrac{2a}{2^2}$ *(M1)*

> **Nota:** Conceda *(M1)* por igualar a $0{,}5$ a *su* derivada con *sus* valores sustituidos correctamente.

$(a =)\ 3$ *(A1)*(ft) *(C2)*

> **Nota:** Corrija con error de arrastre (ft) del apartado (b) siempre y cuando su respuesta **no** es $a = 0$ pues este valor contradice el gráfico.

[2 puntos]

Total [6 puntos]

15. (a) 42 (minutos) ***(A1)*** ***(C1)***

[1 punto]

(b) Stephen está en lo correcto. ***(A1)***

la duración mínima del ensayo es mayor que cero ***(R1)***

O BIEN

ensayó todos los días durante al menos 20 minutos ***(R1)*** ***(C2)***

> **Nota:** No conceda ***(A1)(R0).*** Acepte un razonamiento equivalente que esté basado en el diagrama de caja y bigotes.

[2 puntos]

(c) $0 \le k \le 15,\ k \in \mathbb{N}$ **O BIEN** $0, 1, 2, 3, \ldots, 15$ ***(A1)(A1)(A1)*** ***(C3)***

> **Nota:** Conceda ***(A1)(A1)*** por cada extremo correcto del intervalo; conceda ***(A1)*** por indicar los valores enteros comprendidos entre *sus* extremos. Acepte $k \in \mathbb{Z}$.

[3 puntos]

Total [6 puntos]

Estudios Matemáticos
Nivel Medio
Prueba 2

Martes 19 de noviembre de 2019 (mañana)

1 hora 30 minutos

Instrucciones para los alumnos

- No abra esta prueba hasta que se lo autoricen.
- En esta prueba es necesario usar una calculadora de pantalla gráfica.
- Para esta prueba, se necesita una copia sin anotaciones del **cuadernillo de fórmulas de Estudios Matemáticos NM**.
- Conteste todas las preguntas en el cuadernillo de respuestas provisto.
- Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán darse como valores exactos o con una aproximación de tres cifras significativas.
- La puntuación máxima para esta prueba de examen es **[90 puntos]**.

8819 – 7410

Conteste **todas** las preguntas en el cuadernillo de respuestas provisto. Empiece una página nueva para cada respuesta. Se recomienda que muestre todos los cálculos, siempre que sea posible. Cuando la respuesta sea incorrecta, podrán otorgarse algunos puntos si el método empleado es correcto, siempre que aparezca por escrito. Para los resultados obtenidos con calculadora de pantalla gráfica, deberá reflejarse por escrito el proceso seguido hasta su obtención. Por ejemplo, cuando deba utilizar un gráfico de una calculadora de pantalla gráfica para hallar soluciones, deberá dibujarlo aproximadamente en su respuesta.

1. [Puntuación máxima: 15]

El restaurante Casanova tiene un menú del día en el que el cliente elige **uno** de los siguientes platos: pasta, pescado o marisco.

El encargado hizo una encuesta a 150 clientes y fue anotando la edad del cliente y el plato que había elegido. Los datos se muestran en la siguiente tabla.

	Pasta	**Pescado**	**Marisco**	**Total**
Adultos	24	25	32	81
Niños	20	14	35	69
Total	44	39	67	150

Se realizó una prueba χ^2 a un nivel de significación del 10%. El valor crítico para esta prueba es $4{,}605$.

(a) Indique cuál es la hipótesis nula, H_0, para esta prueba. [1]

(b) Escriba el número de grados de libertad. [1]

(c) Muestre que el número esperado de niños que eligieron marisco es 31, redondeando a dos cifras significativas. [2]

(d) Escriba

(i) El estadístico χ^2;

(ii) El valor del parámetro p. [3]

(e) Indique la conclusión que se puede extraer de esta prueba. Dé una razón que justifique su respuesta. [2]

(f) Se escoge un cliente al azar.

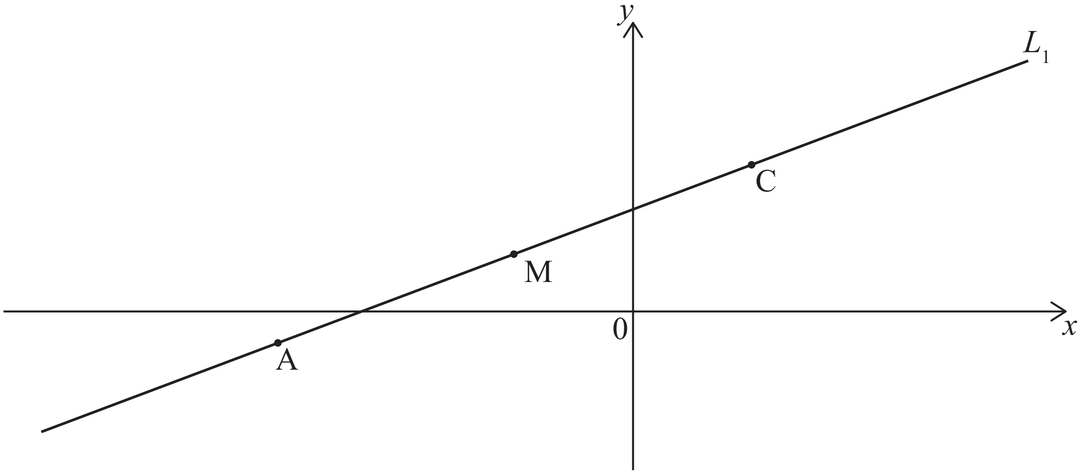
(i) Calcule la probabilidad de que el cliente sea un adulto.

(ii) Calcule la probabilidad de que el cliente sea un adulto o de que el cliente haya elegido marisco.

(iii) Sabiendo que el cliente es un niño, calcule la probabilidad de que haya elegido pasta o pescado. [6]

2. [Puntuación máxima: 13]

La siguiente figura muestra la recta L_1. Los puntos $\mathrm{A}(-9, -1)$, $\mathrm{M}(-3, 2)$ y C pertenecen a L_1.



(a) Halle la pendiente de L_1. [2]

M es el punto medio de AC.

(b) Halle las coordenadas del punto C. [2]

La recta L_2 es perpendicular a L_1 y pasa por el punto M.

(c) Halle la ecuación de L_2. Dé la respuesta en la forma $ax + by + d = 0$, donde a, b, $d \in \mathbb{Z}$. [3]

El punto $\mathrm{N}(k, 4)$ pertenece a L_2.

(d) Halle el valor de k. [2]

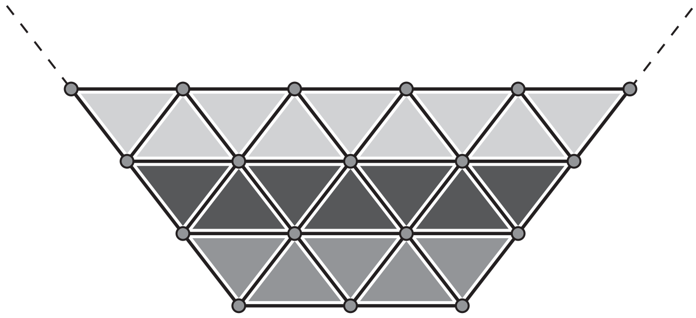
(e) Halle la distancia que hay entre los puntos M y N. [2]

(f) Sabiendo que la longitud de AM es $\sqrt{45}$, halle el área del triángulo ANC. [2]

3. [Puntuación máxima: 15]

Maegan diseña una vidriera decorativa para la fachada de un nuevo Centro de Bellas Artes. La vidriera está hecha de pequeños paneles triangulares. Los tres **primeros** niveles de la vidriera se muestran en la siguiente figura.

la figura no está dibujada a escala



El $1.^{\text{er}}$ nivel —el de la parte inferior de la vidriera— consta de 5 paneles triangulares. En el $2.^{\text{o}}$ nivel hay 7 paneles triangulares y en el $3.^{\text{er}}$ nivel hay 9 paneles triangulares. Cada nivel adicional tiene 2 paneles triangulares más que el nivel que está debajo de él.

(a) Halle el número de paneles triangulares que habrá en el $12.^{\text{o}}$ nivel. [3]

(b) Muestre que el número total de paneles triangulares, S_n, que hay en los primeros n niveles viene dado por:

$$S_n = n^2 + 4n\,.$$ [3]

(c) **A partir de lo anterior**, halle el número total de paneles que habrá en una vidriera de 18 niveles. [2]

Maegan tiene 1000 paneles triangulares para construir la vidriera y no quiere dejar incompleto ningún nivel.

(d) Halle el número máximo de niveles **completos** que podrá hacer Maegan. [3]

El área de cada panel triangular es igual a $1{,}84\,\text{m}^2$.

(e) Halle el área **total** que tendrá la vidriera si se construye con ese número máximo de niveles completos. Exprese el área que ha hallado redondeando al m^2 más cercano. [4]

4. [Puntuación máxima: 16]

El gráfico de la función cuadrática $f(x) = \frac{1}{2}(x-2)(x+8)$ corta al eje y en $(0, c)$.

(a) Halle el valor de c. [2]

El vértice de la función es $(-3 ; -12{,}5)$.

(b) Escriba la ecuación del eje de simetría del gráfico. [2]

La ecuación $f(x) = 12$ tiene dos soluciones. La primera solución es $x = -10$.

(c) **Utilice la simetría** del gráfico para mostrar que la segunda solución es $x = 4$. [1]

(d) Escriba las intersecciones con el eje x del gráfico. [2]

(e) En papel milimetrado, dibuje con precisión el gráfico de $y = f(x)$ para $-10 \le x \le 4$ y $-14 \le y \le 14$. Utilice una escala de $1\,\text{cm}$ para representar 1 unidad en el eje x y $1\,\text{cm}$ para representar 2 unidades en el eje y. [4]

Sea T la tangente en $x = -3$.

(f) (i) Escriba la ecuación de T.

(ii) En el mismo gráfico, dibuje con precisión la tangente T. [3]

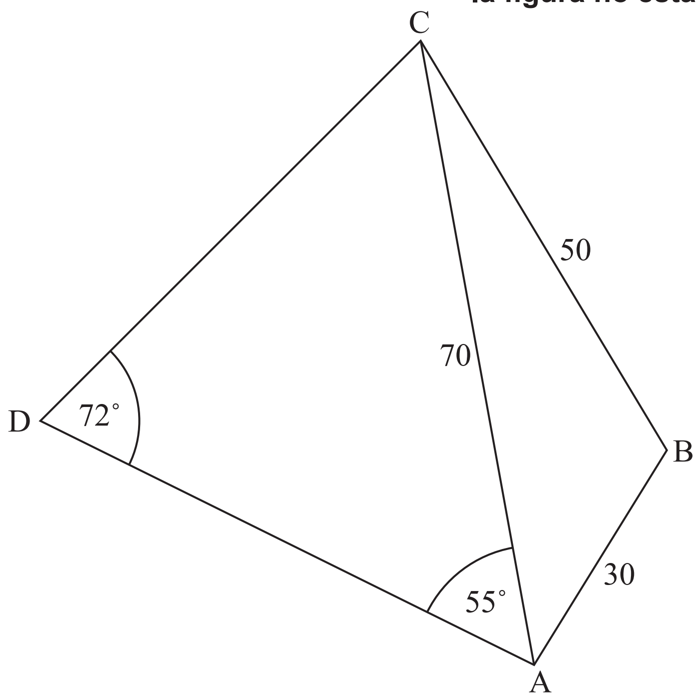
(g) Sabiendo que $f(a) = 5{,}5$ y que $f'(a) = -6$, indique si la función f es creciente o decreciente en $x = a$. Dé una razón que justifique su respuesta. [2]

5. [Puntuación máxima: 15]

Haraya posee dos terrenos que tienen forma triangular: ABC y ACD. AB mide $30\,\mathrm{m}$ de longitud, BC mide $50\,\mathrm{m}$ y AC mide $70\,\mathrm{m}$. El ángulo $\mathrm{D\hat{A}C}$ mide 55° y $\mathrm{A\hat{D}C}$ mide 72°.

La siguiente figura muestra toda esta información.

la figura no está dibujada a escala



(a) Halle la longitud de AD. [4]

(b) Halle el valor de $\mathrm{A\hat{B}C}$. [3]

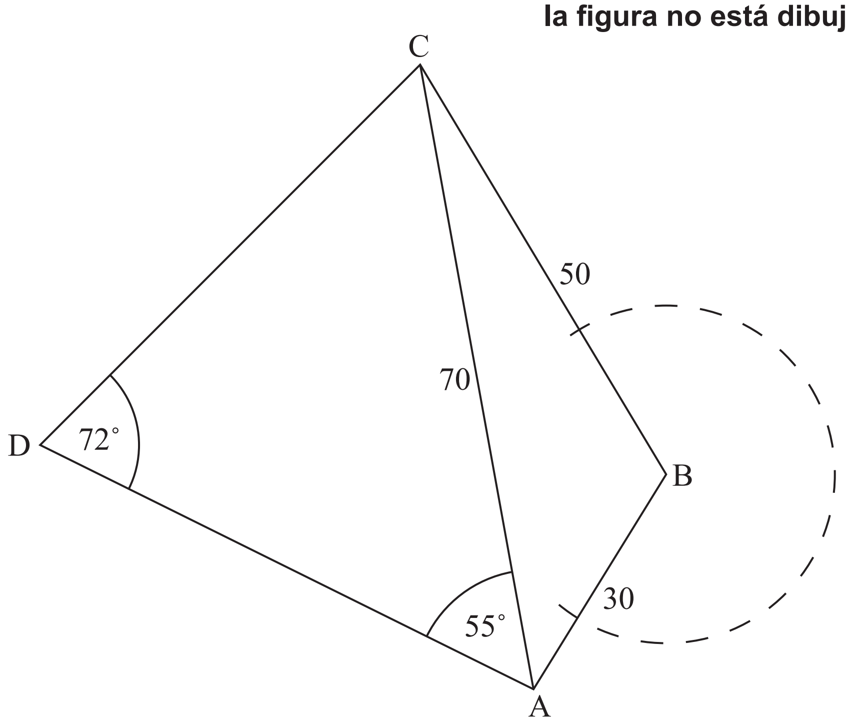
(c) Calcule el área del terreno triangular ABC. [3]

(Esta pregunta continúa en la página siguiente)

(Pregunta 5: continuación)

Haraya ata una cuerda de $20\,\text{m}$ de largo a un poste vertical que se encuentra en el punto B.

la figura no está dibujada a escala

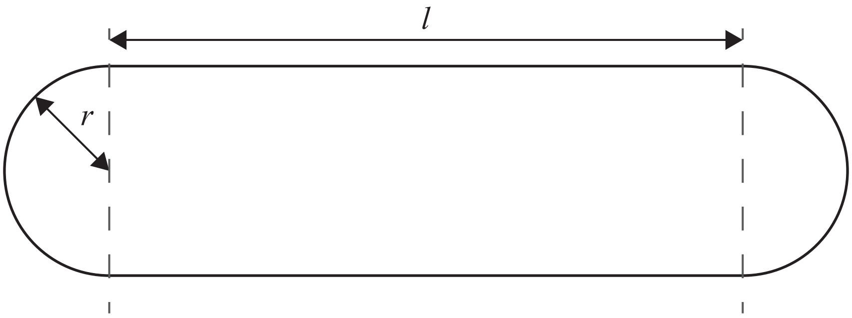


(d) Determine si la cuerda llega hasta el interior del terreno triangular ACD. Justifique su respuesta. [5]

6. [Puntuación máxima: 16]

La empresa Maxwell Ohm Company está diseñando un altavoz Bluetooth portátil. El altavoz tiene forma de cilindro con una semiesfera en cada uno de los extremos del cilindro.

En la siguiente figura se muestran las dimensiones del altavoz en centímetros; r es el radio de la semiesfera y l es la longitud del cilindro, siendo $r > 0$ y $l \geq 0$.



(a) Escriba una expresión para V, el volumen del altavoz en cm^3, en función de r, l y π. [2]

La empresa Maxwell Ohm Company ha decidido que el altavoz tendrá una superficie de $300\,\text{cm}^2$.

(b) Escriba una ecuación que permita calcular la superficie del altavoz en función de r, l y π. [3]

(c) Teniendo en cuenta que el diseño ha de cumplir que $l = \dfrac{150 - 2\pi r^2}{\pi r}$, muestre que $V = 150r - \dfrac{2\pi r^3}{3}$. [2]

(d) Halle $\dfrac{\mathrm{d}V}{\mathrm{d}r}$. [2]

La calidad de sonido del altavoz mejora a medida que V aumenta.

(e) Utilizando la respuesta que dio en el apartado (d), muestre que V es máximo cuando r es igual a $\sqrt{\dfrac{75}{\pi}}\,\text{cm}$. [2]

(f) Halle la longitud del **cilindro** para la cual V es máximo. [2]

(g) Calcule el valor máximo de V. [2]

(h) Utilice la respuesta que dio en el apartado (f) para identificar la forma del altavoz con la que se obtiene la mejor calidad de sonido. [1]

N19/5/MATSD/SP2/SPA/TZ0/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Esquema de calificación

Noviembre de 2019

Estudios Matemáticos

Nivel Medio

Prueba 2

28 páginas

Instrucciones para los examinadores

Observaciones: **En caso de dudas sobre estas instrucciones o temas relacionados con la asignación de puntuaciones, póngase en contacto con su jefe de equipo.**

1 Siglas

M Puntos otorgados por el **método**
A Puntos otorgados por una **respuesta** o por la **precisión**
R Puntos otorgados por un **razonamiento** claro
G Puntos otorgados por soluciones correctas obtenidas mediante la **calculadora de pantalla gráfica**, independientemente del trabajo mostrado.
AG **Respuesta incluida** en la pregunta y, en consecuencia; no se otorgan puntos.
ft Se pueden otorgar puntos por la **coherencia** con resultados previos en la pregunta.

2 Método para corregir

(a) Todas las correcciones deben realizarse en RM Assessor, utilizando las anotaciones de Estudios Matemáticos y de acuerdo con el documento vigente de orientaciones para la corrección electrónica de Estudios Matemáticos NM. Es fundamental leer el documento antes de empezar a corregir.

(b) Si un apartado de una pregunta es del todo correcto use las anotaciones tic con números para otorgar la puntuación máxima. Si un apartado es completamente erróneo use la nota ***A0***, de lo contrario se deben mostrar todas las anotaciones.

(c) No se debe otorgar ningún punto al trabajo tachado por el alumno.

(d) Si el alumno ha dado dos respuestas a una pregunta, solo se debe corregir la primera respuesta.

(e) Si el procedimiento adecuado lleva a la respuesta correcta, pero esta va seguida de más desarrollos que indican una falta de comprensión matemática, entonces **no** se puede otorgar la puntuación máxima. En la mayoría de estos casos se perderá solo el último punto por respuesta. Una excepción podría ser las respuestas numéricas, donde una respuesta exacta que es correcta está seguida de una expresión decimal incorrecta.

Ejemplo:

	Se ve la respuesta correcta	**Hay más desarrollo**	**Acción**
1.	$8\sqrt{2}$	$5{,}65685...$ *(valor decimal incorrecto)*	Otorgue el ultimo ***(A1)*** *(ignore el desarrollo posterior)*
2.	$(x-6)\,(x+1)$	$x=6$ and -1	**No** otorgue el último ***(A1)***

Ejemplo: Calcule la pendiente de la recta que pasa por los puntos $(5;3)$ y $(0;9)$.

Esquema de calificación	**Examen del alumno**	**Corrección**
$\frac{9-3}{0-5}$ ***(M1)*** *Otorgue (M1) por la sustitución correcta en la fórmula de la pendiente* $=-\frac{6}{5}$ ***(A1)***	(i) $\frac{9-3}{0-5}=-\frac{6}{5}$ La pendiente es $=-\frac{6}{5}$ *(Existe una comprensión clara de la pendiente.)* $y=-\frac{6}{5}x+9$ (ii) $\frac{9-3}{0-5}=-\frac{6}{5}$ $y=-\frac{6}{5}x+9$ *(Existe confusión sobre lo requerido.)*	***(M1)*** ***(A1)*** ***(M1)*** ***(A0)***

3 Puntos por la coherencia (ft)

En cualquier paso de una resolución, un error puede afectar a todo el trabajo posterior. Para limitar la gravedad de la penalización, se pueden otorgar puntos por la **coherencia (ft)**. Los esquemas de calificación indicarán, con **"(ft)"**, dónde es adecuado aplicar el criterio de coherencia en una pregunta.

(a) La coherencia se aplica solo de un apartado de una pregunta a otro apartado posterior de la pregunta. No se aplica dentro del mismo apartado.

(b) Si una respuesta que es consecuencia de un procedimiento de coherencia es muy poco realista (por ejemplo, distancias negativas o de un orden de magnitud excesivo) entonces el punto ***A*** final no se debe otorgar.

(c) Si, debido a un error, una pregunta queda transformada en otra **distinta**, **mucho más sencilla**, entonces el criterio de coherencia podría no ser aplicado.

(d) Para otorgar puntos por coherencia en un apartado de una pregunta, **es necesario que haya un trabajo explícito relacionado con ese apartado**. Una respuesta aislada, obtenida por coherencia, pero sin un desarrollo explícito, se considera incorrecta, y no recibirá ningún punto **aunque se aproxime a la respuesta correcta**.

(e) Una excepción a lo anterior sería una pregunta donde lo que se evalúa es el manejo del alumno en el uso de la calculadora de pantalla gráfica, y donde no se pretende que se muestre ningún desarrollo escrito. **En el esquema de calificación vendrá indicado claramente cuando sea el caso.**

(f) El uso inadvertido de radianes será penalizado la primera vez que aparezca. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo un punto por el uso de radianes.

Ejemplo: Cálculo de ángulos y longitudes en trigonometría

Esquema de calificación	Examen del alumno	Corrección
(a) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ ***(M1)(A1)*** *Otorgue **(M1)** por la sustitución en el teorema del seno, **(A1)** por las sustituciones correctas.*	(a) $\frac{\text{sen } A}{4} = \frac{\text{sen } 30}{3}$	***(M1)(A0)*** *(uso del teorema del seno, pero con valores incorrectos)*
$A = 22{,}0^\circ$ $(22{,}0243\ldots)$ ***(A1)(G2)***	$A = 41{,}8^\circ$	***(A0)*** (***Observación**: Aquí, el segundo **(A1)** no ha sido corregido como* **(ft)** *y no se puede otorgar porque existe un error previo en el **mismo** apartado de la pregunta.*)
(b) $x = 7\text{tg}\,(22{,}0243\ldots^\circ)$ ***(M1)*** $= 2{,}83$ $(2{,}83163\ldots)$ ***(A1)*(ft)**	(b) caso (i) $x = 7\text{tg}\,41{,}8^\circ$ $= 6{,}26$ ***pero*** caso (ii) $6{,}26$	***(M1)*** ***(A1)*(ft)** ***(G0)*** *pues no aparece un desarrollo explícito*

4 Uso del Esquema de calificación

(a) Los puntos ***A* dependen** de los puntos ***M*** obtenidos previamente, **no** es posible otorgar ***(M0)(A1)***. Una vez que se otorga un ***(M0)***, se pierden todos los puntos ***A*** que le siguen en ese apartado de la pregunta, incluso si los cálculos son correctos, hasta el siguiente punto ***M***.
La única excepción a lo anterior se hará en el caso de una respuesta donde la precisión se especifique en la pregunta (ver apartado 5).

(b) Los puntos ***A* dependen** de los puntos ***R*** obtenidos, **no** es posible otorgar ***(A1)(R0)***. Así pues, el ***(A1)*** no se puede otorgar a una respuesta que sea correcta cuando no aparece el razonamiento, o este es incorrecto.

(c) En la prueba 2 se espera que los alumnos demuestren su destreza en la comunicación matemática mediante el uso de desarrollos adecuados. Las respuestas que sean correctas, pero no se basen en un desarrollo adecuado **no siempre recibirán la puntuación máxima**. Estas respuestas sin desarrollo que las sustente vienen designadas por ***G*** en el esquema de calificación, como una alternativa a la puntuación máxima. Ejemplo ***(M1)(A1)(A1)(G2).***

Ejemplo: Uso de la trigonometría para el cálculo de un ángulo de un triángulo.

Esquema de calificación	Examen del alumno	Corrección
(a) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ ***(M1)(A1)*** *Otorgue **(M1)** por la sustitución en el teorema del seno, **(A1)** por las sustituciones correctas.* $A = 22{,}0^\circ$ $(22{,}0243\ldots)$ ***(A1)(G2)***	(i) $\frac{\text{sen } A}{3} = \frac{\text{sen } 30}{4}$ $A = 22{,}0^\circ$ (ii) $A = 22{,}0^\circ$ ***Observación:*** *Los puntos **G** se utilizan solo si no se muestra ningún desarrollo, pero la respuesta es correcta.*	***(M1)(A1)*** ***(A1)*** ***(G2)***

(d) Los **métodos alternativos** no siempre se incluyen. Así, si una respuesta es errónea, entonces se debe analizar cuidadosamente el procedimiento, de modo que se asignen puntos por un método distinto de forma coherente con el esquema de calificación.
Cuando en el esquema de calificación se incluyen métodos alternativos para una pregunta, ello viene indicado mediante "**O**" *etc*.

(e) A menos que en la pregunta se especifique lo contrario, se aceptan **expresiones equivalentes**.
Por ejemplo: $\frac{\text{sen}\,\theta}{\cos\theta}$ por $\text{tg}\,\theta$.

En el esquema de calificación, estas expresiones algebraica o numéricamente equivalentes aparecerán a veces escritas entre paréntesis junto a la respuesta requerida.
Cuando se requieran respuestas numéricas como respuesta final de un apartado de una pregunta, en el esquema de calificación se mostrará en este orden:
la respuesta con 3 cifras significativas a partir de la solución completa de la calculadora;
el valor exacto (por ejemplo $\frac{2}{3}$ si corresponde);
la solución completa de la calculadora en la forma $2{,}83163\ldots$ como en el ejemplo anterior.
Cuando se dan las respuestas con 3 cifras significativas y se utilizan después en apartados posteriores de la pregunta llevando a una solución con 3 cifras significativas **distinta**, también se darán estas soluciones.

(f) Dado que este es un examen internacional, se aceptarán todas las **formas de notación alternativas**. Algunos ejemplos:

Comas decimales: 1,7; 1’7; $1 \cdot 7$; 1,7.

Los números decimales menores que 1 podrían aparecer escritos sin un cero en el frente: 0.49 o .49.

Distintas descripciones de un intervalo: $3 < x < 5$; (3; 5);] 3; 5 [.

Distintas formas de notación de las propiedades de los conjuntos (por ejemplo complementario): A'; $\overline{A}$; A^c; $U - A$;$(A$; $U \setminus A$.

Distintas formas de notación lógica: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p$.
$p \Rightarrow q$; $p \to q$; $q \Leftarrow p$.

El nivel de significación podría escribirse como α .

(g) Puntos discrecionales: habrá ocasiones excepcionales en las que el esquema de calificación no cubra el trabajo que aparece. En estos casos se utilizará la nota DM para indicar que el examinador ha utilizado su criterio. La discrecionalidad debe utilizarse con moderación, y si existe duda se debe plantear una excepción a través de RM Assessor al jefe del equipo.

A partir de noviembre de 2011 no habrá una única penalización por prueba por precisión AP, precisión financiera FP y unidades UP. En lugar de ello, estas destrezas serán evaluadas en preguntas específicas y los puntos se otorgarán de acuerdo a lo especificado en los apartados 5, 6 y 7.

5 Precisión de las respuestas

Una precisión incorrecta debe ser penalizada una sola vez en cada pregunta de acuerdo a las siguientes reglas.

Instrucciones: Salvo que se indique lo contrario en la pregunta, todas las respuestas numéricas deberán ser exactas o aproximadas correctamente a 3 cifras significativas.

1. Si el alumno muestra la solución a 4 o más cifras significativas, la cual redondeada a 3 cifras significativas daría la respuesta requerida entonces otorgue ***(A1)*** e ignore cualquier redondeo posterior.
Observación: La solución sin redondear puede estar tanto en la línea de respuesta como en el cuadro de trabajo.

2. Si el alumno **no** muestra la solución sin redondear, entonces otorgue ***(A1)*** si la solución dada está aproximada **correctamente** a 2 o más cifras significativas, y ***(A0)*** en cualquier otro caso.

3. Si una solución aproximada a 2 cifras significativas se utiliza en apartados posteriores, entonces para otorgar más puntos debe aparecer el desarrollo. (Este criterio es el mismo que en el que se aplica coherencia a partir de una respuesta errónea.)

Estos 3 casos (vea los supra índices) se han resumido en la siguiente tabla e ilustrado después con ejemplos.

	Si la solución final de alumno se expresa…				
	Exacta o a 4 o más cifras significativas (y que redondeada **a 3 cifras significativas daría la respuesta correcta**)	**Aproximada incorrectamente a 3 cifras significativas**	Aproximada correctamente a 2 cifras significativas[3]	Aproximada incorrectamente a 2 cifras significativas	Aproximada correcta o incorrectamente a 1 cifra significativa
Aparece la solución sin redondear[1]	Otorgue ***(A1)*** independientemente del redondeo correcto o incorrecto				
No aparece la solución sin redondear[2]	***(A1)***	***(A0)***	***(A1)***	***(A0)***	***(A0)***
Criterio para los apartados posteriores	Como en MS	Como en el criterio de coherencia, solo si se muestra el desarrollo[3]			

Ejemplos:

Esquema de calificación	Examen del alumno	Corrección
$9,43\ (9,43398\ldots)$ ***(A1)***	(i) $9,43398\ldots$ se muestra en el cuadro de trabajo seguido de 9; 9,4; 9,43; 9,434 (redondeados correctamente)	***(A1)***
	(ii) $9,43398\ldots$ se muestra en el cuadro de trabajo seguido de 9,433; 9,44, *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 9,4	***(A1)***
	(iv) 9	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 9,3	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 9,44	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Esquema de calificación	Examen del alumno	Corrección
$7,44\ (7,43798\ldots)$ ***(A1)***	(i) $7,43798\ldots$ se muestra en el cuadro de trabajo seguido de 7; 7,4; 7,44; 7,438 *etc.* (redondeados correctamente)	***(A1)***
	(ii) $7,43798\ldots$ se muestra seguido de 7,437; 7,43 *etc.* (redondeados incorrectamente)	***(A1)***
	(iii) 7,4	***(A1)***
	(iv) 7	***(A0)*** (aproximado correctamente a 1 cifra significativa)
	(v) 7,5	***(A0)*** (aproximado incorrectamente a 2 cifras significativas)
	(vi) 7,43	***(A0)*** (aproximado incorrectamente a 3 cifras significativas)

Ejemplo: ABC es un triángulo rectángulo con el ángulo $\text{ABC} = 90^\circ$, $\text{AC} = 32\text{ cm}$ y $\text{AB} = 30\text{ cm}$. Halle (a) la longitud de BC, (b) el área del triángulo ABC.

<table>
<tr><th>Esquema de calificación</th><th>Examen del alumno</th><th>Corrección</th></tr>
<tr><td>(a) $\text{BC} = \sqrt{32^2 - 30^2}$ (M1)
Otorgue (M1) por la sustitución correcta en el teorema de Pitágoras

$= 11{,}1\left(\sqrt{124}; 11{,}1355...\right)(\text{cm})$ (A1)

(b) $\text{Área} = \frac{1}{2} \times 30 \times 11{,}1355...$ (M1)
Otorgue (M1) por la sustitución correcta en la fórmula del área de un triángulo

$= 167(167{,}032...)(\text{cm}^2)$ (A1)(ft)</td>
<td colspan="2">(a) $\text{BC} = \sqrt{32^2 - 30^2}$ (M1)

$11\,(\text{cm})$ (A1)
(solo se muestra la aproximación a 2 cifras significativas, pero correcta)

(b) caso (i) $\text{Área} = \frac{1}{2} \times 30 \times 11$ (M1)
(se muestra el desarrollo)

$= 165\,(\text{cm}^2)$ (A1)(ft)

caso (ii) $= 165\,(\text{cm}^2)$ (M0)(A0)(ft)
(No se muestra el desarrollo, la solución 11 se trata como procedimiento de coherencia, por lo que no se deben otorgar puntos en este caso)</td></tr>
</table>

La aproximación a 3 cifras significativas de una solución exacta **se debe aceptar si se realiza correctamente**. Las soluciones exactas del tipo $\frac{1}{4}$ se pueden escribir como decimales con menos de 3 cifras significativas si el resultado sigue siendo exacto. La simplificación de una fracción a su expresión irreducible **no** es imprescindible. Las fracciones que contienen un numerador y/o un denominador decimal se aceptan para mostrar sustituciones pero no como respuesta final.

Razones de π y respuestas con expresiones de raíces cuadradas de enteros o cualquier potencia racional de un entero (por ejemplo, $\sqrt{13}, 2^{2/3}, \sqrt[4]{5}$), se pueden aceptar como respuestas exactas. Todas las otras potencias (por ejemplo de no enteros) y valores de funciones transcendentes tales como seno y coseno se deben evaluar.

Si el nivel de precisión viene especificado en la pregunta, se asignarán puntos por dar la respuesta con la precisión requerida. En **todos** estos casos no se obtiene el punto final si el redondeo no sigue las instrucciones dadas en la pregunta. El punto por la precisión especificada se puede considerar como punto por coherencia **(ft)** con independencia de un ***(M0)*** inmediatamente anterior.

6 Nivel de precisión en las preguntas sobre cuestiones financieras

El nivel de precisión de las respuestas se especificará en todas las preguntas que se relacionen con valores en dinero. Generalmente será a la unidad más cercana o a dos cifras decimales. La primera respuesta que no venga dada con el nivel de precisión especificado no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la precisión incorrecta en una pregunta sobre cuestiones financieras.

Ejemplo: En una pregunta sobre cuestiones financieras se pide una precisión de 2 cifras decimales.

Esquema de calificación	Examen del alumno	Corrección
$\$231,62\,(231,6189)$ ***(A1)***	(i) $231,6$	***(A0)***
	(ii) 232	***(A0)*** *(Aproximación correcta con un nivel de precisión distinto del requerido)*
	(iii) $231,61$	***(A0)***
	(iv) $232,00$	***(A0)*** *(Aproximación incorrecta al nivel de precisión requerido)*

7 Unidades de medida en las respuestas

En determinadas preguntas será necesario expresar las unidades de medida, y esto vendrá claramente indicado en el esquema de calificación. La primera respuesta correcta pero sin unidades o con unidades incorrectas no recibirá el punto ***A*** final. En el esquema de calificación se darán instrucciones claras para asegurar que en cada prueba se pierda solo una vez el punto por la falta de unidades o unidades incorrectas.

Las unidades de medida se tendrán en cuenta solamente cuando se ha otorgado ***(A1)*** a la respuesta numérica siguiendo las indicaciones sobre la precisión dadas en el apartado 5.

Ejemplo:

Esquema de calificación	Examen del alumno	Corrección
(a) 37000 m^2 ***(A1)***	(a) 36000 m^2	***(A0)*** *(respuesta incorrecta, por lo que no se consideran las unidades)*
(b) 3200 m^3 ***(A1)***	(b) 3200 m^2	***(A0)*** *(unidades incorrectas)*

8 Calculadoras de pantalla gráfica

Con frecuencia los alumnos van a obtener las soluciones directamente de la calculadora. Deben utilizar la notación matemática y no la notación de la calculadora. No se puede otorgar puntos por método a respuestas incorrectas basadas únicamente en notación de calculadora. No se puede otorgar puntos por método al comentario “usé la calculadora de pantalla gráfica”.

1. (a) $(\mathrm{H}_0:)$ El plato elegido es independiente de la edad (o equivalente) ***(A1)***

> **Nota:** Acepte "no está asociado" o "no depende" en lugar de "es independiente".
> En lugar de "la edad", acepte una alternativa equivalente, como por ejemplo "ser un niño o un adulto".

[1 punto]

(b) 2 ***(A1)***

[1 punto]

(c) $\frac{69}{150}\times\frac{67}{150}\times 150$ **O BIEN** $\frac{69\times 67}{150}$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula de la frecuencia esperada.

$30,82\ \ (30,8)$ ***(A1)***

31 ***(AG)***

> **Nota:** Para conceder el ***(A1)*** el alumno tiene que haber incluido tanto una respuesta sin redondear que conduzca a la respuesta dada en el enunciado como la respuesta redondeada.

[2 puntos]

(d) (i) $(\chi^2_{\text{calc}} =)\ 2,66\ (2,657537\ldots)$ ***(G2)***

(ii) (valor del parámetro $p =$) $0,265\ (0,264803\ldots)$ ***(G1)***

> **Nota:** Conceda ***(G0)(G2)*** sino se ha incluido en la respuesta el estadístico χ^2 o si su valor es incorrecto y el valor del parámetro p es correcto.

[3 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 1

(e) $0,265 > 0,10$ **O BIEN** $2,66 < 4,605$ ***(R1)*(ft)**

No se rechaza la hipótesis nula. ***(A1)*(ft)**

O BIEN

El plato elegido es independiente de la edad (o equivalente). ***(A1)*(ft)**

> **Nota:** Conceda ***(R1)*(ft)** por una comparación correcta de, o bien su estadístico χ^2 con el valor crítico de χ^2, o bien su valor del parámetro p con el nivel de significación. Admita "se acepta" en lugar de "no se rechaza". Se pueden otorgar puntos de arrastre de error desde los apartados (a) y (d).
>
> No conceda ***(A1)(*ft*)(R0)***.

[2 puntos]

(f) (i) $\frac{81}{150}\left(\frac{27}{50},\ 0,54,\ 54\%\right)$ ***(A1)(A1)(G2)***

> **Nota:** Conceda ***(A1)*** por el numerador y ***(A1)*** por el denominador.

(ii) $\frac{116}{150}\left(\frac{58}{75},\ 0,773,\ 0,773333\ldots,\ 77,3\%\right)$ ***(A1)(A1)(G2)***

> **Nota:** Conceda ***(A1)*** por el numerador y ***(A1)*** por el denominador.

(iii) $\frac{34}{69}\left(0,493,\ 0,492753\ldots,\ 49,3\%\right)$ ***(A1)(A1)(G2)***

> **Nota:** Conceda ***(A1)*** por el numerador y ***(A1)*** por el denominador.

[6 puntos]

Total [15 puntos]

2. (a) $\dfrac{2-(-1)}{-3-(-9)}$ ***(M1)***

Nota:	Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula de la pendiente.

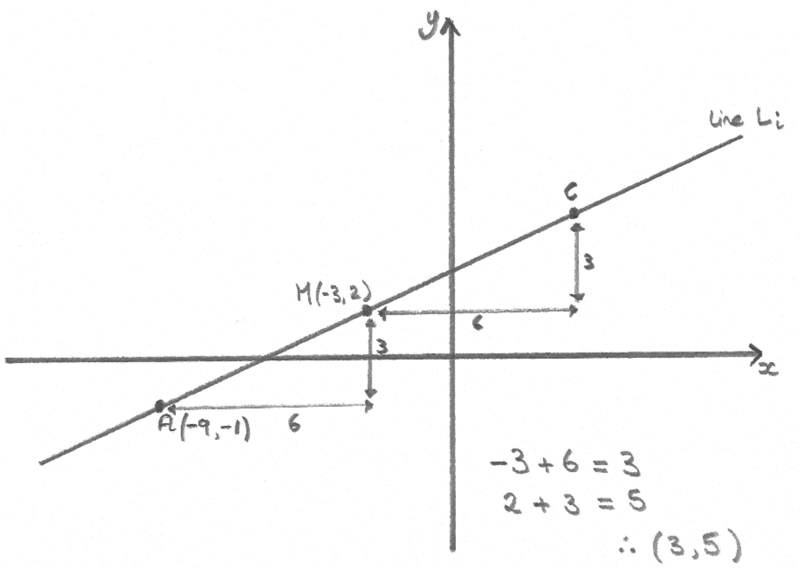
$=\dfrac{1}{2}\left(\dfrac{3}{6},\ 0,5\right)$ ***(A1)(G2)***

[2 puntos]

(b) $-3=\dfrac{-9+x}{2}\ (-6+9=x)$ y $2=\dfrac{-1+y}{2}\ (4+1=y)$ ***(M1)***

Nota:	Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula del punto medio para ambas coordenadas.

O BIEN



(M1)

Nota:	Conceda ***(M1)*** por un bosquejo (dibujo aproximado) donde se muestre que el desplazamiento horizontal entre M y C es igual a 6 y el desplazamiento vertical es igual a 3 **y** también las coordenadas del punto M.

O BIEN

$-3+6=3$ y $2+3=5$ ***(M1)***

Nota:	Conceda ***(M1)*** si ha incluido ecuaciones correctas.

$(3,\ 5)$ ***(A1)(G1)(G1)***

Nota:	Acepte $x=3,\ y=5$. Conceda a lo sumo ***(M1)(A0)*** o ***(G1)(G0)*** si se omiten los paréntesis.

[2 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 2

(c) La pendiente de la normal $= -2$ ***(A1)*(ft)**

Nota: Se pueden otorgar puntos de arrastre de error (*FT*) desde el apartado (a).

$y - 2 = -2(x+3)$ **O BIEN** $2 = -2(-3) + c$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido M correctamente y su pendiente de la recta normal en la fórmula de la recta.

$2x + y + 4 = 0$ (acepte múltiplos enteros) ***(A1)*(ft)*(G3)***

[3 puntos]

(d) $2(k) + 4 + 4 = 0$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido $y = 4$ en la ecuación de su recta normal o por haber sustituido M y $(k, 4)$ en la ecuación de la pendiente de la normal.

$k = -4$ ***(A1)*(ft)*(G2)***

Nota: Se pueden otorgar puntos de arrastre de error (*FT*) desde el apartado (c).

[2 puntos]

(e) $\sqrt{(-4+3)^2 + (4-2)^2}$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente punto M y su N en la fórmula de la distancia.

$\sqrt{5}$ $(2,24;\ 2,23606\ldots)$ ***(A1)*(ft)**

Nota: Se pueden otorgar puntos de arrastre de error desde el apartado (d).

[2 puntos]

(f) $\frac{1}{2} \times \left(2 \times \sqrt{45}\right) \times \sqrt{5}$ ***(M1)***

Nota: Conceda ***(M1)*** por su sustitución correcta en la fórmula del área de un triángulo. Conceda ***(M0)*** por ***su*** $\frac{1}{2} \times \left(\sqrt{45}\right) \times \sqrt{5}$ si no hay pruebas de que haya multiplicado por 2 para hallar la longitud de AC. Acepte cualquier método correcto para hallar el área.

15 ***(A1)*(ft)*(G2)***

Nota: Acepte $15{,}02637\ldots$ si utilizó un valor con 3 c. s. para $\sqrt{5}$.
Se pueden otorgar puntos de arrastre de error (*FT*) desde el apartado (e).

[2 puntos]

Total [13 puntos]

3. (a) $u_{12} = 5 + (12-1) \times (2)$. ***(M1)(A1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido los valores en la fórmula de la progresión aritmética y ***(A1)*** por haberlos sustituido correctamente.

27 ***(A1)(G3)***

[3 puntos]

(b) $S_n = \frac{n}{2}\left(2 \times 5 + (n-1)(2)\right)$ ***(M1)(A1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido valores en la fórmula de la serie aritmética y ***(A1)*** por haber sustituido correctamente.

$S_n = \frac{n}{2}(8 + 2n)$ **O BIEN** $S_n = n(5 + n - 1)$ ***(M1)***

> **Nota:** Conceda ***(M1)*** si hay pruebas de que hubo desarrollo y simplificación, o una división entre 2 que conduzca a la respuesta final.

$S_n = n^2 + 4n$ ***(AG)***

> **Nota:** El alumno tiene que haber incluido esta última línea, sin haber cometido errores en el desarrollo, para conceder el ***(M1)*** final.

[3 puntos]

(c) $(S_{18} =)\ 18^2 + 4 \times 18$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula de S_n.

$(S_{18} =)\ 396$ ***(A1)***

> **Nota:** El uso de "a partir de lo anterior" en el enunciado significa que el alumno ha de utilizar la fórmula de S_n (tomada del apartado (b)).

[2 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 3

(d) $1000 = n^2 + 4n$ **O BIEN** $1000 = \frac{n}{2}\left(10 + (n-1)(2)\right)$ (o equivalente) ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber igualado S_n a 1000 o por igualar a 1000 la fórmula de la suma de los términos de una progresión aritmética correctamente sustituida.

O BIEN

Un bosquejo de $S_n = n^2 + 4n$ y $S_n = 1000$ que muestre el punto de intersección. ***(M1)***

> **Nota:** Conceda ***(M1)*** por un bosquejo (gráfico aproximado) de una cuadrática y una recta horizontal con al menos un punto de intersección.

O BIEN

Un bosquejo de $n^2 + 4n - 1000$ mostrando el corte con el eje x ***(M1)***

> **Nota:** Conceda ***(M1)*** por un bosquejo (gráfico aproximado) de $n^2 + 4n - 1000$ con al menos un punto de corte con el eje x.

$(n =)\ 29,6859\ldots$ **O BIEN** $-2 + 2\sqrt{251}$ ***(A1)***

> **Nota:** Conceda ***(A1)*** si ha incluido $29,6859\ldots$ o bien $-2 + 2\sqrt{251}$. Puede quedar implícito en una respuesta final correcta.

$(n =)\ 29$ ***(A1)*(ft)*(G2)***

> **Nota:** No acepte 30. Conceda como máximo ***(M1)(A1)(A0)*** si ha dado dos respuestas finales. Puede otorgar puntos de arrastre de error (*FT*) desde su respuesta sin redondear.

O BIEN

$S_{30} = 1020$ y $S_{29} = 957$ ***(A2)***

> **Nota:** Conceda ***(A2)*** si ha incluido ambos valores "de cruce". No divida en dos partes esta puntuación ***(A2)***.

$(n =)\ 29$ ***(A1)(G2)***

[3 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 3

(e) $(A =) \left(29^2 + 4\times 29\right)\times\left(1,84\right)$ ***(M1)(M1)***

Nota:	Conceda ***(M1)*** por haber sustituido correctamente sus valores para hallar el número total de paneles triangulares. Conceda ***(M1)*** por haber multiplicado su número de paneles por $1,84$.

O BIEN

$(A =)\ 957\times 1,84$ ***(A1)*****(ft)*****(M1)***

Nota:	Conceda ***(A1)*****(ft)** por haber incluido su 957. Conceda ***(M1)*** por multiplicar su cantidad de paneles por $1,84$. Puede otorgar puntos de arrastre de error (*FT*) desde su apartado (d).

$(A =)\ 1760,88\,(\mathrm{m}^2)$ ***(A1)*****(ft)*****(G2)***

$(A =)\ 1761\,(\mathrm{m}^2)$ ***(A1)*****(ft)*****(G3)***

[4 puntos]

Total [15 puntos]

4. (a) $\frac{1}{2}(0-2)(0+8)$ **O BIEN** $\frac{1}{2}\left(0^2+6(0)-16\right)$ ***(M1)***

Nota: Conceda ***(M1)*** por haber evaluado $f(0)$.

$(c=)\ -8$ ***(A1)(G2)***

Nota: Conceda ***(G2)*** por -8 o por $(0,-8)$.

[2 puntos]

(b) $x=-3$ ***(A1)(A1)***

Nota: Conceda ***(A1)*** por plantear que "$x=$ una constante" y ***(A1)*** si la constante es igual a -3. La respuesta ha de ser una ecuación.

[2 puntos]

(c) $(-3--10)+-3$ ***(M1)***

O BIEN

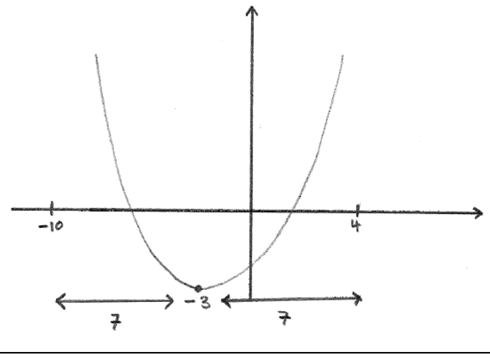
$(-8--10)+2$ ***(M1)***

O BIEN

$\frac{-10+x}{2}=-3$ ***(M1)***

Figura que muestre el eje de simetría y los puntos dados (basta con rotular los valores de la coordenada x: -10, -3 y 4) **y**
alguna indicación de que la distancia horizontal entre el eje de simetría y cada uno de los puntos dados es igual a 7. ***(M1)***

O BIEN



Nota: Conceda ***(M1)*** por un desarrollo correcto utilizando la simetría que hay entre $x=-10$ y $x=-3$. Conceda ***(M0)*** si el alumno ha utilizado $x=-10$ y $x=4$ para mostrar que el eje de simetría es $x=-3$. Conceda ***(M0)*** si el alumno ha resuelto $f(x)=12$ o ha evaluado $f(-10)$ y $f(4)$.

$(x=)\ 4$ ***(AG)***

[1 punto]

Continúa en la pág. siguiente...

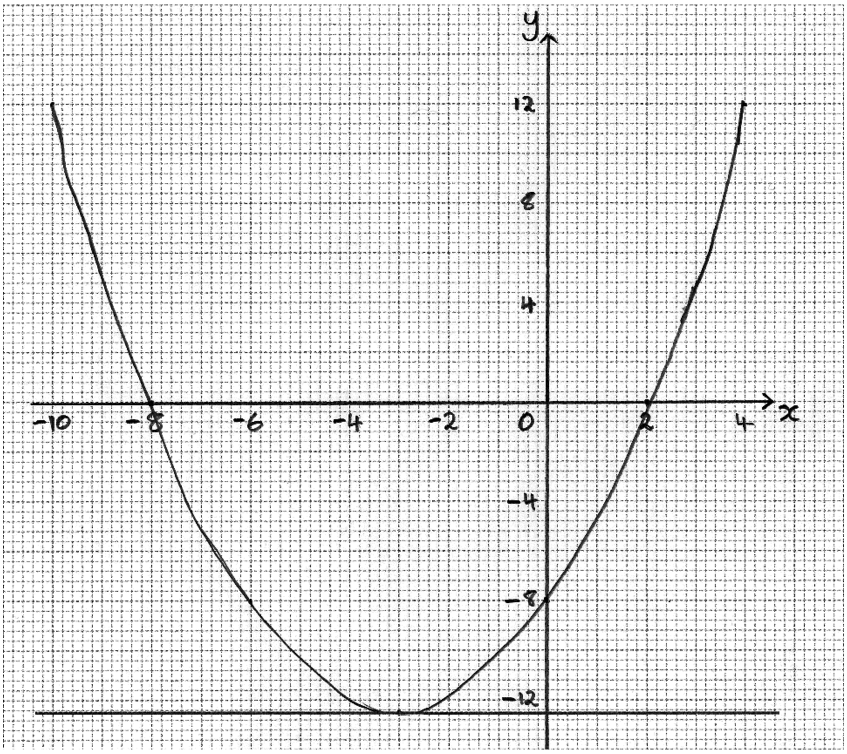
Continuación de la pregunta 4

(d) -8 y 2 ***(A1)(A1)***

Nota:	Acepte $x=-8,\ y=0$ y $x=2,\ y=0$ **o bien** $(-8,\ 0)$ y $(2,\ 0)$; conceda como mucho ***(A0)(A1)*** si ha omitido los paréntesis.

[2 puntos]

(e)



(A1)(A1)(A1)(A1)**(ft)**

Nota:	Conceda ***(A1)*** por la presencia de ejes rotulados con la escala correcta y la “ventana” correcta (los intervalos correctos). Conceda ***(A1)*** si el vértice $(-3,\ -12{,}5)$ está dibujado en la posición correcta. Conceda ***(A1)*** si ha dibujado una curva suave, continua y simétrica respecto a su vértice. Conceda ***(A1)*****(ft)** si la curva atraviesa los ejes x e y en los puntos correctos. Se pueden otorgar puntos de arrastre de error desde su respuesta a los apartados (a) y (d). **Si el gráfico no está hecho en papel milimetrado**, conceda a lo sumo ***(A0)(A0)(A1)(A1)*****(ft)**. Para conceder el último ***(A1)*****(ft)**, su gráfico debe pasar por su -8 y 2.

[4 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 4

(f) (i) $y = -12{,}5$ **O BIEN** $y = 0x - 12{,}5$ ***(A1)(A1)***

Nota:	Conceda ***(A1)*** por plantear que "$y =$ una constante" y ***(A1)*** si la constante es igual a $-12{,}5$. La respuesta ha de ser una ecuación.

(ii) Recta tangente al gráfico dibujada en $x = -3$. ***(A1)*(ft)**

Nota:	Conceda ***(A1)*(ft)** por haber trazado una recta horizontal que sea tangente a la curva en (aproximadamente) $x = -3$. Conceda ***(A0)*** si la trazó sin utilizar la regla. Se pueden otorgar puntos de arrastre de error desde su respuesta al apartado (e).

[3 puntos]

(g) Decreciente ***(A1)***

La pendiente (de la recta tangente) es negativa (en $x = a$) **O BIEN** $f'(a) < 0$ ***(R1)***

Nota:	No acepte "la pendiente (de la recta tangente) es -6". No conceda ***(A1)(R0)***.

[2 puntos]

Total [16 puntos]

5. En el apartado (c) es obligatorio que el alumno incluya las unidades.

(a) $\mathrm{A\hat{C}D} = 53^\circ$ (o equivalente) ***(A1)***

Nota:	Conceda ***(A1)*** si ha incluido 53° (o equivalente).

$$\frac{\mathrm{AD}}{\mathrm{sen}\,53^\circ} = \frac{70}{\mathrm{sen}\,72^\circ}$$ ***(M1)(A1)***

Nota:	Conceda ***(M1)*** por haber sustituido los valores en la fórmula del teorema del seno y ***(A1)*** por haberlos sustituido correctamente.

O BIEN

$$\left(\mathrm{AD}^2 =\right) 60,2915...^2 + 70^2 - 2 \times 70 \times 60,2915... \times \cos 53$$ ***(A1)(M1)(A1)***

Nota:	Conceda ***(A1)*** por haber incluido 53° o $60,2915...$, ***(M1)*** por haber sustituido los valores en la fórmula del teorema del coseno y ***(A1)*** por haberlos sustituido correctamente.

$(\mathrm{AD} =)\ 58,8\,(\mathrm{m})\ (58,7814\ldots)$ ***(A1)(G3)***

[4 puntos]

(b) $$\left(\cos \mathrm{A\hat{B}C}\right) = \frac{30^2 + 50^2 - 70^2}{2 \times 30 \times 50}.$$ ***(M1)(A1)***

Nota:	Conceda ***(M1)*** por haber sustituido los valores en la fórmula del teorema del coseno y ***(A1)*** por haberlos sustituido correctamente.

$(\mathrm{A\hat{B}C} =)\ 120^\circ$ ***(A1)(G2)***

[3 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 5

(c) $A = \frac{1}{2} \times 50 \times 30 \times \text{sen}\,120^{\circ}$ ***(M1)(A1)*****(ft)**

> **Nota:** Conceda ***(M1)*** por haber sustituido los valores en la fórmula del área y ***(A1)*****(ft)** por haber sustituido correctamente. Conceda ***(M0)(A0)(A0)*** por $\frac{1}{2} \times 50 \times 30$.

$(A =)\ 650\ \text{m}^2\ (649{,}519\ldots)$ ***(A1)*****(ft)*****(G2)***

> **Nota:** Se pueden otorgar puntos de arrastre de error desde el apartado (b).

[3 puntos]

(d) **MÉTODO 1 (igualar el resultado del apartado (c) a la expresión del área del triángulo ABC)**

$649{,}519\ldots = \frac{1}{2} \times 70 \times h$ ***(M1)(A1)*****(ft)**

> **Nota:** Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula del área de un triángulo. Conceda ***(A1)*****(ft)** por haber igualado la fórmula del área a su área (la que halló en el apartado (c)).

$(h =)\ 18{,}6\,(\text{m})\ (18{,}5576\ldots)$ ***(A1)*****(ft)**

> **Nota:** Se pueden otorgar puntos de arrastre de error (*FT*) desde su apartado (c).

$20 > 18{,}5576\ldots$ ***(R1)*****(ft)**

> **Nota:** Acepte “la longitud de la cuerda es mayor que la altura del triángulo ABC”.

la cuerda llega hasta el interior del terreno triangular ACD ***(A1)*****(ft)**

> **Nota:** Se pueden otorgar puntos de arrastre de error (*FT*) desde su valor de la altura. La concesión del ***(A1)*** final depende de que se haya concedido antes el ***(R1)***.

Continúa en la pág. siguiente...

Continuación de la pregunta 5

MÉTODO 2 (hallar $\mathrm{C\hat{A}B}$ o bien $\mathrm{A\hat{C}B}$ con el teorema del seno y, a continuación, usar una razón trigonométrica)

$\frac{\mathrm{sen\,C\hat{A}B}}{50} = \frac{\mathrm{sen}\,120^\circ}{70}$ $\left(\mathrm{C\hat{A}B} = 38,2132\ldots^\circ\right)$ ***(M1)***

Nota:	Conceda ***(M1)*** por su sustitución correcta de valores en la fórmula del teorema del seno para hallar $\mathrm{C\hat{A}B}$ o bien $\mathrm{A\hat{C}B}$. Se pueden otorgar puntos de arrastre de error (*ft*) desde su apartado (b).

$(h =)\ 30 \times \mathrm{sen}\,(38,2132\ldots^\circ)$ ***(M1)***

Nota:	Conceda ***(M1)*** por haber sustituido correctamente su valor de $\mathrm{C\hat{A}B}$ o $\mathrm{A\hat{C}B}$ en la fórmula trigonométrica.

$(h =)\ 18,6\,(\mathrm{m})\ (18,5576\ldots)$ ***(A1)*(ft)**

Nota:	Se pueden otorgar puntos de arrastre de error (*FT*) desde su apartado (b).

$20 > 18,5576\ldots$ ***(R1)*(ft)**

Nota:	Acepte “la longitud de la cuerda es mayor que la altura del triángulo ABC”.

la cuerda llega hasta el interior del terreno triangular ACD ***(A1)*(ft)**

Nota:	Se pueden otorgar puntos de arrastre de error (*FT*) desde su valor de la altura. La concesión del ***(A1)*** final depende de que se haya concedido antes el ***(R1)***.

Continúa en la pág. siguiente...

Continuación de la pregunta 5

MÉTODO 3 (hallar $\mathrm{C\hat{A}B}$ o bien $\mathrm{A\hat{C}B}$ con el teorema del coseno y, a continuación, usar una razón trigonométrica)

$\cos \mathrm{A\hat{C}B} = \dfrac{50^2 + 70^2 - 30^2}{2(50)(70)}$ $\left(\mathrm{A\hat{C}B} = 21{,}7867\ldots^\circ\right)$ ***(M1)***

Nota:	Conceda ***(M1)*** por su sustitución correcta de valores en la fórmula del teorema del coseno para hallar $\mathrm{C\hat{A}B}$ o bien $\mathrm{A\hat{C}B}$.

$(h =)\ 50 \times \operatorname{sen}(21{,}7867\ldots^\circ)$ ***(M1)***

Nota:	Conceda ***(M1)*** por haber sustituido correctamente su valor de $\mathrm{C\hat{A}B}$ o $\mathrm{A\hat{C}B}$ en la fórmula trigonométrica.

$(h =)\ 18{,}6\,(\mathrm{m})\ (18{,}5576\ldots)$ ***(A1)*****(ft)**

$20 > 18{,}5576\ldots$ ***(R1)*****(ft)**

Nota:	Acepte “la longitud de la cuerda es mayor que la altura del triángulo ABC”.

la cuerda llega hasta el interior del terreno triangular ACD ***(A1)*****(ft)**

Nota:	Se pueden otorgar puntos de arrastre de error (*FT)* desde su valor de la altura. La concesión del ***(A1)*** final depende de que se haya concedido antes el ***(R1)***.

Continúa en la pág. siguiente...

Continuación de la pregunta 5

MÉTODO 4 (hallar el área del triángulo de altura 20, justificando la contradicción)

$A = \frac{1}{2}(70)(20) = 700\,(\mathrm{m}^2)$ ***(M1)(A1)***

Nota:	Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula del área de un triángulo para un triángulo de altura 20 y base 70. Conceda ***(A1)*** por el 700. Conceda ***(M0)(A0)*** por un 700 no respaldado por razonamientos/cálculos, a no ser que en un razonamiento posterior se explique cómo se obtuvo ese 700.

$700 > 649{,}519\ldots$ ***(R1)***

si la cuerda toca exactamente el lado AC, entonces este triángulo tiene un área mayor que ABC y, dado que la distancia AC es fija, la altura tiene que ser menor que 20 ***(R1)***

O BIEN

$\frac{1}{2}(70)(20) > \frac{1}{2}(70)$ (la altura perpendicular a AC) y, por consiguiente,

$20 >$ la altura perpendicular a AC ***(R1)*****(ft)**

Nota:	Conceda ***(R1)*** por una explicación donde se diga que el triángulo de partida ABC y este nuevo triángulo tienen la misma base (70) y que, por consiguiente, la altura del triángulo ABC es menor que 20.

Por consiguiente, la cuerda llega hasta el interior del terreno triangular ACD. ***(A1)*****(ft)**

Nota:	Es posible utilizar otros métodos, además de los ya enumerados aquí. Todos estos métodos se pueden clasificar en dos grandes grupos: el primero se basa en hallar la altura del triángulo y compararla con 20; el segundo, en crear un triángulo artificial que tenga altura igual a 20 y explicar por qué este triángulo no es ABC haciendo referencia al área **y** a las longitudes de los lados dadas en el enunciado.

[5 puntos]

Total [15 puntos]

6. (a) $(V =)\ \dfrac{4\pi r^3}{3} + \pi r^2 l$ (o equivalente) ***(A1)(A1)***

> **Nota:** Conceda ***(A1)*** por la fórmula del volumen de una semiesfera multiplicada por 2, o bien por la fórmula del volumen de un cilindro; conceda ***(A1)*** por dar una expresión que sea completamente correcta. Acepte expresiones equivalentes. Conceda a lo sumo ***(A1)(A0)*** si usan h en lugar de l.

[2 puntos]

(b) $300 = 4\pi r^2 + 2\pi r l$ ***(A1)(A1)(A1)***

> **Nota:** Conceda ***(A1)*** por dar la superficie de una semiesfera multiplicada por 2. Conceda ***(A1)*** por dar la superficie de un cilindro. Conceda ***(A1)*** por sumar sus fórmulas e igualarlas a 300.
> Conceda a lo sumo ***(A1)(A1)(A0)*** si usan h en lugar de l a menos que ya se haya penalizado en el apartado (a).

[3 puntos]

(c) $V = \dfrac{4\pi r^3}{3} + \pi r^2 \left(\dfrac{150 - 2\pi r^2}{\pi r}\right)$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por haber sustituido correctamente los valores en su fórmula de V.

$V = \dfrac{4\pi r^3}{3} + 150r - 2\pi r^3$ ***(M1)***

> **Nota:** Conceda ***(M1)*** por desarrollar el paréntesis y simplificar la expresión del cilindro en V de una manera correcta que conduzca a la respuesta final.

$V = 150r - \dfrac{2\pi r^3}{3}$ ***(AG)***

> **Nota:** El alumno tiene que haber incluido esta última línea, sin haber cometido errores en el desarrollo, para conceder el segundo ***(M1)***.

[2 puntos]

Continúa en la pág. siguiente...

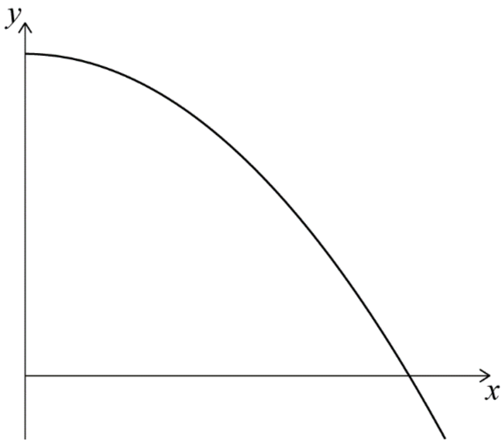
Continuación de la pregunta 6

(d) $\left(\frac{\mathrm{d}V}{\mathrm{d}r}=\right) 150-2\pi r^2$ ***(A1)(A1)***

Nota:	Conceda ***(A1)*** por el 150. Conceda ***(A1)*** por $-2\pi r^2$. Conceda como máximo ***(A1)(A0)*** si ha incluido términos adicionales.

[2 puntos]

(e) $150-2\pi r^2=0$ **O BIEN** $\frac{\mathrm{d}V}{\mathrm{d}r}=0$ **O BIEN** un bosquejo (dibujo aproximado) de $\frac{\mathrm{d}V}{\mathrm{d}r}$ donde esté indicada la intersección con el eje x ***(M1)***



Nota:	Conceda ***(M1)*** por igualar su derivada a cero o por un bosquejo (dibujo aproximado) de su derivada donde esté indicada la intersección con el eje x.

$r=\sqrt{\frac{150}{2\pi}}$ **O BIEN** $r^2=\frac{150}{2\pi}$ ***(A1)***

$r=\sqrt{\frac{75}{\pi}}$ ***(AG)***

Nota:	El alumno tiene que haber incluido la línea ***(AG)*** para conceder el ***(A1)*** anterior.

[2 puntos]

Continúa en la pág. siguiente...

Continuación de la pregunta 6

(f) $(l=)\ \dfrac{150-2\pi\left(\sqrt{\dfrac{75}{\pi}}\right)^2}{\pi\left(\sqrt{\dfrac{75}{\pi}}\right)}$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula (dada en el enunciado) de la longitud del cilindro.

$(l=)\ 0\,(\text{cm})$ ***(A1)(G2)***

Nota: Conceda ***(M1)(A1)*** por haber sustituido correctamente la aproximación de 3 c. s. 4,89, y haber llegado a la respuesta correcta de cero.

[2 puntos]

(g) $V=150\left(\sqrt{\dfrac{75}{\pi}}\right)-\dfrac{2\pi\left(\sqrt{\dfrac{75}{\pi}}\right)^3}{3}$ **O BIEN** $V=\dfrac{4\pi\left(\sqrt{\dfrac{75}{\pi}}\right)^3}{3}$ ***(M1)***

Nota: Conceda ***(M1)*** por haber sustituido correctamente los valores en la fórmula del volumen del altavoz o del volumen de una esfera.

$489\left(488{,}602\ldots,\ 100\sqrt{\dfrac{75}{\pi}}\right)(\text{cm}^3)$ ***(A1)*(ft)*(G2)***

Nota: Acepte 489,795… si utilizó valores de $\sqrt{\dfrac{75}{\pi}}$ dados con 3 c. s. Conceda ***(M1)(A1)*(ft)** por haber sustituido correctamente en su volumen del altavoz. Se pueden otorgar puntos de arrastre de error (*FT*) desde sus apartados (a) y (f).

[2 puntos]

(h) Esfera (esférica) ***(A1)*(ft)**

Nota: En esta pregunta hay que utilizar el resultado del apartado (f), así que si el alumno dejó sin contestar el apartado (f), al apartado (h) hay que concederle ***(A0)***. Se pueden otorgar puntos de arrastre de error desde su $l>0$.

[1 punto]
Total [16 puntos]